CBSE • CLASS 10 • MATHEMATICS

# MATHEMATICS BEYOND THE FORMULA

## CHAPTER 2 • POLYNOMIALS

*Don't Memorize Mathematics. Understand, Visualise and Discover It.*

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# 01 Chapter Introduction

**8D • DISCOVER — WHY SHOULD I LEARN THIS?**

Throw a ball into the air. It rises, pauses for a heartbeat, and falls back to the ground. That graceful curve it draws in the sky is not just physics — it is a **polynomial** drawing itself. The arch of a bridge, the path of water from a fountain, the profit curve of a small business — nature and life keep writing the same kind of mathematics: **polynomials**.

In this chapter you will learn to **see** a polynomial before you calculate with it. Its graph will tell you its secrets at a glance: where it touches the ground (its **zeroes**), how many such touching points are possible, and how the zeroes are secretly wired to the coefficients — a bridge so reliable that the board examiner tests it every single year.

### What you will master in this chapter

- What a polynomial is: degree, types (linear, quadratic, cubic) and the value p(a).
- The **geometrical meaning of zeroes** — zeroes are the x-coordinates where the graph meets the x-axis (NCERT Exercise 2.1).
- Finding zeroes by **splitting the middle term**, with every split justified.
- The zeroes–coefficients bridge for quadratics: $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$ — **derived** in Section 06, then verified on every NCERT question.
- The same bridge for **cubic** polynomials: three relations, one of them with a famous minus sign.
- **Forming** a quadratic polynomial when the sum and product of its zeroes are given.
- The **division algorithm**: $p(x) = g(x) \times q(x) + r(x)$, worked by full long division.

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# Your Chapter Map — 18 Stops, One Journey

Tick each stop as you finish it. The Plus blocks are your exam-fitness training — do not skip them.

| Discover & Visualise | Understand & Derive | Solve, Apply, Recall |
|---|---|---|
| ☐ 01 Chapter Introduction<br>☐ 02 Prerequisites<br>☐ 03 Real-Life Story<br>☐ 04 Visual Concepts (CPA) | ☐ 05 Mathematical Foundations<br>☐ 06 Formula Derivations<br>☐ 08 Multiple Methods<br>☐ 10 Common Mistakes | ☐ 07 Detailed Solved Examples<br>☐ 09 Hands-on Activities<br>☐ 11 Graded Practice (5 Levels)<br>☐ 12 Mathematical Reasoning |
| ☐ 13 Real-Life Applications<br>☐ 14 Competency-Based Questions<br>☐ 15 Olympiad Challenges | ☐ PLUS: Concept MCQs (25)<br>☐ PLUS: Find the Error (10)<br>☐ PLUS: Board PYQs (solved) | ☐ 16 Chapter Mind Map<br>☐ Pin-Point Flash Cards<br>☐ 17 Active Recall<br>☐ 18 Mastery Tracker<br>☐ PLUS: 3 Model Tests |

## How to use this chapter (read this first!)

1. **Never read a solution passively.** Cover the steps, try yourself, then uncover one step at a time.
2. Every solution shows **Step 1, Step 2, Step 3…** with a small *Why* note in purple handwriting — the reason behind the step. Examiners award marks for these steps, not only for the final answer.
3. After Section 07, test yourself in Sections 11–15 and the Plus blocks **before** looking at the Answer Key.
4. Finish with the Mind Map (16), the Flash Cards, the closed-book Recall (17) and your Mastery Tracker (18). Come back on Day 3, 7, 14, 21 and 30 — the Smart Memory schedule, with its Day 7 / 21 / 30 mini-tests, tells you exactly what to do each day.

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# 02 Prerequisites

EARLIER CLASSES RECAP — THE TOOLS YOU ALREADY OWN

Polynomials grow out of four ideas you already know: algebraic expressions, equations, factorisation and graphs. Refresh them here.

## 1. Expressions, terms and substitution

An **algebraic expression** is built from variables and constants using +, −, ×, ÷. In $3x^2 - 2x + 1$, the terms are $3x^2$, $-2x$ and 1. **Substituting** a value means replacing the variable everywhere: for $p(x) = 3x^2 - 2x + 1$, $p(1) = 3 - 2 + 1 = 2$ and $p(0) = 1$. This chapter calls such a result the **value of the polynomial**.

## 2. Solving a linear equation

To solve $2x - 6 = 0$, you undo operations in reverse: $2x = 6$, so $x = 3$. Every "find the zeroes" step in this chapter ends with exactly this move, applied to each factor separately: if $(x - 4)(x + 2) = 0$, then $x - 4 = 0$ OR $x + 2 = 0$. A product is zero only when at least one factor is zero — this is called the **zero-product rule**.

## 3. Factorisation by splitting the middle term

To factorise $x^2 - 3x - 4$, find two numbers whose **sum** is −3 and whose **product** is −4: they are −4 and 1. Split: $x^2 - 4x + x - 4 = x(x - 4) + 1(x - 4) = (x - 4)(x + 1)$. For $ax^2 + bx + c$ with $a \neq 1$, the two numbers must multiply to **a × c** — Section 07 uses this in every solution.

## 4. Coordinates and graphs

A point (x, y) sits x units across and y units up. The graph of $y = p(x)$ is the picture of all points whose y-value equals $p(x)$. Where this picture meets the **x-axis**, the y-value is 0 — and those x-values are about to become the heroes of this chapter: the **zeroes**.

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# Quick Check — Are You Ready? (5 Questions)

Try all five without looking back. Answers are given right below — cover them with your hand first!

**Q1.** Find the value of $p(x) = 2x^2 + 5x - 3$ at $x = 1$ and at $x = 0$.

**Q2.** Solve: (a) $3x - 9 = 0$ (b) $2x + 7 = 0$.

**Q3.** Factorise $x^2 + 5x + 6$.

**Q4.** Factorise $2x^2 + 5x + 3$. (Careful: here $a = 2$, so the split numbers must multiply to $a \times c = 6$.)

**Q5.** The graph of $y = x - 3$ meets the x-axis at which point? What is the y-value there?

### Answers — check only after trying

**Q1.** $p(1) = 2 + 5 - 3 = 4$. $p(0) = -3$ (only the constant term survives).

**Q2.** (a) $3x = 9$, so $x = 3$. (b) $2x = -7$, so $x = -7/2$.

**Q3.** Split numbers: 2 and 3 (sum 5, product 6). $x^2 + 2x + 3x + 6 = x(x + 2) + 3(x + 2) = (x + 2)(x + 3)$.

**Q4.** Split numbers: 2 and 3 (sum 5, product $6 = a \times c$). $2x^2 + 2x + 3x + 3 = 2x(x + 1) + 3(x + 1) = (x + 1)(2x + 3)$.

**Q5.** It meets the x-axis at (3, 0); the y-value on the x-axis is always 0. Setting $y = 0$ gives $x = 3$ — you have just found your first zero of a polynomial!

**TOPPER TIP:** *Got 4 or 5 right? You are fully ready. Got fewer? Re-read the four cards on the previous page once, then come back — do not skip this step.*

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## 03 Real-Life Mathematical Story

**DISCOVER • VISUALISE — THE ARCH THAT TOUCHED THE GROUND**

Before any formula, live the problem. Read this story slowly — the mathematics will walk out of the story by itself.

*The town school is getting a new entrance, and the young architect **Anitha** has drawn something beautiful: a smooth, curved **arch** over the main gate, shaped like an upside-down U.*

*At the site, the head mason scratches his head. "Madam, my workers build with measurements, not with curves. At every point, I must know **how high** the arch is. And first of all — **where exactly do the two feet of the arch stand?** How wide is the opening?"*

*Anitha smiles and opens her notebook. She draws a ground line and marks the left foot of the arch as her starting point. "Measure x metres from the left foot along the ground. Then the height of the arch above that point, in metres, is given by one small rule:*

$$\textbf{height} = -x^2 + 4x$$

*Try it, she says. At x = 1, the height is −1 + 4 = 3 metres. At x = 2, the height is −4 + 8 = 4 metres — the top of the arch. At x = 3, the height is −9 + 12 = 3 metres again. The arch rises, peaks, and comes down — perfectly symmetric.*

*"But madam," the mason asks, "where does it come back to the **ground**? Where is the second foot?"*

*That question — where does the height become zero? — is the question of this whole chapter. In mathematics, the rule $-x^2 + 4x$ is called a **polynomial**, and the values of x where it becomes 0 are called its **zeroes**.*

Anitha has a second puzzle in her notebook too. Behind the school, a rectangular garden plot is being planned. Its length is 3 metres more than a chosen width number x, and its breadth is 4 metres more than x — so its area works out to $x^2 + 7x + 12$ square metres. The gardener wants the two side-lengths written separately, not hidden inside one formula. Both puzzles are solved on the next two pages — one step at a time.

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# Solving the Story — Problem 1: Where Does the Arch Land?

**STORY PROBLEM 1**

**The height of the arch at distance x metres from its left foot is $h(x) = -x^2 + 4x$ metres. Find where the arch meets the ground again, and hence the width of the opening.**

**THINK FIRST:** Whenever a story asks 'where does it touch / land / become zero', it is asking for the ZEROES of a polynomial.

1. "Meets the ground" means the height is 0, so we must solve $h(x) = 0$, that is, $-x^2 + 4x = 0$.
   *Why: A zero of a polynomial is exactly an input that makes the output zero.*

2. Take out the common factor x: $-x^2 + 4x = x(-x + 4) = 0$. Factorising the same expression another way: $x(4 - x) = 0$.
   *Why: Both terms contain x, so x comes out as a common factor.*

3. By the zero-product rule, either $x = 0$ or $4 - x = 0$.
   *Why: A product can be zero only if one of its factors is zero.*

4. From $4 - x = 0$, we get $x = 4$.

5. So $h(x)$ has two zeroes: $x = 0$ and $x = 4$. The first is the left foot (where we started measuring); the second is the right foot.
   *Why: Zeroes are positions here — the two feet of the arch.*

6. Width of the opening = distance between the feet = $4 - 0$ = **4 metres**.

**ANSWER: The arch meets the ground at x = 0 m (left foot) and x = 4 m (right foot). The opening is 4 metres wide.**

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# Solving the Story — Problem 2: The Garden's Two Sides

**STORY PROBLEM 2**

**The area of the rectangular garden is $A(x) = x^2 + 7x + 12$ m$^2$. Write the area as a product of the two side-lengths. If x = 2, find the actual length, breadth and area.**

**THINK FIRST:** An area written as one polynomial often hides two side-lengths. Factorising is how you set them free.

1. We need two numbers whose sum is 7 (the coefficient of x) and whose product is 12 (the constant term).
   *Why: This is the splitting-the-middle-term hunt from Section 02.*

2. Try pairs multiplying to 12: 1 × 12 (sum 13), 2 × 6 (sum 8), 3 × 4 (sum 7). The pair 3 and 4 fits both conditions.
   *Why: Check BOTH conditions every time — sum and product.*

3. Split the middle term: $x^2 + 7x + 12 = x^2 + 3x + 4x + 12$.

4. Group and factorise: $x(x + 3) + 4(x + 3) = (x + 3)(x + 4)$.
   *Why: The common bracket (x + 3) is the signal that the grouping worked.*

5. So the two sides are (x + 3) m and (x + 4) m. Check by expanding: $(x + 3)(x + 4) = x^2 + 4x + 3x + 12 = x^2 + 7x + 12$. Correct.
   *Why: Always expand back to verify a factorisation.*

6. Put x = 2: length = 6 m, breadth = 5 m, area = 30 m$^2$. Cross-check: $A(2) = 4 + 14 + 12 = 30$ m$^2$. Same.
   *Why: Two routes, one answer — real verification.*

**ANSWER: Area = (x + 3)(x + 4) m$^2$, so the sides are (x + 3) m and (x + 4) m. For x = 2: 6 m by 5 m, area 30 m$^2$.**

## The two story signals of this chapter — pin these in memory

**Touches the ground / lands / height becomes zero / break-even** → set the polynomial equal to 0 and find its **zeroes**.
**Area given as one expression, sides wanted → factorise** the polynomial; the factors are the sides. You will meet both signals again in Sections 13 and 14 — and in the board exam.

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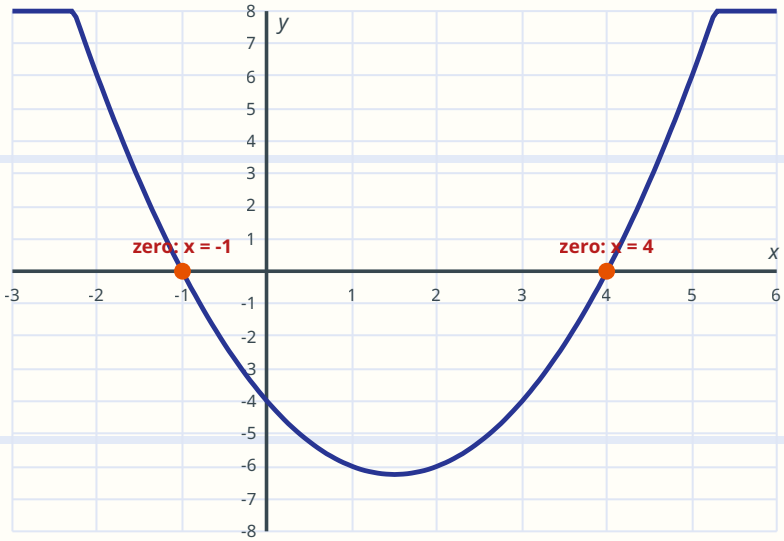
# 04 Visual Concept Explanation

SINGAPORE CPA — CONCRETE → PICTORIAL → ABSTRACT

**Concrete:** the arch and the garden of Section 03 were real objects. **Pictorial:** now we draw the polynomial as a picture — a graph. **Abstract:** only after the picture is clear do we touch formulas. Here is the hero picture of this chapter: the graph of $y = x^2 - 3x - 4$, built point by point from this table:

| **x** | −2 | −1 | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|
| **$y = x^2 - 3x - 4$** | 6 | **0** | −4 | −6 | −6 | −4 | **0** | 6 |




Look at the two orange dots. The curve crosses the x-axis at **x = −1** and **x = 4** — exactly the two table entries where y became 0. That is the whole secret, visible to the eye:

**PIN POINT**

**The ZEROES of a polynomial p(x) are the x-coordinates of the points where the graph of y = p(x) meets the x-axis. Count the meetings, and you have counted the zeroes.**

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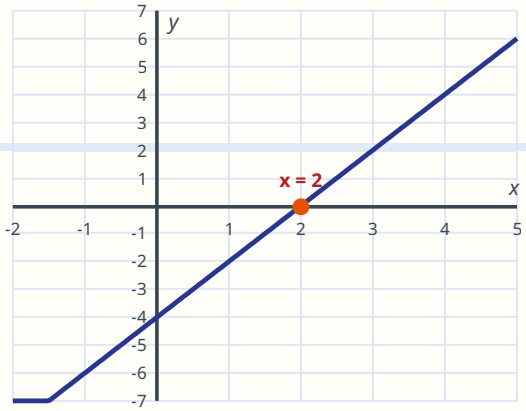
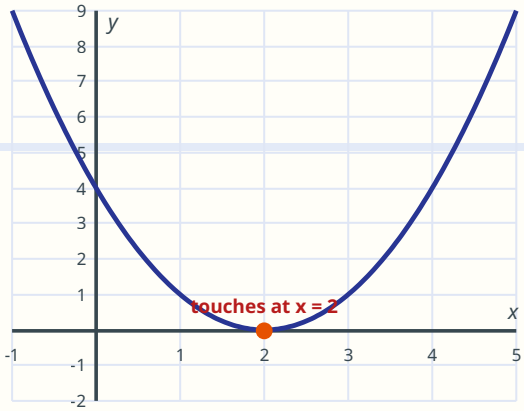
# Four Pictures You Must Recognise Instantly

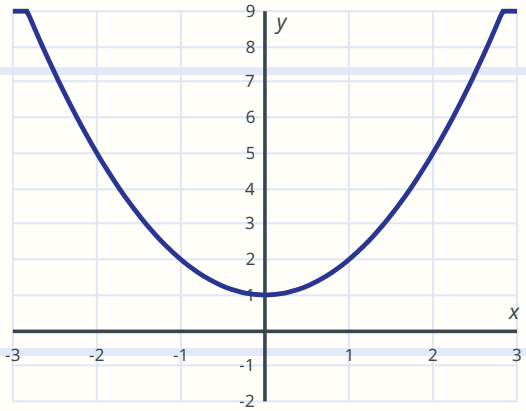
Every polynomial graph you will ever meet in Class 10 is one of these shapes. Read each picture the same way: **where does it meet the x-axis?**



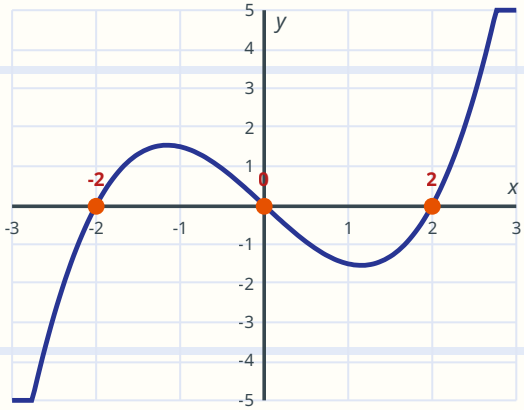

**Linear** $y = 2x - 4$: a straight line. Crosses once → **1 zero** ($x = 2$).




**Quadratic** $y = (x - 2)^2$: a parabola that **touches** the axis at one point → **1 zero**, counted twice ($x = 2, 2$).




**Quadratic** $y = x^2 + 1$: floats above the axis, never meets it → **0 zeroes** (no real zeroes).




**Cubic** $y = (x^3 - 4x)/2$: an S-curve crossing three times → **3 zeroes** ($x = -2, 0, 2$).

# The Degree Rule — How Many Zeroes Can There Be?

Compare the four pictures with the degrees of their polynomials and a law appears:

| Polynomial type | Degree | Graph shape | Zeroes possible |
|---|---|---|---|
| Linear, e.g. $2x - 4$ | 1 | Straight line | Exactly 1 |
| Quadratic, e.g. $x^2 - 3x - 4$ | 2 | Parabola (U or ∩) | 0, 1 (touching) or 2 |
| Cubic, e.g. $x^3 - 4x$ | 3 | S-shaped curve | 1, 2 (one touching) or 3 |
| Degree n (general) | n | — | **At most n** |

**Why "at most"?** Each zero a gives a factor $(x - a)$ of the polynomial. A degree-n polynomial has room for at most n such linear factors — so it can have at most n zeroes. Section 12 proves this properly. For the exam, the working form is:

**PIN POINT**

**A polynomial of degree n has AT MOST n zeroes. Quadratic → at most 2. Cubic → at most 3. If your answer ever shows more zeroes than the degree, your answer is certainly wrong — recheck.**

One more reading skill: the parabola of a quadratic opens **upwards** when the coefficient of $x^2$ is positive (like $x^2 - 3x - 4$) and **downwards** when it is negative (like the arch, $-x^2 + 4x$). The lowest (or highest) point of the parabola is called its **vertex** — for the arch, the vertex (2, 4) was the top of the gate, 4 metres high.

