CBSE · CLASS 10 · MATHEMATICS

MATHEMATICS BEYOND THE FORMULA

CHAPTER 1 · REAL NUMBERS

Don't Memorize Mathematics. Understand, Visualise and Discover It.

The KalviYogi 8D Way: Discover · Visualise · Understand · Derive · Solve · Apply · Challenge · Recall

Every solution with COMPLETE steps — one by one, nothing skipped

Fundamental Theorem of Arithmetic · HCF & LCM · Irrational Numbers

Five-Level Practice · Smart Memory System · Mastery Tracker

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even when no teacher is beside you.

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01 Chapter Introduction

8D • DISCOVER — WHY SHOULD I LEARN THIS?

*Look around you. A shopkeeper packs sweets into boxes so that **nothing is left over**. Two buses leave a stand at different intervals and sometimes **leave together**. A tile-layer covers a floor with **equal square tiles** without cutting any tile. Behind every one of these everyday puzzles sits the hero of this chapter — the **Real Numbers** and their prime building blocks.*

*In Class 9 you met the number family. In this chapter you go one level deeper: you will discover that **every whole number is built from prime numbers in exactly one way** — and that single discovery gives you machines for finding HCF and LCM, for deciding whether a number can end in 0, and even for proving that numbers like √2 and √5 can never be written as a fraction.*

What you will master in this chapter

- *The Fundamental Theorem of Arithmetic — and why "unique" is the magic word.*
- *Prime factorisation of any number, using a factor tree and a division ladder.*
- *HCF and LCM by prime factorisation — for two numbers and for three.*
- *The identity HCF × LCM = a × b — **derived**, not just memorised, and used to find a missing value.*
- *Ending-digit and composite-number proofs (can 6^n end in 0?).*
- *Proof by contradiction — the master key for proving √2, √5, 3 + 2√5 and friends irrational.*
- *Story problems: when a story is secretly asking for HCF, and when for LCM.*

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Your Chapter Map — 18 Stops, One Journey

This chapter is not a list of formulas. It is a journey with 18 stops. Tick each stop as you finish it.

Discover & Visualise	Understand & Derive	Solve, Apply, Recall
☐ 01 Chapter Introduction ☐ 02 Prerequisite Knowledge ☐ 03 Real-Life Story ☐ 04 Visual Concepts (CPA)	☐ 05 Mathematical Foundations ☐ 06 Formula Derivations ☐ 08 Multiple Solution Methods ☐ 10 Common Mistakes	☐ 07 Detailed Solved Examples ☐ 09 Hands-on Activities ☐ 11 Graded Practice (5 Levels) ☐ 12 Mathematical Reasoning
☐ 13 Real-Life Applications ☐ 14 Competency-Based Questions	☐ 15 Olympiad Challenges ☐ 16 Chapter Mind Map	☐ 17 Active Recall ☐ 18 Mastery Tracker

How to use this chapter (read this first!)

1. **Never read a solution passively.** Cover the steps, try yourself, then uncover one step at a time.
2. Every solution shows **Step 1, Step 2, Step 3...** with a small Why note in purple — the reason behind the step. The board examiner gives marks for these steps, not just the final answer.
3. After Section 07, test yourself in Sections 11–15 **before** looking at the Answer Key.
4. Finish with the Mind Map (16), the closed-book Recall (17) and your Mastery Tracker (18). Come back on Day 3, 7, 14, 21 and 30 — the Smart Memory schedule on the tracker page tells you exactly what to do each day.

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02 Prerequisite Knowledge

CLASS 9 RECAP — THE TOOLS YOU ALREADY OWN

Everything in this chapter is built from five Class 9 ideas. Refresh them here — two minutes now will save you twenty minutes later.

1. The number family

Natural numbers: 1, 2, 3, ... **Whole numbers:** 0, 1, 2, 3, ... **Integers:** ... −2, −1, 0, 1, 2, ...
Rational numbers: numbers that can be written as p/q, where p and q are integers and $q \neq 0$.
Irrational numbers: real numbers that can NOT be written as p/q — their decimals never end and never repeat (for example $\sqrt{2}$, $\sqrt{3}$, π).
Real numbers = rationals + irrationals together — every point on the number line.

2. Prime and composite numbers

A **prime** has exactly two factors: 1 and itself (2, 3, 5, 7, 11, 13, ...). A **composite** number has more than two factors (4, 6, 8, 9, 10, ...). The number **1 is neither** prime nor composite — a favourite examiner trap!

3. Factors and multiples

A **factor** of a number divides it exactly (factors of 12: 1, 2, 3, 4, 6, 12). A **multiple** is what you get on multiplying (multiples of 12: 12, 24, 36, ...). HCF works with factors; LCM works with multiples.

4. Co-prime numbers

Two numbers are **co-prime** when their only common factor is 1 — that is, HCF = 1. Example: 14 and 25 are co-prime. This idea is the heart of every irrationality proof in Section 07.

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Quick Check — Are You Ready? (5 Questions)

Try all five without looking back. Answers are given right below — cover them with your hand first!

Q1. Is 29 a prime number? How do you check?

Q2. Write 48 as a product of prime numbers.

Q3. Write the first five multiples of 7.

Q4. Write all the factors of 36.

Q5. Are 14 and 25 co-prime? Give a reason.

Answers — check only after trying

Q1. Yes. Test divisibility by primes up to $\sqrt{29} \approx 5.4$, i.e. 2, 3, 5: 29 is divisible by none of them, so 29 is prime.

Q2. $48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$.

Q3. 7, 14, 21, 28, 35.

Q4. 1, 2, 3, 4, 6, 9, 12, 18, 36.

Q5. Yes. Factors of 14: 1, 2, 7, 14. Factors of 25: 1, 5, 25. The only common factor is 1, so HCF(14, 25) = 1 — they are co-prime.

TOPPER TIP: Got 4 or 5 right? You are fully ready. Got fewer? Re-read the four cards on the previous page once, then come back — do not skip this step.

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03 Real-Life Mathematical Story

DISCOVER • VISUALISE — THE SWEET SHOP PUZZLE

Before any formula, live the problem. Read this story slowly — the mathematics will walk out of the story by itself.

It is the week before Diwali, and **Murugan Sweet House** is glowing with lights. This morning, the kitchen has produced two fresh batches: **168 coconut barfis** and **252 milk laddus**.

Murugan's daughter **Kavya**, who studies in Class 10, is in charge of the gift boxes. Her father's rules are strict:

- Every gift box must contain **only one type** of sweet.
- Every box must contain the **same number** of sweets.
- **No sweet may be left over** — every barfi and every laddu must be packed.
- To save cardboard, the boxes should be **as large as possible**.

Kavya tries 10 sweets per box: 168 ÷ 10 leaves a remainder. She tries 12: the barfis pack perfectly (14 boxes), but 252 ÷ 12 = 21 boxes — wait, that works too! She tries 20: laddus are left over. She tries 24, 28, 42... some numbers work, some fail. She needs the **largest** number that divides **both** 168 and 252 exactly.

Meanwhile, a second problem is waiting. The **ghee supplier** visits the shop **every 12 days**, and the **sugar supplier** visits **every 18 days**. Today, by luck, **both suppliers are at the shop together**. Murugan wants to plan a big joint order. "Kavya, after how many days will both of them stand in my shop together again?"

*Two questions. Two famous ideas. The first asks for the largest number hiding inside both 168 and 252 — that is the **HCF**. The second asks for the smallest day-count that both 12 and 18 can reach together — that is the **LCM**. Let us solve both, one step at a time.*

Solving the Story — Step by Step

STORY PROBLEM 1

Gift boxes: 168 barfis and 252 laddus. Every box holds one type only, all boxes hold equal sweets, nothing is left over, and boxes are as large as possible. How many sweets per box, and how many boxes of each type?

THINK FIRST: The words 'as large as possible', 'equal' and 'nothing left over' are the story's way of saying HCF.

1. Write both numbers as products of primes. $168 = 2^3 \times 3 \times 7$ and $252 = 2^2 \times 3^2 \times 7$.
   Why: Prime factorisation is the X-ray of a number — it shows every divisor hiding inside.

2. The sweets-per-box number must divide BOTH 168 and 252, so it must be built only from the **common** primes: 2, 3 and 7.
   Why: A common divisor cannot use a prime that one of the numbers does not have.

3. For the LARGEST such number, take each common prime to the **smaller** of its two powers: 2^2 (smaller of 2^3, 2^2), 3^1 (smaller of 3^1, 3^2), 7^1.
   Why: Taking a bigger power of 2, like $2^3 = 8$, would fail to divide 252 — 252 only has 2^2.

4. Multiply: $2^2 \times 3 \times 7 = 4 \times 3 \times 7 = \mathbf{84}$. So each box holds 84 sweets. This number is the HCF of 168 and 252.

5. Count the boxes: barfi boxes = $168 \div 84 = \mathbf{2}$; laddu boxes = $252 \div 84 = \mathbf{3}$. Total 5 gift boxes.
   Why: Dividing each batch by the box size gives the number of boxes — and both divisions come out exact, as promised.

ANSWER: 84 sweets in each box; 2 boxes of barfis and 3 boxes of laddus (5 boxes in all).

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Solving the Story — Problem 2: The Suppliers

STORY PROBLEM 2

The ghee supplier visits every 12 days and the sugar supplier every 18 days. Both visited today. After how many days will they visit together again?

THINK FIRST: *The words 'together again' and 'after how many days' are the story's way of saying LCM. Remember these two signals — they return in Exercise 1.1 Q7 and in Section 13.*

1. List when the ghee supplier comes (multiples of 12): 12, 24, **36**, 48, ...
   Why: Visit days are multiples of the gap between visits.
2. List when the sugar supplier comes (multiples of 18): 18, **36**, 54, ...
3. The first day appearing in BOTH lists is 36 — the **smallest common multiple**, the LCM of 12 and 18.
   Why: We want the FIRST time they coincide, so we take the smallest common multiple, not just any one.
4. Quick check by primes: $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$, so LCM $= 2^2 \times 3^2 = 36$. Same answer.
   Why: For LCM, take every prime at its GREATEST power — Section 05 will make this rule precise.

ANSWER: Both suppliers will be at the shop together again after 36 days.

The two story signals — pin these in your memory

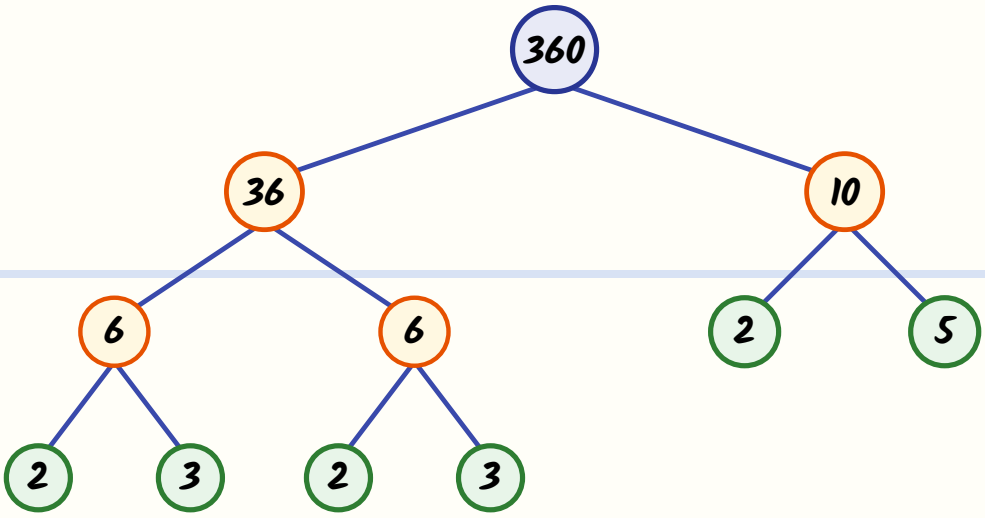
Largest / equal / nothing left over / pack / cut → *the answer is a DIVISOR of the given numbers → take the greatest one:* **HCF**.
Together again / meet again / repeat / after how long → *the answer is a MULTIPLE of the given numbers → take the smallest one:* **LCM**.
Nearly every word problem in this chapter — and in the board exam — is one of these two signals wearing a costume.

