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How this book is organised: Every chapter follows the same 19-stop journey — from a real-life story and visual concepts, through derivations and fully-solved NCERT examples, to five levels of practice, flash cards, active recall and your personal Mastery Tracker. Finish one chapter fully before starting the next.

Front matter: page 3 explains how to use this book and the 8D system; page 4 explains the Smart Memory System and the KalviYogi Digital Companion.

How to Use This Book — The 8D System

This is not a guide to copy answers from. It is a **self-learning system**: a teacher beside you, inside the book. Work with a pencil. Never read a solution before trying the question yourself for at least five minutes.

The KalviYogi 8D Mathematical Learning Framework

Dimension	The question in your mind	Where it happens
1. Discover	Why should I learn this?	Chapter Introduction + Real-Life Story
2. Visualise	Can I see it as a picture?	Visual Concept Explanation (Concrete → Pictorial → Abstract)
3. Understand	What is the core concept?	Mathematical Foundations
4. Derive	How did this formula come?	Formula Derivations — nothing is just "given"
5. Solve	How do I solve it, step by step?	Detailed Solved Examples + Multiple Methods
6. Apply	Where is it used in life?	Real-Life Applications + Competency Questions
7. Challenge	Can I solve it another way? Harder?	Reasoning, Olympiad & Genius-level problems
8. Recall	Can I do it without the book?	Flash Cards, Active Recall, Smart Memory tests

Every chapter, the same 19 stops

01 Introduction · 02 Prerequisites · 03 Real-Life Story · 04 Visual Concepts · 05 Foundations · 06 Derivations · 07 Solved Examples (complete NCERT) · 08 Multiple Methods · 09 Activities & Digital · 10 Common Mistakes · 11 Graded Practice (5 levels) · 12 Reasoning · 13 Applications · 14 Competency · 15 Olympiad Challenges · 16 Mind Map · 17 Pin-Point Flash Cards · 18 Active Recall · 19 Mastery Tracker — plus Practice Plus: 25 MCQs, Assertion–Reason, Find-the-Error, Previous-Year Questions and 3 Model Tests in every chapter.

Teacher's note: the yellow Pin-Point Flash Cards are the heart of revision. Cut them out, keep them in your pocket, and test yourself whenever you have two free minutes.

The KalviYogi Smart Memory System

Learning once is not learning. Your brain keeps what it recalls at spaced intervals. Every chapter ends with a Mastery Tracker built on this schedule:

Day	What you do
Day 0	Pre-learning assessment — try the chapter's starter questions cold. Note what you cannot do.
Day 1	Learn the chapter fully and practise. Fill your Error Tracker honestly.
Day 3	Without-book recall: flash cards + Active Recall questions, book closed.
Day 7	Day 7 Mini-Test (inside the chapter). Independent problem-solving only.
Day 14	Mixed practice: redo every question in your Error Tracker.
Day 21	Day 21 Mini-Test — long-term recall check.
Day 30	Day 30 Mastery Test + Concept Mastery Score. Below 80%? Revisit the sections marked in your tracker.

KalviYogi Digital Companion

Every chapter connects to video lessons, interactive simulations and online tests in the KalviYogi app / website. **QR codes and links activate with the KalviYogi app/website at print.** Print teaches you to think; digital lets you see it move.

A note on honesty

Marking-scheme lines in this book follow the official CBSE pattern where verified, and are labelled "CBSE-Style" elsewhere. Previous-year questions carry a year only when verified against CBSE papers. Answers in this book were recomputed independently — where popular guides disagree with this book, check the working: that is exactly the habit this book is teaching you.

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CBSE · CLASS 10 · MATHEMATICS

MATHEMATICS BEYOND THE FORMULA

CHAPTER 1 · REAL NUMBERS

Don't Memorize Mathematics. Understand, Visualise and Discover It.

The KalviYogi 8D Way: Discover · Visualise · Understand · Derive · Solve · Apply · Challenge · Recall

Every solution with COMPLETE steps — one by one, nothing skipped

Fundamental Theorem of Arithmetic · HCF & LCM · Irrational Numbers

Five-Level Practice · Smart Memory System · Mastery Tracker

Written like a topper's own notebook — so you can learn

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01 Chapter Introduction

8D • DISCOVER — WHY SHOULD I LEARN THIS?