*TOPPER TIP: In Exercise 2.1 the board gives you ONLY pictures. Practise the reflex now: picture → count x-axis meetings → that count is your answer. Nothing else is asked.*

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# 05 Mathematical Foundations

UNDERSTAND — THE PRECISE IDEAS, WORD BY WORD

## 1. What is a polynomial?

A **polynomial** in x is an expression in which x appears only with **whole-number powers** (0, 1, 2, 3, ...) and the coefficients are real numbers. Examples: $4x + 5$; $x^2 - 3x - 4$; $2x^3 - 7x + 1$; 9 (a constant is a polynomial too).

**Not** polynomials: $x^{-1} + 2$ (negative power), $\sqrt{x} + 3$ (fractional power), $1/x + x$ (x in a denominator). The powers must never be negative or fractional.

## 2. Degree — the polynomial's rank

The **degree** is the highest power of x in the polynomial. $5x^3 - 4x^2 + 7$ has degree 3. A non-zero constant (like 9) has degree 0. The **zero polynomial** 0 is special: its degree is **not defined** — a favourite one-mark trap.

| Degree | Name | General form | Example |
|---|---|---|---|
| 1 | Linear | $ax + b$, $a \neq 0$ | $2x - 4$ |
| 2 | Quadratic | $ax^2 + bx + c$, $a \neq 0$ | $x^2 - 3x - 4$ |
| 3 | Cubic | $ax^3 + bx^2 + cx + d$, $a \neq 0$ | $x^3 - 4x$ |

By number of terms: one term = **monomial** ($7x^2$), two terms = **binomial** ($x + 5$), three terms = **trinomial** ($x^2 - 3x - 4$).

## 3. Value of a polynomial — p(a)

p(a) means: replace x by a everywhere and calculate. For $p(x) = x^2 - 3x - 4$: $p(4) = 16 - 12 - 4 = 0$; $p(0) = -4$; $p(-1) = 1 + 3 - 4 = 0$. Keep $p(4) = 0$ and $p(-1) = 0$ in mind — they are about to earn a name.

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# Zero of a Polynomial — The Key Definition

## 4. Zero of a polynomial

A real number **a** is called a **zero** of the polynomial p(x) if **p(a) = 0**. So 4 and −1 are zeroes of $x^2 - 3x - 4$, because p(4) = 0 and p(−1) = 0. Read it both ways: "a is a zero" and "p(a) = 0" are the same statement wearing two coats.

**PIN POINT**

**a is a ZERO of p(x) ⇔ p(a) = 0 ⇔ the graph of y = p(x) meets the x-axis at x = a ⇔ (x − a) is a factor of p(x). Four faces of one fact — the exam tests each face separately.**

## 5. The zeroes–coefficients bridge (statements — derived in Section 06)

If α and β are the zeroes of the quadratic $ax^2 + bx + c$ ($a \neq 0$), then
**α + β = −b/a** and **αβ = c/a**.

If α, β, γ are the zeroes of the cubic $ax^3 + bx^2 + cx + d$ ($a \neq 0$), then
**α + β + γ = −b/a**, **αβ + βγ + γα = c/a**, **αβγ = −d/a**.

Notice the pattern of signs: minus, plus, minus — they alternate, starting with minus. Section 06 shows you WHY, so you will never need to memorise blindly.

## 6. The division algorithm (foundation for Section 07)

Just as 17 ÷ 5 gives quotient 3 and remainder 2 with 17 = 5 × 3 + 2, polynomials divide the same way. For any two polynomials p(x) and g(x) (g(x) ≠ 0), there exist q(x) and r(x) with
**p(x) = g(x) × q(x) + r(x)**, where r(x) = 0 or **degree of r(x) < degree of g(x)**.

You divide until the remainder's degree is smaller than the divisor's degree — then you must stop. Two fully worked long divisions appear in Section 07.

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# 06 Formula Derivations

8D • DERIVE — NEVER MEMORISE WHAT YOU CAN DERIVE

## Derivation 1 — The quadratic bridge: α + β = −b/a, αβ = c/a

Let α and β be the zeroes of $p(x) = ax^2 + bx + c$, with $a \neq 0$. We build the bridge in five steps.

1. Since α and β are the zeroes, $(x - \alpha)$ and $(x - \beta)$ are factors of p(x). So $p(x) = a(x - \alpha)(x - \beta)$ for some constant multiple — and the multiple must be a, to match the $x^2$ coefficient.
   *Why: A zero a always brings its factor (x − a) with it (Section 05, Pin Point).*

2. First expand the brackets: $(x - \alpha)(x - \beta) = x^2 - \beta x - \alpha x + \alpha\beta = x^2 - (\alpha + \beta)x + \alpha\beta$.
   *Why: Multiply term by term; collect the two middle terms.*

3. Multiply by a: $p(x) = ax^2 - a(\alpha + \beta)x + a\alpha\beta$.

4. But p(x) is also $ax^2 + bx + c$. Two writings of one polynomial must match coefficient by coefficient: coefficient of x gives $b = -a(\alpha + \beta)$, and the constant term gives $c = a\alpha\beta$.
   *Why: Equal polynomials have equal coefficients — this comparison is the engine of the derivation.*

5. Divide: from $b = -a(\alpha + \beta)$, we get **$\alpha + \beta = -b/a$**. From $c = a\alpha\beta$, we get **$\alpha\beta = c/a$**. The bridge is built.
   *Why: Notice where the minus sign is born: it comes from the −(α + β) in the expansion. That is why the sum formula carries a minus and the product formula does not.*

**Instant check** on an old friend, $x^2 - 3x - 4$ (zeroes −1 and 4): sum of zeroes = −1 + 4 = 3, and $-b/a = -(-3)/1 = 3$. Product = −1 × 4 = −4, and $c/a = -4/1 = -4$. Both match — the bridge holds.

**PIN POINT**

**SUM of zeroes = −(coefficient of x) / (coefficient of $x^2$). PRODUCT of zeroes = (constant term) / (coefficient of $x^2$). The minus lives ONLY in the sum.**

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# Derivation 2 — The Cubic Bridge (Same Logic, One More Zero)

Let α, β, γ be the zeroes of $p(x) = ax^3 + bx^2 + cx + d$. Then $p(x) = a(x - \alpha)(x - \beta)(x - \gamma)$. We expand in two stages.

1. Stage 1: $(x - \alpha)(x - \beta) = x^2 - (\alpha + \beta)x + \alpha\beta$ — exactly as in Derivation 1.

2. Stage 2: multiply the result by $(x - \gamma)$. The $x^3$ term: $x^2 \times x = x^3$. The $x^2$ terms: $-(\alpha + \beta)x \times x$ and $x^2 \times (-\gamma)$ give $-(\alpha + \beta + \gamma)x^2$.
*Why: Track one power at a time — that keeps a long expansion error-free.*

3. The x terms: $\alpha\beta \times x$ and $-(\alpha + \beta)x \times (-\gamma)$ give $(\alpha\beta + \beta\gamma + \gamma\alpha)x$. The constant term: $\alpha\beta \times (-\gamma) = -\alpha\beta\gamma$.
*Why: Each x-term pairs two zeroes; the constant pairs all three — and collects a minus sign from the single (−γ).*

4. So $(x - \alpha)(x - \beta)(x - \gamma) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$. Multiply by a and compare with $ax^3 + bx^2 + cx + d$.

5. Coefficient of $x^2$: $b = -a(\alpha + \beta + \gamma) \Rightarrow$ **$\alpha + \beta + \gamma = -b/a$**. Coefficient of x: $c = a(\alpha\beta + \beta\gamma + \gamma\alpha) \Rightarrow$ **$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$**. Constant: $d = -a\alpha\beta\gamma \Rightarrow$ **$\alpha\beta\gamma = -d/a$**.
*Why: Signs alternate −, +, − — one minus for each bracket you multiply by. Now the pattern explains itself.*

**Instant check** on $x^3 - 6x^2 + 11x - 6$, whose zeroes are 1, 2, 3: sum = 6 and $-b/a = 6/1 = 6$. Pairs: $1\cdot2 + 2\cdot3 + 3\cdot1 = 2 + 6 + 3 = 11$ and $c/a = 11$. Product: $1 \cdot 2 \cdot 3 = 6$ and $-d/a = -(-6)/1 = 6$. All three match.

**PIN POINT**

**CUBIC: $\alpha + \beta + \gamma = -b/a$; $\alpha\beta + \beta\gamma + \gamma\alpha = c/a$; $\alpha\beta\gamma = -d/a$. The product of three zeroes carries a MINUS — the most-lost mark in this chapter.**

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# Derivation 3 — Building a Polynomial from Sum and Product

The bridge also works in reverse. Suppose a quadratic has zeroes α, β with **sum S = α + β** and **product P = αβ**. What is the polynomial?

1. From Derivation 1, the polynomial is $a(x - \alpha)(x - \beta) = a[x^2 - (\alpha + \beta)x + \alpha\beta]$.
   *Why: We reuse the expansion — no new work needed.*

2. Replace α + β by S and αβ by P: $p(x) = a[x^2 - Sx + P]$, where a can be ANY non-zero number.
   *Why: That is why Exercise 2.2 Q2 says 'find A quadratic polynomial' — there are infinitely many, all multiples of one another.*

3. Choose a to clear fractions: if S and P are whole numbers, a = 1 is fine. If S = 1/4 and P = −1, choose a = 4 to get $4x^2 - x - 4$ — no fractions anywhere.
   *Why: Multiplying a polynomial by a non-zero constant never changes its zeroes.*

**Mini-example:** sum = 5, product = 6. Then $p(x) = x^2 - 5x + 6$. Check: its zeroes are 2 and 3 (factorise: (x − 2)(x − 3)), and indeed 2 + 3 = 5, 2 × 3 = 6.

**PIN POINT**

**FORMING A QUADRATIC: $p(x) = k[x^2 - (\text{sum of zeroes})x + (\text{product of zeroes})]$, $k \neq 0$. Minus before the sum, plus before the product — every time.**

*TOPPER TIP: Derivations 1–3 are one single idea viewed from three sides: expand $a(x - \alpha)(x - \beta)$ and compare. If you can redo that expansion, you own every formula in this chapter.*

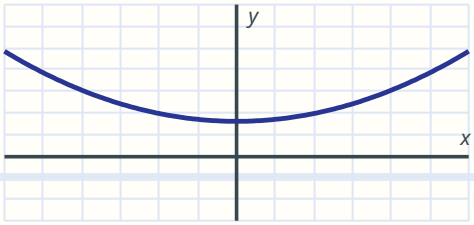
# 07 Detailed Solved Examples

**8D • SOLVE — NCERT EXERCISES 2.1 & 2.2, EVERY STEP SHOWN**

**Exercise 2.1** has just one question with six graphs: *"The graphs of y = p(x) are given below, for some polynomials p(x). Find the number of zeroes of p(x), in each case."* The method never changes: **count the x-axis meetings**. We still show the steps — because in the exam hall, a calm method beats a quick guess.

**EXERCISE 2.1 • GRAPH (i)**

**Find the number of zeroes of p(x) from the graph of y = p(x).**




**1** Look only at the x-axis and count the meetings. The curve stays completely above the x-axis. It neither crosses nor touches it — the count of meetings is 0.

*Why: Zeroes live on the x-axis and nowhere else (Section 04).*

**ANSWER: The number of zeroes of p(x) is 0.**

**CBSE MARKING SCHEME:** 1 mark — correct count of x-axis intersections.

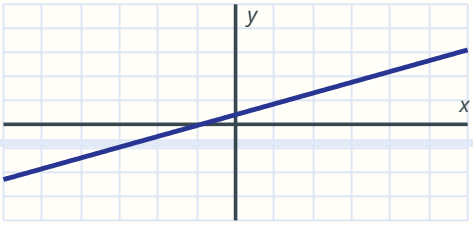
### Before you turn the page — predict!

For each of the five graphs ahead, cover the answer and say the count out loud *before* reading the step. Predicting first — even when you are wrong — makes the correction stick. That is the Discover habit of the 8D system, and it costs you ten seconds a graph.

# Exercise 2.1 — Graphs (ii) and (iii)

**EXERCISE 2.1 • GRAPH (ii)**

**Find the number of zeroes of p(x) from the graph of y = p(x).**




**1** Look only at the x-axis and count the meetings. The straight line cuts the x-axis at exactly one point — the count of meetings is 1.

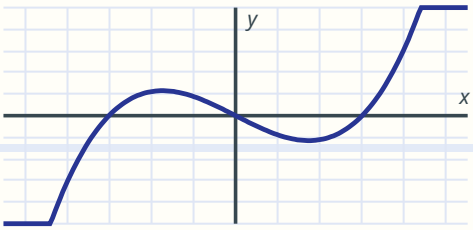
*Why: Zeroes live on the x-axis and nowhere else (Section 04).*

**ANSWER: The number of zeroes of p(x) is 1.**

**CBSE MARKING SCHEME:** 1 mark — correct count of x-axis intersections.

**EXERCISE 2.1 • GRAPH (iii)**

**Find the number of zeroes of p(x) from the graph of y = p(x).**




**1** Look only at the x-axis and count the meetings. The curve weaves across the axis: it crosses once on the left, once near the middle and once on the right — the count of meetings is 3.

*Why: Zeroes live on the x-axis and nowhere else (Section 04).*

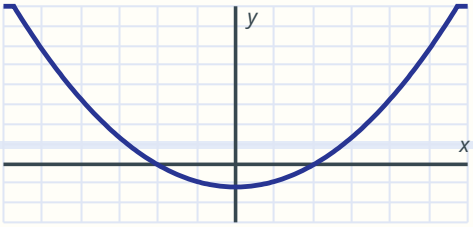
**ANSWER: The number of zeroes of p(x) is 3.**

**CBSE MARKING SCHEME:** 1 mark — correct count of x-axis intersections.

# Exercise 2.1 — Graphs (iv) and (v)

**EXERCISE 2.1 • GRAPH (iv)**

**Find the number of zeroes of p(x) from the graph of y = p(x).**




**1** Look only at the x-axis and count the meetings. The parabola dips below the axis and comes back up, crossing it at two points — the count of meetings is 2.

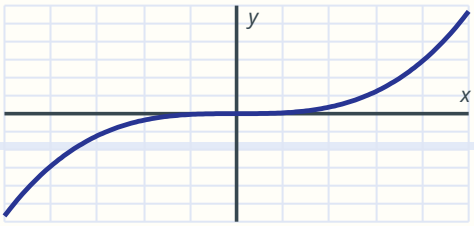
*Why: Zeroes live on the x-axis and nowhere else (Section 04).*

**ANSWER: The number of zeroes of p(x) is 2.**

**CBSE MARKING SCHEME:** 1 mark — correct count of x-axis intersections.

**EXERCISE 2.1 • GRAPH (v)**

**Find the number of zeroes of p(x) from the graph of y = p(x).**




**1** Look only at the x-axis and count the meetings. The curve flattens as it passes through the origin but meets the axis only there — the count of meetings is 1.

*Why: Zeroes live on the x-axis and nowhere else (Section 04).*

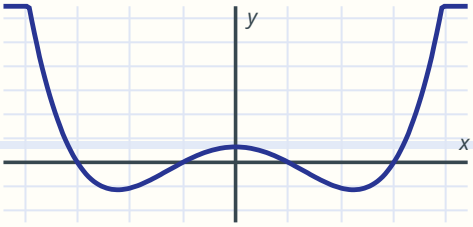
**ANSWER: The number of zeroes of p(x) is 1.**

**CBSE MARKING SCHEME:** 1 mark — correct count of x-axis intersections.

# Exercise 2.1 — Graph (vi) and the Summary

**EXERCISE 2.1 • GRAPH (vi)**

**Find the number of zeroes of p(x) from the graph of y = p(x).**




**1** Look only at the x-axis and count the meetings. The wavy curve crosses the axis four times — twice on the left of the origin and twice on the right — the count of meetings is 4.

*Why: Zeroes live on the x-axis and nowhere else (Section 04).*

**ANSWER: The number of zeroes of p(x) is 4.**

**CBSE MARKING SCHEME:** 1 mark — correct count of x-axis intersections.

## Exercise 2.1 — all six answers at a glance

(i) 0 • (ii) 1 • (iii) 3 • (iv) 2 • (v) 1 • (vi) 4. Graph (vi)'s four zeroes force degree at least 4 — the degree rule in reverse. Exam trap: never count the Y-axis meeting point — that is p(0), not a zero.

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# 07B Exercise 2.2 Q1 — Find Zeroes and Verify

THE BOARD'S FAVOURITE QUESTION TYPE • 3 MARKS EACH

**NCERT Exercise 2.2, Q1:** *"Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients."* Every part follows the same five-beat rhythm: **read a, b, c → split the middle term → factorise → zeroes → verify sum → verify product**. Learn the rhythm once; play it six times.

**EX 2.2 Q1 (i)**

**Find the zeroes of $x^2 - 2x - 8$ and verify the relationship between the zeroes and the coefficients.**

**THINK FIRST:** Split, factorise, then verify. The split pair must multiply to a × c = −8 and add to b = −2.

1. Read the coefficients: a = 1, b = −2, c = −8. So a × c = −8.
   *Why: Always begin by naming a, b, c — with their signs.*
2. Find the split pair: two numbers with product −8 and sum −2. They are −4 and +2 (since −4 × 2 = −8 and −4 + 2 = −2).
   *Why: For a × c negative, the two numbers have opposite signs.*
3. Split and group: $x^2 - 4x + 2x - 8 = x(x - 4) + 2(x - 4) = (x - 4)(x + 2)$.
4. Zero-product rule: x − 4 = 0 or x + 2 = 0, so the zeroes are α = 4 and β = −2.
5. Verify the sum: α + β = 4 + (−2) = 2. And −b/a = −(−2)/1 = 2. Equal.
   *Why: Write both sides explicitly — the verification IS the second half of the question.*
6. Verify the product: αβ = 4 × (−2) = −8. And c/a = −8/1 = −8. Equal.

