UNIT VII · LESSON 01

DC, AC & Three-Phase Circuits

Ohm & Kirchhoff laws · network theorems · transients · AC phasors · power & power factor · resonance · three-phase systems · two-port networks

20
questions in Unit VII
Circuits · Machines · Sensors · Power Electronics

DC BASIC LAWS

- **Ohm's law** $V = IR$; $R = \rho L/A$; $R_t = R_0(1 + \alpha t)$
- Series: $R = \Sigma R$, same current; **voltage divider** $V_1 = V\,R_1/(R_1 + R_2)$
- Parallel: $1/R = \Sigma 1/R$; two resistors $R_1R_2/(R_1 + R_2)$; **current divider** $I_1 = I\,R_2/(R_1 + R_2)$
- **KCL**: Σ currents at a node = 0 (charge conservation)
- **KVL**: Σ voltages around a loop = 0 (energy conservation)
- Power $P = VI = I^2R = V^2/R$; energy $W = Pt$ (1 kWh = 3.6 MJ)
- **Star ↔ delta**: $R_Y = R_aR_b/(R_a + R_b + R_c)$; $R_\Delta = (\Sigma R_iR_j)/R_{opposite}$; equal: $\mathbf{R_Y = R_\Delta/3}$
- Mesh analysis (KVL, loops = b − n + 1), nodal analysis (KCL, n − 1 equations)
- Ideal voltage source: zero internal resistance; ideal current source: infinite

RL · RC · RLC TRANSIENTS

- RC: $\boldsymbol{\tau = RC}$; charging $v_C = V(1 - e^{-t/\tau})$; discharging $V\,e^{-t/\tau}$
- RL: $\boldsymbol{\tau = L/R}$; $i = (V/R)(1 - e^{-t/\tau})$
- After **1τ: 63.2 %**, 5τ ≈ steady state (99.3 %)

MUST KNOW NETWORK THEOREMS

Theorem	Statement / use
Superposition	response = sum of responses to each source alone (others: V short, I open); **linear** only, not for power
Thevenin	any linear network = V_{th} in series with R_{th}
Norton	= I_N in parallel with R_N; $\mathbf{I_N = V_{th}/R_{th}}$
Maximum power transfer	$R_L = R_{th} \rightarrow \mathbf{P_{max} = V_{th}^2/4R_{th}}$, efficiency **50 %**
AC max power	$Z_L = Z_{th}^*$ (conjugate)
Reciprocity	V/I ratio unchanged when source and meter swap (single source, passive)
Millman	$V = \Sigma(V_i/R_i) / \Sigma(1/R_i)$ for parallel sources
Tellegen	$\Sigma\, v_k i_k = 0$ (any lumped network)
Substitution, compensation	replace branch by source

- Example: V_{th} = 12 V, R_{th} = 4 Ω → P_{max} = 144/16 = **9 W** at R_L = 4 Ω

RL · RC · RLC TRANSIENTS (CONTD.)

- Capacitor voltage and inductor current **cannot change instantaneously**
- At $t = 0^+$: uncharged C = short, L = open; at $t = \infty$ (dc): C = open, L = short
- Series RLC: $\alpha = R/2L$, $\omega_0 = 1/\sqrt{(LC)}$; overdamped $\alpha > \omega_0$, **critically damped $R = 2\sqrt{(L/C)}$**, underdamped $\alpha < \omega_0$
- Energy: $\frac{1}{2}CV^2$ (capacitor), $\frac{1}{2}LI^2$ (inductor)

SINUSOIDS AC FUNDAMENTALS

- $v = V_m \sin(\omega t + \varphi)$, $\omega = 2\pi f$; India: **50 Hz**, 230 V rms (peak 325 V)

Waveform	RMS	Average (half cycle)	Form factor	Peak factor
Sine	$V_m/\sqrt{2} = 0.707V_m$	$2V_m/\pi = 0.637V_m$	**1.11**	**1.414**
Square	V_m	V_m	1	1
Triangular	$V_m/\sqrt{3}$	$V_m/2$	1.155	1.732
Half-wave rectified sine	$V_m/2$	V_m/π	1.57	2

- Form factor = RMS/average; peak (crest) factor = peak/RMS

PHASORS IMPEDANCE

Element	Impedance	Phase of I w.r.t. V
R	R	in phase
L	$jX_L = j\omega L$	**lags** by 90°
C	$-jX_C = 1/(j\omega C)$	**leads** by 90°

- Series RL: $Z = \sqrt{(R^2 + X_L^2)}$, $\varphi = \tan^{-1}(X_L/R)$
- Admittance $Y = 1/Z = G + jB$ (conductance, susceptance)
- Memory aid **"ELI the ICE man"**: voltage leads current in L; current leads voltage in C

PHASORS IMPEDANCE (CONTD.)