Look around you. A shopkeeper packs sweets into boxes so that **nothing is left over**. Two buses leave a stand at different intervals and sometimes **leave together**. A tile-layer covers a floor with **equal square tiles** without cutting any tile. Behind every one of these everyday puzzles sits the hero of this chapter — the **Real Numbers** and their prime building blocks.

In Class 9 you met the number family. In this chapter you go one level deeper: you will discover that **every whole number is built from prime numbers in exactly one way** — and that single discovery gives you machines for finding HCF and LCM, for deciding whether a number can end in 0, and even for proving that numbers like √2 and √5 can never be written as a fraction.

What you will master in this chapter

- The Fundamental Theorem of Arithmetic — and why "unique" is the magic word.
- Prime factorisation of any number, using a factor tree and a division ladder.
- HCF and LCM by prime factorisation — for two numbers and for three.
- The identity HCF × LCM = a × b — **derived**, not just memorised, and used to find a missing value.
- Ending-digit and composite-number proofs (can 6^n end in 0?).
- Proof by contradiction — the master key for proving √2, √5, 3 + 2√5 and friends irrational.
- Story problems: when a story is secretly asking for HCF, and when for LCM.

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Your Chapter Map — 19 Stops, One Journey

This chapter is not a list of formulas. It is a journey with 19 stops. Tick each stop as you finish it.

Discover & Visualise	Understand & Derive	Solve, Apply, Recall
☐ 01 Chapter Introduction ☐ 02 Prerequisite Knowledge ☐ 03 Real-Life Story ☐ 04 Visual Concepts (CPA)	☐ 05 Mathematical Foundations ☐ 06 Formula Derivations ☐ 08 Multiple Solution Methods ☐ 10 Common Mistakes	☐ 07 Detailed Solved Examples ☐ 09 Hands-on Activities ☐ 11 Graded Practice (5 Levels) ☐ 12 Mathematical Reasoning
☐ 13 Real-Life Applications ☐ 14 Competency-Based Questions	☐ 15 Olympiad Challenges ☐ 16 Chapter Mind Map	☐ 17 Pin-Point Flash Cards ☐ 18 Active Recall ☐ 19 Mastery Tracker

How to use this chapter (read this first!)

1. **Never read a solution passively.** Cover the steps, try yourself, then uncover one step at a time.
2. Every solution shows **Step 1, Step 2, Step 3…** with a small *Why* note in purple — the reason behind the step. The board examiner gives marks for these steps, not just the final answer.
3. After Section 07, test yourself in Sections 11–15 **before** looking at the Answer Key.
4. Finish with the Mind Map (16), the Flash Cards (17), the closed-book Recall (18) and your Mastery Tracker (19). Come back on Day 3, 7, 14, 21 and 30 — the Smart Memory schedule on the tracker page tells you exactly what to do each day.

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02 Prerequisite Knowledge

CLASS 9 RECAP — THE TOOLS YOU ALREADY OWN

Everything in this chapter is built from five Class 9 ideas. Refresh them here — two minutes now will save you twenty minutes later.

1. The number family

Natural numbers: 1, 2, 3, ... **Whole numbers:** 0, 1, 2, 3, ... **Integers:** ... −2, −1, 0, 1, 2, ...
Rational numbers: numbers that can be written as p/q, where p and q are integers and q ≠ 0. **Irrational numbers:** real numbers that can NOT be written as p/q — their decimals never end and never repeat (for example √2, √3, π).
Real numbers = rationals + irrationals together — every point on the number line.

2. Prime and composite numbers

A **prime** has exactly two factors: 1 and itself (2, 3, 5, 7, 11, 13, ...). A **composite** number has more than two factors (4, 6, 8, 9, 10, ...). The number **1 is neither** prime nor composite — a favourite examiner trap!

3. Factors and multiples

A **factor** of a number divides it exactly (factors of 12: 1, 2, 3, 4, 6, 12). A **multiple** is what you get on multiplying (multiples of 12: 12, 24, 36, ...). HCF works with factors; LCM works with multiples.