**ANSWER: Zeroes: 4 and −2. Sum = 2 = −b/a and product = −8 = c/a — relationship verified.**

**CBSE MARKING SCHEME:** 1 mark for correct factorisation and zeroes • 1 mark for verifying the sum • 1 mark for verifying the product.

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# Exercise 2.2 Q1 (ii) — The Equal-Zeroes Case

**EX 2.2 Q1 (ii)**

**Find the zeroes of $4s^2 - 4s + 1$ and verify the relationship between the zeroes and the coefficients. (Variable is s.)**

**THINK FIRST:** Here a × c = 4. The split pair must multiply to +4 and add to −4 — suspect a repeated pair.

1. Read the coefficients: a = 4, b = −4, c = 1. So a × c = 4.
2. Find the split pair: −2 and −2 (product +4, sum −4).
   *Why: A repeated split pair is the fingerprint of equal zeroes.*
3. Split and group: $4s^2 - 2s - 2s + 1 = 2s(2s - 1) - 1(2s - 1) = (2s - 1)(2s - 1) = (2s - 1)^2$.
4. Zero-product rule: 2s − 1 = 0 (twice), so s = 1/2. The zeroes are α = 1/2 and β = 1/2 — equal zeroes.
   *Why: Equal zeroes mean the graph TOUCHES the x-axis at s = 1/2, like picture (b) of Section 04.*
5. Verify the sum: α + β = 1/2 + 1/2 = 1. And −b/a = −(−4)/4 = 1. Equal.
6. Verify the product: αβ = 1/2 × 1/2 = 1/4. And c/a = 1/4. Equal.

**ANSWER: Zeroes: 1/2 and 1/2 (equal). Sum = 1 = −b/a and product = 1/4 = c/a — relationship verified.**

**CBSE MARKING SCHEME:** 1 mark for correct factorisation and zeroes • 1 mark for verifying the sum • 1 mark for verifying the product.

# Exercise 2.2 Q1 (iii) — First, Arrange in Standard Form

**EX 2.2 Q1 (iii)**

**Find the zeroes of $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients.**

**THINK FIRST:** NCERT has shuffled the terms on purpose. Your first step is not splitting — it is housekeeping: write the polynomial in the standard order $ax^2 + bx + c$.

1. Rewrite in standard form: $6x^2 - 7x - 3$. Now read: $a = 6$, $b = -7$, $c = -3$, so $a \times c = -18$.
   *Why: Reading b = 7 from the shuffled form is the classic error here (see Find-the-Error E7). The coefficient of x is −7.*
2. Find the split pair: product $-18$, sum $-7$. They are $-9$ and $+2$.
3. Split and group: $6x^2 - 9x + 2x - 3 = 3x(2x - 3) + 1(2x - 3) = (2x - 3)(3x + 1)$.
4. Zero-product rule: $2x - 3 = 0$ gives $x = 3/2$; $3x + 1 = 0$ gives $x = -1/3$. Zeroes: $\alpha = 3/2$, $\beta = -1/3$.
5. Verify the sum: $3/2 + (-1/3) = 9/6 - 2/6 = 7/6$. And $-b/a = -(-7)/6 = 7/6$. Equal.
   *Why: Fraction addition: use LCM 6 — show it, do not jump.*
6. Verify the product: $(3/2) \times (-1/3) = -1/2$. And $c/a = -3/6 = -1/2$. Equal.

**ANSWER: Zeroes: 3/2 and −1/3. Sum = 7/6 = −b/a and product = −1/2 = c/a — relationship verified.**

**CBSE MARKING SCHEME:** 1 mark for correct factorisation and zeroes • 1 mark for verifying the sum • 1 mark for verifying the product.

# Exercise 2.2 Q1 (iv) — When the Constant Term Is Zero

**EX 2.2 Q1 (iv)**

**Find the zeroes of $4u^2 + 8u$ and verify the relationship between the zeroes and the coefficients. (Variable is u.)**

**THINK FIRST:** No constant term — so no splitting hunt is needed. A common factor is staring at you.

1. Read the coefficients: a = 4, b = 8, c = 0.
   *Why: c = 0 is information, not a gap: it promises that one zero will be 0.*
2. Take out the common factor: $4u^2 + 8u = 4u(u + 2)$.
   *Why: Both terms contain 4u.*
3. Zero-product rule: 4u = 0 gives u = 0; u + 2 = 0 gives u = −2. Zeroes: α = 0, β = −2.
4. Verify the sum: 0 + (−2) = −2. And −b/a = −8/4 = −2. Equal.
5. Verify the product: 0 × (−2) = 0. And c/a = 0/4 = 0. Equal.
   *Why: Product zero ⇔ at least one zero is zero. The bridge agrees with common sense.*

**ANSWER: Zeroes: 0 and −2. Sum = −2 = −b/a and product = 0 = c/a — relationship verified.**

**CBSE MARKING SCHEME:** 1 mark for correct factorisation and zeroes • 1 mark for verifying the sum • 1 mark for verifying the product.

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# Exercise 2.2 Q1 (v) — The Difference of Squares

**EX 2.2 Q1 (v)**

**Find the zeroes of $t^2 - 15$ and verify the relationship between the zeroes and the coefficients. (Variable is t.)**

**THINK FIRST:** There is no middle term (b = 0), so splitting has nothing to split. Recognise the shape: a square minus a number — the difference-of-squares pattern.

1. Read the coefficients: a = 1, b = 0, c = −15.
2. Write 15 as a square: $15 = (\sqrt{15})^2$. So $t^2 - 15 = t^2 - (\sqrt{15})^2$.
   *Why: Any positive number is the square of its square root — this trick unlocks every '$x^2$ − number' polynomial.*
3. Apply $a^2 - b^2 = (a - b)(a + b)$: $t^2 - 15 = (t - \sqrt{15})(t + \sqrt{15})$.
4. Zero-product rule: $t = \sqrt{15}$ or $t = -\sqrt{15}$. Zeroes: $\alpha = \sqrt{15}$, $\beta = -\sqrt{15}$.
   *Why: Never report only the positive root — see Find-the-Error ES. A quadratic with b = 0 has zeroes in ± pairs.*
5. Verify the sum: $\sqrt{15} + (-\sqrt{15}) = 0$. And $-b/a = -0/1 = 0$. Equal.
6. Verify the product: $\sqrt{15} \times (-\sqrt{15}) = -15$. And $c/a = -15/1 = -15$. Equal.
   *Why: √15 × √15 = 15 — a root times itself returns the number.*

**ANSWER: Zeroes: $\sqrt{15}$ and $-\sqrt{15}$. Sum = 0 = −b/a and product = −15 = c/a — relationship verified.**

**CBSE MARKING SCHEME:** 1 mark for correct factorisation and zeroes • 1 mark for verifying the sum • 1 mark for verifying the product.

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# Exercise 2.2 Q1 (vi) — The Full Rhythm, One Last Time

**EX 2.2 Q1 (vi)**

**Find the zeroes of $3x^2 - x - 4$ and verify the relationship between the zeroes and the coefficients.**

**THINK FIRST:** $a \times c = -12$ and $b = -1$. Hunt for the pair calmly.

1. Read the coefficients: $a = 3$, $b = -1$, $c = -4$. So $a \times c = -12$.
2. Find the split pair: product $-12$, sum $-1$. They are $-4$ and $+3$.
3. Split and group: $3x^2 - 4x + 3x - 4 = x(3x - 4) + 1(3x - 4) = (3x - 4)(x + 1)$.
4. Zero-product rule: $3x - 4 = 0$ gives $x = 4/3$; $x + 1 = 0$ gives $x = -1$. Zeroes: $\alpha = 4/3$, $\beta = -1$.
5. Verify the sum: $4/3 + (-1) = 1/3$. And $-b/a = -(-1)/3 = 1/3$. Equal.
6. Verify the product: $(4/3) \times (-1) = -4/3$. And $c/a = -4/3$. Equal.

**ANSWER: Zeroes: 4/3 and −1. Sum = 1/3 = −b/a and product = −4/3 = c/a — relationship verified.**

**CBSE MARKING SCHEME:** 1 mark for correct factorisation and zeroes • 1 mark for verifying the sum • 1 mark for verifying the product.

***TOPPER TIP:*** *Six questions, one rhythm. In the board exam this exact question type appeared in 2023 AND 2024 papers (see the PYQ section). Master the rhythm here and those marks are already yours.*

# 07C Exercise 2.2 Q2 — Form a Polynomial from Sum and Product

THE BRIDGE, CROSSED BACKWARDS

**NCERT Exercise 2.2, Q2:** *"Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively."* Six parts, one formula. Watch the signs: **minus** the sum, **plus** the product.

**EX 2.2 Q2 (i)**

**Find a quadratic polynomial whose sum of zeroes is 1/4 and product of zeroes is −1.**

**THINK FIRST:** Use the forming formula from Derivation 3: $p(x) = k[x^2 - Sx + P]$. Then choose k to clear fractions.

1. Write S = 1/4 and P = −1. The monic form is $x^2 - (1/4)x + (-1) = x^2 - (1/4)x - 1$.
   *Why: The product P = −1 enters with its own sign: + (−1) = −1.*

2. Clear the fraction by choosing k = 4: multiply every term by 4: $4x^2 - x - 4$.
   *Why: Multiplying by 4 changes the polynomial's size, never its zeroes.*

3. Self-check: for $4x^2 - x - 4$, sum of zeroes = −(−1)/4 = 1/4 and product = −4/4 = −1. Both as given.
   *Why: Ten seconds of checking protects two full marks.*

**ANSWER: $4x^2 - x - 4$ (any non-zero multiple is equally correct).**

**CBSE MARKING SCHEME:** 1 mark for writing $x^2 - Sx + P$ with correct signs • 1 mark for the final polynomial with fractions cleared.

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# Exercise 2.2 Q2 — Parts (ii) and (iii)

**EX 2.2 Q2 (ii)**

**Find a quadratic polynomial whose sum of zeroes is √2 and product of zeroes is 1/3.**

**THINK FIRST:** Use the forming formula from Derivation 3: $p(x) = k[x^2 - Sx + P]$. Then choose k to clear fractions.

1. Write $S = \sqrt{2}$ and $P = 1/3$. The monic form is $x^2 - \sqrt{2}\,x + 1/3$.
2. Clear the fraction by choosing $k = 3$: $3x^2 - 3\sqrt{2}\,x + 1$.
   *Why: Only the 1/3 needed clearing; √2 is a perfectly respectable coefficient.*
3. Self-check: sum $= -(-3\sqrt{2})/3 = \sqrt{2}$; product $= 1/3$. Both as given.

**ANSWER: $3x^2 - 3\sqrt{2}\,x + 1$.**

**CBSE MARKING SCHEME:** 1 mark for writing $x^2 - Sx + P$ with correct signs • 1 mark for the final polynomial with fractions cleared.

**EX 2.2 Q2 (iii)**

**Find a quadratic polynomial whose sum of zeroes is 0 and product of zeroes is √5.**

**THINK FIRST:** Use the forming formula from Derivation 3: $p(x) = k[x^2 - Sx + P]$. Then choose k to clear fractions.

1. Write $S = 0$ and $P = \sqrt{5}$. The monic form is $x^2 - 0 \cdot x + \sqrt{5} = x^2 + \sqrt{5}$.
   *Why: Sum zero means the x-term vanishes — the zeroes will be opposites of each other, like ± pairs.*
2. No fractions to clear, so $k = 1$: the polynomial is $x^2 + \sqrt{5}$.
3. Self-check: sum $= -0/1 = 0$; product $= \sqrt{5}/1 = \sqrt{5}$. Both as given.

**ANSWER: $x^2 + \sqrt{5}$.**

**CBSE MARKING SCHEME:** 1 mark for writing $x^2 - Sx + P$ with correct signs • 1 mark for the final polynomial with fractions cleared.

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# Exercise 2.2 Q2 — Parts (iv) and (v)

**EX 2.2 Q2 (iv)**

**Find a quadratic polynomial whose sum of zeroes is 1 and product of zeroes is 1.**

**THINK FIRST:** Use the forming formula from Derivation 3: $p(x) = k[x^2 - Sx + P]$. Then choose k to clear fractions.

1. Write S = 1 and P = 1. The monic form is $x^2 - 1 \cdot x + 1 = x^2 - x + 1$.
2. No fractions, k = 1: the polynomial is $x^2 - x + 1$.
3. Self-check: sum = $-(-1)/1 = 1$; product = $1/1 = 1$. Both as given.
   *Why: Note: this polynomial's graph never touches the x-axis — its 'zeroes' are not real numbers. The forming formula still works; Class 11 will explain why.*

**ANSWER: $x^2 - x + 1$.**

**CBSE MARKING SCHEME:** 1 mark for writing $x^2 - Sx + P$ with correct signs • 1 mark for the final polynomial with fractions cleared.

**EX 2.2 Q2 (v)**

**Find a quadratic polynomial whose sum of zeroes is −1/4 and product of zeroes is 1/4.**

**THINK FIRST:** Use the forming formula from Derivation 3: $p(x) = k[x^2 - Sx + P]$. Then choose k to clear fractions.

1. Write S = −1/4 and P = 1/4. The monic form is $x^2 - (-1/4)x + 1/4 = x^2 + (1/4)x + 1/4$.
   *Why: Minus a negative is plus — this is THE sign trap of this exercise. Slow down here.*
2. Clear fractions with k = 4: $4x^2 + x + 1$.
3. Self-check: sum = −1/4; product = 1/4. Both as given.

**ANSWER: $4x^2 + x + 1$.**

**CBSE MARKING SCHEME:** 1 mark for writing $x^2 - Sx + P$ with correct signs • 1 mark for the final polynomial with fractions cleared.

# Exercise 2.2 Q2 — Part (vi) and the Method

**EX 2.2 Q2 (vi)**

**Find a quadratic polynomial whose sum of zeroes is 4 and product of zeroes is 1.**

**THINK FIRST:** Use the forming formula from Derivation 3: $p(x) = k[x^2 - Sx + P]$. Then choose k to clear fractions.

1. Write S = 4 and P = 1. The monic form is $x^2 - 4x + 1$.
2. No fractions, k = 1: the polynomial is $x^2 - 4x + 1$.
3. Self-check: sum = $-(-4)/1 = 4$; product = $1/1 = 1$. Both as given.

**ANSWER: $x^2 - 4x + 1$.**

**CBSE MARKING SCHEME:** 1 mark for writing $x^2 - Sx + P$ with correct signs • 1 mark for the final polynomial with fractions cleared.

***TOPPER TIP:*** *Finished all six? Notice the pattern of your own work: read S and P → write $x^2 - Sx + P$ → clear fractions → self-check. Four moves, every time. That is a method, and methods are what toppers actually memorise.*

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# Beyond the Exercise — The Division Algorithm at Work

**ENRICHMENT • FULLY WORKED**

The rationalised NCERT exercise list no longer asks for polynomial long division, but the **division algorithm** (Section 05, idea 6) remains in the chapter's foundations — and it returns in Class 10's later chapters and in Class 11. Learn it here, completely, with two examples. In the working below, x^2 means $x^2$ and x^3 means $x^3$.

**DIVISION EXAMPLE 1**

**Divide $2x^2 + 3x + 1$ by $x + 2$. Find the quotient and the remainder.**

**THINK FIRST:** Treat it exactly like school long division of numbers: divide, multiply, subtract, bring down — but with degrees instead of digits.

1. Ask: what must multiply $(x + 2)$ to give the first term $2x^2$? Answer: $2x$. Write $2x$ as the first term of the quotient.
   *Why: Divide leading term by leading term: $2x^2 \div x = 2x$.*
2. Multiply back: $2x \times (x + 2) = 2x^2 + 4x$. Subtract from the dividend: $(2x^2 + 3x) - (2x^2 + 4x) = -x$. Bring down the +1: new dividend $-x + 1$.
   *Why: Subtraction kills the $x^2$ term — that is the point of the step.*
3. Repeat: $-x \div x = -1$, the next quotient term. Multiply back: $-1 \times (x + 2) = -x - 2$. Subtract: $(-x + 1) - (-x - 2) = 3$.
4. The remainder 3 has degree 0, which is less than the divisor's degree 1 — so we STOP. Quotient $q(x) = 2x - 1$, remainder $r(x) = 3$.
   *Why: The stopping rule is about DEGREES, not about the remainder being zero.*
5. Verify with the algorithm: $g(x)q(x) + r(x) = (x + 2)(2x - 1) + 3 = (2x^2 + 3x - 2) + 3 = 2x^2 + 3x + 1 = p(x)$. Correct.
   *Why: Never trust a division you have not multiplied back.*

**ANSWER: Quotient = $2x - 1$, remainder = 3.**

# The Written Layout — How a Long Division Looks on Paper

Here is Division Example 1 written out in the traditional layout. Read it with the five steps of the previous page: each "divide – multiply – subtract" cycle produces one line of the quotient and one subtraction block.

```
           2x - 1            <- quotient
         ____________
x + 2  ) 2x^2 + 3x + 1
         2x^2 + 4x
         ----------
               -x + 1
               -x - 2
               ------
                    3        <- remainder
```

### Reading the layout — three habits

1. **Align by degree:** $x^2$ terms above $x^2$ terms, x terms above x terms — misalignment is the number-one cause of subtraction errors.
2. **Subtract the WHOLE row:** when you subtract $2x^2 + 4x$, the sign of every term flips — this is where the $-x$ in the working comes from $(3x - 4x)$.
3. **The remainder sits alone at the end** only when its degree has dropped below the divisor's. Here 3 (degree 0) < x + 2 (degree 1) — stop.

**TOPPER TIP:** *On rough paper, rule faint vertical guide-lines under each degree column before you begin. Thirty seconds of drawing saves three minutes of error-hunting.*

# Division Example 2 — A Cubic Divided by a Quadratic

**DIVISION EXAMPLE 2**

**Divide $x^3 - 3x^2 + 5x - 3$ by $x^2 - 2$. Find the quotient and the remainder.**

**THINK FIRST:** The divisor has degree 2, so the quotient will have degree $3 - 2 = 1$, and the remainder's degree must drop below 2.

1. Write the dividend with all powers present: $x^3 - 3x^2 + 5x - 3$ (no term is missing here). Divide leading terms: $x^3 \div x^2 = x$. First quotient term: x.

2. Multiply back: $x \times (x^2 - 2) = x^3 - 2x$. Subtract: $(x^3 - 3x^2 + 5x) - (x^3 - 2x) = -3x^2 + 7x$. Bring down $-3$: new dividend $-3x^2 + 7x - 3$.
   *Why: Watch the x-term: 5x − (−2x) = 7x. Sign slips live here.*

3. Repeat: $-3x^2 \div x^2 = -3$. Multiply back: $-3 \times (x^2 - 2) = -3x^2 + 6$. Subtract: $(-3x^2 + 7x - 3) - (-3x^2 + 6) = 7x - 9$.