04 Visual Concept Explanation

SINGAPORE CPA — CONCRETE → PICTORIAL → ABSTRACT

Concrete first. *In the story, Kavya physically grouped sweets into boxes. Grouping, sharing and arranging real objects is always the first stage of understanding a number idea.*

Now the picture. *A composite number can be "opened up" like a tree branching downwards. Keep splitting every branch until every leaf is prime. The leaves can never change — that is the whole secret of this chapter. Here is 360, opened up:*



Read the green leaves from left to right: 2, 3, 2, 3, 2, 5. Collect them: ***360 = $2^3 \times 3^2 \times 5$****. Any other way of splitting 360 gives exactly the same leaves — try it in Section 09.*

Then the abstract. *Once the picture is clear, we write the short symbolic form: $360 = 2^3 \times 3^2 \times 5$. Symbols are just the picture, compressed. Never memorise the symbol before you can see the picture.*

> ***TOPPER TIP:*** *Examiners can tell the difference: a student who has SEEN the tree never stops a factorisation halfway, because an unsplit branch looks unfinished.*

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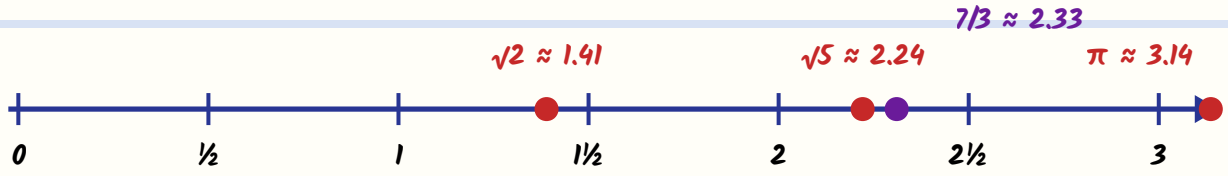
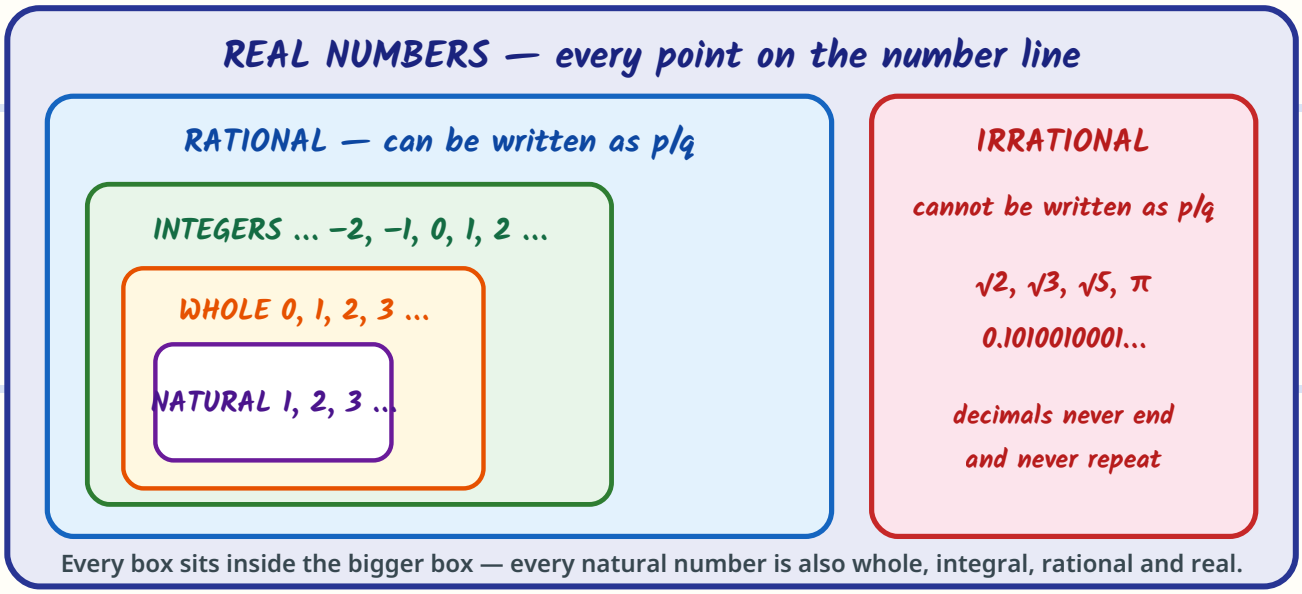
Visual Concepts — The Number Line and the Number Family

Rational numbers sit at exact fraction points. Irrational numbers also sit at exact points — √2 is a perfectly definite point, even though its decimal never ends. ***Both families together fill the whole line with no gaps: that is why they are called REAL numbers.***



Notice: √2 (about 1.41) sits just left of 1½, and √5 (about 2.24) sits just left of 7/3 (about 2.33). An irrational number is not "rough" or "unknown" — only its decimal is endless.

The number family — boxes inside boxes



Every box sits inside the bigger box — every natural number is also whole, integral, rational and real.

This chapter works in two corners of the picture: the ***natural numbers*** *(for HCF and LCM) and the border between* ***rational and irrational*** *(for the proof questions).*

05 Mathematical Foundations

UNDERSTAND — THE PRECISE IDEAS, WORD BY WORD

Theorem — Fundamental Theorem of Arithmetic

Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

*Read it in two halves. Half 1: a prime factorisation always **exists** — keep splitting and you always reach primes. Half 2: it is **unique** — 360 is $2^3 \times 3^2 \times 5$ today, tomorrow, and for every student on Earth. No other combination of primes can ever multiply to 360.*

Worked example — factorise 360 by the division ladder, one step at a time

1. $360 \div 2 = 180$. Write 2 on the left of the ladder.
   Why: Start with the smallest prime. 360 is even, so 2 divides it.
2. $180 \div 2 = 90$.
   Why: 180 is still even — stay with 2.
3. $90 \div 2 = 45$.
   Why: 90 is even once more.
4. $45 \div 3 = 15$.
   Why: 45 is odd, so move to the next prime, 3. Digit sum 4 + 5 = 9 confirms 3 divides 45.
5. $15 \div 3 = 5$.
   Why: 15 = 3 × 5, so 3 divides once more.
6. $5 \div 5 = 1$. Stop.
   Why: We stop at 1 — the ladder is complete and every divisor used is prime.
7. Collect the left column: $360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = \mathbf{2^3 \times 3^2 \times 5}$.
   Why: Counting repeats and writing powers is the abstract (symbol) form of the ladder picture.
8. Check by multiplying back: $2^3 \times 3^2 \times 5 = 8 \times 9 \times 5 = 360$.
   Why: Always multiply back. Ten seconds of checking protects the full mark.

2	360
2	180
2	90
3	45
3	15
5	5

Foundations — HCF and LCM, Defined Precisely

HCF — Highest Common Factor

The HCF of two or more numbers is the **largest number that divides each of them exactly**. By prime factorisation: **take only the primes common to all the numbers, each raised to its SMALLEST power.**

LCM — Lowest Common Multiple

The LCM of two or more numbers is the **smallest number that is a multiple of each of them.** By prime factorisation: **take every prime that appears in any of the numbers, each raised to its GREATEST power.**

Worked example — HCF and LCM of 12 and 18, step by step

1. Factorise: $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$.
   Why: Always factorise first — HCF and LCM are read off the factorisations, never guessed.
2. Common primes are 2 and 3. For HCF take the SMALLER powers: 2^1 and 3^1.
   Why: A common factor cannot contain 2^2, because 18 has only one 2.
3. HCF = $2 \times 3 = \mathbf{6}$.
4. For LCM take every prime at its GREATER power: 2^2 and 3^2.
   Why: A common multiple must contain enough of each prime to cover both numbers.
5. LCM = $4 \times 9 = \mathbf{36}$.
6. Golden check: HCF × LCM = 6 × 36 = 216, and 12 × 18 = 216. They match.
   Why: This check works for TWO numbers. Section 06 derives WHY it is true.

Rational and irrational — the exact definitions

A real number is **rational** if it can be written as p/q with p, q integers and $q \neq 0$ (examples: 5, $-3/4$, 0.25, 2.333...). A real number is **irrational** if it can NOT be written in that form (examples: $\sqrt{2}$, $\sqrt{5}$, π, 0.1010010001...). A decimal that terminates or repeats is always rational; a decimal that neither terminates nor repeats is always irrational.

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06 Formula Derivations

8D • DERIVE — NEVER MEMORISE WHAT YOU CAN DERIVE

Derivation A — Why is HCF × LCM = a × b?

*You have used this identity since Class 6. Today you will see **why** it must be true. Take the pair 12 and 18 and watch ONE prime at a time.*

1. Write the factorisations: $12 = 2^2 \times 3^1$ and $18 = 2^1 \times 3^2$.
   Why: The whole proof lives inside these two lines.
2. Look only at the prime 2. Its exponents in the two numbers are **2** (in 12) and **1** (in 18).
   Why: Forget the whole numbers — follow a single prime.
3. HCF takes the SMALLER exponent of 2, which is 1. LCM takes the LARGER exponent of 2, which is 2.
   Why: That is exactly how HCF and LCM were defined in Section 05.
4. Add them: 1 + 2 = 3. But also 2 + 1 = 3 — the SUM of the two original exponents.
   Why: For any two numbers x and y: min(x, y) + max(x, y) = x + y, because one of them IS the min and the other IS the max. Nothing is lost, nothing is doubled.
5. Now the prime 3: exponents are 1 (in 12) and 2 (in 18). HCF takes 1, LCM takes 2, and 1 + 2 = 3 = the sum of the original exponents again.
   Why: The same thing happens for EVERY prime, in EVERY pair of numbers.
6. So in the product HCF × LCM, every prime's exponent equals the sum of its exponents in a and in b.
7. But in the product a × b, a prime's exponent is ALSO the sum of its exponents in a and b — that is what multiplying does to powers.
   Why: Compare the two products prime by prime: they are identical.
8. Two numbers with identical prime factorisations are the same number (Fundamental Theorem — uniqueness). Therefore **HCF × LCM = a × b**. Proved, not memorised.

WARNING — TWO NUMBERS ONLY: For three numbers the trick collapses, because with three exponents the middle one is lost: min + max ≠ sum of all three. Example: for 12, 15, 21, HCF × LCM = 3 × 420 = 1260, but 12 × 15 × 21 = 3780. Never use the identity for three numbers.

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Derivation B — The Full Proof that √2 is Irrational

*This is the most beautiful argument in the chapter. Its weapon is **proof by contradiction**: assume the opposite of what you want, walk forward honestly, and crash into an impossibility. Follow every step — Exercise 1.2 is this same proof wearing different clothes.*

1. **Assume the opposite:** suppose √2 IS rational.
   Why: In a contradiction proof, we temporarily believe the enemy's claim and show it self-destructs.
2. Then $\sqrt{2} = a/b$, where a and b are **co-prime** positive integers and $b \neq 0$.
   Why: Any fraction can be reduced to lowest form, so we may assume a and b share no common factor except 1. This condition is the trap we are setting.
3. Square both sides: $2 = a^2/b^2$, so $\mathbf{a^2 = 2b^2}$.
   Why: Squaring removes the root — that is why we square first in every such proof.
4. So a^2 is exactly twice an integer — meaning **2 divides a^2**.
   Why: $a^2 = 2 \times (b^2)$, and b^2 is an integer.
5. **Lemma:** if a prime p divides a^2, then p divides a. Apply it with p = 2: **2 divides a**.
   Why: The lemma is proved on the next page. It is the engine of the whole argument.
6. Write $a = 2c$ for some integer c, and substitute into $a^2 = 2b^2$: $(2c)^2 = 2b^2$, so $4c^2 = 2b^2$, so $\mathbf{b^2 = 2c^2}$.
   Why: Naming $a = 2c$ is how we USE the fact '2 divides a' — a is double something.
7. So b^2 is also twice an integer: **2 divides b^2**, and by the lemma again, **2 divides b**.
   Why: The same engine fires a second time, now on b.
8. But now 2 is a common factor of a AND b — while Step 2 said a and b are co-prime, with NO common factor except 1. **Contradiction.**
   Why: The assumption has crashed. Every step was forced, so the only possible fault is the assumption itself.
9. Therefore the assumption is false: **√2 is irrational.** ∎
   Why: This completes the proof. Notice the rhythm: Assume → Square → p divides a → Substitute → p divides b → Contradiction.

TOPPER TIP: Learn the RHYTHM, not the lines. With prime 5 in place of 2, the identical rhythm proves √5 irrational — that is Exercise 1.2 Q1, solved fully in Section 07.

The Lemma — and the Pattern for √3 and √5

Lemma: if prime p divides a^2, then p divides a — why?