- Example: R = 30 Ω, X_L = 40 Ω → Z = **50 Ω**, pf = 0.6 lagging

TRIANGLE POWER IN AC

Quantity	Formula	Unit
Active (real) power P	$VI\cos\varphi = I^2R$	W
Reactive power Q	$VI\sin\varphi$	VAR
Apparent power S	$VI = \sqrt{(P^2 + Q^2)}$	VA
Complex power	$S = VI^* = P + jQ$	
Power factor	$\cos\varphi = P/S = R/Z$	

TRIANGLE POWER IN AC (CONTD.)

- Low pf → higher current, losses, larger equipment → **pf correction** with shunt capacitors
- Capacitor kVAR = $P(\tan \varphi_1 - \tan \varphi_2)$
- Example: 100 kW at pf 0.6 → 0.9: Q_C = 100(1.333 − 0.484) = **84.9 kVAR**

3Φ THREE-PHASE SYSTEMS

Quantity	Star (Y)	Delta (Δ)
Line voltage	$\mathbf{V_L = \sqrt{3}\, V_{ph}}$	$V_L = V_{ph}$
Line current	$I_L = I_{ph}$	$\mathbf{I_L = \sqrt{3}\, I_{ph}}$
Power	$\mathbf{P = \sqrt{3}\, V_L I_L \cos \varphi}$ (both)	

- Phase sequence RYB, 120° apart; line voltage leads phase voltage by **30°** (star)
- Balanced load: neutral current = 0
- **Two-wattmeter method**: $P = W_1 + W_2$; $\mathbf{\tan \varphi = \sqrt{3}(W_1 - W_2)/(W_1 + W_2)}$
- pf = 1 → $W_1 = W_2$; pf = 0.5 → one reading **zero**; pf < 0.5 → one reading **negative**
- Example: 400 V, 10 A line, pf 0.8 → P = √3 × 400 × 10 × 0.8 = **5.54 kW**
- Δ load converted to Y: $Z_Y = Z_\Delta/3$

PARAMETERS TWO-PORT NETWORKS (CONTD.)

- Reciprocal: $Z_{12} = Z_{21}$, $Y_{12} = Y_{21}$, **AD − BC = 1**, $h_{12} = -h_{21}$
- Symmetrical: $Z_{11} = Z_{22}$, **A = D**

SERIES & PARALLEL RESONANCE

Feature	Series RLC	Parallel (tank)
f_r	$1/(2\pi\sqrt{(LC)})$	≈ $1/(2\pi\sqrt{(LC)})$ (ideal)
Impedance at f_r	**minimum = R**	**maximum = L/(CR)** (dynamic)
Current at f_r	maximum	minimum (line)
Called	acceptor circuit	rejector circuit
Magnification	voltage across L, C = Q V	current magnification

- $\mathbf{Q = \omega_0 L/R = (1/R)\sqrt{(L/C)}}$ (series); bandwidth $\mathbf{BW = f_r/Q = R/(2\pi L)}$
- Half-power frequencies f_1, f_2: $f_r = \sqrt{(f_1 f_2)}$
- Example L = 10 mH, C = 1 µF, R = 10 Ω → f_r ≈ **1592 Hz**, Q = 10, BW ≈ 159 Hz

PARAMETERS TWO-PORT NETWORKS

Parameters	Equations	Use
Z (open circuit)	V = Z I	series connection
Y (short circuit)	I = Y V	parallel connection
h (hybrid)	$V_1 = h_{11} I_1 + h_{12} V_2$; $I_2 = h_{21} I_1 + h_{22} V_2$	transistors
ABCD (transmission)	$V_1 = AV_2 - BI_2$; $I_1 = CV_2 - DI_2$	**cascade** (multiply matrices), lines

PARAMETERS TWO-PORT NETWORKS (CONTD.)

- Graph theory: tree has **n − 1** branches; links = b − n + 1; incidence, tie-set, cut-set matrices

TRAP EXAM TRAPS

- Superposition cannot be used to compute power directly
- In delta, line voltage = phase voltage; in star, line current = phase current
- Parallel resonance gives **maximum** impedance; series gives minimum
- Power factor of a purely reactive load is 0, not 1
- Max power transfer ≠ maximum efficiency

Q → A RAPID RECALL

Equivalent star of 3 Ω delta	1 Ω each
Max power efficiency	50 %
Norton current	V_{th}/R_{th}
Time constant of RC circuit	RC
Charge after one time constant	63.2 %
RMS of sine with peak 100 V	70.7 V
Form factor of sine	1.11
Current in pure inductor	lags by 90°
Unit of reactive power	VAR
Impedance of series RLC at resonance	R (minimum)
Line voltage in star, phase 230 V	≈ 400 V
Wattmeter readings at pf 0.5	one zero
Reciprocity condition for ABCD	AD − BC = 1