4. The remainder $7x - 9$ has degree $1 < 2$ (the divisor's degree) — STOP. Quotient $q(x) = x - 3$, remainder $r(x) = 7x - 9$.

5. Verify: $(x^2 - 2)(x - 3) + (7x - 9) = (x^3 - 3x^2 - 2x + 6) + 7x - 9 = x^3 - 3x^2 + 5x - 3 = p(x)$. Correct.

**ANSWER: Quotient = x − 3, remainder = 7x − 9.**

```
          x - 3            <- quotient
        _________________
x^2 - 2 ) x^3 - 3x^2 + 5x - 3
          x^3       - 2x
          -----------------
               -3x^2 + 7x - 3
               -3x^2      + 6
               --------------
                        7x - 9   <- remainder
```

**PIN POINT**

**DIVISION ALGORITHM: p(x) = g(x) × q(x) + r(x), where r(x) = 0 OR deg r(x) < deg g(x). If r(x) = 0, then g(x) is a FACTOR of p(x) — and every zero of g(x) is a zero of p(x).**

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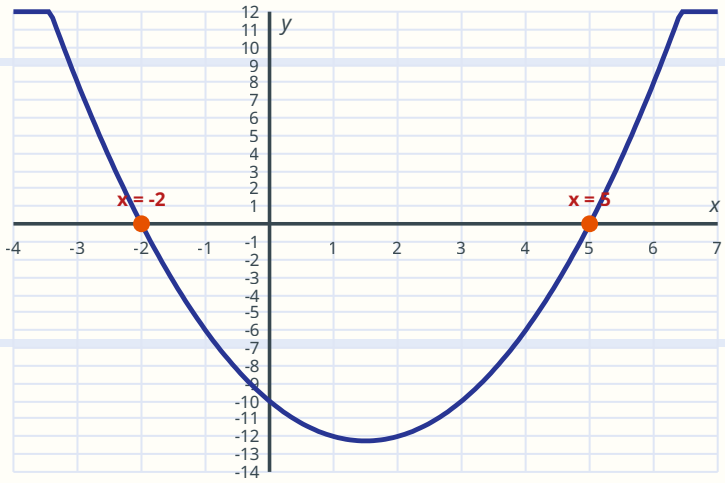
# 08 Multiple Methods

8D • CHALLENGE — ONE PROBLEM, THREE ROADS

**The problem:** find the zeroes of $p(x) = x^2 - 3x - 10$. A one-method student is stuck when that method fails. A three-method student simply changes roads. Learn all three.

## Method A — Read the graph (the Visual road)




The parabola crosses the x-axis at $x = -2$ and $x = 5$, so the zeroes are **$-2$ and 5**. Strength: instant understanding. Weakness: needs an accurate graph — useless for zeroes like 4/3.

## Method B — Factorise by splitting (the Algebra road)

1. $a \times c = -10$, $b = -3$. Split pair: $-5$ and $+2$.
2. $x^2 - 5x + 2x - 10 = x(x - 5) + 2(x - 5) = (x - 5)(x + 2)$.
3. Zero-product rule: $x = 5$ or $x = -2$.

Strength: exact, works in the exam hall with no graph paper. This is the board's expected road.

# Method C — and Choosing Your Road

## Method C — The sum-product detective (the Bridge road)

1. From the coefficients, the zeroes (call them α, β) must satisfy α + β = −b/a = 3 and αβ = c/a = −10.
   *Why: The bridge tells us the zeroes' fingerprints before we know the zeroes.*
2. Hunt two numbers with sum 3 and product −10: factor pairs of −10 are (1, −10), (−1, 10), (2, −5), (−2, 5). Only −2 and 5 add to 3.
3. So the zeroes are −2 and 5 — agreeing with Methods A and B.
   *Why: Three roads, one destination. When two roads agree, you can walk into the answer box with confidence.*

## Forming a polynomial — two roads

**Problem:** form a quadratic whose zeroes are 3 and −2.

**Road 1 (factors):** $(x - 3)(x + 2) = x^2 - x - 6$.

**Road 2 (formula):** $S = 3 + (-2) = 1$, $P = 3 \times (-2) = -6$, so $p(x) = x^2 - Sx + P = x^2 - x - 6$. Same road-end — Road 2 is faster when only S and P are given; Road 1 is faster when the zeroes themselves are given.

| Situation | Best road |
|---|---|
| Graph is drawn / picture question | Method A — read the x-axis meetings |
| Exam question: "find the zeroes" | Method B — split and factorise |
| Only sum/product conditions given | Method C — detective work on the bridge |
| Checking any answer | Use a DIFFERENT road from the one that produced it |

> **TOPPER TIP:** *The golden rule of Method-choosing: solve by one road, verify by another. A graph check takes twenty seconds and has saved more board marks than any other habit in this chapter.*

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# 09 Interactive Simulations & Hands-on Activities

TRY IT YOURSELF — LEARN BY DOING

Graph paper ready? In this chapter, your hand must learn what your eye saw in Section 04. Complete all three activities before opening the answers.

## Activity 1 — The graph-plotting lab: $y = x^2 - 3x - 4$

Complete the table, plot the eight points on graph paper, join them with a smooth curve, and read off the zeroes. (This is the same polynomial as Section 04 — now YOU draw it.)

| x | −2 | −1 | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|
| y | ... | ... | −4 | ... | −6 | ... | ... | ... |

Questions: (a) At which two x-values is y = 0? (b) What is the lowest point (vertex) of your curve? (c) The curve is symmetric about a vertical line — which line?

## Activity 2 — Predict first, plot second

For each polynomial, FIRST write your prediction ("it will cross / touch / miss the x-axis because..."), THEN compute the table and plot.

| Polynomial | Table (x, y) | My prediction |
|---|---|---|
| $y = x^2 - 4x + 4$ | x = 0, 1, 2, 3, 4 → y = 4, ..., ..., ..., 4 | |
| $y = x^2 + 1$ | x = −2, −1, 0, 1, 2 → y = 5, ..., ..., ..., 5 | |

## Activity 3 — The zero hunt

A number is a zero only if substituting it makes the polynomial exactly 0. Test each candidate and mark Zero / Not a zero.

| Polynomial | Candidate | Value on substituting | Zero? |
|---|---|---|---|
| $p(x) = x^2 - 5x + 6$ | x = 1, 2, 3, 4 | | |
| $q(x) = 2x^2 + 3x - 2$ | x = 1/2, −2, 1 | | |
| $r(x) = x^2 - 2x - 3$ | x = −1, 2, 3 | | |

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# Activity Answers — Check After Trying

**Activity 1:** y-values: 6, 0, −4, −6, −6, −4, 0, 6. (a) y = 0 at x = −1 and x = 4 — the zeroes. (b) Vertex: the lowest value is y = −6, reached at x = 1 and x = 2 — the true vertex sits between them at x = 3/2, y = −25/4 (the midpoint of the zeroes −1 and 4 is 3/2 — Section 12 explains why). (c) The line x = 3/2.

**Activity 2:** $y = x^2 - 4x + 4$: values 4, 1, 0, 1, 4 — the parabola TOUCHES the axis at x = 2 (one zero, counted twice: it is $(x - 2)^2$). $y = x^2 + 1$: values 5, 2, 1, 2, 5 — the smallest value is 1, never 0, so the curve MISSES the axis: no real zeroes. If your prediction used the words “the values never reach 0”, you were thinking like a mathematician.

**Activity 3:** p(x): p(1) = 2 (not), p(2) = 0 (ZERO), p(3) = 0 (ZERO), p(4) = 2 (not). q(x): q(1/2) = 1/2 + 3/2 − 2 = 0 (ZERO), q(−2) = 8 − 6 − 2 = 0 (ZERO), q(1) = 3 (not). r(x): r(−1) = 0 (ZERO), r(2) = −3 (not), r(3) = 0 (ZERO).

**TOPPER TIP:** *Notice Activity 3's hidden lesson: p(1) = 2 and p(4) = 2 — equal values, neither zero. Only the exact value 0 earns the title ‘zero of the polynomial’. Close is not zero.*

**THE KALVIYOGI DIGITAL COMPANION** — this chapter continues in the KalviYogi app/website:
- **Video lesson:** “Zeroes You Can See” — the graph method, animated.
- **Simulation:** Graph Explorer — drag a parabola up and down and watch its zeroes appear, merge and vanish.
- **Simulation:** Split-the-Middle-Term Trainer — endless factorisation practice with instant feedback.
- **Online test:** Chapter 2 test — 20 questions, marked instantly, errors explained.
- **Podcast:** “The Arch That Touched the Ground” — the Section 03 story as audio revision.
- **Active Recall deck:** 15 flashcards built from this chapter's Pin Points.

QR code and link activate with the KalviYogi app/website at print.

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# 10 Common Mistakes

LEARN FROM OTHER STUDENTS' ERRORS — BEFORE THEY BECOME YOURS

**MISTAKE 1 — Confusing 'zero of the polynomial' with 'the value at zero'.**

**Wrong way:** "The zero of $x^2 - 3x - 4$ is $p(0) = -4$."

**Why wrong:** $p(0)$ is the value of the polynomial AT $x = 0$ — here it is the constant term, $-4$. A **zero** is an input that makes the value zero.

**Right way:** Solve $p(x) = 0$: zeroes are $-1$ and $4$, because $p(-1) = 0$ and $p(4) = 0$. Keep the two phrases in separate boxes in your mind: 'value at 0' vs 'zero of $p(x)$'.

**MISTAKE 2 — Losing the minus sign in the sum of zeroes.**

**Wrong way:** For $2x^2 - 7x + 3$: "sum of zeroes = $b/a = -7/2$."

**Why wrong:** The formula is $\alpha + \beta = -b/a$. Here $b = -7$, so $-b/a = -(-7)/2 = +7/2$. Check against the actual zeroes 3 and 1/2: their sum is 7/2, positive.

**Right way:** Say the formula aloud with its minus: "sum = minus-b-by-a". Then substitute b WITH its sign.

**MISTAKE 3 — Reading b from a shuffled polynomial.**

**Wrong way:** For $6x^2 - 3 - 7x$, taking $b = 7$ (or $c = -7$) straight from the printed order.

**Why wrong:** The letters a, b, c are defined by POSITION in the standard form $ax^2 + bx + c$ — a with $x^2$, b with x, c constant. In the shuffled polynomial the x-term is $-7x$.

**Right way:** First rewrite: $6x^2 - 7x - 3$. Only then read $a = 6$, $b = -7$, $c = -3$. (NCERT Q1(iii) is designed to catch exactly this.)

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# Common Mistakes 4–6

**MISTAKE 4 — Counting the wrong meetings in graph questions.**

**Wrong way:** Counting where the curve crosses the y-axis, or counting its turning points, as zeroes.

**Why wrong:** A zero needs y = 0 — the point must lie ON the x-axis. A y-axis crossing has x = 0, not y = 0.

**Right way:** Run your finger along the X-AXIS only, and count touches/crossings there. Nothing else counts.

**MISTAKE 5 — Sign slip when forming a polynomial.**

**Wrong way:** Sum = −1/4, product = 1/4 → student writes $x^2 - (1/4)x + 1/4$.

**Why wrong:** The formula subtracts the sum AS IT IS: $x^2 - Sx + P$ with S = −1/4 gives $x^2 - (-1/4)x + 1/4 = x^2 + (1/4)x + 1/4$. The student subtracted +1/4 instead of −1/4.

**Right way:** Write S in brackets first — $x^2 - (-1/4)x + (1/4)$ — then simplify the signs. (This is NCERT Q2(v), and it appears in Find-the-Error E3's family.)

**MISTAKE 6 — A split pair that satisfies only ONE condition.**

**Wrong way:** For $3x^2 - x - 4$, splitting with −3 and +2 because −3 + 2 = −1 = b.

**Why wrong:** The pair must ALSO multiply to a × c = −12. But −3 × 2 = −6. The grouping then fails — no common bracket appears — and students force a wrong factorisation.

**Right way:** The pair is −4 and +3: sum −1, product −12. Check BOTH conditions before writing a single term. If the grouping produces no common bracket, your pair was wrong — go back, do not force it.

***TOPPER TIP:*** *Three of these six mistakes are pure SIGN mistakes. In this chapter, marks are rarely lost on ideas — they are lost on minus signs. Slow down at every −b/a, every −(−...), every split.*

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# 11 Graded Practice Problems

FIVE LEVELS — CLIMB ONE LEVEL AT A TIME • ANSWERS IN THE ANSWER KEY

Rules of the climb: finish a level with at most one error before moving up. Write full steps for Level 2 onwards — answers alone earn nothing in the board exam.

## LEVEL 1 — FOUNDATION (direct concepts)

**L1.1** For $p(x) = 2x^2 - 3x + 5$, find $p(1)$.

**L1.2** Is $x = 2$ a zero of $x^2 - 5x + 6$? Justify in one line.

**L1.3** Find the zeroes of $x^2 - 9$.

**L1.4** For $5x^2 - 3x + 2$, write down the sum of zeroes and the product of zeroes. (Do not find the zeroes.)

**L1.5** Find the zeroes of $x^2 - 7x + 12$.

**L1.6** Write a quadratic polynomial whose zeroes are 2 and 3.

## LEVEL 2 — SKILL BUILDER (textbook standard)

**L2.1** Find the zeroes of $2x^2 - 7x + 3$ and verify the relationship between the zeroes and the coefficients.

**L2.2** Find the zeroes of $4x^2 + 4x - 3$ and verify the relationship between the zeroes and the coefficients.

**L2.3** Find a quadratic polynomial whose sum of zeroes is $-3$ and product of zeroes is 2.

**L2.4** Find a quadratic polynomial whose sum of zeroes is 0 and product of zeroes is $-4$.

**L2.5** Find the zeroes of $x^2 + 7x + 10$ and verify the relationship between the zeroes and the coefficients.

**L2.6** Find the zeroes of $3x^2 + 5x - 2$ and verify the relationship between the zeroes and the coefficients.

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# Graded Practice — Levels 3, 4 and 5

## LEVEL 3 — APPLICATION (real-life problems)

**L3.1** A ball's height after t seconds is $h(t) = -5t^2 + 30t$ metres. After how many seconds does it hit the ground? (Ignore t = 0, the moment of throwing.)

**L3.2** A rectangular lawn has area $A(x) = x^2 + 11x + 30$ $m^2$. Express the area as a product of the two side-lengths.

**L3.3** An arch has height $h(x) = -x^2 + 6x - 5$ metres at distance x from a fixed pillar. Find where the arch meets the ground, and hence the width of the opening.

**L3.4** Two numbers have sum 11 and product 30. Form a quadratic polynomial with these as sum and product of zeroes, and hence find the two numbers.

## LEVEL 4 — ADVANCED REASONING (multi-concept)

**L4.1** If α and β are the zeroes of $x^2 - 5x + 6$, find $\alpha^2 + \beta^2$ — without finding α and β separately.

**L4.2** One zero of $3x^2 - 8x + (2k + 1)$ is seven times the other. Find the value of k.

**L4.3** If α, β are the zeroes of $ax^2 + bx + c$ and $\alpha + \beta = \alpha\beta$, show that $b + c = 0$.

## LEVEL 5 — MATHEMATICAL GENIUS (olympiad flavour)

**L5.1** Find all values of k for which $x^2 + kx + 9$ has equal zeroes.

**L5.2** If α, β are the zeroes of $x^2 - 6x + 5$, form a quadratic polynomial whose zeroes are α + 1 and β + 1.

> **TOPPER TIP:** *Stuck at a level? Section 07 contains a worked twin of almost every question here. Find the twin, study its steps, close the book, and try again.*

# Practice Plus — Concept MCQs (1 of 3)

**25 one-mark questions** in true board style. Choose the ONE correct option. The Answer Key gives the key AND a one-line explanation for every question — read the explanation even when you are right.

**M1.** The degree of the polynomial $5x^3 - 4x^2 + 7x - 2$ is

(A) 2 (B) 4 (C) 3 (D) 1

**M2.** The zeroes of $x^2 - 3x - 4$ are

(A) −4 and 1 (B) 4 and −1 (C) 4 and 1 (D) −4 and −1

**M3.** Which of the following is a zero of $x^2 - 7x + 10$?

(A) 3 (B) 2 (C) 4 (D) −2

**M4.** The sum of the zeroes of $3x^2 - 5x + 2$ is

(A) 5/3 (B) −5/3 (C) 2/3 (D) −2/3

**M5.** The product of the zeroes of $2x^2 + 3x - 5$ is

(A) 5/2 (B) −3/2 (C) −5/2 (D) 3/5

**M6.** A quadratic polynomial whose zeroes are −3 and 4 is

(A) $x^2 + x - 12$ (B) $x^2 - x + 12$ (C) $x^2 + x + 12$ (D) $x^2 - x - 12$

**M7.** The maximum number of zeroes a cubic polynomial can have is

(A) 2 (B) 3 (C) 4 (D) 1

**M8.** The graph of $y = p(x)$ is a parabola touching the x-axis at exactly one point. The number of distinct zeroes of $p(x)$ is

(A) 1 (B) 2 (C) 0 (D) 3

**M9.** If $\alpha + \beta = 0$ and one zero of a quadratic is 3, the other zero is

(A) 0 (B) 3 (C) −3 (D) 6

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# Practice Plus — Concept MCQs (2 of 3)

**M10.** The number of real zeroes of $x^2 + 1$ is

(A) 0 (B) 1 (C) 2 (D) 4

**M11.** If $x = 1$ is a zero of $x^2 + kx - 6$, then $k$ equals

(A) $-5$ (B) 6 (C) $-6$ (D) 5

**M12.** The sum of the zeroes of $-x^2 + 5x - 6$ is

(A) $-5$ (B) 5 (C) 6 (D) $-6$

**M13.** The product of the zeroes of $4x^2 - 9$ is

(A) 9/4 (B) $-4/9$ (C) $-9/4$ (D) 0

**M14.** The zeroes of $6x^2 - 7x - 3$ are

(A) 3/2 and $-1/3$ (B) $-3/2$ and 1/3 (C) 3 and $-1/3$ (D) 3/2 and 1/3

**M15.** A linear polynomial has at most how many zeroes?

(A) 0 (B) 1 (C) 2 (D) 3

**M16.** If $\alpha, \beta$ are the zeroes of $x^2 - 6x + 8$, then $\alpha + \beta + \alpha\beta$ equals

(A) 6 (B) 8 (C) 2 (D) 14

**M17.** A polynomial of degree 4 has at most how many zeroes?

(A) 1 (B) 2 (C) 4 (D) 5

**M18.** For $p(x) = x^2 - 5x + 6$, $p(x) = 0$ when $x$ equals

(A) 1 (B) 3 (C) 4 (D) 6

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# Practice Plus — Concept MCQs (3 of 3)

**M19.** The zeroes of $x^2 - 2x$ are

(A) 0 and 2 (B) 0 and −2 (C) 2 and −2 (D) only 2

**M20.** If the zeroes of $x^2 + px + q$ are 2 and 3, then q equals

(A) 5 (B) −5 (C) 6 (D) −6

**M21.** A parabola crosses the x-axis at two distinct points. The number of zeroes of the corresponding polynomial is

(A) 0 (B) 1 (C) 2 (D) 3

**M22.** If α and β are the zeroes of a quadratic and αβ = 0, then

(A) at least one zero is 0 (B) both zeroes are 0 (C) α + β = 0 (D) the polynomial has no zeroes

**M23.** A quadratic polynomial with sum of zeroes −1/2 and product of zeroes −3 is

(A) $2x^2 - x - 6$ (B) $x^2 - (1/2)x + 3$ (C) $2x^2 + x + 6$ (D) $2x^2 + x - 6$

**M24.** The zero of the polynomial $3x - 6$ is

(A) 6 (B) 2 (C) −2 (D) 3

**M25.** If α, β are the zeroes of $x^2 - x - 6$, then $1/\alpha + 1/\beta$ equals

(A) 1/6 (B) −5/6 (C) −1/6 (D) 5/6

**TOPPER TIP:** *Scoring guide: 22–25 — board-ready on concepts. 16–21 — revise the sections named in the key's explanations. Below 16 — redo Sections 04–06, then retake on Day 3.*

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# 12 Mathematical Reasoning

JUSTIFY AND PROVE — FULL SOLUTIONS

**REASONING 1**

**If α and β are the zeroes of $ax^2 + bx + c$ ($a \neq 0$, $c \neq 0$), prove that $1/\alpha + 1/\beta = -b/c$.**

**THINK FIRST:** Almost every 'prove' question in this chapter is the bridge wearing a disguise. Undress it: rewrite the target using α + β and αβ, then substitute.

