UNIT V · LESSON 04

Random Processes & Noise

probability · random variables · distributions · stationarity · ergodicity · autocorrelation & PSD · noise sources · noise figure · noise in communication systems

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questions in Unit V
Signals · DSP · Analog & Digital Comm

RULES PROBABILITY BASICS

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$; independent: $P(A \cap B) = P(A)P(B)$
- Conditional: $P(A|B) = P(A \cap B)/P(B)$; **Bayes**: $P(A|B) = P(B|A)P(A)/P(B)$
- Total probability: $P(B) = \Sigma\, P(B|A_i)P(A_i)$
- Binary channel: $P(0) = 0.6$, error $p = 0.1$ → $P(\text{receive } 1) = 0.6\times0.1 + 0.4\times0.9 =$ **0.42**

CDF · PDF RANDOM VARIABLES

- **CDF** $F_X(x) = P(X \le x)$: non-decreasing, 0 → 1
- **PDF** $f_X(x) = dF/dx \ge 0$, $\int f\,dx = 1$
- Mean $E[X] = \int x\, f(x)\, dx$; **variance** $\sigma^2 = E[X^2] - (E[X])^2$
- $E[aX + b] = aE[X] + b$; $Var(aX + b) = a^2\, Var(X)$
- Independent X, Y: $Var(X + Y) = Var\ X + Var\ Y$; pdf of sum = **convolution** of pdfs
- **Central limit theorem**: sum of many independent RVs → **Gaussian**
- Characteristic function, moments; covariance; correlation coefficient $\rho \in [-1, 1]$

COMMON DISTRIBUTIONS

Distribution	Mean	Variance	Where
Uniform (a, b)	$(a + b)/2$	$(b - a)^2/12$	quantisation error, random phase
Gaussian $N(\mu, \sigma^2)$	μ	σ^2	thermal noise
Rayleigh	$\sigma\sqrt{(\pi/2)}$	$(2 - \pi/2)\sigma^2$	envelope of narrowband noise, fading
Rician	—	—	fading with line-of-sight
Exponential (λ)	$1/\lambda$	$1/\lambda^2$	inter-arrival times
Poisson (λ)	λ	λ	counts (shot noise, arrivals)
Binomial (n, p)	np	$np(1 - p)$	bit errors in a block

- Gaussian: 68 % within ±σ, 95.4 % within ±2σ, 99.7 % within ±3σ

COMMON DISTRIBUTIONS (CONTD.)

- **Q-function**: $Q(x) = P(Z > x) = \frac{1}{2}\,\mathrm{erfc}(x/\sqrt{2})$; $Q(0) = 0.5$, $Q(3) \approx 1.35 \times 10^{-3}$

STATISTICS RANDOM PROCESSES

- Random process X(t, s): ensemble of sample functions
- Mean $m_X(t) = E[X(t)]$; **autocorrelation** $R_X(t_1, t_2) = E[X(t_1)X(t_2)]$
- **Strict-sense stationary (SSS)**: all statistics shift-invariant
- **Wide-sense stationary (WSS)**: constant mean and R_X depends only on $\tau = t_2 - t_1$
- **Ergodic**: time averages = ensemble averages (ergodic ⇒ stationary)
- Example $X(t) = A\cos(\omega_c t + \Theta)$, Θ uniform $(0, 2\pi)$ → WSS, $R_X(\tau) = (A^2/2)\cos\omega_c\tau$

TYPES NOISE SOURCES

Noise	Cause	Spectrum
Thermal (Johnson)	random electron motion in resistors	white; P = **kTB**, $V_{rms} = \sqrt{(4kTRB)}$
Shot	discrete charge carriers crossing a junction	white; I_n^2 = **2qIB**
Flicker (1/f)	surface/contact effects	∝ 1/f, low frequency
Partition	current division between electrodes	white-ish
Transit-time	carrier transit ≈ signal period	high frequency
External	atmospheric, cosmic (galactic), man-made	