4. Co-prime numbers

Two numbers are **co-prime** when their only common factor is 1 — that is, HCF = 1. Example: 14 and 25 are co-prime. This idea is the heart of every irrationality proof in Section 07.

5. Powers and exponents

a^2 means a × a, and a^3 means a × a × a. The small raised number is the **exponent** (power). Two facts you will use constantly: squaring a number **doubles** every exponent in its prime factorisation (if $a = 2^3 \times 5$, then $a^2 = 2^6 \times 5^2$), and multiplying numbers **adds** exponents of the same prime. The lemma of Section 06 is built entirely on the first fact.

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Quick Check — Are You Ready? (5 Questions)

Try all five without looking back. Answers are given right below — cover them with your hand first!

Q1. Is 29 a prime number? How do you check?

Q2. Write 48 as a product of prime numbers.

Q3. Write the first five multiples of 7.

Q4. Write all the factors of 36.

Q5. Are 14 and 25 co-prime? Give a reason.

Answers — check only after trying

Q1. Yes. Test divisibility by primes up to $\sqrt{29} \approx 5.4$, i.e. 2, 3, 5: 29 is divisible by none of them, so 29 is prime.

Q2. $48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$.

Q3. 7, 14, 21, 28, 35.

Q4. 1, 2, 3, 4, 6, 9, 12, 18, 36.

Q5. Yes. Factors of 14: 1, 2, 7, 14. Factors of 25: 1, 5, 25. The only common factor is 1, so HCF(14, 25) = 1 — they are co-prime.

TOPPER TIP: *Got 4 or 5 right? You are fully ready. Got fewer? Re-read the five cards on the previous page once, then come back — do not skip this step.*

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03 Real-Life Mathematical Story

DISCOVER • VISUALISE — THE SWEET SHOP PUZZLE

Before any formula, live the problem. Read this story slowly — the mathematics will walk out of the story by itself.

*It is the week before Diwali, and **Murugan Sweet House** is glowing with lights. This morning, the kitchen has produced two fresh batches: **168 coconut barfis** and **252 milk laddus**.*

*Murugan's daughter **Kavya**, who studies in Class 10, is in charge of the gift boxes. Her father's rules are strict:*

- *Every gift box must contain **only one type** of sweet.*
- *Every box must contain the **same number** of sweets.*
- ***No sweet may be left over** — every barfi and every laddu must be packed.*
- *To save cardboard, the boxes should be **as large as possible**.*

*Kavya tries 10 sweets per box: 168 ÷ 10 leaves a remainder. She tries 12: the barfis pack perfectly (14 boxes), but 252 ÷ 12 = 21 boxes — wait, that works too! She tries 20: laddus are left over. She tries 24, 28, 42... some numbers work, some fail. She needs the **largest** number that divides **both** 168 and 252 exactly.*

*Meanwhile, a second problem is waiting. The **ghee supplier** visits the shop **every 12 days**, and the **sugar supplier** visits **every 18 days**. Today, by luck, **both suppliers are at the shop together**. Murugan wants to plan a big joint order. "Kavya, after how many days will both of them stand in my shop together again?"*

Two questions. Two famous ideas. The first asks for the largest number hiding inside both 168 and 252 — that is the **HCF**. The second asks for the smallest day-count that both 12 and 18 can reach together — that is the **LCM**. Let us solve both, one step at a time.

Solving the Story — Step by Step

STORY PROBLEM 1

Gift boxes: 168 barfis and 252 laddus. Every box holds one type only, all boxes hold equal sweets, nothing is left over, and boxes are as large as possible. How many sweets per box, and how many boxes of each type?

THINK FIRST: The words 'as large as possible', 'equal' and 'nothing left over' are the story's way of saying HCF.

1. Write both numbers as products of primes. $168 = 2^3 \times 3 \times 7$ and $252 = 2^2 \times 3^2 \times 7$.

   Why: Prime factorisation is the X-ray of a number — it shows every divisor hiding inside.

2. The sweets-per-box number must divide BOTH 168 and 252, so it must be built only from the **common** primes: 2, 3 and 7.

   Why: A common divisor cannot use a prime that one of the numbers does not have.