1. Combine the fractions on the left: $1/\alpha + 1/\beta = (\beta + \alpha) / (\alpha\beta) = (\alpha + \beta) / (\alpha\beta)$.
   *Why: Add fractions by the common denominator αβ — and note the numerator becomes the SUM of zeroes.*
2. Substitute the bridge values: $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$. So $1/\alpha + 1/\beta = (-b/a) / (c/a)$.
3. Dividing by a fraction multiplies by its reciprocal: $(-b/a) \times (a/c) = -b/c$. The a cancels.
   *Why: That cancellation is why the answer is so clean — a disappears completely.*
4. Hence $1/\alpha + 1/\beta = -b/c$. Proved. ∎

**ANSWER: $1/\alpha + 1/\beta = (\alpha + \beta)/(\alpha\beta) = -b/c$. ∎**

**REASONING 2**

**Prove that a quadratic polynomial cannot have more than two zeroes.**

1. Suppose α is one zero of the quadratic p(x). Then (x − α) is a factor of p(x), so p(x) = (x − α) × (a linear polynomial), say p(x) = (x − α)(mx + n).
   *Why: Dividing a degree-2 polynomial by a degree-1 factor leaves a degree-1 quotient (division algorithm, Section 05).*
2. Let β be any other zero, $\beta \neq \alpha$. Then p(β) = 0 gives (β − α)(mβ + n) = 0.
3. Since $\beta \neq \alpha$, the first factor β − α is not zero. So the second factor must be: mβ + n = 0, giving $\beta = -n/m$.
   *Why: The zero-product rule again — one factor MUST be zero, and we have ruled out the first.*
4. −n/m is a single fixed number. So besides α, there is room for only ONE more zero. Total: at most 2 zeroes. Proved. ∎
   *Why: The same argument, repeated, shows a degree-n polynomial has at most n zeroes — the degree rule of Section 04 is a theorem, not a coincidence.*

**ANSWER: A quadratic polynomial has at most two zeroes. ∎**

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# Reasoning 3 — The Parabola's Mirror Line

**REASONING 3**

**Show that the axis of symmetry of the parabola $y = ax^2 + bx + c$ is the line $x = -b/(2a)$. Verify for $y = x^2 - 3x - 4$.**

1. Let the zeroes of the polynomial be α and β (assume two real zeroes). The graph meets the x-axis at (α, 0) and (β, 0).

2. A parabola is symmetric: its axis is the vertical line passing midway between any two points at the same height — in particular, midway between the two x-axis meetings. So the axis is $x = (\alpha + \beta)/2$.
*Why: Symmetry means the two zeroes stand at equal distances from the mirror line.*

3. Substitute the bridge: $\alpha + \beta = -b/a$, so the axis is $x = (-b/a)/2 = -b/(2a)$. Proved. ∎

4. Verify on $x^2 - 3x - 4$: zeroes are $-1$ and $4$, midpoint $= (-1 + 4)/2 = 3/2$. And $-b/(2a) = 3/2$. Match.
*Why: This is exactly the symmetry line you found by drawing in Activity 1 — the picture and the algebra agree.*

5. Bonus: the vertex sits on this line, so its x-coordinate is $-b/(2a)$. For our parabola the vertex is $(3/2, -25/4)$.

**ANSWER: The axis of symmetry is $x = -b/(2a)$; for $x^2 - 3x - 4$ it is $x = 3/2$. ∎**

***TOPPER TIP:*** *Reasoning questions reward structure: state what you use, substitute, conclude. Write 'Proved' (or the symbol ∎) at the end — it tells the examiner the argument is complete.*

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# 13 Real-Life Applications

8D • APPLY — THE CHAPTER AT WORK IN THE WORLD

**APPLICATION 1**

**A cricketer hits a ball. Its height after t seconds is $h(t) = -5t^2 + 20t$ metres. (a) After how long does the ball land? (b) How high is it after 1 second?**

**THINK FIRST:** One polynomial, two question types: 'when is it zero' (solve) and 'what is it at time t' (substitute). Read the question to know which.

1. (a) Landing means height = 0: solve $-5t^2 + 20t = 0$.
   *Why: The story signal 'lands' = zeroes, as in Section 03.*
2. Factor out $-5t$: $-5t(t - 4) = 0$.
3. Zero-product rule: $t = 0$ or $t = 4$. $t = 0$ is the moment of the hit, so the ball lands at $t = 4$ seconds.
   *Why: Both zeroes are real events — launch and landing. The story chooses which one answers the question.*
4. (b) Substitute $t = 1$: $h(1) = -5 + 20 = 15$ metres.

**ANSWER: (a) The ball lands after 4 seconds. (b) After 1 second it is 15 m high.**

**APPLICATION 2**

**A rectangular flower bed has area $A(x) = x^2 + 8x + 15$ m$^2$. (a) Write the area as a product of the two side-lengths. (b) If $x = 4$, find the sides and verify the area.**

1. (a) Split hunt: two numbers with sum 8 and product 15 — they are 3 and 5.
2. $x^2 + 8x + 15 = x^2 + 3x + 5x + 15 = x(x + 3) + 5(x + 3) = (x + 3)(x + 5)$. So the sides are $(x + 3)$ m and $(x + 5)$ m.
3. (b) Put $x = 4$: sides are 7 m and 9 m. Area $= 7 \times 9 = 63$ m$^2$.
4. Verify from the formula: $A(4) = 16 + 32 + 15 = 63$ m$^2$. Same.

**ANSWER: (a) Sides: $(x + 3)$ m and $(x + 5)$ m. (b) For $x = 4$: 7 m by 9 m, area 63 m$^2$.**

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# Application 3 — The Arch Again

**APPLICATION 3**

**Remember Anitha's arch from Section 03, with height $h(x) = -x^2 + 4x$. A delivery lorry 3.75 m tall must pass through the centre of the arch. Can it pass, and with how much clearance?**

**THINK FIRST:** 'Centre of the arch' is a midpoint-of-zeroes question; 'can it pass' is a comparison. Two small ideas, one real decision.

1. The arch's feet (zeroes) are at x = 0 and x = 4, so its centre is at x = (0 + 4)/2 = 2.
   *Why: The centre of a symmetric arch is the midpoint of its zeroes — Reasoning 3 in disguise.*
2. Height at the centre: h(2) = −4 + 8 = 4 metres.
3. Compare: arch height 4 m > lorry height 3.75 m. Clearance = 4 − 3.75 = 0.25 m.

**ANSWER: Yes, the lorry passes, with 0.25 m (25 cm) of clearance at the centre.**

## The story signals of this chapter — collected

**Lands / touches down / returns to the ground** → zeroes of the height polynomial.
**Area as one expression, sides wanted** → factorise.
**Centre / topmost point** → midpoint of the zeroes, then substitute.
**Break-even / no profit no loss** → zeroes of the profit polynomial (next page).
Every application in this chapter — and nearly every one in the board exam — is one of these four signals in a costume.

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# Application 4 — A Business Breaks Even

**APPLICATION 4**

**A small business models its daily profit (in rupees) on selling x items as $P(x) = -x^2 + 40x - 300$. Find the break-even points — the sales levels at which profit is exactly zero.**

**THINK FIRST:** Break-even = zeroes of the profit polynomial. Whenever a story says 'no profit, no loss', set the polynomial to zero.

1. Break-even means $P(x) = 0$: solve $-x^2 + 40x - 300 = 0$. Multiply by $-1$: $x^2 - 40x + 300 = 0$.
   *Why: Multiplying an equation by −1 keeps its zeroes — and makes the splitting hunt friendlier.*
2. Split hunt: two numbers with sum $-40$ and product $300$ — they are $-30$ and $-10$.
3. $x^2 - 30x - 10x + 300 = x(x - 30) - 10(x - 30) = (x - 30)(x - 10) = 0$.
4. Zero-product rule: $x = 30$ or $x = 10$.
5. Interpret: below 10 items the business loses money (setting-up costs dominate); between 10 and 30 it profits; beyond 30, discounting and overtime pull profit back to zero at 30.
   *Why: A downward parabola is positive BETWEEN its zeroes — the graph from Section 04 is doing business analysis.*

**ANSWER: Break-even at 10 items and at 30 items. Profit is positive for sales strictly between 10 and 30 items.**

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# 14 Competency-Based Questions

FIVE CASE STUDIES • ASSERTION–REASON • BOARD EXAM STYLE, FULLY SOLVED

## Case Study 1 — The Bridge Arch

A river bridge has a parabolic arch. Taking the left bank at x = 0, the height of the arch above water at distance x metres is $p(x) = -x^2 + 8x - 12$ metres. Answer the four questions.

**(i)** Find the zeroes of p(x).

1. Set $p(x) = 0$ and multiply by $-1$: $x^2 - 8x + 12 = 0$.
2. Split: $-2$ and $-6$ (sum $-8$, product 12): $(x - 2)(x - 6) = 0$.
3. Zeroes: $x = 2$ and $x = 6$.

**ANSWER: 2 and 6.**

**(ii)** What is the width of the arch at water level?

1. The arch meets the water at its zeroes, $x = 2$ and $x = 6$.
   *Why: Water level is height 0 — the zeroes ARE the meeting points.*
2. Width $= 6 - 2 = 4$ metres.

**ANSWER: 4 metres.**

# Case Study 1 (continued) and Case Study 2

**(iii)** Verify that the sum and product of the zeroes match the coefficients of p(x).

1. Sum of zeroes = 2 + 6 = 8. From coefficients (a = −1, b = 8): −b/a = −8/−1 = 8. Match.
   *Why: Careful: a is −1 here, not 1 — read coefficients from p(x) as written.*
2. Product of zeroes = 2 × 6 = 12. And c/a = −12/−1 = 12. Match.

**ANSWER: Sum = 8 = −b/a; product = 12 = c/a. Verified.**

**(iv)** Find the greatest height of the arch above the water.

1. The vertex stands midway between the zeroes: x = (2 + 6)/2 = 4.
   *Why: Reasoning 3: the axis of symmetry passes through the midpoint.*
2. p(4) = −16 + 32 − 12 = 4 metres.

**ANSWER: 4 metres, at the centre x = 4.**

## Case Study 2 — The Juggler's Ball

A juggler throws a ball straight up. Its height after t seconds is $h(t) = -5t^2 + 25t$ metres.

**(i)** Find the zeroes of h(t).

1. Set h(t) = 0: $-5t^2 + 25t = 0$. Factor out −5t: $-5t(t - 5) = 0$.
2. Zeroes: t = 0 and t = 5.

**ANSWER: 0 and 5.**

**(ii)** For how long is the ball in the air?

1. It leaves the hand at t = 0 and returns to hand height at the next zero, t = 5.

**ANSWER: 5 seconds.**

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# Case Studies 2 (continued) and 3

**(iii)** How high is the ball after 2 seconds?

1. Substitute t = 2: h(2) = −5(4) + 25(2) = −20 + 50 = 30 metres.

**ANSWER: 30 metres.**

**(iv)** Use the coefficients of h(t) to predict the sum of its zeroes, and check against part (i).

1. a = −5, b = 25, so −b/a = −25/−5 = 5.
2. From part (i): 0 + 5 = 5. Match.

**ANSWER: Predicted sum 5; actual sum 5. Verified.**

## Case Study 3 — The School Garden Plot

A rectangular garden plot has area $A(x) = x^2 + 9x + 20$ m$^2$, where x is a design number chosen by the gardener.

**(i)** Express A(x) as a product of the two side-lengths.

1. Split hunt: sum 9, product 20 — the pair is 4 and 5.
2. $x^2 + 9x + 20 = (x + 4)(x + 5)$. Sides: (x + 4) m and (x + 5) m.

**ANSWER: (x + 4) m and (x + 5) m.**

**(ii)** If the gardener chooses x = 3, find the two sides.

1. Substitute: 3 + 4 = 7 m and 3 + 5 = 8 m.

**ANSWER: 7 m and 8 m.**

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# Case Studies 3 (continued) and 4

**(iii)** For x = 3, verify the area in two ways.

1. From the sides: $7 \times 8 = 56$ m$^2$.
2. From the formula: $A(3) = 9 + 27 + 20 = 56$ m$^2$. Same.

**ANSWER: 56 m$^2$ by both routes.**

**(iv)** The zeroes of A(x) are −4 and −5. Why can the gardener not choose x = −4, even though it is a zero?

1. At x = −4, one side becomes x + 4 = 0 and the other is −1 — a plot with a zero or negative side does not exist.
   *Why: Zeroes belong to algebra; the STORY decides which numbers are allowed. Lengths must be positive.*
2. So x must be positive in this context (in fact x > 0).

**ANSWER: Because side-lengths must be positive; at x = −4 the sides would be 0 m and −1 m, which is impossible for a real plot.**

## Case Study 4 — The Snack Stall

A school-fair snack stall models its profit (in rupees) on selling x items as $P(x) = -2x^2 + 32x - 96$.

**(i)** Find the break-even sales levels (profit exactly zero).

1. Set P(x) = 0 and divide by −2: $x^2 - 16x + 48 = 0$.
2. Split: −4 and −12 (sum −16, product 48): $(x - 4)(x - 12) = 0$.
3. Break-even at x = 4 and x = 12 items.

**ANSWER: 4 items and 12 items.**

**(ii)** What is the profit on selling 8 items?

1. $P(8) = -2(64) + 32(8) - 96 = -128 + 256 - 96 = 32$.

**ANSWER: Rs. 32 profit.**

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# Case Studies 4 (continued) and 5

**(iii)** Predict the sum of the zeroes from the coefficients of P(x), and check.

1. $-b/a = -32/-2 = 16$.
2. Actual: 4 + 12 = 16. Match.

**ANSWER: 16, matching 4 + 12.**

**(iv)** Between which sales levels does the stall make a profit?

1. P(x) is a downward parabola, positive between its zeroes 4 and 12.
   *Why: Section 04's picture, doing business: between the zeroes the curve is above the axis.*
2. So profit is positive for $4 < x < 12$ items.

**ANSWER: Strictly between 4 and 12 items.**

## Case Study 5 — The Fountain Jet

Water leaves a fountain nozzle at ground level. At horizontal distance x metres, the jet's height is $y = -x^2 + 6x$ metres.

**(i)** How far from the nozzle does the water land?

1. Landing means $y = 0$: $-x^2 + 6x = 0$, i.e. $x(6 - x) = 0$ (after taking out $-x$, signs arranged).
2. Zeroes: $x = 0$ (the nozzle) and $x = 6$. The water lands 6 m away.

**ANSWER: 6 metres.**

# Case Study 5 (continued)

**(ii)** Find the product of the zeroes from the coefficients, and check it directly.

**1** c/a = 0/−1 = 0. Directly: 0 × 6 = 0. Match.

*Why: c = 0 always forces a zero at x = 0 — here, the nozzle itself.*

**ANSWER: 0, matching 0 × 6.**

**(iii)** What is the greatest height of the jet, and where?

**1** Midpoint of zeroes: x = (0 + 6)/2 = 3. Height there: y = −9 + 18 = 9 metres.

**ANSWER: 9 metres, at x = 3 m from the nozzle.**

**TOPPER TIP:** *Five case studies, one skeleton: read the story → name the polynomial → zeroes or substitute → answer in the story's words. CBSE's competency questions reward exactly this skeleton — the context changes, the skeleton does not.*

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# Assertion–Reason Questions (1 of 2)

**Options:** (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is NOT the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.

**AR1. Assertion (A):** The graph of $y = x^2 - 4$ crosses the x-axis at two points.
**Reason (R):** The number of zeroes of a polynomial equals the number of points where its graph meets the x-axis.

**AR2. Assertion (A):** The sum of the zeroes of $3x^2 - 6x + 2$ is 2.
**Reason (R):** The sum of the zeroes of $ax^2 + bx + c$ is $c/a$.

**AR3. Assertion (A):** $x = 2$ is a zero of $x^2 - 5x + 6$.
**Reason (R):** A number a is a zero of p(x) if and only if p(a) = 0, and here p(2) = 0.

**AR4. Assertion (A):** A quadratic polynomial can have three distinct zeroes.
**Reason (R):** A polynomial of degree n has at most n zeroes.

**AR5. Assertion (A):** The product of the zeroes of $2x^2 - 3x + 7$ is 7/2.
**Reason (R):** The sum of the zeroes of $ax^2 + bx + c$ is $-b/a$.

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# Assertion–Reason Questions (2 of 2)

**Options:** (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is NOT the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.