Step 1: Write a as a product of primes, say $a = 2 \times 3 \times 3 \times 7$ (the argument works for any factorisation).
Step 2: Then $a^2 = a \times a$ has every prime of a appearing **twice as many times**: $a^2 = 2^2 \times 3^4 \times 7^2$. Squaring only DOUBLES exponents — it can never create a brand-new prime.
Step 3: So if a prime p divides a^2, p must be one of the primes inside a^2's factorisation — hence one of the primes inside a itself.
Step 4: Therefore p divides a. ∎ (This uses uniqueness of prime factorisation — the Fundamental Theorem again.)

The 5-beat pattern — one proof, many numbers

To prove $\sqrt{p}$ irrational for any prime p, repeat Derivation B with p in place of 2:
Beat 1: Assume $\sqrt{p} = a/b$, co-prime. **Beat 2:** Square: $a^2 = p \cdot b^2$, so $p \mid a^2$.
Beat 3: Lemma: $p \mid a$, so $a = p \cdot c$. **Beat 4:** Substitute back: $b^2 = p \cdot c^2$, so $p \mid b$.
Beat 5: p divides both a and b — contradiction. Hence $\sqrt{p}$ is irrational.

With $p = 3$ this proves **$\sqrt{3}$ irrational**; with $p = 5$ it proves **$\sqrt{5}$ irrational**. You will write the $\sqrt{5}$ proof in full in Section 07 (Ex 1.2 Q1) and the $\sqrt{3}$ proof in Section 12.

TOPPER TIP: In the board exam, this proof is worth 3 marks and appears again and again. The marks are given beat by beat — which is why this book never compresses a proof into one paragraph.

07 Detailed Solved Examples

8D • SOLVE — NCERT EXERCISES 1.1 & 1.2, EVERY STEP SHOWN

*Here are the complete NCERT exercises for this chapter, solved the KalviYogi way: **every step numbered, every step justified, nothing skipped**. The small printed line under each solution shows how the CBSE examiner splits the marks — read it to see exactly where marks live.*

EXERCISE 1.1 • QUESTION 1 (i) and (ii)

Express each number as a product of its prime factors: (i) 140 (ii) 156

THINK FIRST: Use the division ladder: divide by the smallest prime that works, again and again, until 1 is reached. Every division is one step.

1. **(i) 140:** 140 ÷ 2 = 70.
   Why: 140 is even, so start with the smallest prime, 2.
2. 70 ÷ 2 = 35.
   Why: 70 is still even.
3. 35 ÷ 5 = 7.
   Why: 35 is odd; it ends in 5, so 5 divides it.
4. 7 ÷ 7 = 1. The ladder is complete.
   Why: 7 is prime; dividing by itself gives 1, our stop signal.
5. Collect the divisors: 140 = 2 × 2 × 5 × 7 = **$2^2 \times 5 \times 7$**. Check: 4 × 5 × 7 = 140.
   Why: The check confirms no divisor was missed.
6. **(ii) 156:** 156 ÷ 2 = 78.
   Why: 156 is even.
7. 78 ÷ 2 = 39.
   Why: 78 is even once more.
8. 39 ÷ 3 = 13.
   Why: 39 is odd; digit sum 3 + 9 = 12, so 3 divides it.
9. 13 ÷ 13 = 1. The ladder is complete.
   Why: 13 is prime.
10. Collect: 156 = 2 × 2 × 3 × 13 = **$2^2 \times 3 \times 13$**. Check: 4 × 3 × 13 = 156.

ANSWER: (i) 140 = $2^2 \times 5 \times 7$ (ii) 156 = $2^2 \times 3 \times 13$

CBSE MARKING SCHEME: 1 mark for each correct factorisation. The ladder working must be visible — a bare final answer earns nothing if one prime is wrong.

COMMON MISTAKE: *Stopping at 140 = 2 × 70. The number 70 is composite — the ladder must continue until EVERY divisor is prime and 1 is reached.*

Exercise 1.1 Q1 (iii) — 3825

EXERCISE 1.1 • QUESTION 1 (iii)

Express 3825 as a product of its prime factors.

THINK FIRST: *Same ladder. For a big number, divisibility tests choose the prime at each step.*

1. $3825 \div 3 = 1275$.
   Why: Digit sum 3 + 8 + 2 + 5 = 18 is divisible by 3, so 3 divides first.
2. $1275 \div 3 = 425$.
   Why: Digit sum of 1275 is 15, again divisible by 3.
3. $425 \div 5 = 85$.
   Why: 425 is odd; it ends in 5, so move to 5.
4. $85 \div 5 = 17$.
   Why: 85 ends in 5 once more.
5. $17 \div 17 = 1$. The ladder is complete.
   Why: 17 is prime.
6. Collect the divisors: $3825 = 3 \times 3 \times 5 \times 5 \times 17 = \mathbf{3^2 \times 5^2 \times 17}$.
   Why: Group equal primes and write them as powers.
7. Check: $3^2 \times 5^2 \times 17 = 9 \times 25 \times 17 = 225 \times 17 = 3825$.
   Why: Multiply back in chunks: 9 × 25 = 225 first.

ANSWER: $3825 = 3^2 \times 5^2 \times 17$

CBSE MARKING SCHEME: 1 mark. Divisibility reasoning (digit-sum test) shown in the ladder is the method mark.

TOPPER TIP: *Numbers ending in 25 always have 5 at least twice in their factorisation — a handy prediction to check your ladder against.*

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Exercise 1.1 Q1 (iv) and (v)

EXERCISE 1.1 • QUESTION 1 (iv) and (v)

Express each number as a product of its prime factors: (iv) 5005 (v) 7429

THINK FIRST: (iv) is friendly. (v) is the chapter's patience test: several primes fail before one works. Show every trial — the trials ARE the working.

1. **(iv) 5005:** 5005 ÷ 5 = 1001.
   Why: 5005 ends in 5, so 5 divides it.
2. 1001 ÷ 7 = 143.
   Why: 1001 is a famous number: 1001 = 7 × 11 × 13. Worth remembering!
3. 143 ÷ 11 = 13.
   Why: 143 = 11 × 13 — another pair worth recognising on sight.
4. 13 ÷ 13 = 1. Collect: 5005 = **5 × 7 × 11 × 13**. Check: 5 × 7 × 11 × 13 = 35 × 143 = 5005.
5. **(v) 7429:** Test 2, 3, 5 first: 7429 is odd (not divisible by 2); digit sum 7 + 4 + 2 + 9 = 22 (not divisible by 3); it does not end in 0 or 5 (not divisible by 5).
   Why: Rule out the small primes with quick tests before long division.
6. Test 7: 7 × 1061 = 7427 ≠ 7429. Test 11: 11 × 675 = 7425 ≠ 7429. Test 13: 13 × 571 = 7423 ≠ 7429. None works.
   Why: Multiply the prime by a trial quotient and compare — near misses prove the prime does NOT divide.
7. Test 17: 17 × 437 = 7429. Yes! So 7429 ÷ 17 = 437.
   Why: Patience pays — 17 is the first prime factor.
8. Now factorise 437: test 17 again (17 × 25 = 425 ≠ 437), then 19: 19 × 23 = 437. Yes! So 437 = 19 × 23, and 23 is prime.
   Why: 437 looks prime but is not — always keep testing until the quotient itself is prime or 1.
9. Collect: 7429 = **17 × 19 × 23**. Check: 17 × 19 = 323; 323 × 23 = 7429.

ANSWER: (iv) 5005 = 5 × 7 × 11 × 13 (v) 7429 = 17 × 19 × 23

CBSE MARKING SCHEME: 1 mark each. In (v), visible trial divisions earn the method mark even though they take space.

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Exercise 1.1 Q2 (i) — HCF and LCM of 26 and 91

EXERCISE 1.1 • QUESTION 2 (i)

Find the LCM and HCF of 26 and 91, and verify that LCM × HCF = product of the two numbers.

THINK FIRST: Factorise both numbers completely first. Then HCF = common primes, smallest powers; LCM = all primes, greatest powers.

1. Factorise 26: $26 \div 2 = 13$; $13 \div 13 = 1$. So $26 = 2 \times 13$.
2. Factorise 91: $91 \div 7 = 13$; $13 \div 13 = 1$. So $91 = 7 \times 13$.
   Why: 91 is odd and doesn't end in 5; 7 works: 7 × 13 = 91. Many students wrongly call 91 prime!
3. The only common prime is 13, with power 1 in both. HCF = **13**.
   Why: HCF uses common primes only, at the smaller power.
4. LCM takes every prime at its greatest power: $2^1 \times 7^1 \times 13^1 =$ **182**.
   Why: 2 appears only in 26 and 7 only in 91 — LCM still needs both.
5. Verify: LCM × HCF = 182 × 13 = 2366.
6. Product of the numbers: 26 × 91 = 2366.
7. 2366 = 2366. The identity is verified.
   Why: This verification is a required part of the question — never skip it.

ANSWER: HCF(26, 91) = 13 and LCM(26, 91) = 182; LCM × HCF = 2366 = 26 × 91. Verified.

CBSE MARKING SCHEME: 3 marks: 1 for both factorisations, 1 for HCF and LCM, 1 for the verification.

COMMON MISTAKE: Calling 91 a prime number. 91 = 7 × 13 is the most famous 'fake prime' in this chapter — always test 7.

Exercise 1.1 Q2 (ii) — HCF and LCM of 510 and 92

EXERCISE 1.1 • QUESTION 2 (ii)

Find the LCM and HCF of 510 and 92, and verify that LCM × HCF = product of the two numbers.

THINK FIRST: Same method. The LCM will be large — build it prime by prime and multiply in stages.

1. Factorise 510: $510 \div 2 = 255$.
   Why: 510 is even.
2. $255 \div 3 = 85$.
   Why: Digit sum 2 + 5 + 5 = 12, so 3 divides 255.
3. $85 \div 5 = 17$.
   Why: 85 ends in 5.
4. $17 \div 17 = 1$. So $510 = 2 \times 3 \times 5 \times 17$.
5. Factorise 92: $92 \div 2 = 46$; $46 \div 2 = 23$; $23 \div 23 = 1$. So $92 = 2^2 \times 23$.
6. Common prime: only 2. Smaller power is 2^1. HCF = **2**.
   Why: 23, 3, 5, 17 are not common, so they cannot enter the HCF.
7. LCM takes every prime at its greatest power: $2^2 \times 3 \times 5 \times 17 \times 23$.
8. Multiply in stages: $2^2 \times 3 = 12$; $12 \times 5 = 60$; $60 \times 17 = 1020$; $1020 \times 23 =$ **23460**. So LCM = 23460.
   Why: Stage-wise multiplication prevents arithmetic slips with big LCMs.
9. Verify: LCM × HCF = 23460 × 2 = 46920.
10. Product of the numbers: 510 × 92 = 46920.
    Why: 510 × 92 = 510 × 90 + 510 × 2 = 45900 + 1020 = 46920.
11. 46920 = 46920. The identity is verified.

ANSWER: HCF(510, 92) = 2 and LCM(510, 92) = 23460; LCM × HCF = 46920 = 510 × 92. Verified.

CBSE MARKING SCHEME: 3 marks: 1 for factorisations, 1 for HCF and LCM, 1 for verification.

Exercise 1.1 Q2 (iii) — HCF and LCM of 336 and 54

EXERCISE 1.1 • QUESTION 2 (iii)

Find the LCM and HCF of 336 and 54, and verify that LCM × HCF = product of the two numbers.

THINK FIRST: 336 has a long ladder of 2s. Take it one division at a time.

1. Factorise 336: $336 \div 2 = 168$.
2. $168 \div 2 = 84$.
3. $84 \div 2 = 42$.
4. $42 \div 2 = 21$.
   Why: Four divisions by 2 in a row — count them carefully: that will become 2^4.
5. $21 \div 3 = 7$.
   Why: 21 is odd; digit sum 3, so 3 divides it.
6. $7 \div 7 = 1$. So $336 = 2^4 \times 3 \times 7$.
7. Factorise 54: $54 \div 2 = 27$; $27 \div 3 = 9$; $9 \div 3 = 3$; $3 \div 3 = 1$. So $54 = 2 \times 3^3$.
8. Common primes: 2 and 3. Smaller powers: 2^1 and 3^1. HCF $= 2 \times 3 = \mathbf{6}$.
9. LCM: every prime at its greatest power: $2^4 \times 3^3 \times 7$. Multiply in stages: $16 \times 27 = 432$; $432 \times 7 = \mathbf{3024}$.
10. Verify: LCM × HCF $= 3024 \times 6 = 18144$.
11. Product of the numbers: $336 \times 54 = 18144$.
    Why: $336 \times 54 = 336 \times 50 + 336 \times 4 = 16800 + 1344 = 18144$.
12. $18144 = 18144$. The identity is verified.