3. For the LARGEST such number, take each common prime to the **smaller** of its two powers: 2^2 (smaller of 2^3, 2^2), 3^1 (smaller of 3^1, 3^2), 7^1.

   Why: Taking a bigger power of 2, like $2^3 = 8$, would fail to divide 252 — 252 only has 2^2.

4. Multiply: $2^2 \times 3 \times 7 = 4 \times 3 \times 7 =$ **84**. So each box holds 84 sweets. This number is the HCF of 168 and 252.

5. Count the boxes: barfi boxes = 168 ÷ 84 = **2**; laddu boxes = 252 ÷ 84 = **3**. Total 5 gift boxes.

   Why: Dividing each batch by the box size gives the number of boxes — and both divisions come out exact, as promised.

ANSWER: 84 sweets in each box; 2 boxes of barfis and 3 boxes of laddus (5 boxes in all).

Solving the Story — Problem 2: The Suppliers

STORY PROBLEM 2

The ghee supplier visits every 12 days and the sugar supplier every 18 days. Both visited today. After how many days will they visit together again?

THINK FIRST: The words 'together again' and 'after how many days' are the story's way of saying LCM. Remember these two signals — they return in Exercise 1.1 Q7 and in Section 13.

1. List when the ghee supplier comes (multiples of 12): 12, 24, **36**, 48, ...

   Why: Visit days are multiples of the gap between visits.

2. List when the sugar supplier comes (multiples of 18): 18, **36**, 54, ...

3. The first day appearing in BOTH lists is 36 — the **smallest common multiple**, the LCM of 12 and 18.

   Why: We want the FIRST time they coincide, so we take the smallest common multiple, not just any one.

4. Quick check by primes: $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$, so LCM = $2^2 \times 3^2 = 36$. Same answer.

   Why: For LCM, take every prime at its GREATEST power — Section 05 will make this rule precise.

ANSWER: Both suppliers will be at the shop together again after 36 days.

The two story signals — pin these in your memory

Largest / equal / nothing left over / pack / cut → the answer is a DIVISOR of the given numbers → take the greatest one: **HCF**.
Together again / meet again / repeat / after how long → the answer is a MULTIPLE of the given numbers → take the smallest one: **LCM**.
Nearly every word problem in this chapter — and in the board exam — is one of these two signals wearing a costume.

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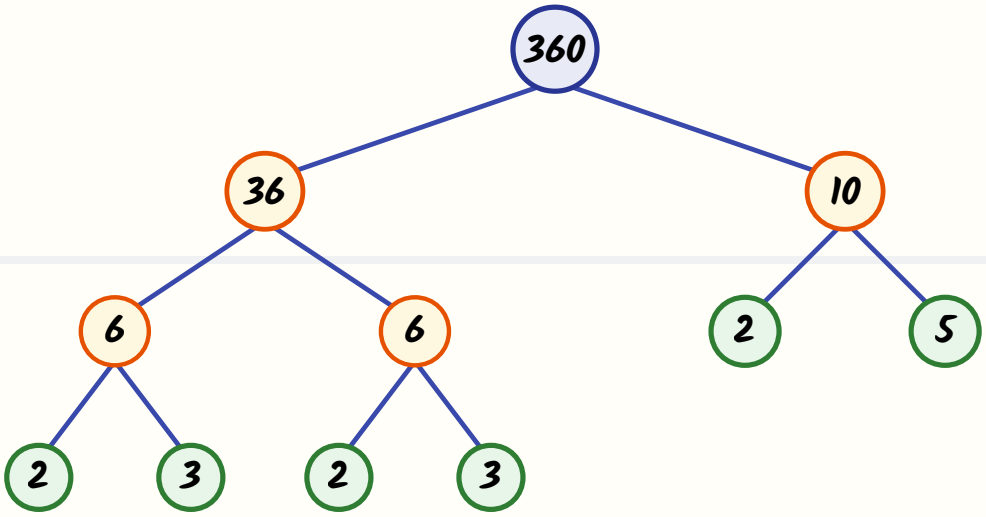
04 Visual Concept Explanation

SINGAPORE CPA — CONCRETE → PICTORIAL → ABSTRACT

Concrete first. In the story, Kavya physically grouped sweets into boxes. Grouping, sharing and arranging real objects is always the first stage of understanding a number idea.