**AR6. Assertion (A):** If $\alpha + \beta = 5$ and $\alpha\beta = 6$, then a quadratic polynomial with zeroes $\alpha$, $\beta$ is $x^2 - 5x + 6$.

**Reason (R):** A quadratic polynomial with sum of zeroes S and product P is $k[x^2 - Sx + P]$.

**AR7. Assertion (A):** The zeroes of $4x^2 - 4x + 1$ are 1/2 and 1/2.

**Reason (R):** $4x^2 - 4x + 1 = (2x - 1)^2$.

**AR8. Assertion (A):** $p(x) = x^2 + 4$ has two real zeroes.

**Reason (R):** The equation $x^2 = -4$ has no real solution.

**AR9. Assertion (A):** The sum of the zeroes of $x^2 - 2x - 8$ is 2.

**Reason (R):** The sum of the zeroes of a quadratic polynomial equals b/a.

**AR10. Assertion (A):** The zeroes of $x^3 - 6x^2 + 11x - 6$ are 1, 2 and 3.

**Reason (R):** The sum of the zeroes of this cubic is $-b/a = 6$, and $1 + 2 + 3 = 6$.

> ***TOPPER TIP:*** *Answers with one-line justifications are in the Answer Key. Remember: option (a) needs R to EXPLAIN A — 'both true' alone is only option (b). AR10 is the classic trap: a matching sum is necessary, but it does not by itself prove those are the zeroes.*

# 15 Olympiad & Foundation Challenges

8D • CHALLENGE — TEN PROBLEMS THAT STRETCH THE BRIDGE

**CHALLENGE 1**

**If α, β are the zeroes of $x^2 + px + q$, express $\alpha^3 + \beta^3$ in terms of p and q.**

**THINK FIRST:** Olympiad rule of this chapter: never find α, β themselves. Work with their sum and product as two single numbers.

1. Read the bridge: $\alpha + \beta = -p$ and $\alpha\beta = q$.
2. Use the identity $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$.
   *Why: Cubes of a sum split into a cube of the sum minus a correction — the correction is where αβ enters.*
3. Substitute: $(-p)^3 - 3(q)(-p) = -p^3 + 3pq$.

**ANSWER: $\alpha^3 + \beta^3 = 3pq - p^3$.**

**CHALLENGE 2**

**The zeroes of $x^2 - kx + 6$ are in the ratio 2 : 3. Find k.**

1. Let the zeroes be 2m and 3m for some number m.
   *Why: Ratio problems: give the parts a common unit m.*
2. Product: $(2m)(3m) = 6m^2 = c/a = 6$, so $m^2 = 1$ and $m = \pm 1$.
3. Sum: $2m + 3m = 5m = -b/a = k$. So $k = 5$ (when $m = 1$) or $k = -5$ (when $m = -1$).
   *Why: Both signs are genuine: zeroes (2, 3) give k = 5; zeroes (−2, −3) give k = −5.*

**ANSWER: k = 5 or k = −5.**

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# Olympiad Challenges 3 and 4

**CHALLENGE 3**

**Show that 1, 2, 3 are the zeroes of $x^3 - 6x^2 + 11x - 6$, and verify all three cubic relations.**

1. Test $x = 1$: $p(1) = 1 - 6 + 11 - 6 = 0$. So $(x - 1)$ is a factor.
   *Why: Always test $x = 1$ and $x = -1$ first in a cubic — they are the cheapest tests in algebra.*
2. Divide $p(x)$ by $(x - 1)$ (division algorithm): quotient $x^2 - 5x + 6$, remainder 0.
3. Factorise the quotient: $x^2 - 5x + 6 = (x - 2)(x - 3)$. So $p(x) = (x - 1)(x - 2)(x - 3)$ and the zeroes are 1, 2, 3.
4. Verify: sum $1 + 2 + 3 = 6 = -b/a = 6$. Pairs: $2 + 6 + 3 = 11 = c/a$. Product: $6 = -d/a = 6$. All three relations hold.

**ANSWER: Zeroes 1, 2, 3; sum = 6 = −b/a, pairwise sum = 11 = c/a, product = 6 = −d/a. All verified.**

**CHALLENGE 4**

**If α, β are the zeroes of $x^2 - 5x + 6$, find $\alpha^4 + \beta^4$.**

1. Bridge: $\alpha + \beta = 5$, $\alpha\beta = 6$.
2. First squares: $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 25 - 12 = 13$.
3. Now treat $\alpha^2$, $\beta^2$ as a new pair with sum 13 and product $(\alpha\beta)^2 = 36$: $\alpha^4 + \beta^4 = 13^2 - 2(36) = 169 - 72 = 97$.
   *Why: The same identity, climbed twice — like stairs.*
4. Sanity check with the actual zeroes 2 and 3: $16 + 81 = 97$. Match.

**ANSWER: $\alpha^4 + \beta^4 = 97$.**

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# Olympiad Challenges 5 and 6

**CHALLENGE 5**

**The zeroes of $x^2 - 7x + k$ differ by 3. Find k.**

1. Bridge: $\alpha + \beta = 7$ and $\alpha\beta = k$.
2. Use $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$: we get $3^2 = 49 - 4k$.
   *Why: 'Differ by' questions always enter through this identity.*
3. So $4k = 49 - 9 = 40$, and $k = 10$.
4. Check: $x^2 - 7x + 10$ has zeroes 2 and 5, which indeed differ by 3.

**ANSWER: k = 10.**

**CHALLENGE 6**

**If α, β are the zeroes of $x^2 - 8x + k$ and $\alpha + \beta = \alpha\beta$, find k.**

1. Bridge: $\alpha + \beta = 8$ and $\alpha\beta = k$.
   *Why: Read both values straight off the coefficients.*
2. The condition says $8 = k$.
   *Why: Sometimes the 'hard' question is one line long — if your bridge is strong.*

**ANSWER: k = 8.**

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# Olympiad Challenges 7 and 8

**CHALLENGE 7**

**Form a quadratic polynomial whose zeroes are the SQUARES of the zeroes of $x^2 - 5x + 6$.**

1. For the old polynomial: $\alpha + \beta = 5$, $\alpha\beta = 6$.
2. The new zeroes are $\alpha^2$ and $\beta^2$. Their sum: $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 25 - 12 = 13$.
3. Their product: $\alpha^2\beta^2 = (\alpha\beta)^2 = 36$.
4. Form it: $x^2 - 13x + 36$.

   *Why: Check with the actual zeroes 2, 3: squares are 4, 9; $(x - 4)(x - 9) = x^2 - 13x + 36$. Match.*

**ANSWER: $x^2 - 13x + 36$.**

**CHALLENGE 8**

**One zero of $x^3 - 3x^2 - 10x + 24$ is 2. Find all its zeroes.**

1. Since 2 is a zero, $(x - 2)$ is a factor. Divide $p(x)$ by $(x - 2)$: quotient $x^2 - x - 12$, remainder 0.

   *Why: The division algorithm turns one known zero into a quadratic problem.*
2. Factorise: $x^2 - x - 12 = (x - 4)(x + 3)$, giving zeroes 4 and $-3$.
3. All zeroes: 2, 4, $-3$. Quick check of the sum: $2 + 4 - 3 = 3 = -b/a = 3$. Consistent.

**ANSWER: The zeroes are 2, 4 and $-3$.**

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# Olympiad Challenges 9 and 10

**CHALLENGE 9**

**If α, β are the zeroes of $2x^2 - 7x + 3$, find α/β + β/α.**

1. Combine: $\alpha/\beta + \beta/\alpha = (\alpha^2 + \beta^2) / (\alpha\beta)$.
2. Bridge: $\alpha + \beta = 7/2$, $\alpha\beta = 3/2$. So $\alpha^2 + \beta^2 = (7/2)^2 - 2(3/2) = 49/4 - 3 = 37/4$.
3. Divide: $(37/4) / (3/2) = (37/4) \times (2/3) = 37/6$.

**ANSWER: α/β + β/α = 37/6.**

**CHALLENGE 10**

**Find all values of k for which $x^2 + kx + k$ has equal zeroes.**

1. Equal zeroes means α = β. Then $(\alpha - \beta)^2 = 0$, i.e. $(\alpha + \beta)^2 = 4\alpha\beta$.
   *Why: Equal-zeroes questions always pass through this gate.*
2. Bridge: $\alpha + \beta = -k$, $\alpha\beta = k$. Substitute: $(-k)^2 = 4k$, so $k^2 - 4k = 0$, i.e. $k(k - 4) = 0$.
3. So k = 0 or k = 4. Check k = 4: $x^2 + 4x + 4 = (x + 2)^2$ — equal zeroes −2, −2. Check k = 0: the polynomial becomes $x^2$, whose zeroes are 0 and 0 — also equal, and still a genuine quadratic (coefficient of $x^2$ is 1 ≠ 0).
   *Why: Do not throw away k = 0 carelessly — examine what it does to the polynomial before rejecting it.*

**ANSWER: k = 0 or k = 4.**

***TOPPER TIP:*** *Ten challenges, and not one of them needed the actual zeroes found first. That is the Olympiad lesson of this chapter: sum and product are handles strong enough to lift the whole polynomial.*

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# Practice Plus — Find the Error (1 of 3)

Each item shows a student's written solution (in handwriting) containing **exactly one error**. Your job: spot it, name it, correct it. This is how examiners think — learn to mark like one.

**E1.** Find the sum of the zeroes of $2x^2 - 5x + 3$.

*Sum of zeroes = b/a = −5/2.*

**Find the error** — then write the corrected solution. (Corrections: Answer Key.)

**E2.** Find the zeroes of $x^2 - 2x - 8$.

*$x^2 - 2x - 8 = (x - 4)(x + 2)$. So $x - 4 = 0$ or $x + 2 = 0$, giving $x = 4$ or $x = 2$.*

**Find the error** — then write the corrected solution. (Corrections: Answer Key.)

**E3.** Find a quadratic polynomial with sum of zeroes 1/4 and product of zeroes −1.

*$p(x) = x^2 + (1/4)x - 1$.*

**Find the error** — then write the corrected solution. (Corrections: Answer Key.)

**E4.** Find the product of the zeroes of $3x^2 + 6x - 9$.

*Product of zeroes = −c/a = −(−9)/3 = 3.*

**Find the error** — then write the corrected solution. (Corrections: Answer Key.)

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# Practice Plus — Find the Error (2 of 3)

**E5.** Find the zeroes of $t^2 - 15$.

*$t^2 = 15$, so $t = \sqrt{15}$. The zero is $\sqrt{15}$.*

**Find the error** — then write the corrected solution. (Corrections: Answer Key.)

**E6.** A graph meets the y-axis at (0, 3) and meets the x-axis at x = 1 and x = 4. How many zeroes does p(x) have?

*The graph meets the axes at 3 points in total, so p(x) has 3 zeroes.*

**Find the error** — then write the corrected solution. (Corrections: Answer Key.)

**E7.** Verify the sum of zeroes for $6x^2 - 3 - 7x$, whose zeroes are 3/2 and −1/3.

*Here $b = 7$, so $-b/a = -7/6$. But $3/2 - 1/3 = 7/6$. The relationship fails!*

**Find the error** — then write the corrected solution. (Corrections: Answer Key.)

**E8.** Divide $x^2 + 5x + 6$ by $x + 2$.

*Quotient = $x + 3$, remainder = 6.*

**Find the error** — then write the corrected solution. (Corrections: Answer Key.)

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# Practice Plus — Find the Error (3 of 3)

**E9.** Is $-2$ a zero of $p(x) = x^2 - 3x + 2$?

*$p(-2) = (-2)^2 - 3(-2) + 2 = 4 - 6 + 2 = 0$. Yes, $-2$ is a zero.*

**Find the error** — then write the corrected solution. (Corrections: Answer Key.)

**E10.** State the sum of the zeroes of $x^3 - 6x^2 + 11x - 6$.

*Sum of zeroes $= b/a = -6$.*

**Find the error** — then write the corrected solution. (Corrections: Answer Key.)

**TOPPER TIP:** *If you found all ten quickly, your error-detector is exam-grade. These ten errors are the ten most common real errors in this chapter — examiners meet them thousands of times.*

# Board Exam Hall — Previous Year Questions (1 of 6)

Every question below is a **real CBSE board examination question** on Polynomials. Where the examination paper and year are verified, we print them; where they are not, the question is labelled simply *CBSE Board Exam Question*. We never invent a year. Solve each before reading the steps — this is the real thing.

**CBSE 2024 • STANDARD • MCQ**

**PYQ 1**

**If α and β (α > β) are the zeroes of the polynomial $-x^2 + 8x + 9$, then (α − β) is equal to: (A) −10 (B) 10 (C) ±10 (D) 8**

**THINK FIRST:** The condition α > β removes the ± confusion — but only if you find both zeroes with their signs.

1. Factorise: $-x^2 + 8x + 9 = -(x^2 - 8x - 9) = -(x - 9)(x + 1)$.
   *Why: Take −1 out first; then split −8x with −9 and +1.*
2. Zeroes: x = 9 and x = −1. Since α > β: α = 9, β = −1.
3. α − β = 9 − (−1) = 10.
   *Why: Subtracting a negative adds — the last trap of the question.*

**ANSWER: Option (B): α − β = 10.**

**CBSE-STYLE MARKING SCHEME:** 1 mark — correct option. (Zeroes 1/2 + difference 1/2 in a worked version.)

# Previous Year Questions (2 of 6)

**CBSE 2024 • BASIC**

**PYQ 2**

**If α, β are the zeroes of the quadratic polynomial $2x^2 + 7x + 5$, find the value of $\alpha^2 + \beta^2 + \alpha\beta$.**

**THINK FIRST:** Do NOT find α, β. Build $\alpha^2 + \beta^2$ from the bridge, then add αβ.

1. Bridge: $\alpha + \beta = -7/2$ and $\alpha\beta = 5/2$.
2. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 49/4 - 5 = 29/4$.
3. Add αβ: $29/4 + 5/2 = 29/4 + 10/4 = 39/4$.

**ANSWER: $\alpha^2 + \beta^2 + \alpha\beta = 39/4$.**

**CBSE-STYLE MARKING SCHEME:** 1 mark for sum and product from coefficients • 1 mark for the identity • 1 mark for the final value.

**CBSE 2024 • COMPARTMENT**

**PYQ 3**

**If α and β are the zeroes of the polynomial $p(x) = x^2 - (k + 5)x + (5k + 1)$ such that $\alpha + \beta = \alpha\beta/3$, find the value of k.**

**THINK FIRST:** Translate the condition into an equation in k using the bridge — one equation, one unknown.

1. Bridge: $\alpha + \beta = k + 5$ and $\alpha\beta = 5k + 1$.
   *Why: Read −b/a and c/a carefully: $b = -(k + 5)$, so $-b/a = k + 5$.*
2. The condition: $k + 5 = (5k + 1)/3$.
3. Multiply by 3: $3k + 15 = 5k + 1$. So $15 - 1 = 5k - 3k$, giving $14 = 2k$, hence $k = 7$.

**ANSWER: $k = 7$.**

**CBSE-STYLE MARKING SCHEME:** 1 mark for sum and product • 1 mark for the equation • 1 mark for $k = 7$.

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# Previous Year Questions (3 of 6)

**CBSE 2023 • STANDARD**

**PYQ 4**

**If one zero of the polynomial $p(x) = 6x^2 + 37x - (k - 2)$ is reciprocal of the other, find the value of k.**

**THINK FIRST:** 'Reciprocal of the other' is a product condition in disguise: if the zeroes are α and 1/α, their product is 1.

1. Let the zeroes be α and 1/α. Their product is α × (1/α) = 1.
2. But product of zeroes = c/a = −(k − 2)/6. So −(k − 2)/6 = 1.
   *Why: Watch the constant term: it is −(k − 2), brackets and all.*
3. Solve: −(k − 2) = 6, so k − 2 = −6, hence k = −4.

**ANSWER: k = −4.**

**CBSE-STYLE MARKING SCHEME:** 1 mark for product = 1 • 1 mark for the equation • 1 mark for k = −4.

**CBSE 2023 • BASIC**

**PYQ 5**

**Find the value of k such that the polynomial $p(x) = 3x^2 + 2kx + x - k - 5$ has the sum of its zeroes equal to half of their product.**

**THINK FIRST:** First tidy the polynomial: group the x-terms, group the constants.

1. Rewrite: $p(x) = 3x^2 + (2k + 1)x - (k + 5)$. So a = 3, b = 2k + 1, c = −(k + 5).
   *Why: Grouping like terms is step zero here.*
2. Sum of zeroes = −(2k + 1)/3. Product of zeroes = −(k + 5)/3.
3. Condition: −(2k + 1)/3 = (1/2)(−(k + 5)/3). Multiply by −6: 2(2k + 1) = k + 5, so 4k + 2 = k + 5, giving 3k = 3 and k = 1.

**ANSWER: k = 1.**

**CBSE-STYLE MARKING SCHEME:** 1 mark for tidying and reading a, b, c • 1 mark for the equation • 1 mark for k = 1.

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# Previous Year Questions (4 of 6)

**CBSE 2023 • BASIC • MCQ**

**PYQ 6**

**If one zero of the quadratic polynomial $kx^2 + 3x + k$ is 2, then the value of k is: (A) −6/5 (B) 6/5 (C) 5/6 (D) −5/6**

**THINK FIRST:** '2 is a zero' means p(2) = 0. Substitute — one line.

1. Substitute x = 2: k(4) + 3(2) + k = 0, so 5k + 6 = 0.
2. Hence k = −6/5.

**ANSWER: Option (A): k = −6/5.**

**CBSE-STYLE MARKING SCHEME:** 1 mark — correct option.

**CBSE 2023 • COMPARTMENT**

**PYQ 7**

**Find the zeroes of the polynomial $4x^2 + 17x − 15$ and verify the relationship between the zeroes and the coefficients.**

**THINK FIRST:** The Section 07 rhythm, in a real paper. a × c = −60.

1. Split pair: product −60, sum 17 — they are +20 and −3.
2. $4x^2 + 20x − 3x − 15 = 4x(x + 5) − 3(x + 5) = (4x − 3)(x + 5)$.
3. Zeroes: x = 3/4 and x = −5.
4. Verify sum: 3/4 − 5 = −17/4, and −b/a = −17/4. Verify product: (3/4)(−5) = −15/4, and c/a = −15/4. Both match.