ANSWER: HCF(336, 54) = 6 and LCM(336, 54) = 3024; LCM × HCF = 18144 = 336 × 54. Verified.

CBSE MARKING SCHEME: 3 marks: 1 for factorisations, 1 for HCF and LCM, 1 for verification.

TOPPER TIP: When a ladder has repeated divisions by the same prime, tick each one off with your pen as you count the power — losing one '2' is the commonest slip in this exercise.

Exercise 1.1 Q3 (i) — Three Numbers: 12, 15 and 21

EXERCISE 1.1 • QUESTION 3 (i)

Find the LCM and HCF of 12, 15 and 21 by applying the prime factorisation method.

THINK FIRST: Same rules, one extra care: for the HCF of three numbers, a prime must be present in ALL THREE to qualify.

1. Factorise each number: $12 = 2^2 \times 3$; $15 = 3 \times 5$; $21 = 3 \times 7$.
   Why: Short ladders: 12 = 2×2×3, 15 = 3×5, 21 = 3×7.
2. Look for a prime present in all three factorisations: only **3** appears in 12, 15 and 21.
   Why: 2 is missing from 15 and 21; 5 and 7 appear in only one number each.
3. HCF takes 3 at its smallest power anywhere, which is 3^1. HCF = **3**.
4. LCM takes every prime that appears anywhere, at its greatest power: 2^2 (from 12), 3^1, 5^1, 7^1.
5. Multiply in stages: $4 \times 3 = 12$; $12 \times 5 = 60$; $60 \times 7 = \mathbf{420}$. So LCM = 420.
6. Sanity check: $420 \div 12 = 35$, $420 \div 15 = 28$, $420 \div 21 = 20$ — all exact, so 420 is truly a common multiple; and no smaller candidate survives all three.
   Why: Dividing the LCM back by each number is a fast honesty check.

ANSWER: HCF(12, 15, 21) = 3 and LCM(12, 15, 21) = 420.

CBSE MARKING SCHEME: 2 marks: 1 for the three factorisations, 1 for HCF and LCM.

COMMON MISTAKE: Trying to verify with HCF × LCM = product here. That identity is for TWO numbers only (Derivation A, Section 06): $3 \times 420 = 1260 \neq 12 \times 15 \times 21 = 3780$ — and nothing has gone wrong!

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Exercise 1.1 Q3 (ii) and (iii)

EXERCISE 1.1 • QUESTION 3 (ii) and (iii)

Find the LCM and HCF of the following integers by the prime factorisation method: (ii) 17, 23 and 29 (iii) 8, 9 and 25

THINK FIRST: Both triples have NO prime common to all three. Watch what that forces the HCF to be.

1. **(ii)** 17, 23 and 29 are each prime themselves, so their factorisations are just 17, 23 and 29.
   Why: Test each: none of 17, 23, 29 has any divisor besides 1 and itself.
2. No prime is common to all three (in fact, no prime is shared by even two of them). So HCF = **1**.
   Why: When nothing is common, the HCF falls back to 1 – the numbers are pairwise co-prime.
3. LCM = 17 × 23 × 29. Multiply in stages: 17 × 23 = 391; 391 × 29 = **11339**.
   Why: With no shared primes, the LCM is simply the full product.
4. **(iii)** Factorise: $8 = 2^3$; $9 = 3^2$; $25 = 5^2$.
5. The three factorisations use completely different primes (2, 3 and 5). No prime is common to all three, so HCF = **1**.
   Why: The numbers need not be prime for HCF to be 1 – they only need to share no prime.
6. LCM takes every prime at its greatest power: $2^3 \times 3^2 \times 5^2 = 8 \times 9 \times 25$. Multiply: 8 × 9 = 72; 72 × 25 = **1800**.

ANSWER: (ii) HCF = 1, LCM = 11339 (iii) HCF = 1, LCM = 1800

CBSE MARKING SCHEME: 2 marks for each part: factorisations + correct HCF and LCM.

TOPPER TIP: HCF = 1 does NOT mean the numbers are prime. It means their prime factorisations share nothing — compare 8, 9, 25: all composite, yet HCF = 1.

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Exercise 1.1 Q4 and Q5

EXERCISE 1.1 • QUESTION 4

Given that HCF (306, 657) = 9, find LCM (306, 657).

THINK FIRST: This is Derivation A at work: for two numbers, HCF × LCM = product of the numbers. We are given the HCF, so the LCM is one division away.

1. Write the identity with the known values: 9 × LCM (306, 657) = 306 × 657.
   Why: The identity holds for any TWO positive integers — Section 06 proved it.
2. Compute the product: 306 × 657 = 201042.
   Why: 306 × 657 = 306 × 600 + 306 × 57 = 183600 + 17442 = 201042.
3. Divide both sides by 9: LCM = 201042 ÷ 9 = **22338**.
   Why: Isolating the unknown is just undoing the multiplication.
4. Honesty check: 22338 ÷ 306 = 73 and 22338 ÷ 657 = 34 — both exact, so 22338 really is a common multiple of 306 and 657.
   Why: And since HCF × LCM matched the product exactly, it is the SMALLEST such multiple.

ANSWER: LCM (306, 657) = 22338.

CBSE MARKING SCHEME: 2 marks: 1 for using the identity correctly, 1 for the arithmetic.

EXERCISE 1.1 • QUESTION 5

Check whether 6^n can end with the digit 0 for any natural number n.

THINK FIRST: Think about what 'ending in 0' forces. A number ends in 0 exactly when 10 divides it — and 10 = 2 × 5. So the prime 5 must appear in the number's factorisation.

1. Suppose, if possible, that 6^n ends with the digit 0 for some natural number n.
   Why: We test the claim by examining the prime factorisation — it cannot lie.
2. Then 10 divides 6^n, so both 2 and 5 are prime factors of 6^n.
   Why: 10 = 2 × 5, and a divisor's primes must all live inside the number (Fundamental Theorem).
3. But factorise 6^n: $6^n = (2 \times 3)^n = 2^n \times 3^n$. Its only prime factors are 2 and 3 — for every n.
   Why: Powers never create new primes: 6^n is built from 2s and 3s alone, however large n is.
4. The prime 5 is missing, so 10 can never divide 6^n.
   Why: Contradiction with Step 2 — the supposition collapses.
5. Therefore 6^n can NEVER end with the digit 0, for any natural number n.
   Why: In fact 6^n always ends in 6: 6, 36, 216, 1296, ...

ANSWER: No — 6^n can never end with the digit 0 for any natural number n.

CBSE MARKING SCHEME: 2 marks: 1 for linking 'ends in 0' to the prime factor 5, 1 for the factorisation argument.

Exercise 1.1 Q6 — Proving Numbers Composite

EXERCISE 1.1 • QUESTION 6

Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

THINK FIRST: A composite number is one that has a factor besides 1 and itself. Strategy: do NOT multiply everything out. Instead, spot the common factor in the two terms and factor it out — the two resulting factors expose the number as composite.

1. **First number:** $7 \times 11 \times 13 + 13$. Both terms contain the factor 13: the first term is $13 \times (7 \times 11)$ and the second term is 13×1.
   Why: Look for a factor shared by BOTH terms of the sum — here it is 13.

2. Factor 13 out: $13 \times (7 \times 11 + 1) = 13 \times (77 + 1) = 13 \times 78$.
   Why: This is just the distributive law run backwards.

3. Now the number is written as 13×78, where BOTH factors are greater than 1 and smaller than the number itself (the number is 1014).
   Why: A number expressed as a product of two smaller factors greater than 1 is composite by definition.

4. Hence $7 \times 11 \times 13 + 13$ is composite. (Its full factorisation is $2 \times 3 \times 13^2$, if ever needed.)

ANSWER: $7 \times 11 \times 13 + 13 = 13 \times 78$ — composite.

CBSE MARKING SCHEME: 1 mark for this number — awarded for the factorisation, not for the final value.

Exercise 1.1 Q6 (continued) — The Second Number

EXERCISE 1.1 • QUESTION 6 — SECOND NUMBER

Explain why 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 is a composite number.

1. Both terms contain the factor 5: the long product contains × 5 × inside it, and the second term is 5 × 1.
   Why: The shared factor this time is 5, hiding inside the long multiplication.
2. Factor 5 out: 5 × (7 × 6 × 4 × 3 × 2 × 1 + 1) = 5 × (1008 + 1) = 5 × 1009.
   Why: Inside the bracket, the × 5 × has been removed from the product: 7 × 6 × 4 × 3 × 2 × 1 = 1008.
3. Both factors, 5 and 1009, are greater than 1 and smaller than the number itself (which is 5045).
4. Hence 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 is also composite.

ANSWER: 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 = 5 × 1009 — composite. (Together with the first number, Q6 is complete: 1 mark was for each factorisation.)

COMMON MISTAKE: Multiplying everything out first (getting 1014 and 5045) and then hunting for factors. That is the slow road — and for bigger numbers it is a dead end. Factor out FIRST.

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Exercise 1.1 Q7 — Sonia and Ravi

EXERCISE 1.1 • QUESTION 7

There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

THINK FIRST: *This is Story Problem 2 wearing sports clothes. They meet at the start when the elapsed time is a whole number of rounds for BOTH — a common multiple of 18 and 12 — and we want the first such time: the LCM.*

1. Sonia is back at the start after every multiple of 18 minutes: 18, 36, 54, ...

   Why: Each of her rounds takes exactly 18 minutes.

2. Ravi is back at the start after every multiple of 12 minutes: 12, 24, 36, 48, ...

3. They are at the start TOGETHER at the times common to both lists: 36, 72, ... The first such time is 36 minutes.

   Why: Common multiples of 18 and 12 — and 'again' means the smallest one greater than zero.

4. Confirm by prime factorisation: $18 = 2 \times 3^2$ and $12 = 2^2 \times 3$, so LCM $= 2^2 \times 3^2 = 4 \times 9 = 36$.

   Why: The prime method and the listing method agree, as they must.

5. Check the meaning: in 36 minutes Sonia completes $36 \div 18 = 2$ rounds and Ravi completes $36 \div 12 = 3$ rounds — both finish a whole number of rounds at the same instant.

   Why: Translating the answer back into the story is the final, often-forgotten step.

ANSWER: They will meet again at the starting point after 36 minutes.

CBSE MARKING SCHEME: 3 marks: 1 for recognising LCM, 1 for computing it, 1 for the concluding statement in context.

COMMON MISTAKE: *Using HCF here (answer 6) because the numbers 'feel' like a sharing problem. Ask instead: do I need a divisor (sharing/cutting) or a multiple (meeting/repeating)? Meeting = multiples = LCM.*

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Exercise 1.2 Q1 — Proving √5 is Irrational

EXERCISE 1.2 • QUESTION 1

Prove that √5 is irrational.

THINK FIRST: Run the 5-beat rhythm of Derivation B with p = 5. Every beat is one step — write them all.

1. Assume, to the contrary, that √5 is rational.
   Why: Proof by contradiction begins by assuming the opposite of the claim.
2. Then √5 = a/b, where a and b are co-prime positive integers and b ≠ 0.
   Why: Reduce the fraction to lowest form — this co-prime condition is what will be contradicted.
3. Square both sides: $5 = a^2/b^2$, so $a^2 = 5b^2$.
   Why: Squaring clears the root.
4. So 5 divides a^2, because $a^2 = 5 \times b^2$.
5. By the lemma (prime p dividing a^2 must divide a), 5 divides a.
   Why: Section 06 proved this lemma from the Fundamental Theorem.
6. Write a = 5c for some integer c. Substitute into $a^2 = 5b^2$: $(5c)^2 = 5b^2$, so $25c^2 = 5b^2$, so $b^2 = 5c^2$.
   Why: Divide $25c^2 = 5b^2$ by 5 to get $b^2 = 5c^2$.
7. So 5 divides b^2, and by the lemma again, 5 divides b.
   Why: The engine fires a second time, now on b.
8. Thus 5 is a common factor of a and b — contradicting Step 2, where a and b were co-prime.
   Why: Every step was forced; the only weak point is the opening assumption.
9. The assumption is false. Therefore √5 is irrational. ∎

ANSWER: √5 is irrational.