Now the picture. A composite number can be "opened up" like a tree branching downwards. Keep splitting every branch until every leaf is prime. The leaves can never change — that is the whole secret of this chapter. Here is 360, opened up:



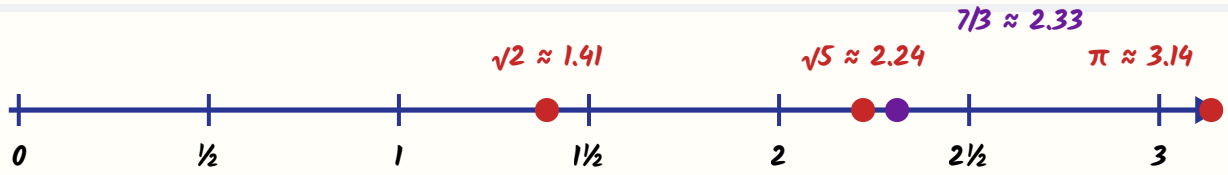
Read the green leaves from left to right: 2, 3, 2, 3, 2, 5. Collect them: **$360 = 2^3 \times 3^2 \times 5$**. Any other way of splitting 360 gives exactly the same leaves — try it in Section 09.

Then the abstract. Once the picture is clear, we write the short symbolic form: $360 = 2^3 \times 3^2 \times 5$. Symbols are just the picture, compressed. Never memorise the symbol before you can see the picture.

> **TOPPER TIP:** *Examiners can tell the difference: a student who has SEEN the tree never stops a factorisation halfway, because an unsplit branch looks unfinished.*

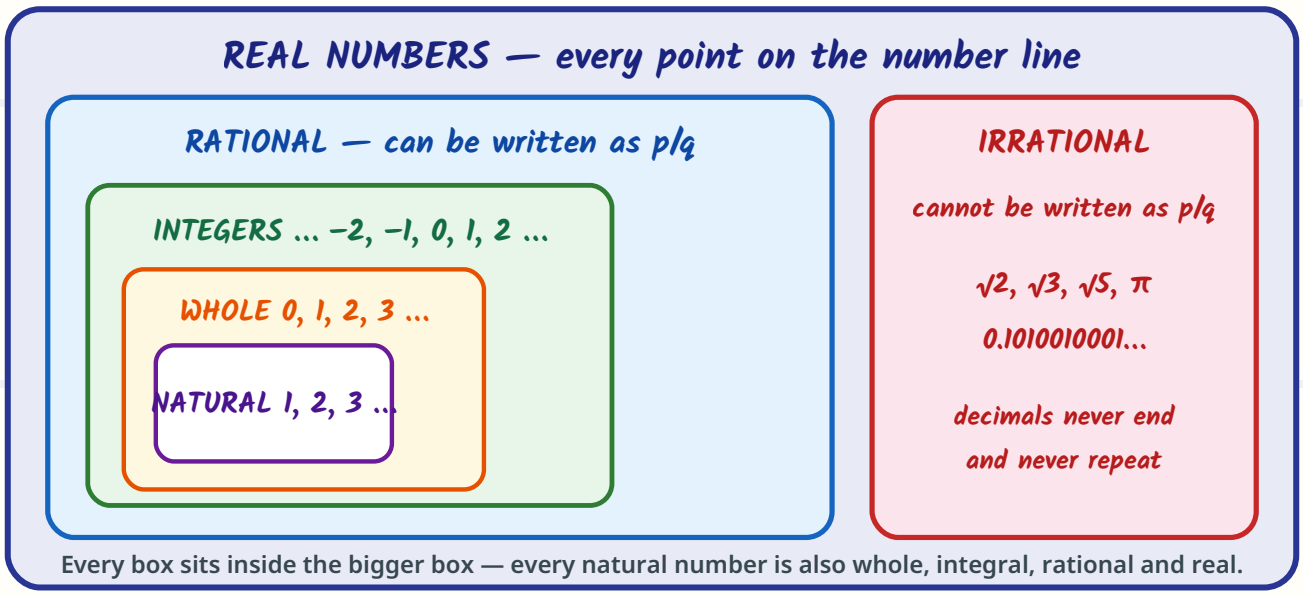
Visual Concepts — The Number Line and the Number Family