**ANSWER: Zeroes: 3/4 and −5; sum = −17/4 = −b/a, product = −15/4 = c/a. Verified.**

**CBSE-STYLE MARKING SCHEME:** 1 mark for zeroes • 1 mark for sum verification • 1 mark for product verification.

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# Previous Year Questions (5 of 6)

**CBSE 2023 • COMPARTMENT**

**PYQ 8**

**Find the zeroes of the polynomial $x^2 - 11x + 18$ and verify the relationship between the zeroes and the coefficients.**

**THINK FIRST:** Split pair: product 18, sum −11.

1. $x^2 - 11x + 18 = (x - 2)(x - 9)$, so the zeroes are 2 and 9.
2. Verify sum: $2 + 9 = 11 = -(-11)/1$. Verify product: $2 \times 9 = 18 = 18/1$. Both match.

**ANSWER: Zeroes: 2 and 9; sum = 11 = −b/a, product = 18 = c/a. Verified.**

**CBSE-STYLE MARKING SCHEME:** 1 mark for zeroes • 1 mark for sum verification • 1 mark for product verification.

**CBSE BOARD EXAM QUESTION**

**PYQ 9**

**If α and β are the zeroes of the polynomial $x^2 - 3x + k$ such that $\alpha - \beta = 1$, find the value of k.**

**THINK FIRST:** Use the identity $(\alpha + \beta)^2 - (\alpha - \beta)^2 = 4\alpha\beta$ — it connects all three quantities at once.

1. Bridge: $\alpha + \beta = 3$ and $\alpha\beta = k$.
2. Substitute into the identity: $3^2 - 1^2 = 4k$, so $8 = 4k$.
3. Hence $k = 2$. (Check: $x^2 - 3x + 2$ has zeroes 2 and 1, differing by 1.)

**ANSWER: k = 2.**

**CBSE-STYLE MARKING SCHEME:** 1 mark for sum and product • 1 mark for the identity • 1 mark for k = 2.

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# Previous Year Questions (6 of 6)

**CBSE BOARD EXAM QUESTION**

**PYQ 10**

**If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - p(x + 1) - c$, show that $(\alpha + 1)(\beta + 1) = 1 - c$.**

**THINK FIRST:** First expand f(x) into standard form — the coefficients are hiding inside the brackets. (Here p and c are constants, not the polynomial.)

1. Expand: $f(x) = x^2 - px - p - c$. So the coefficient of x is $-p$ and the constant term is $-(p + c)$.
2. Bridge: $\alpha + \beta = p$ and $\alpha\beta = -(p + c)$.
3. Expand the target: $(\alpha + 1)(\beta + 1) = \alpha\beta + \alpha + \beta + 1$.
4. Substitute: $-(p + c) + p + 1 = -p - c + p + 1 = 1 - c$. Shown. ∎
   *Why: The p's cancel — the examiner designed it so.*

**ANSWER: $(\alpha + 1)(\beta + 1) = 1 - c$. ∎**

**CBSE-STYLE MARKING SCHEME:** 1 mark for standard form • 1 mark for sum and product • 1 mark for the expansion and conclusion.

# 16 Chapter Mind Map

THE WHOLE CHAPTER ON ONE PAGE — REVISE FROM HERE

## POLYNOMIALS

Heart of the chapter: **the zeroes–coefficients bridge** — the zeroes of a polynomial and its coefficients determine each other.

### Zeroes — the Picture

Zeroes = x-coordinates where y = p(x) meets the x-axis. Linear: 1. Quadratic: 0, 1 (touch) or 2. Cubic: up to 3. Degree n → at most n zeroes.

### Finding Zeroes

Split the middle term (pair: sum b, product ac) → factorise → zero-product rule. b = 0? Use the difference of squares — zeroes come in ± pairs.

### The Quadratic Bridge

$\alpha + \beta = -b/a$; $\alpha\beta = c/a$. Derived by expanding $a(x - \alpha)(x - \beta)$ and comparing coefficients.

### The Cubic Bridge

$\alpha + \beta + \gamma = -b/a$; $\alpha\beta + \beta\gamma + \gamma\alpha = c/a$; $\alpha\beta\gamma = -d/a$. Signs: −, +, −.

### Forming a Polynomial

$k[x^2 - Sx + P]$. Minus the sum, plus the product. Choose k to clear fractions.

### Division Algorithm

$p(x) = g(x)q(x) + r(x)$; stop when deg r < deg g. Remainder 0 ⇒ g(x) is a factor.

### Bridge Gymnastics (PYQ & Olympiad)

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$; $1/\alpha + 1/\beta = -b/c$; $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$; reciprocal zeroes ⇒ $\alpha\beta = 1$; equal zeroes ⇒ $(\alpha + \beta)^2 = 4\alpha\beta$.

### Story Signals

Lands / touches / break-even → solve p(x) = 0. Area as one expression → factorise for the sides. Centre / highest point → midpoint of zeroes.

### Key Results to Quote

$x^2 - 2x - 8$: zeroes 4, −2 • $4s^2 - 4s + 1$: equal zeroes 1/2, 1/2 • $t^2 - 15$: $\pm\sqrt{15}$ • Sum 1/4, product −1 → $4x^2 - x - 4$ • $(x^3 - 3x^2 + 5x - 3) \div (x^2 - 2)$: quotient $x - 3$, remainder $7x - 9$

**TOPPER TIP:** *Revise actively: cover a box, speak its contents aloud from memory, then uncover and check. A mind map you can REPRODUCE from memory is worth ten readings.*

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# Pin-Point Flash Cards (1 of 2) — Cut Them Out

**How to use these cards:** cut along the dashed borders (ask an adult to help with scissors if needed). Keep the deck in your pocket or pencil box. On every Smart Memory day (Day 1, 3, 7, 14, 21, 30), shuffle the deck, read a card's title, and say the pin-point aloud BEFORE flipping your eyes to the fact. Cards you miss twice go into your Error Tracker. Sixteen cards — the sixteen most exam-important facts of this chapter.

### Card 1 · Zero of a Polynomial

**a is a zero of p(x) ⇔ p(a) = 0 ⇔ the graph meets the x-axis at x = a ⇔ (x − a) is a factor of p(x).**

### Card 2 · The Degree Rule

**A polynomial of degree n has AT MOST n zeroes. Quadratic: at most 2. Cubic: at most 3. More zeroes than the degree = a wrong answer, always.**

### Card 3 · Quadratic — Sum of Zeroes

**$\alpha + \beta = -b/a = -$(coefficient of x) / (coefficient of $x^2$). The MINUS belongs to the sum.**

### Card 4 · Quadratic — Product of Zeroes

**$\alpha\beta = c/a$ = (constant term) / (coefficient of $x^2$). NO minus here — do not invent one.**

### Card 5 · Forming a Quadratic

**$p(x) = k[x^2 -$ (sum of zeroes)x + (product of zeroes)], for any $k \neq 0$. Minus the sum, plus the product.**

### Card 6 · Cubic — Sum of Zeroes

**For $ax^3 + bx^2 + cx + d$: $\alpha + \beta + \gamma = -b/a$.**

### Card 7 · Cubic — Pairwise Sum

**$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$. Three pairs, each pair once, all plus.**

### Card 8 · Cubic — Product of Zeroes

**$\alpha\beta\gamma = -d/a$. The product of THREE zeroes carries a MINUS — the most-lost mark in this chapter.**

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# Pin-Point Flash Cards (2 of 2)

### Card 9 · Graph Shapes

Linear → straight line. Quadratic → parabola (U or ∩). Cubic → S-curve. Zeroes = meetings with the X-axis only.

### Card 10 · No Touch, No Zero

If the graph never meets the x-axis — like $y = x^2 + 1$, whose smallest value is 1 — the polynomial has NO real zeroes.

### Card 11 · Equal Zeroes

Zeroes are equal ⇔ the graph TOUCHES the x-axis at one point ⇔ $(\alpha + \beta)^2 = 4\alpha\beta$. Example: $4s^2 - 4s + 1$, zeroes 1/2, 1/2.

### Card 12 · Splitting the Middle Term

For $ax^2 + bx + c$: split b into two parts whose SUM is b and whose PRODUCT is a × c. Check both — then group.

### Card 13 · Difference of Squares

$x^2 - m = (x - \sqrt{m})(x + \sqrt{m})$. When b = 0, zeroes come as a ± pair: $t^2 - 15 \rightarrow \pm\sqrt{15}$. Never report only the plus.

### Card 14 · Division Algorithm

$p(x) = g(x) \times q(x) + r(x)$, where r(x) = 0 or deg r(x) < deg g(x). Remainder 0 ⇒ g(x) is a factor of p(x).

### Card 15 · Reciprocal Zeroes

If one zero is the reciprocal of the other, their product is 1: c/a = 1. (CBSE 2023 asked exactly this.)

### Card 16 · Standard Form First

ALWAYS rewrite in the form $ax^2 + bx + c$ before reading a, b, c. In $6x^2 - 3 - 7x$, b is −7 — not 7.

**TOPPER TIP:** *Mastered all 16? Test a friend. Teaching a card to someone else is the deepest form of revision — if you can explain Card 8's minus sign, you truly own it.*

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# 17 Active Recall

8D • RECALL — CLOSE THE BOOK, THEN ANSWER

**How this works:** close every other page. Answer all ten questions on rough paper, from memory alone. Only then open the Answer Key. Anything you could not recall is exactly what your Day-3 revision must target — write it in the Error Tracker (Section 18).

## Ten rapid questions

**R1.** What is the degree of $7x^3 - 4x + 9$?

**R2.** Find the zeroes of $x^2 - 5x + 6$.

**R3.** Find the sum of the zeroes of $2x^2 - 3x + 7$.

**R4.** Find the product of the zeroes of $5x^2 + 9x - 2$.

**R5.** What is the maximum number of zeroes of a cubic polynomial?

**R6.** For $p(x) = x^2 - 4x + 3$, find $p(0)$.

**R7.** Form a quadratic polynomial with sum of zeroes 3 and product of zeroes $-4$.

**R8.** Find the zeroes of $x^2 - 16$.

**R9.** For the polynomial $2x^2 + bx + c$, the sum of the zeroes is 5. Find b.

**R10.** Find k if $x = 1$ is a zero of $x^2 + kx - 5$.

## Five recall prompts — say each answer aloud, in a full sentence

**P1.** Define a zero of a polynomial.

**P2.** State both zeroes–coefficients relations for a quadratic $ax^2 + bx + c$.

**P3.** State the geometrical meaning of the zeroes of a polynomial.

**P4.** State the division algorithm for polynomials, including the condition on the remainder.

**P5.** Explain how to form a quadratic polynomial when the sum S and product P of its zeroes are given.

> **TOPPER TIP:** *Scoring: 13–15 correct — chapter mastered, move on. 9–12 — revise the missed topics, retake in 3 days. Below 9 — re-study Sections 05–07, then retake. Record your score in the Mastery Tracker.*

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# 18 Self-Assessment & Mastery Tracker

KALVIYOGI SMART MEMORY SYSTEM — LEARN IT, THEN KEEP IT

Memory fades on a predictable curve — unless you revisit at the right moments. This schedule is your revisit plan for THIS chapter. Tick each day off when done. The Day 7, Day 21 and Day 30 mini-tests follow this section.

| Day | Your activity for Chapter 2 | Done |
|---|---|---|
| **Day 0** | Pre-learning assessment: attempt the Quick Check (Section 02) and Rapid Questions R1–R10 (Section 17) BEFORE studying. Note your score. | ☐ |
| **Day 1** | Learn the chapter fully (Sections 01–10) and complete Level 1–3 practice plus the 25 MCQs. | ☐ |
| **Day 3** | Without-book recall: reproduce the Mind Map (Section 16) on a blank page; shuffle the Flash Cards; finish Level 4. | ☐ |
| **Day 7** | Independent problem-solving: Level 5 + Olympiad Challenges, timed, book closed. Then take **Mini-Test 1** (next pages). | ☐ |
| **Day 14** | Mixed practice: re-attempt every question in your Error Tracker; redo the PYQ set (Section: Board Exam Hall) fresh. | ☐ |
| **Day 21** | Long-term recall test: take **Mini-Test 2**; write the quadratic and cubic bridge derivations from memory, then verify line by line. | ☐ |
| **Day 30** | Chapter mastery assessment: take **Mini-Test 3** and one full Model Test under exam conditions; update your Concept Mastery Score below. | ☐ |

## Formula Recall Tracker — tick when you can say it with the book closed

| Formula / Statement | Day 1 | Day 3 | Day 7 | Day 21 |
|---|---|---|---|---|
| Zeroes = x-axis meetings of the graph | ☐ | ☐ | ☐ | ☐ |
| Quadratic: $\alpha + \beta = -b/a$; $\alpha\beta = c/a$ | ☐ | ☐ | ☐ | ☐ |
| Cubic: $-b/a$; $c/a$; $-d/a$ (signs −, +, −) | ☐ | ☐ | ☐ | ☐ |
| Forming: $k[x^2 - Sx + P]$ | ☐ | ☐ | ☐ | ☐ |
| Division algorithm + degree rule for r(x) | ☐ | ☐ | ☐ | ☐ |
| Degree rule: degree n → at most n zeroes | ☐ | ☐ | ☐ | ☐ |

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# Mastery Tracker — Errors and Concept Scores

## Error Tracker — every mistake becomes a lesson

| Date | Question | What went wrong | The correct idea |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

## Concept Mastery Score — rate yourself honestly, out of 5

| Concept | Score / 5 | Status |
|---|---|---|
| Reading zeroes from graphs (Ex 2.1 skill) | ___ / 5 | ☐ Weak ☐ Okay ☐ Mastered |
| Splitting the middle term & factorising | ___ / 5 | ☐ Weak ☐ Okay ☐ Mastered |
| Verifying the bridge (Ex 2.2 Q1 skill) | ___ / 5 | ☐ Weak ☐ Okay ☐ Mastered |
| Forming polynomials from sum & product | ___ / 5 | ☐ Weak ☐ Okay ☐ Mastered |
| Cubic relations (all three, signs correct) | ___ / 5 | ☐ Weak ☐ Okay ☐ Mastered |
| Division algorithm (long division) | ___ / 5 | ☐ Weak ☐ Okay ☐ Mastered |
| Bridge gymnastics ($\alpha^2 + \beta^2$, $1/\alpha + 1/\beta$, $\alpha - \beta$) | ___ / 5 | ☐ Weak ☐ Okay ☐ Mastered |

## Learning Time Tracker

| Day / Date | Minutes studied | What I studied |
|---|---|---|
| | | |
| | | |
| | | |

Mastery rule: a concept counts as MASTERED only when you score 4 or 5 AND can reproduce it on Day 21 without the book. Anything less — revisit, don't just re-read.

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## MINI-TEST 1 • DAY 7 — CORE SKILLS • 10 QUESTIONS • BOOK CLOSED

Answer all ten on rough paper. Marks and answers are in the Answer Key — check only after finishing. Record your score in the Mastery Tracker.

**T1.** Find the zeroes of $x^2 - 9x + 20$.

**T2.** Find the sum of the zeroes of $7x^2 - 3x + 1$.

**T3.** For $p(x) = x^3 - x + 2$, find $p(2)$.

**T4.** Form a quadratic polynomial with sum of zeroes 2 and product of zeroes $-15$.

**T5.** Find the product of the zeroes of $6x^2 - x - 2$.

**T6.** Find the zeroes of $2x^2 + 3x - 2$.

**T7.** Is $-1$ a zero of $x^2 + 3x + 2$? Justify in one line.

**T8.** A parabola touches the x-axis at exactly one point. How many distinct zeroes does its polynomial have?

**T9.** Find k if $x = 1$ is a zero of $x^2 + kx - 4$.

**T10.** Find the zeroes of $x^2 - 2x - 3$.

> ***TOPPER TIP:*** *Target: 8/10 or more. Below that, your Day 14 work is decided for you: Sections 05–07 again, slowly.*

## MINI-TEST 2 • DAY 21 — MIXED RECALL • 10 QUESTIONS • BOOK CLOSED

Answer all ten on rough paper. Marks and answers are in the Answer Key — check only after finishing. Record your score in the Mastery Tracker.

**T1.** For $4x^2 + 8x$, write the sum and the product of the zeroes.

**T2.** Find the zeroes of $5x^2 - 20$.

**T3.** Form a quadratic polynomial whose zeroes are $-5$ and $1$.

**T4.** If $\alpha$, $\beta$ are the zeroes of $x^2 - 7x + 12$, find $1/\alpha + 1/\beta$.

**T5.** Divide $x^2 + 6x + 8$ by $x + 4$. Give the quotient and remainder.

**T6.** If $\alpha$, $\beta$ are the zeroes of $x^2 - x - 6$, find $\alpha^2 + \beta^2$.

**T7.** Find the sum of the zeroes of the cubic $x^3 - 2x^2 + 3x - 4$.

**T8.** The product of the zeroes of $x^2 - kx + 9$ is 9 and their sum is 6. Find k.

**T9.** Find the zeroes of $3x^2 - 12x$.

**T10.** A polynomial's graph does not meet the x-axis at all. How many real zeroes does it have?

> ***TOPPER TIP:*** *Target: 8/10. Any cubic or division slip goes straight into the Error Tracker — those are Day-30 deciders.*

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## MINI-TEST 3 • DAY 30 — MASTERY CHECK • 10 QUESTIONS • BOOK CLOSED

Answer all ten on rough paper. Marks and answers are in the Answer Key — check only after finishing. Record your score in the Mastery Tracker.

**T1.** If α, β are the zeroes of $x^2 - 2x - 3$, find $\alpha^3 + \beta^3$.

**T2.** Find k so that $x^2 - 10x + k$ has equal zeroes.

**T3.** The zeroes of $x^2 - 8x + k$ differ by 2. Find k.

**T4.** If α, β are the zeroes of $x^2 - 4x + 3$, form a quadratic with zeroes 2α and 2β.

**T5.** For the cubic $x^3 + 3x^2 - 5x + 2$, write the value of αβ + βγ + γα.

**T6.** For which k do the zeroes of $4x^2 - 3x + k$ become reciprocals of each other?

**T7.** If α, β are the zeroes of $x^2 - 2x - 8$, find α − β (taking α > β).

**T8.** Divide $x^3 + 1$ by x + 1. Give the quotient and remainder.

**T9.** Form a quadratic polynomial whose zeroes satisfy α + β = αβ = 4.

**T10.** What is the maximum number of zeroes of a polynomial of degree 5?

> ***TOPPER TIP:*** *Target: 9/10 — this is board standard. Score it honestly, update the Mastery Tracker, and close the chapter.*

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# Chapter Model Test A — Basics

**CHAPTER MODEL TEST A • BASICS • BUILD THE FOUNDATIONS • MAX. MARKS: 20 • TIME: 45 MINUTES**

**General instructions:** All questions are compulsory. Section A: 5 MCQs × 1 mark. Section B: 3 questions × 2 marks. Section C: 3 questions × 3 marks. Show all steps — step marks apply, exactly as in the board exam. Answers: Answer Key.