CBSE MARKING SCHEME: 3 marks, awarded beat by beat: assumption with co-prime condition (1), 5 | a and 5 | b (1), contradiction and conclusion (1).

TOPPER TIP: Examiners deduct when the co-prime condition is not stated in Step 2 — without it, there is nothing to contradict in Step 8.

Exercise 1.2 Q2 — Proving 3 + 2√5 is Irrational

EXERCISE 1.2 • QUESTION 2

Prove that 3 + 2√5 is irrational.

THINK FIRST: New shape, same weapon. Assume it is rational, then use ordinary algebra to drag the √5 out alone. If √5 is forced to equal a rational expression, we have a contradiction — because Q1 just proved √5 irrational.

1. Assume, to the contrary, that $3 + 2\sqrt{5}$ is rational.
2. Then $3 + 2\sqrt{5} = a/b$ for some integers a and b with $b \neq 0$.
   Why: This is simply the definition of a rational number — no co-prime condition is needed for this style of proof.
3. Subtract 3 from both sides: $2\sqrt{5} = a/b - 3$.
   Why: Begin isolating the irrational part.
4. Write the right side as a single fraction: $2\sqrt{5} = (a - 3b)/b$.
5. Divide both sides by 2: $\sqrt{5} = (a - 3b)/(2b)$.
   Why: Now √5 stands completely alone on the left.
6. The right side, $(a - 3b)/(2b)$, is a ratio of two integers with a non-zero denominator — so it is a rational number.
   Why: a, b integers ⇒ a − 3b and 2b are integers, and $2b \neq 0$ since $b \neq 0$.
7. So √5 would be rational — but Exercise 1.2 Q1 proved √5 is irrational. Contradiction.
   Why: The new proof stands on the shoulders of the previous one — that is why the exercise order matters.
8. Therefore our assumption is false: $3 + 2\sqrt{5}$ is irrational. ∎

ANSWER: $3 + 2\sqrt{5}$ is irrational.

CBSE MARKING SCHEME: 3 marks: assumption (1/2), isolating √5 by correct rearrangement (1½), contradiction citing Q1 (1).

TOPPER TIP: The general law you just used: rational ± irrational = irrational, and (non-zero rational) × irrational = irrational. Section 05's definition card plus this proof pattern settle every question of this family.

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Exercise 1.2 Q3 — Three More Irrationals

EXERCISE 1.2 • QUESTION 3 (i)

Prove that $1/\sqrt{2}$ is irrational.

THINK FIRST: Assume it is rational and turn it upside down: if $1/\sqrt{2}$ were a tidy fraction, then $\sqrt{2}$ would be one too.

1. Assume, to the contrary, that $1/\sqrt{2}$ is rational.
2. Then $1/\sqrt{2} = a/b$ for integers a, b with $a \neq 0$ and $b \neq 0$.
   Why: $a \neq 0$ because $1/\sqrt{2}$ is not zero.
3. Take reciprocals of both sides: $\sqrt{2} = b/a$.
   Why: Flipping a fraction is legal since neither side is zero.
4. b/a is a ratio of integers with non-zero denominator, so $\sqrt{2}$ would be rational.
5. But $\sqrt{2}$ is irrational (proved in Section 06, Derivation B). Contradiction.
6. Therefore $1/\sqrt{2}$ is irrational. ∎

ANSWER: $1/\sqrt{2}$ is irrational.

CBSE MARKING SCHEME: 1 mark: the reciprocal step plus citing √2's irrationality.

EXERCISE 1.2 • QUESTION 3 (ii) and (iii)

Prove that the following are irrationals: (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$

THINK FIRST: Same isolation trick as Q2: assume rational, rearrange until $\sqrt{5}$ or $\sqrt{2}$ stands alone, and let Q1 / Section 06 deliver the contradiction.

1. **(ii)** Assume $7\sqrt{5}$ is rational, say $7\sqrt{5} = a/b$ with $b \neq 0$.
2. Divide both sides by 7: $\sqrt{5} = a/(7b)$.
   Why: Isolating the root is the whole game.
3. $a/(7b)$ is rational, so $\sqrt{5}$ would be rational — contradicting Q1. Hence $7\sqrt{5}$ is irrational. ∎
4. **(iii)** Assume $6 + \sqrt{2}$ is rational, say $6 + \sqrt{2} = a/b$ with $b \neq 0$.
5. Subtract 6 from both sides: $\sqrt{2} = a/b - 6 = (a - 6b)/b$.
6. $(a - 6b)/b$ is rational, so $\sqrt{2}$ would be rational — contradicting Section 06. Hence $6 + \sqrt{2}$ is irrational. ∎

ANSWER: (ii) $7\sqrt{5}$ is irrational (iii) $6 + \sqrt{2}$ is irrational

CBSE MARKING SCHEME: 1 mark each: rearrangement isolating the root (1/2) and the contradiction citing the known irrational (1/2).

08 Multiple Solution Methods

8D • CHALLENGE — ONE PROBLEM, TWO ROADS

*A mathematical thinker owns more than one road to every answer. Take the pair **48 and 120** and find its HCF by two completely different methods.*

Method 1 — Prime factorisation

1. Factorise 48: $48 \div 2 = 24$; $24 \div 2 = 12$; $12 \div 2 = 6$; $6 \div 2 = 3$; $3 \div 3 = 1$. So $48 = 2^4 \times 3$.
2. Factorise 120: $120 \div 2 = 60$; $60 \div 2 = 30$; $30 \div 2 = 15$; $15 \div 3 = 5$; $5 \div 5 = 1$. So $120 = 2^3 \times 3 \times 5$.
3. HCF: common primes are 2 and 3; take smaller powers: $2^3 \times 3 = 8 \times 3 = \mathbf{24}$.
4. Bonus — the same factorisations also give the LCM: $2^4 \times 3 \times 5 = \mathbf{240}$.

   Why: The prime road hands you HCF and LCM together.

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Multiple Methods — Method 2: Euclid's Division

The second road needs no factorisation at all — only division with remainder, repeated until the remainder is 0.

1. Divide the bigger by the smaller: $120 = 48 \times 2 + 24$.
   Why: We only care about the remainder, 24.
2. Now divide the previous divisor by the remainder: $48 = 24 \times 2 + 0$.
   Why: Each step replaces the pair by (divisor, remainder) — the HCF never changes.
3. The remainder is 0, so the last divisor is the HCF: **24**.
   Why: Euclid's method stops exactly when the remainder hits 0.
4. Euclid gives only the HCF. Get the LCM from the identity: LCM = (48 × 120) ÷ 24 = 5760 ÷ 24 = **240**.
   Why: Derivation A (Section 06) is the bridge between the two methods.

When to use which?

Prime factorisation — best when numbers are small or medium, when you need HCF and LCM, and for THREE numbers (Euclid handles only pairs). It is also the method NCERT Exercise 1.1 asks for by name.
Euclid's division — best for very large numbers whose factors are hard to spot (try factorising 7429 again!). It needs no factorisation at all — just division with remainder.

09 Interactive Simulations & Hands-on Activities

TRY IT YOURSELF — LEARN BY DOING

Put the pen in your own hand. Complete these three activities before checking the answers at the bottom of the next page.

Activity 1 — Factor-tree challenge

Draw the complete factor tree for each number and write the prime factorisation. The first branch is started for you.

Number	First split	Prime factorisation (you write it)
72	72 = 8 × 9	72 = ...
180	180 = 18 × 10	180 = ...
294	294 = 2 × 147	294 = ...

Activity 2 — Plot it on the number line

On a hand-drawn number line from 0 to 3, mark these five numbers as accurately as you can: 3/2, √2, 7/3, √5, 5/2. Use the approximations √2 ≈ 1.414 and √5 ≈ 2.236. Then answer: which is larger, √5 or 7/3? How does the picture prove it?

DIGITAL: Scan the chapter QR code (on the book's contents page) to open the KalviYogi Number-Line Simulation — drag √2 and √5 onto the line and watch their decimals grow digit by digit.

Activity 3 — The HCF × LCM verification lab

For each pair, find the HCF and LCM yourself, then complete the last two columns. The final column must match the product column — if it doesn't, your HCF or LCM is wrong. The lab catches its own errors!

Pair	HCF	LCM	HCF × LCM	Product of the pair	Match?
16 and 20				320	☐
25 and 40				1000	☐
14 and 35				490	☐

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Activity Answers — Check After Trying

Activity 1: $72 = 2^3 \times 3^2$ ($72 = 8 \times 9 = 2^3 \times 3^2$). $180 = 2^2 \times 3^2 \times 5$. $294 = 2 \times 3 \times 7^2$ ($294 = 2 \times 147 = 2 \times 3 \times 49$).

Activity 2: Order on the line: $\sqrt{2}$ (1.414) < 3/2 (1.5) < $\sqrt{5}$ (2.236) < 7/3 (2.333) < 5/2 (2.5). So **7/3 is larger than $\sqrt{5}$** — its point lies to the right.

Activity 3: 16 and 20: HCF 4, LCM 80, $4 \times 80 = 320$. 25 and 40: HCF 5, LCM 200, $5 \times 200 = 1000$. 14 and 35: HCF 7, LCM 70, $7 \times 70 = 490$. All three match.

TOPPER TIP: If your lab row did not match, the error is almost always a missed prime in one factorisation. Re-run the ladder for that number — do not just adjust the answer.

DIGITAL: Scan the chapter QR code for the KalviYogi Factor-Tree Simulator — split numbers yourself, race the clock, and try to find a factorisation of 360 DIFFERENT from $2^3 \times 3^2 \times 5$. (You will fail. That failure IS the Fundamental Theorem.)

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10 Common Mistakes

LEARN FROM OTHER STUDENTS' ERRORS — BEFORE THEY BECOME YOURS

MISTAKE 1 — Using HCF × LCM = a × b for THREE numbers.

Wrong way: For 12, 15, 21, writing "HCF × LCM = 12 × 15 × 21, so if HCF = 3 then LCM = 1260."

Why wrong: The identity was derived (Section 06) from min + max = sum of TWO exponents. With three exponents, the middle value is lost.

Right way: The identity is for exactly two numbers. For three numbers, build the LCM directly from prime powers: $2^2 \times 3 \times 5 \times 7 = 420$.

MISTAKE 2 — Stopping the factorisation at a composite number.

Wrong way: $156 = 12 \times 13$, full stop.

Why wrong: 12 is composite ($12 = 2^2 \times 3$). The answer is not yet in primes, so HCF/LCM read from it will be wrong.

Right way: Continue until every factor is prime: $156 = 2^2 \times 3 \times 13$. If any factor can be split, split it.

MISTAKE 3 — Writing "a = 5c" from $a^2 = 5b^2$ without justification.

Wrong way: "$a^2 = 5b^2$, so $a = 5c$" — jumping straight to the conclusion.

Why wrong: The step uses TWO facts: 5 divides a^2 (from the equation), and the lemma that a prime dividing a^2 divides a. Skipping them loses the method mark — and hides the logic from yourself.

Right way: Write both links: "$5 \mid a^2 \Rightarrow 5 \mid a$ (lemma) $\Rightarrow a = 5c$."

Common Mistakes 4–6

MISTAKE 4 — Forgetting the co-prime condition in an irrationality proof.
Wrong way: "Let $\sqrt{5} = a/b$. Then $a^2 = 5b^2$ … so 5 divides a and b. Hence irrational."
Why wrong: "5 divides both" is only a contradiction if a and b were promised to share no common factor. Without the promise, there is no contradiction and the proof proves nothing.
Right way: Always open with: "$\sqrt{5} = a/b$ where a, b are **co-prime**." Then the final line lands: "… contradicting that a and b are co-prime."

MISTAKE 5 — Choosing LCM for a 'largest equal pieces' problem.
Wrong way: Ribbons of 96 cm and 120 cm are cut into equal pieces of the greatest possible length; a student computes LCM = 480 cm — longer than both ribbons!
Why wrong: A piece length must DIVIDE both ribbons. LCM is a multiple — it is bigger than the ribbons and cannot be cut out of them.
Right way: Cutting/sharing/packing into equal largest parts = HCF (here 24 cm). Meeting/repeating together = LCM. Let the story's action decide.