Rational numbers sit at exact fraction points. Irrational numbers also sit at exact points — √2 is a perfectly definite point, even though its decimal never ends. **Both families together fill the whole line with no gaps: that is why they are called REAL numbers.**



Notice: √2 (about 1.41) sits just left of 1½, and √5 (about 2.24) sits just left of 7/3 (about 2.33). An irrational number is not "rough" or "unknown" — only its decimal is endless.

The number family — boxes inside boxes



Every box sits inside the bigger box — every natural number is also whole, integral, rational and real.

This chapter works in two corners of the picture: the **natural numbers** (for HCF and LCM) and the border between **rational and irrational** (for the proof questions).

05 Mathematical Foundations

UNDERSTAND — THE PRECISE IDEAS, WORD BY WORD

Theorem — Fundamental Theorem of Arithmetic

Every integer greater than 1 can be expressed as a product of prime numbers in a unique way, apart from the order in which the prime factors occur. (For a prime, the "product" is the prime itself.)

Two halves. Half 1: a prime factorisation always **exists** — keep splitting and you reach primes. Half 2: it is **unique** — 360 is $2^3 \times 3^2 \times 5$ for every student, forever.

PIN POINT: Unique factorisation powers this whole chapter — HCF, LCM, proofs.

Worked example — factorise 360 by the division ladder, one step at a time

1. 360 ÷ 2 = 180. Write 2 on the left of the ladder.
   Why: Start with the smallest prime. 360 is even, so 2 divides it.
2. 180 ÷ 2 = 90.
   Why: 180 is still even — stay with 2.
3. 90 ÷ 2 = 45.
   Why: 90 is even once more.
4. 45 ÷ 3 = 15.
   Why: 45 is odd — move to the next prime, 3 (digit sum 9 confirms it).
5. 15 ÷ 3 = 5.
   Why: 15 = 3 × 5, so 3 divides once more.
6. 5 ÷ 5 = 1. Stop.
   Why: We stop at 1 — the ladder is complete and every divisor used is prime.
7. Collect the left column: 360 = 2 × 2 × 2 × 3 × 3 × 5 = $\mathbf{2^3 \times 3^2 \times 5}$.
   Why: Counting repeats and writing powers is the abstract (symbol) form of the ladder picture.
8. Check by multiplying back: $2^3 \times 3^2 \times 5 = 8 \times 9 \times 5 = 360$.
   Why: Always multiply back. Ten seconds of checking protects the full mark.

The ladder as one strip: 2 | 360 → 2 | 180 → 2 | 90 → 3 | 45 → 3 | 15 → 5 | 5 → 1.

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Foundations — HCF and LCM, Defined Precisely

HCF — Highest Common Factor

The HCF of two or more numbers is the **largest number that divides each of them exactly**. By prime factorisation: **take only the primes common to all the numbers, each raised to its SMALLEST power.**

LCM — Lowest Common Multiple

The LCM of two or more numbers is the **smallest number that is a multiple of each of them**. By prime factorisation: **take every prime that appears in any of the numbers, each raised to its GREATEST power.**

Worked example — HCF and LCM of 12 and 18, step by step

1. Factorise: $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$.
   Why: Always factorise first — HCF and LCM are read off the factorisations, never guessed.
2. Common primes are 2 and 3. For HCF take the SMALLER powers: 2^1 and 3^1.
   Why: A common factor cannot contain 2^2, because 18 has only one 2.
3. HCF = $2 \times 3 = \mathbf{6}$.
4. For LCM take every prime at its GREATER power: 2^2 and 3^2.
   Why: A common multiple must contain enough of each prime to cover both numbers.
5. LCM = $4 \times 9 = \mathbf{36}$.
6. Golden check: HCF × LCM = 6 × 36 = 216, and 12 × 18 = 216. They match.
   Why: This check works for TWO numbers. Section 06 derives WHY it is true.