## Section A — Multiple Choice (1 mark each)

**A1.** The degree of $4x^2 - 3x + 7$ is

(A) 1 (B) 2 (C) 3 (D) 7

**A2.** The zero of the polynomial $2x - 4$ is

(A) 4 (B) $-2$ (C) 2 (D) 0

**A3.** The sum of the zeroes of $x^2 - 5x + 6$ is

(A) 6 (B) $-5$ (C) $-6$ (D) 5

**A4.** The product of the zeroes of $3x^2 + 2x - 1$ is

(A) 1/3 (B) $-1/3$ (C) $-2/3$ (D) 3

**A5.** The number of zeroes of a linear polynomial is

(A) 2 (B) 0 (C) 3 (D) 1

## Section B — Short Answers (2 marks each)

**A6.** For $p(x) = x^2 - 4x + 3$, find $p(1)$ and $p(-1)$.

**A7.** Find the zeroes of $x^2 - 8x + 15$.

**A8.** Write the sum and the product of the zeroes of $5x^2 - 9x + 4$.

## Section C — Long Answers (3 marks each)

**A9.** Find the zeroes of $2x^2 - 7x + 3$ and verify the relationship between the zeroes and the coefficients.

**A10.** Find a quadratic polynomial whose sum of zeroes is 7 and product of zeroes is 12.

**A11.** The graph of $y = p(x)$ crosses the x-axis at $x = -1$, $x = 0$ and $x = 3$, and nowhere else. How many zeroes does $p(x)$ have, and what are they?

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# Chapter Model Test B — Standard

**CHAPTER MODEL TEST B • STANDARD • BOARD PATTERN • MAX. MARKS: 20 • TIME: 45 MINUTES**

**General instructions:** All questions are compulsory. Section A: 5 MCQs × 1 mark. Section B: 3 questions × 2 marks. Section C: 3 questions × 3 marks. Show all steps — step marks apply, exactly as in the board exam. Answers: Answer Key.

## Section A — Multiple Choice (1 mark each)

**B1.** The zeroes of $x^2 + 5x + 6$ are

(A) 2 and 3 (B) −2 and −3 (C) −2 and 3 (D) 2 and −3

**B2.** A quadratic polynomial with sum of zeroes −3 and product of zeroes 2 is

(A) $x^2 + 3x - 2$ (B) $x^2 - 3x + 2$ (C) $x^2 + 3x + 2$ (D) $x^2 - 3x - 2$

**B3.** The product of the zeroes of $-4x^2 + x + 3$ is

(A) 3/4 (B) 1/4 (C) −1/4 (D) −3/4

**B4.** If $x = 3$ is a zero of $x^2 - kx + 6$, then $k$ equals

(A) 3 (B) 5 (C) −5 (D) 2

**B5.** The zeroes of $9x^2 - 9$ are

(A) 1 and −1 (B) 1 (C) −1 (D) 3 and −3

## Section B — Short Answers (2 marks each)

**B6.** Find the zeroes of $4x^2 - 4x - 3$.

**B7.** Form a quadratic polynomial whose zeroes are 1/2 and −1/2.

**B8.** If $\alpha$, $\beta$ are the zeroes of $x^2 - 4x + 3$, find $\alpha + \beta - \alpha\beta$.

## Section C — Long Answers (3 marks each)

**B9.** Find the zeroes of $3x^2 - x - 4$ and verify the relationship between the zeroes and the coefficients.

**B10.** Divide $x^2 + 7x + 10$ by $x + 2$. Give the quotient and the remainder, and verify your answer.

**B11.** The sum of the zeroes of $x^2 - kx + 8$ is 6. Find $k$, and hence find the zeroes.

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# Chapter Model Test C — Advanced

**CHAPTER MODEL TEST C • ADVANCED • FOR THE 95+ CLUB • MAX. MARKS: 20 • TIME: 45 MINUTES**

**General instructions:** All questions are compulsory. Section A: 5 MCQs × 1 mark. Section B: 3 questions × 2 marks. Section C: 3 questions × 3 marks. Show all steps — step marks apply, exactly as in the board exam. Answers: Answer Key.

## Section A — Multiple Choice (1 mark each)

**C1.** If α, β are the zeroes of $x^2 - 3x + 2$, then $\alpha^2 + \beta^2$ equals

(A) 9 (B) 13 (C) 3 (D) 5

**C2.** The value of k for which $x^2 + 6x + k$ has equal zeroes is

(A) 6 (B) 36 (C) 9 (D) −9

**C3.** If α, β are the zeroes of $x^2 - 5x + 6$, then $1/\alpha + 1/\beta$ equals

(A) 6/5 (B) 5/6 (C) 1/6 (D) −5/6

**C4.** The zeroes of $x^3 - 4x$ are

(A) 0 only (B) 0, 2 and −2 (C) 2 and −2 (D) 0 and 4

**C5.** The product of the zeroes of $2x^2 + 5x + 2$ is

(A) 5/2 (B) 1 (C) 2/5 (D) −1

## Section B — Short Answers (2 marks each)

**C6.** If α, β are the zeroes of $x^2 - 6x + 5$ with $\alpha > \beta$, find $\alpha - \beta$.

**C7.** If α, β are the zeroes of $x^2 - 3x + 2$, form a quadratic polynomial whose zeroes are $\alpha + 2$ and $\beta + 2$.

**C8.** One zero of $x^2 + 3x + k$ is the reciprocal of the other. Find k.

## Section C — Long Answers (3 marks each)

**C9.** If α, β are the zeroes of $x^2 - 4x + 1$, find $\alpha^3 + \beta^3$.

**C10.** Divide $x^3 - 3x^2 + 5x - 3$ by $x^2 - 2$. Give the quotient and the remainder, and verify.

**C11.** If α, β are the zeroes of $x^2 - px + q$, prove that $\alpha^2 + \beta^2 = p^2 - 2q$.

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# Answer Key — Graded Practice, Levels 1 and 2

## Level 1 — Foundation

**L1.1** $p(1) = 2 - 3 + 5 = 4$. **L1.2** Yes: $p(2) = 4 - 10 + 6 = 0$. **L1.3** $x^2 - 9 = (x - 3)(x + 3)$, zeroes 3 and −3. **L1.4** Sum = $-(-3)/5 = 3/5$; product = 2/5. **L1.5** $(x - 3)(x - 4)$, zeroes 3 and 4. **L1.6** $x^2 - 5x + 6$ (or any non-zero multiple).

## Level 2 — Skill Builder

**L2.1** $2x^2 - 7x + 3 = (2x - 1)(x - 3)$; zeroes 3 and 1/2. Sum = $7/2 = -(-7)/2$; product = 3/2. Verified.

**L2.2** $4x^2 + 4x - 3 = (2x - 1)(2x + 3)$; zeroes 1/2 and −3/2. Sum = $-1 = -4/4$; product = −3/4. Verified.

**L2.3** $x^2 - (-3)x + 2 = x^2 + 3x + 2$. **L2.4** $x^2 - 0x + (-4) = x^2 - 4$.

**L2.5** $(x + 2)(x + 5)$; zeroes −2 and −5. Sum = −7; product = 10. Verified. **L2.6** $(3x - 1)(x + 2)$; zeroes 1/3 and −2. Sum = −5/3; product = −2/3. Verified.

**TOPPER TIP:** *Marked yourself wrong at Level 2? Compare your working with the twin solution in Section 07 step by step — find the FIRST step where you differ. That step is the lesson.*

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# Answer Key — Levels 3, 4, 5 and Active Recall

## Level 3 — Application

**L3.1** $-5t(t-6)=0 \rightarrow t=6$ seconds (ignoring $t=0$). **L3.2** $(x+5)(x+6)$ — split pair 5, 6. **L3.3** $-x^2+6x-5=0 \rightarrow x^2-6x+5=0 \rightarrow (x-1)(x-5)$; meets the ground at $x=1$ and $x=5$; width = 4 m. **L3.4** $x^2-11x+30=(x-5)(x-6)$; the numbers are 5 and 6.

## Level 4 — Advanced Reasoning

**L4.1** $\alpha+\beta=5$, $\alpha\beta=6$, so $\alpha^2+\beta^2=25-12=13$. **L4.2** Let the zeroes be m and 7m: sum $8m=8/3 \rightarrow m=1/3$; product $7m^2=7/9=(2k+1)/3 \rightarrow 2k+1=7/3 \rightarrow k=2/3$. **L4.3** $\alpha+\beta=-b/a$ and $\alpha\beta=c/a$; the condition gives $-b/a=c/a$, so $-b=c$, i.e. $b+c=0$. ∎

## Level 5 — Mathematical Genius

**L5.1** Equal zeroes: $(\alpha+\beta)^2=4\alpha\beta$ gives $k^2=36$, so $k=6$ or $k=-6$ (polynomials $(x+3)^2$ and $(x-3)^2$ up to scale). **L5.2** New sum $=(\alpha+\beta)+2=8$; new product $=\alpha\beta+(\alpha+\beta)+1=5+6+1=12$; polynomial $x^2-8x+12$.

## Active Recall (Section 17)

**R1** 3. **R2** 2 and 3. **R3** 3/2. **R4** −2/5. **R5** 3. **R6** 3. **R7** $x^2-3x-4$. **R8** 4 and −4. **R9** $-b/2=5 \rightarrow b=-10$. **R10** $1+k-5=0 \rightarrow k=4$.

**P1** A zero of p(x) is a real number a with p(a) = 0. **P2** $\alpha+\beta=-b/a$ and $\alpha\beta=c/a$. **P3** The zeroes are the x-coordinates of the points where the graph of y = p(x) meets the x-axis. **P4** $p(x)=g(x)q(x)+r(x)$ with $r(x)=0$ or $\deg r(x)<\deg g(x)$. **P5** $p(x)=k[x^2-Sx+P]$ for any non-zero k.

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# Answer Key — Concept MCQs with Explanations (1 of 2)

**M1 (C)** Degree = highest power of x = 3. **M2 (B)** (x − 4)(x + 1) → zeroes 4, −1. **M3 (B)** p(2) = 4 − 14 + 10 = 0. **M4 (A)** −b/a = 5/3. **M5 (C)** c/a = −5/2. **M6 (D)** Sum = 1, product = −12 → $x^2$ − x − 12. **M7 (B)** Degree rule: at most 3. **M8 (A)** One touching point = one distinct zero (counted twice). **M9 (C)** Zeroes summing to 0 are opposites: −3.

**M10 (A)** $x^2$ + 1 ≥ 1 > 0 always; the graph never meets the axis. **M11 (D)** p(1) = 0 → 1 + k − 6 = 0 → k = 5. **M12 (B)** −b/a = −5/−1 = 5. **M13 (C)** c/a = −9/4.

# Answer Key — Concept MCQs with Explanations (2 of 2)

**M14 (A)** Split −9, +2: (2x − 3)(3x + 1) → 3/2, −1/3. **M15 (B)** A line meets the x-axis exactly once. **M16 (D)** (α + β) + αβ = 6 + 8 = 14. **M17 (C)** Degree rule. **M18 (B)** p(3) = 9 − 15 + 6 = 0.

**M19 (A)** x(x − 2) = 0 → 0, 2. **M20 (C)** q = product of zeroes = 2 × 3 = 6. **M21 (B)** Two crossings = two zeroes. **M22 (A)** A product is zero only if a factor is zero. **M23 (D)** k[$x^2$ + (1/2)x − 3] with k = 2. **M24 (B)** 3x = 6 → x = 2. **M25 (C)** (α + β)/(αβ) = 1/−6 = −1/6.

## Assertion–Reason — answers

**AR1 (a)** $x^2$ − 4 = (x − 2)(x + 2) crosses at ±2, and R is the very definition used. **AR2 (c)** Sum = −(−6)/3 = 2 is true; R states the product formula, so R is false. **AR3 (a)** p(2) = 0, and R is the definition that settles it. **AR4 (d)** A is false (at most 2); R is the true degree rule. **AR5 (b)** Both true, but R speaks of the SUM — it does not explain the product in A.

**AR6 (a)** R is the forming formula applied directly. **AR7 (a)** $(2x - 1)^2$ = 0 gives x = 1/2 twice — R explains A. **AR8 (d)** A is false; R is true. **AR9 (c)** The sum is indeed 2, but R is false: the sum is −b/a, not b/a. **AR10 (b)** Both true — but a matching sum alone cannot prove the zeroes (many triples sum to 6), so R is not the correct explanation.

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# Answer Key — Find-the-Error Corrections

**E1.** The formula carries a minus: sum = −b/a = −(−5)/2 = **5/2**. **E2.** x + 2 = 0 gives x = **−2**, not 2. Zeroes: 4 and −2. **E3.** The sum is SUBTRACTED: $x^2 - (1/4)x - 1$ (i.e. $4x^2 - x - 4$). **E4.** Product = c/a with no extra minus: −9/3 = **−3**. **E5.** t = **±√15** — two zeroes, not one.

**E6.** Only X-axis meetings count: the zeroes are **2** (at x = 1 and x = 4); the y-axis meeting is p(0), not a zero. **E7.** Standard form first: $6x^2 - 7x - 3$, so b = −7 and −b/a = 7/6 — the relationship HOLDS. **E8.** $(x + 2)(x + 3) = x^2 + 5x + 6$ exactly, so the remainder is **0**. **E9.** −3 × (−2) = +6, so p(−2) = 4 + 6 + 2 = **12 ≠ 0** — −2 is NOT a zero. **E10.** Sum of zeroes of a cubic = **−b/a = 6**, not b/a.

## Smart Memory Mini-Tests — answers

**Mini-Test 1 (Day 7):** 1. 4, 5. 2. 3/7. 3. 8. 4. $x^2 - 2x - 15$. 5. −1/3. 6. 1/2, −2. 7. Yes — p(−1) = 1 − 3 + 2 = 0. 8. One distinct zero. 9. k = 3. 10. 3, −1.

**Mini-Test 2 (Day 21):** 1. Sum −2, product 0. 2. 2, −2. 3. $x^2 + 4x - 5$. 4. 7/12. 5. Quotient x + 2, remainder 0. 6. 13. 7. 2. 8. k = 6. 9. 0, 4. 10. Zero real zeroes.

**Mini-Test 3 (Day 30):** 1. 26. 2. k = 25. 3. k = 15. 4. $x^2 - 8x + 12$. 5. −5. 6. k = 4. 7. 6. 8. Quotient $x^2 - x + 1$, remainder 0. 9. $x^2 - 4x + 4$. 10. Five.

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# Answer Key — Model Tests A, B and C

## Model Test A — Basics

**Section A:** A1 (B) 2. A2 (C) 2. A3 (D) 5. A4 (B) −1/3. A5 (D) 1.

**Section B:** A6 $p(1) = 0$; $p(-1) = 8$. A7 $(x - 3)(x - 5)$: zeroes 3, 5. A8 Sum 9/5; product 4/5.

**Section C:** A9 Zeroes 3 and 1/2; sum $7/2 = -b/a$, product $3/2 = c/a$ — verified. A10 $x^2 - 7x + 12$. A11 Three zeroes: −1, 0 and 3 (every x-axis crossing is a zero).

## Model Test B — Standard

**Section A:** B1 (B) −2 and −3. B2 (C) $x^2 + 3x + 2$. B3 (D) −3/4. B4 (B) 5. B5 (A) 1 and −1.

**Section B:** B6 $(2x - 3)(2x + 1)$: zeroes 3/2, −1/2. B7 $x^2 - 1/4$, i.e. $4x^2 - 1$. B8 $(\alpha + \beta) - \alpha\beta = 4 - 3 = 1$.

**Section C:** B9 Zeroes 4/3 and −1; sum 1/3, product −4/3 — verified. B10 Quotient $x + 5$, remainder 0; check $(x + 2)(x + 5) = x^2 + 7x + 10$. B11 $-b/a = k = 6$; then $x^2 - 6x + 8 = (x - 2)(x - 4)$: zeroes 2, 4.

## Model Test C — Advanced

**Section A:** C1 (D) 5. C2 (C) 9. C3 (B) 5/6. C4 (B) 0, 2 and −2. C5 (B) 1.

**Section B:** C6 $(\alpha - \beta)^2 = 36 - 20 = 16$, so $\alpha - \beta = 4$. C7 New sum $= 3 + 4 = 7$; new product $= 2 + 2(3) + 4 = 12$; $x^2 - 7x + 12$. C8 Product $= 1 \rightarrow k/1 = 1 \rightarrow k = 1$.

**Section C:** C9 $\alpha^3 + \beta^3 = 4^3 - 3(1)(4) = 64 - 12 = 52$. C10 Quotient $x - 3$, remainder $7x - 9$ (verify by multiplying back, as in Division Example 2). C11 $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = p^2 - 2q$, since $\alpha + \beta = p$ and $\alpha\beta = q$ for $x^2 - px + q$. ∎

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# Revision Card — Chapter 2 on One Page

## The one picture

**Zeroes of p(x)** = x-coordinates where the graph of y = p(x) meets the x-axis. Degree n → at most n zeroes. Parabola up if a > 0, down if a < 0; vertex on the line x = −b/(2a), midway between the zeroes.

## The bridge

**Quadratic** $ax^2 + bx + c$: α + β = −b/a, αβ = c/a. **Cubic** $ax^3 + bx^2 + cx + d$: sum −b/a, pairs c/a, product −d/a. Both come from expanding a(x − α)(x − β)… and comparing coefficients.

## The two recipes

**Finding zeroes:** standard form → split pair (sum b, product ac) → group → zero-product rule. **Forming:** $k[x^2 - Sx + P]$, k chosen to clear fractions. **Division:** p = gq + r, stop when deg r < deg g; always multiply back to check.

## Story signals

**Lands / touches / break-even / height zero** → solve p(x) = 0. **Area as one expression** → factorise for sides. **Centre / highest point** → midpoint of the zeroes. **Reciprocal zeroes** → product = 1. **Equal zeroes** → $(\alpha + \beta)^2 = 4\alpha\beta$.

## Exam-eve checklist

☐ Standard form written before reading a, b, c ☐ Minus sign spoken in −b/a ☐ Both verifications written in Q1-type questions ☐ ± pair reported when b = 0 ☐ Cubic product has its minus ☐ Answer translated back into the story's words

**Chapter 2 complete. You did not memorise Polynomials — you saw them, derived them, and mastered them.**

Next: Chapter 3 — Pair of Linear Equations in Two Variables, in the same 8D format. • KalviYogi • Mathematics – Beyond the Formula • 369 Tesla × KalviYogi • © Edunivra Pvt Ltd