WIENER–KHINCHINE AUTOCORRELATION & PSD

- **PSD $S_X(f)$ = FT of $R_X(\tau)$** (Wiener–Khinchine theorem)
- $R_X(0) = E[X^2]$ = **average power** = $\int S_X(f)\,df$
- $R_X(\tau)$ even, max at $\tau = 0$; $S_X(f)$ real, even, ≥ 0
- LTI filter: **$S_Y(f) = |H(f)|^2 S_X(f)$**; mean $m_Y = m_X H(0)$
- Cross-correlation, cross-PSD; uncorrelated vs orthogonal vs independent

MODELS WHITE & BAND-LIMITED NOISE

- **White noise**: $S(f) = N_0/2$ (two-sided) for all f; $R(\tau) = (N_0/2)\,\delta(\tau)$
- **AWGN**: additive, white, Gaussian — standard channel model
- White noise through ideal LPF of bandwidth B: power = **N_0B**
- RC low-pass filter with white input: output power = **kT/C** (for thermal noise), noise bandwidth = 1/(4RC)
- **Noise-equivalent bandwidth** $B_N = \int|H(f)|^2\,df / |H(0)|^2$
- Narrowband noise: $n(t) = n_I(t)\cos\omega_c t - n_Q(t)\sin\omega_c t$; envelope Rayleigh, phase uniform

TYPES NOISE SOURCES (CONTD.)

- $k = 1.38 \times 10^{-23}$ J/K; at 290 K, kT = **−174 dBm/Hz**
- Example: R = 1 kΩ, T = 300 K, B = 10 kHz → $V_{rms} \approx 0.41$ μV

FIGURE OF MERIT NOISE IN ANALOG MODULATION

System	$\gamma = (SNR)_o/(SNR)_c$	Note
DSB-SC (coherent)	**1**	same as baseband
SSB-SC (coherent)	**1**	half bandwidth
AM (envelope, tone, μ)	$\mu^2/(2+\mu^2)$ → max **1/3** at μ = 1	threshold effect at low CNR
FM (tone, β)	**(3/2) β^2**	threshold ≈ 10–13 dB CNR

- FM noise PSD at output ∝ f^2 (parabolic) → **pre-emphasis / de-emphasis** (75 μs in FM broadcast)
- FM gives SNR improvement at the cost of bandwidth (β = 5 → γ = 37.5)
- **Threshold effect**: below threshold, output SNR drops sharply (AM envelope, FM)

Q → A RAPID RECALL (CONTD.)

Equivalent temperature for F = 2	290 K
Stage dominating overall noise figure	first stage

AMPLIFIERS NOISE FIGURE

- **Noise factor $F = (S/N)_{in} / (S/N)_{out} \geq 1$**; noise figure NF = 10 log F dB
- **Equivalent noise temperature $T_e = (F - 1)\,T_0$**, T_0 = 290 K
- **Friis formula**: $F = F_1 + (F_2 - 1)/G_1 + (F_3 - 1)/(G_1G_2) + \ldots$
- $T_e = T_1 + T_2/G_1 + T_3/(G_1G_2) + \ldots$
- **First stage dominates** → use a low-noise, high-gain amplifier (LNA) first
- Example: $F_1 = 2$ (3 dB), $G_1 = 10$; $F_2 = 5$ → F = 2 + 4/10 = **2.4** (3.8 dB)
- Lossy line (loss L): F = L, gain 1/L
- Noise figure 3 dB → $T_e = 290$ K

Q → A RAPID RECALL

Mean of uniform (0, 2)	1
Variance of uniform (−Δ/2, Δ/2)	$\Delta^2/12$
Distribution of thermal noise	Gaussian
Envelope of narrowband Gaussian noise	Rayleigh
PSD is FT of	autocorrelation
Output PSD of LTI system	$\lvert H(f)\rvert^2 S_X(f)$
Thermal noise power	kTB
Shot noise mean-square current	2qIB
Noise figure of an ideal amplifier	0 dB (F = 1)

Q → A RAPID RECALL (CONTD.)

FM figure of merit, β = 2	6
Max figure of merit of AM	1/3

TRAP EXAM TRAPS

- Independent ⇒ uncorrelated, but uncorrelated ⇒ independent only for jointly Gaussian
- WSS does not imply SSS (except Gaussian processes)
- Noise figure is a ratio of SNRs, **not** of noise powers alone
- Friis: later stages are divided by the **product of earlier gains** (linear, not dB)
- Two-sided PSD $N_0/2$ vs one-sided N_0 — check which is given