MISTAKE 6 — "$\sqrt{2}$ = 1.414, so it is rational because I can write its decimal."
Wrong way: Treating a rounded approximation as the number itself.
Why wrong: 1.414 is not $\sqrt{2}$; it is a terminating decimal that sits very close to $\sqrt{2}$. The true decimal of $\sqrt{2}$ never terminates and never repeats — that is exactly what makes it irrational.
Right way: Judge rationality by the DEFINITION (can it be written as p/q?), never by a calculator display. Section 06 settles $\sqrt{2}$ by proof, not by approximation.

TOPPER TIP: Before submitting any answer in this chapter, run the three-point check: (1) Is every factor prime? (2) Did I use HCF/LCM for the right story-signal? (3) In a proof, did I state the co-prime condition and reach an explicit contradiction?

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11 Graded Practice Problems

FIVE LEVELS — CLIMB ONE LEVEL AT A TIME • ANSWERS IN THE ANSWER KEY

Rules of the climb: finish a level with at most one error before moving up. Write full steps for Level 2 onwards — answers alone earn nothing in the board exam.

LEVEL 1 — FOUNDATION (direct concepts)

L1.1 Express 225 as a product of prime factors.

L1.2 Express 360 as a product of prime factors.

L1.3 Find the HCF of 18 and 48.

L1.4 Find the LCM of 14 and 20.

L1.5 Find the HCF and the LCM of 9 and 25. What special thing do you notice about the HCF?

L1.6 Using prime factorisation, find the HCF of 24 and 36.

LEVEL 2 — SKILL BUILDER (textbook standard)

L2.1 Find the HCF and LCM of 90 and 144, and verify that HCF × LCM = product of the two numbers.

L2.2 Find the HCF of 96 and 404 by prime factorisation. Hence, find their LCM.

L2.3 Find the HCF and LCM of 8, 36 and 90 by the prime factorisation method.

L2.4 Find the HCF and LCM of 867 and 255.

L2.5 Check whether 12^n can end with the digit 0 or the digit 5 for any natural number n. Give reasons.

L2.6 Explain why $3 \times 5 \times 7 + 7$ is a composite number.

Graded Practice — Levels 3, 4 and 5

LEVEL 3 — APPLICATION (real-life problems)

L3.1 Two traffic lights at a crossing change every 48 seconds and every 72 seconds. They change together at 7:00 a.m. At what time will they next change together?

L3.2 A hall floor is 18 m long and 12 m wide. Square tiles of the largest possible size are to be laid on it without cutting any tile. Find the side length of each tile and the number of tiles needed.

L3.3 A school takes 156 Class-9 students and 234 Class-10 students on a field trip. Each bus carries students of only one class, every bus carries the same number of students, and that number is as large as possible. How many students ride in each bus, and how many buses are needed in all?

L3.4 Three bells toll at intervals of 9, 12 and 15 minutes. They toll together at 8:00 a.m. When will they next toll together?

LEVEL 4 — ADVANCED REASONING (multi-concept)

L4.1 Two numbers are in the ratio 3 : 4 and their LCM is 180. Find the two numbers and their HCF.

L4.2 Find the prime factorisation of $5 \times 7 \times 11 + 11$, and hence explain why the number is composite.

L4.3 Prove that the product of any three consecutive positive integers is always divisible by 6. (Hint: among three consecutive integers, one is a multiple of 3 and at least one is even.)

LEVEL 5 — MATHEMATICAL GENIUS (olympiad flavour)

L5.1 A sweet seller packs sweets in groups of 12, in groups of 16, or in groups of 18 — and in every case exactly 5 sweets are left unpacked. What is the smallest possible number of sweets he has?

L5.2 Find the smallest number that leaves a remainder of 1 when divided by 5, by 6 or by 7, and is exactly divisible by 11.

> **TOPPER TIP:** Stuck at a level? The worked examples of Section 07 contain a twin of almost every question here. Find the twin, study its steps, close the book, and try again.

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12 Mathematical Reasoning

JUSTIFY AND PROVE — FULL SOLUTIONS

REASONING 1

Prove that √3 is irrational.

THINK FIRST: Compare this proof with Ex 1.2 Q1 (√5) line by line. Only the prime changed — the logic is one single machine.

1. Assume, to the contrary, that √3 is rational.
   Why: The 5-beat rhythm of Section 06, now with p = 3.
2. Then √3 = a/b, where a and b are co-prime positive integers and b ≠ 0.
3. Square both sides: $3 = a^2/b^2$, so $a^2 = 3b^2$.
4. So 3 divides a^2, since $a^2 = 3 \times b^2$.
5. By the lemma, 3 divides a. Write a = 3c for some integer c.
   Why: Prime dividing the square must divide the number itself.
6. Substitute: $(3c)^2 = 3b^2$, so $9c^2 = 3b^2$, so $b^2 = 3c^2$.
   Why: Divide $9c^2 = 3b^2$ by 3.
7. So 3 divides b^2, and by the lemma again, 3 divides b.
8. Now 3 divides both a and b, contradicting the co-prime condition of Step 2.
9. The assumption is false. Therefore √3 is irrational. ∎

ANSWER: √3 is irrational.

Reasoning 2 — A Divisibility Proof by Cases

REASONING 2

Prove that the product of any two consecutive positive integers is always divisible by 2.

THINK FIRST: When a statement says 'any' or 'always', examples can only support it — never prove it. Cases or algebra must do the work.

1. Let the two consecutive integers be n and $n + 1$, where n is any positive integer.
   Why: Naming the unknown is always Step 1 of a general proof — we must prove it for EVERY n at once.
2. Every positive integer is either even or odd, so exactly two cases are possible. We settle both.
   Why: A proof by cases is complete only when the cases cover everything and each case is finished.
3. **Case 1: n is even.** Then $n = 2k$ for some integer k, so $n(n + 1) = 2k(n + 1)$ — visibly twice an integer, hence divisible by 2.
   Why: The factor n itself supplies the 2.
4. **Case 2: n is odd.** Then $n + 1$ is even (odd + 1 = even), so $n + 1 = 2k$ for some integer k, and $n(n + 1) = n \times 2k$ — again twice an integer, hence divisible by 2.
   Why: This time the OTHER factor supplies the 2. One of the two neighbours is always even.
5. In both cases $n(n + 1)$ is divisible by 2, and there are no other cases.
6. Therefore the product of any two consecutive positive integers is always divisible by 2. ∎
   Why: Check with examples: 7 × 8 = 56, 12 × 13 = 156 — both even, as proved.

ANSWER: $n(n + 1)$ is always divisible by 2.

Reasoning 3 — A Stronger Divisibility Proof

REASONING 3

Prove that $n^3 - n$ is divisible by 6 for every positive integer n.

THINK FIRST: Notice the ladder of ideas: factorise → recognise consecutive integers → use remainders → combine co-prime divisors. Each rung is a tool you now own.

1. Factorise the expression: $n^3 - n = n(n^2 - 1) = n(n - 1)(n + 1)$.
   Why: Use the identity $a^2 - 1 = (a - 1)(a + 1)$ with $a = n$.

2. Reorder the factors: $n^3 - n = (n - 1) \times n \times (n + 1)$ — a product of THREE consecutive integers.
   Why: The factorisation has revealed hidden structure: three neighbours in a row.

3. Among any three consecutive integers, one is divisible by 3: their remainders on division by 3 are 0, 1, 2 in some order, and the one with remainder 0 is a multiple of 3. So 3 divides the product.
   Why: Remainders cycle 0, 1, 2, 0, 1, 2, ... — any three consecutive integers must contain a full cycle.

4. Among any two consecutive integers, one is even (Reasoning 2), so among three consecutive integers at least one is even. Hence 2 divides the product.
   Why: We reuse the previous result — proofs build on proofs.

5. So the product is divisible by both 2 and 3. Since 2 and 3 are co-prime, the product is divisible by $2 \times 3 = 6$.
   Why: Divisibility by two co-prime numbers combines into divisibility by their product.

6. Therefore $n^3 - n$ is divisible by 6 for every positive integer n. ∎
   Why: Test: $n = 4$ gives $64 - 4 = 60 = 6 \times 10$; $n = 5$ gives $125 - 5 = 120 = 6 \times 20$.

ANSWER: $n^3 - n$ is always divisible by 6.

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13 Real-Life Applications

8D • APPLY — THE CHAPTER AT WORK IN THE WORLD

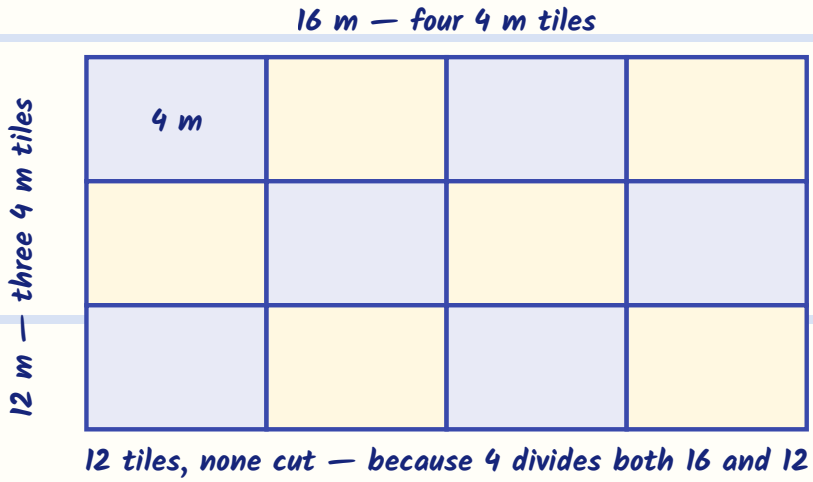
APPLICATION 1 — TILING A FLOOR (HCF)

A rectangular floor is 16 m long and 12 m wide. It is to be covered with square tiles of the largest possible size, without cutting any tile. Find the side of each tile and the number of tiles needed.

THINK FIRST: *Cutting/packing/covering with the largest equal size → HCF. Always.*

1. The tile side must divide BOTH 16 and 12 exactly — otherwise a tile would have to be cut at the edges.
   Why: No cutting allowed means the side is a common divisor of length and width.
2. 'Largest possible' tells us to take the greatest common divisor: the HCF.
   Why: Same signal as the gift boxes in the story.
3. Factorise: $16 = 2^4$ and $12 = 2^2 \times 3$. HCF = $2^2 = 4$. So each tile is 4 m × 4 m.
   Why: Common prime is 2; take the smaller power.
4. Tiles along the length: $16 \div 4 = 4$. Tiles along the width: $12 \div 4 = 3$.
5. Total tiles = $4 \times 3 = 12$.
   Why: Multiply rows by columns — and note how the HCF made both divisions exact.

ANSWER: Each tile is 4 m square, and 12 tiles are needed.



Real-Life Application 2 — Buses Leaving Together

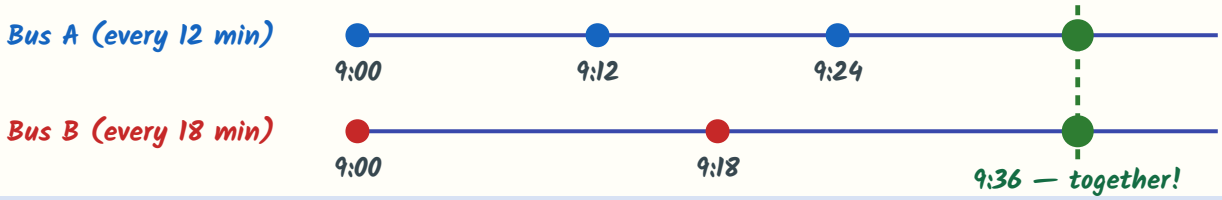
APPLICATION 2 — BUSES LEAVING TOGETHER (LCM)

From a bus stand, Bus A departs every 12 minutes and Bus B departs every 18 minutes. Both departed together at 9:00 a.m. At what time will they next depart together?

THINK FIRST: Repeating events meeting again → LCM. Always.

1. Bus A departs at multiples of 12 minutes after 9:00; Bus B at multiples of 18 minutes.
   Why: Departure times are multiples of the intervals.
2. They coincide at common multiples; the NEXT time together is the smallest one: the LCM.
   Why: Same signal as the suppliers in the story.
3. Factorise: $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$. LCM $= 2^2 \times 3^2 = 36$.
   Why: Every prime at its greatest power.
4. 9:00 a.m. + 36 minutes = **9:36 a.m.**
   Why: Translate the number back into the story's language — a time, not just 36.

ANSWER: They will next depart together at 9:36 a.m.



See it: the two rows of dots march at different spacings, and the first place they line up again is 36 minutes — the LCM, drawn as a picture.

Real-Life Application 3 — Distributing Fruit

APPLICATION 3 — DISTRIBUTING SWEETS EQUALLY (HCF)

For a school celebration, 250 oranges and 350 apples are packed into fruit boxes. Each box contains only one kind of fruit, every box contains the same number of fruits, no fruit is left over, and the boxes are as large as possible. Find the number of fruits per box and the total number of boxes.

THINK FIRST: *One method, many costumes: sweets, fruits, tiles, ribbons — the HCF does not care what the objects are.*

1. The fruits-per-box number must divide both 250 and 350 exactly, and must be the largest such number: the HCF.
   Why: The story signal is identical to Application 1 — only the objects changed.
2. Factorise: $250 = 2 \times 5^3$ and $350 = 2 \times 5^2 \times 7$.
   Why: $250 = 2 \times 125$; $350 = 2 \times 175 = 2 \times 25 \times 7$.
3. Common primes: 2 and 5. Smaller powers: 2^1 and 5^2. HCF $= 2 \times 25 =$ **50** fruits per box.
4. Orange boxes: $250 \div 50 = 5$. Apple boxes: $350 \div 50 = 7$.
5. Total boxes $= 5 + 7 =$ **12**.

ANSWER: 50 fruits in each box; 12 boxes in all (5 of oranges, 7 of apples).

Real-Life Application 4 — Cutting Ribbons

APPLICATION 4 — CUTTING RIBBONS (HCF)

A red ribbon is 96 cm long and a gold ribbon is 120 cm long. Both are cut into pieces of equal length, the greatest possible length, with nothing wasted. Find the length of each piece and the total number of pieces.

THINK FIRST: *If your answer for a cutting problem is LARGER than the ribbons themselves, you have computed an LCM by mistake — see Common Mistake 5.*

1. Each piece length must divide both 96 and 120 exactly — a common divisor; 'greatest possible' makes it the HCF.
   Why: Cutting into equal pieces is the third costume of the same idea.
2. Factorise: $96 = 2^5 \times 3$ and $120 = 2^3 \times 3 \times 5$.
   Why: 96 = 32 × 3; 120 = 8 × 15.
3. Common primes: 2 and 3. Smaller powers: 2^3 and 3^1. HCF = 8 × 3 = **24** cm per piece.
4. Red pieces: 96 ÷ 24 = 4. Gold pieces: 120 ÷ 24 = 5.
5. Total pieces = 4 + 5 = **9**.

ANSWER: Each piece is 24 cm long; there are 9 pieces in all (4 red, 5 gold).

Section 13 in one breath

*Four stories, two ideas. Tiling, distributing and cutting all asked for a **divisor** — the greatest one (HCF). Buses departing together asked for a **multiple** — the smallest common one (LCM). When you meet a new story, do not ask "which formula?" Ask: **"Is the answer a divisor of the given numbers, or a multiple of them?"** The story always tells you.*

14 Competency-Based Questions

CASE STUDY & ASSERTION-REASON — BOARD EXAM STYLE, FULLY SOLVED

Case Study — The School Garden Project. *Green Valley School is building a kitchen garden. The gardening club has received* ***126 rose saplings*** *and* ***210 marigold saplings****. The club decides that in each row, only one kind of sapling will be planted; every row must contain the* ***same number of saplings****; all saplings must be planted; and to keep the rows easy to maintain, the number of saplings in each row should be the* ***maximum possible****.*

CASE QUESTION (i)

What is the maximum number of saplings that can be planted in each row?

1. The row size must divide both 126 and 210 exactly, and must be maximum: the HCF of 126 and 210.
   Why: Recognise the story signal first — equal rows, nothing left, maximum size.
2. Factorise 126: $126 \div 2 = 63$; $63 \div 3 = 21$; $21 \div 3 = 7$; $7 \div 7 = 1$. So $126 = 2 \times 3^2 \times 7$.
3. Factorise 210: $210 \div 2 = 105$; $105 \div 3 = 35$; $35 \div 5 = 7$; $7 \div 7 = 1$. So $210 = 2 \times 3 \times 5 \times 7$.
4. Common primes: 2, 3 and 7. Smaller powers: 2^1, 3^1, 7^1. HCF $= 2 \times 3 \times 7 =$ **42**.

ANSWER: 42 saplings in each row.

CASE QUESTION (ii)

How many rows of each kind will the garden have, and how many rows in total?

1. Rose rows $= 126 \div 42 =$ **3**.
   Why: Divide each stock by the row size.
2. Marigold rows $= 210 \div 42 =$ **5**.
3. Total rows $= 3 + 5 =$ **8**.

ANSWER: 3 rows of roses and 5 rows of marigolds — 8 rows in all.

Competency — Case (continued) and Assertion–Reason

CASE QUESTION (iii)

The rose bed is watered every 6 days and the marigold bed every 8 days. If both beds were watered today, after how many days will both be watered on the same day again?

1. Watering days are multiples of 6 and of 8; the first common day is the LCM.
   Why: 'Together again' – the LCM signal.
2. $6 = 2 \times 3$ and $8 = 2^3$, so LCM $= 2^3 \times 3 = \mathbf{24}$.

ANSWER: Both beds will be watered together again after 24 days.

CASE QUESTION (iv)

Verify the identity HCF × LCM = product of the numbers for 126 and 210.

1. From part (i), HCF = 42. Find the LCM from the factorisations: every prime at its greatest power: $2 \times 3^2 \times 5 \times 7 = 630$.
   Why: $126 = 2 \times 3^2 \times 7$ and $210 = 2 \times 3 \times 5 \times 7$ – collect the greatest powers.
2. HCF × LCM $= 42 \times 630 = 26460$.
3. Product of the numbers: $126 \times 210 = 26460$.
4. $26460 = 26460$ — the identity holds, as Derivation A guarantees for any two numbers.

ANSWER: HCF × LCM = 26460 = 126 × 210. Verified.

Assertion–Reason (choose: (a) both true, R explains A (b) both true, R does not explain A (c) A true, R false (d) A false, R true)

AR 1. Assertion (A): The HCF of 34 and 51 is 17. **Reason (R):** The HCF of two numbers is the product of the smallest powers of their common prime factors.
Solution: $34 = 2 \times 17$ and $51 = 3 \times 17$; the only common prime is 17, at power 1, so HCF = 17 — A is true. R states the exact rule used, and it is true. **Answer: (a)** — both true, and R is the correct explanation of A.

AR 2. Assertion (A): $\sqrt{2}$ is an irrational number. **Reason (R):** If a prime p divides a^2, then p divides a.
Solution: A is true (Derivation B, Section 06). R is true (the lemma, proved in Section 06) and it is the key step on which the proof of A stands. **Answer: (a)** — both true, and R is the correct explanation of A.

15 Olympiad & Foundation Challenges

8D • CHALLENGE — FOR STUDENTS WHO FINISH EARLY AND THINK DEEP

CHALLENGE 1

Find the smallest number that leaves a remainder of 1 when divided by 5, by 6 or by 7, and is exactly divisible by 11.

THINK FIRST: *Convert a remainder condition into a divisibility condition (subtract the remainder), then let the LCM generate all candidates at once.*

1. A number leaving remainder 1 on division by 5, 6 and 7 must be exactly 1 more than a common multiple of 5, 6 and 7.
   Why: Remainder 1 means: subtract 1 and the number becomes divisible by all three.
2. The common multiples of 5, 6, 7 are the multiples of their LCM. Compute it: 5 = 5; 6 = 2 × 3; 7 = 7, so LCM = 2 × 3 × 5 × 7 = 210.
   Why: No prime is shared, so the LCM is the full product.
3. So our number has the form 210k + 1 for some positive integer k: 211, 421, 631, ...
4. We also need 11 to divide 210k + 1. Split 210 using 11: 210 = 11 × 19 + 1, so 210k + 1 = 11 × 19k + (k + 1).
   Why: The first part is automatically divisible by 11 — so everything depends on the small remainder k + 1.
5. Thus 11 must divide k + 1. The smallest possibility is k + 1 = 11, i.e. k = 10.
   Why: Choosing the smallest k gives the smallest number, as asked.
6. The number is 210 × 10 + 1 = **2101**.
7. Full check: 2101 = 11 × 191 exactly; 2101 ÷ 5 = 420 remainder 1; 2101 ÷ 6 = 350 remainder 1; 2101 ÷ 7 = 300 remainder 1. All four conditions hold.
   Why: A challenge answer is not finished until it is checked against EVERY condition.

ANSWER: The smallest such number is 2101.

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Olympiad Challenge 2

CHALLENGE 2

Find the greatest number that divides 245 and 2053, leaving a remainder of 5 in each case.

THINK FIRST: *Remainder problems: first remove the remainder, then the problem becomes a familiar HCF/LCM question in disguise.*

1. Call the required number d. Dividing 245 by d leaves remainder 5, so d divides $245 - 5 = 240$ exactly.
   Why: Subtracting the remainder converts the problem into an exact-division problem.
2. Similarly, d divides $2053 - 5 = 2048$ exactly.
3. So d is a common divisor of 240 and 2048; the GREATEST such number is their HCF.
   Why: 'Greatest' + 'common divisor' = HCF, as always.
4. Factorise: $240 = 2^4 \times 3 \times 5$. And $2048 = 2^{11}$ (keep doubling: 2, 4, 8, ..., 1024, 2048 — eleven 2s).
   Why: 2048 is a pure power of 2 — recognising powers of 2 up to $2^{11} = 2048$ is a handy olympiad skill.
5. The only common prime is 2; take the smaller power: HCF $= 2^4 =$ **16**.
6. Verify: $245 = 16 \times 15 + 5$ and $2053 = 16 \times 128 + 5$. Both leave remainder 5, and no larger number can work, because any such number must divide both 240 and 2048.
   Why: The verification uses the ORIGINAL numbers, not the reduced ones.

ANSWER: The greatest such number is 16.

Olympiad Challenge 3 — The Grand Finale

CHALLENGE 3

Prove that √2 + √3 is irrational. (You may first prove that √6 is irrational.)

THINK FIRST: Big proofs are built from small proved blocks. When a problem looks too hard, ask: what smaller fact would make this easy? Prove THAT first.

1. **Part 1 — Lemma: √6 is irrational.** Assume $\sqrt{6} = a/b$ with a, b co-prime. Squaring: $a^2 = 6b^2$.
   Why: We need √6 as a tool. Build the tool first — this is how real mathematics is assembled.
2. Since $a^2 = 2 \times (3b^2)$, 2 divides a^2, so 2 divides a (lemma). Write $a = 2c$. Then $4c^2 = 6b^2$, so $2c^2 = 3b^2$.
3. So 3 divides $2c^2$. The prime 3 does not divide 2, so 3 must divide c^2 (3 can only enter through c's factorisation), hence 3 divides c. Write $c = 3d$.
   Why: Watch where the prime can possibly hide — factorisation leaves it nowhere else to go.
4. Then $a = 2c = 6d$, and from $a^2 = 6b^2$ we get $36d^2 = 6b^2$, so $b^2 = 6d^2$. Hence 3 divides b^2, so 3 divides b.
5. Now 3 divides both a and b — contradicting co-prime. So √6 is irrational. ∎
6. **Part 2 — the main proof.** Assume, to the contrary, that √2 + √3 is rational; call it r.
7. Square both sides: $r^2 = (\sqrt{2} + \sqrt{3})^2 = 2 + 2\sqrt{6} + 3 = 5 + 2\sqrt{6}$.
   Why: Use $(x + y)^2 = x^2 + 2xy + y^2$ with $xy = \sqrt{2} \times \sqrt{3} = \sqrt{6}$.
8. Rearrange: $2\sqrt{6} = r^2 - 5$, so $\sqrt{6} = (r^2 - 5)/2$.
   Why: Isolate the root — the same finishing move as Ex 1.2 Q2 and Q3.
9. Since r is rational, r^2 is rational, so $(r^2 - 5)/2$ is rational. That would make √6 rational.
10. But Part 1 proved √6 is irrational. Contradiction.
11. Therefore √2 + √3 is irrational. ∎
    Why: Two contradiction proofs locked together — the lemma does the heavy lifting inside the main proof.

ANSWER: √2 + √3 is irrational.

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16 Chapter Mind Map

THE WHOLE CHAPTER ON ONE PAGE — REVISE FROM HERE

REAL NUMBERS

Heart of the chapter: ***Fundamental Theorem of Arithmetic*** *— every composite number is a product of primes in exactly ONE way.*

Prime Factorisation

Factor tree or division ladder. Divide by the smallest prime; stop at 1. Multiply back to check.

→

HCF

Common primes only, SMALLEST powers. Story signal: largest, equal, cut, pack, no leftovers.

LCM

All primes, GREATEST powers. Story signal: meet again, together, repeat, smallest that fits all.

The Golden Identity

HCF × LCM = a × b — TWO numbers only. Derived from min + max = sum of exponents.

Ending-Digit Proofs

Ending in 0 needs primes 2 AND 5. If 5 is missing from the factorisation, the number can never end in 0. (6^n, 12^n, 8^n)

Composite Proofs

Do NOT multiply out. Factor out the common factor: 7×11×13 + 13 = 13 × 78.

Irrational Numbers — Proof by Contradiction

5 beats: Assume rational a/b, co-prime → Square → $p \mid a^2$ so $p \mid a$ → Substitute, get $p \mid b$ → Contradiction. Engine: the lemma "$p \mid a^2 \Rightarrow p \mid a$".

→

The Family

$\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ irrational → also $3 + 2\sqrt{5}$, $1/\sqrt{2}$, $7\sqrt{5}$, $6 + \sqrt{2}$ — by isolating the root.

Key Results to Quote

HCF(336, 54) = 6, LCM = 3024 · LCM(306, 657) = 22338 from HCF 9 · Sonia–Ravi meet after 36 min · $3825 = 3^2 \times 5^2 \times 17$ · $7429 = 17 \times 19 \times 23$

TOPPER TIP: *Revise actively: cover a box, speak its contents aloud from memory, then uncover and check. A mind map you can REPRODUCE from memory is worth ten readings.*

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17 Active Recall

8D • RECALL — CLOSE THE BOOK, THEN ANSWER

How this works: *close every other page. Answer all ten questions on rough paper, from memory alone. Only then open the Answer Key. Anything you could not recall is exactly what your Day-3 revision must target — write it in the Error Tracker (Section 18).*

Ten rapid questions

R1. State the Fundamental Theorem of Arithmetic.

R2. What is the HCF of 12 and 18?

R3. What is the LCM of 12 and 18?

R4. Write 100 as a product of prime factors.

R5. Find the HCF and the LCM of 20 and 28.

R6. Can 8^n end with the digit 0 for any natural number n? Why, or why not?

R7. Is $1/\sqrt{2}$ rational or irrational?

R8. Find the HCF and the LCM of 7, 11 and 13.

R9. If HCF(16, x) = 4 and LCM(16, x) = 48, find x.

R10. Is $7 \times 11 \times 13 + 13$ prime or composite? Justify in one line.

Five recall prompts — say each answer aloud, in a full sentence

P1. Define HCF in one sentence.

P2. Define LCM in one sentence.

P3. State the identity connecting HCF, LCM and two numbers — and state its limit.

P4. Define an irrational number.

P5. State the lemma that powers every irrationality proof in this chapter.

> **TOPPER TIP:** Scoring: 13–15 correct — chapter mastered, move on. 9–12 — revise the missed topics, retake in 3 days. Below 9 — re-study Sections 05–07, then retake. Record your score in the Mastery Tracker.

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18 Self-Assessment & Mastery Tracker

KALVIYOGI SMART MEMORY SYSTEM — LEARN IT, THEN KEEP IT

Memory fades on a predictable curve — unless you revisit at the right moments. This schedule is your revisit plan for THIS chapter. Tick each day off when done.

Day	Your activity for Chapter 1	Done
Day 0	Pre-learning assessment: attempt the Quick Check (Section 02) and Rapid Questions R1–R10 (Section 17) BEFORE studying. Note your score.	☐
Day 1	Learn the chapter fully (Sections 01–10) and complete Level 1–3 practice.	☐
Day 3	Without-book recall: reproduce the Mind Map (Section 16) on a blank page; redo Rapid Questions; finish Level 4.	☐
Day 7	Independent problem-solving: Level 5 + Olympiad Challenges, timed, book closed.	☐
Day 14	Mixed practice: re-attempt every question in your Error Tracker; solve Ex 1.1 Q2 and Q3 fresh.	☐
Day 21	Long-term recall test: write the $\sqrt{5}$ proof and the HCF×LCM derivation from memory, then verify line by line.	☐
Day 30	Chapter mastery assessment: full self-test under exam conditions; update your Concept Mastery Score below.	☐

Formula Recall Tracker — tick when you can say it with the book closed

Formula / Statement	Day 1	Day 3	Day 7	Day 21
Fundamental Theorem of Arithmetic	☐	☐	☐	☐
HCF rule (common primes, smallest powers)	☐	☐	☐	☐
LCM rule (all primes, greatest powers)	☐	☐	☐	☐
HCF × LCM = a × b (two numbers only)	☐	☐	☐	☐
Lemma: $p \mid a^2 \Rightarrow p \mid a$	☐	☐	☐	☐
The 5-beat contradiction pattern	☐	☐	☐	☐

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Mastery Tracker — Errors and Concept Scores

Error Tracker — every mistake becomes a lesson

Date	Question	What went wrong	The correct idea

Concept Mastery Score — rate yourself honestly, out of 5

Concept	Score / 5	Status
Prime factorisation (tree and ladder)	___ / 5	☐ Weak ☐ Okay ☐ Mastered
HCF & LCM — two numbers	___ / 5	☐ Weak ☐ Okay ☐ Mastered
HCF & LCM — three numbers	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Ending-digit & composite-number proofs	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Irrationality proofs	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Word problems (choosing HCF vs LCM)	___ / 5	☐ Weak ☐ Okay ☐ Mastered

Learning Time Tracker

Day / Date	Minutes studied	What I studied

Mastery rule: a concept counts as MASTERED only when you score 4 or 5 AND can reproduce it on Day 21 without the book. Anything less — revisit, don't just re-read.

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Answer Key — Graded Practice, Levels 1 and 2

Level 1 — Foundation

L1.1 $225 = 3^2 \times 5^2$. **L1.2** $360 = 2^3 \times 3^2 \times 5$. **L1.3** HCF(18, 48) = 6. **L1.4** LCM(14, 20) = 140. **L1.5** HCF(9, 25) = 1 and LCM(9, 25) = 225; the special point: 9 and 25 are co-prime — their factorisations (3^2 and 5^2) share no prime. **L1.6** HCF(24, 36) = 12.

Level 2 — Skill Builder

L2.1 $90 = 2 \times 3^2 \times 5$; $144 = 2^4 \times 3^2$. HCF = $2 \times 3^2 = 18$; LCM = $2^4 \times 3^2 \times 5 = 720$. Check: $18 \times 720 = 12960 = 90 \times 144$.

L2.2 $96 = 2^i \times 3$; $404 = 2^2 \times 101$. HCF = $2^2 = 4$. LCM = $(96 \times 404) \div 4 = 38784 \div 4 = 9696$.

L2.3 $8 = 2^3$; $36 = 2^2 \times 3^2$; $90 = 2 \times 3^2 \times 5$. HCF = $2^1 = 2$ (only prime in all three is 2 — 3 is missing from 8). LCM = $2^3 \times 3^2 \times 5 = 360$.

L2.4 $867 = 3 \times 17^2$; $255 = 3 \times 5 \times 17$. HCF = $3 \times 17 = 51$. LCM = $(867 \times 255) \div 51 = 867 \times 5 = 4335$.

L2.5 No. $12^n = 2^{2n} \times 3^n$ — its only primes are 2 and 3. Ending in 0 requires the prime 5 ($10 = 2 \times 5$), and ending in 5 also requires the factor 5 (the number would be a multiple of 5). Since 5 never appears, neither ending is possible.

L2.6 $3 \times 5 \times 7 + 7 = 7 \times (3 \times 5 + 1) = 7 \times 16 = 112$. It is a product of two integers both greater than 1, so it is composite.

TOPPER TIP: Marked yourself wrong at Level 2? Compare your working with the twin solution in Section 07 step by step — find the FIRST step where you differ. That step is the lesson.

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Answer Key — Levels 3, 4, 5 and Active Recall

Level 3 — Application

L3.1 LCM(48, 72) = 144 seconds = 2 min 24 s, so they change together again at **7:02:24 a.m.**

L3.2 Tile side = HCF(18, 12) = 6 m. Tiles = (18 ÷ 6) × (12 ÷ 6) = 3 × 2 = **6 tiles**.

L3.3 Students per bus = HCF(156, 234) = 78. Buses: 156 ÷ 78 = 2 and 234 ÷ 78 = 3, so **5 buses**.

L3.4 LCM(9, 12, 15) = 180 minutes = 3 hours, so they toll together again at **11:00 a.m.**

Level 4 — Advanced Reasoning

L4.1 Write the numbers as 3k and 4k. Since 3 and 4 are co-prime, LCM = 3 × 4 × k = 12k = 180, so k = 15. The numbers are **45 and 60**, and HCF = k = **15**. (Check: HCF × LCM = 15 × 180 = 2700 = 45 × 60.)

L4.2 $5 \times 7 \times 11 + 11 = 11 \times (5 \times 7 + 1) = 11 \times 36 = 2^2 \times 3^2 \times 11$. It has factors other than 1 and itself (for example 11 and 36), so it is composite.

L4.3 Let the integers be n, n + 1, n + 2. Among any three consecutive integers, the remainders on division by 3 are 0, 1, 2 in some order, so one of them is divisible by 3. Among the first two, one is even, so the product is divisible by 2 as well. A number divisible by both 2 and 3 (co-prime) is divisible by 6. ∎

Level 5 — Mathematical Genius

L5.1 The number of sweets is 5 more than a common multiple of 12, 16, 18. LCM(12, 16, 18) = $2^4 \times 3^2$ = 144. Smallest such number = 144 + 5 = **149 sweets**.

L5.2 2101. (Candidates are 210k + 1, since LCM(5, 6, 7) = 210. Because 210 = 11 × 19 + 1, we need 11 | (k + 1); smallest k = 10 gives 2101 = 11 × 191. Full solution: Challenge 1, Section 15.)

Active Recall (Section 17)

R1 Every composite number can be expressed as a product of primes, and the factorisation is unique apart from order. **R2** 6. **R3** 36. **R4** $100 = 2^2 \times 5^2$. **R5** HCF = 4, LCM = 140. **R6** No — $8^n = 2^{3n}$ has no prime factor 5, and ending in 0 requires the factor 5. **R7** Irrational. **R8** HCF = 1, LCM = 1001. **R9** x = (4 × 48) ÷ 16 = 12. **R10** Composite — it equals 13 × 78.

P1 The HCF is the largest number that divides each of the given numbers exactly. **P2** The LCM is the smallest number that is a multiple of each of the given numbers. **P3** HCF × LCM = a × b — valid for two numbers only. **P4** A real number that cannot be written as p/q with p, q integers, $q \neq 0$. **P5** If a prime p divides a^2, then p divides a.

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Revision Card — Chapter 1 on One Page

The one theorem

Fundamental Theorem of Arithmetic: every composite number = a product of primes, in exactly one way (apart from order).

The two rules

HCF = common primes, smallest powers. **LCM** = all primes, greatest powers. **Identity:** HCF × LCM = a × b (two numbers only — derived from min + max = sum).

The proof machine (irrationality)

Assume rational a/b, co-prime → square: $a^2 = p \cdot b^2$ → $p \mid a^2 \Rightarrow p \mid a$ (lemma) → $a = pc$, substitute → $p \mid b$ → contradiction. Works for $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$. For sums/products ($3 + 2\sqrt{5}$, $7\sqrt{5}$, $6 + \sqrt{2}$, $1/\sqrt{2}$): isolate the root and cite the known irrational.

Story signals

HCF: largest / greatest / equal pieces / cut / pack / no leftovers. **LCM:** meet again / together again / repeat / smallest that fits all. **Ending in 0:** needs prime 5 in the factorisation. **Composite proof:** factor out the common factor — never multiply out.

Exam-eve checklist

- ☐ Ladder shown for every factorisation
- ☐ Verification line written in Q2-type questions
- ☐ Co-prime condition stated in every proof
- ☐ Final answer translated back into the story's words
- ☐ 91 = 7 × 13 — not prime!

Chapter 1 complete. You did not memorise Real Numbers — you discovered, visualised, derived and mastered them.

Next: Chapter 2 — Polynomials, in the same 8D format. · KalviYogi · Mathematics – Beyond the Formula · 369 Tesla × KalviYogi · © Edunivra Pvt Ltd