Rational and irrational — the exact definitions

A real number is **rational** if it can be written as p/q with p, q integers and $q \neq 0$ (examples: 5, −3/4, 0.25, 2.333…). A real number is **irrational** if it can NOT be written in that form (examples: √2, √5, π, 0.1010010001…). A decimal that terminates or repeats is always rational; a decimal that neither terminates nor repeats is always irrational.

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06 Formula Derivations

8D • DERIVE — NEVER MEMORISE WHAT YOU CAN DERIVE

Derivation A — Why is HCF × LCM = a × b?

You have used this identity since Class 6. Today you will see **why** it must be true. Take the pair 12 and 18 and watch ONE prime at a time.

1. Write the factorisations: $12 = 2^2 \times 3^1$ and $18 = 2^1 \times 3^2$.
   Why: The whole proof lives inside these two lines.
2. Look only at the prime 2. Its exponents in the two numbers are **2** (in 12) and **1** (in 18).
   Why: Forget the whole numbers — follow a single prime.
3. HCF takes the SMALLER exponent of 2, which is 1. LCM takes the LARGER exponent of 2, which is 2.
   Why: That is exactly how HCF and LCM were defined in Section 05.
4. Add them: 1 + 2 = 3. But also 2 + 1 = 3 — the SUM of the two original exponents.
   Why: For any two numbers x and y: min(x, y) + max(x, y) = x + y, because one of them IS the min and the other IS the max. Nothing is lost, nothing is doubled.
5. Now the prime 3: exponents are 1 (in 12) and 2 (in 18). HCF takes 1, LCM takes 2, and 1 + 2 = 3 = the sum of the original exponents again.
   Why: The same thing happens for EVERY prime, in EVERY pair of numbers.
6. So in the product HCF × LCM, every prime's exponent equals the sum of its exponents in a and in b.
7. But in the product a × b, a prime's exponent is ALSO the sum of its exponents in a and b — that is what multiplying does to powers.
   Why: Compare the two products prime by prime: they are identical.
8. Two numbers with identical prime factorisations are the same number (Fundamental Theorem — uniqueness). Therefore **HCF × LCM = a × b**. Proved, not memorised.

WARNING — TWO NUMBERS ONLY: For three numbers the trick collapses, because with three exponents the middle one is lost: min + max ≠ sum of all three. Example: for 12, 15, 21, HCF × LCM = 3 × 420 = 1260, but 12 × 15 × 21 = 3780. Never use the identity for three numbers.

PIN POINT: min + max = sum of the two exponents — that single line IS the whole proof of HCF × LCM = a × b.

Derivation B — The Full Proof that √2 is Irrational

This is the most beautiful argument in the chapter. Its weapon is **proof by contradiction**: assume the opposite of what you want, walk forward honestly, and crash into an impossibility. Follow every step — Exercise 1.2 is this same proof wearing different clothes.

1. **Assume the opposite:** suppose √2 IS rational.
   Why: In a contradiction proof, we temporarily believe the enemy's claim and show it self-destructs.

2. Then √2 = a/b, where a and b are **co-prime** positive integers and b ≠ 0.
   Why: Any fraction can be reduced to lowest form; a and b share no common factor except 1. This is the trap.

3. Square both sides: $2 = a^2/b^2$, so $\mathbf{a^2 = 2b^2}$.
   Why: Squaring removes the root — that is why we square first in every such proof.

4. So a^2 is exactly twice an integer — meaning **2 divides a^2**.
   Why: $a^2 = 2 \times (b^2)$, and b^2 is an integer.

5. **Lemma:** if a prime p divides a^2, then p divides a. Apply it with p = 2: **2 divides a**.
   Why: The lemma is proved on the next page. It is the engine of the whole argument.

6. Write a = 2c for some integer c, and substitute into $a^2 = 2b^2$: $(2c)^2 = 2b^2$, so $4c^2 = 2b^2$, so $\mathbf{b^2 = 2c^2}$.
   Why: Naming a = 2c is how we USE the fact '2 divides a' — a is double something.

7. So b^2 is also twice an integer: **2 divides b^2**, and by the lemma again, **2 divides b**.
   Why: The same engine fires a second time, now on b.

8. But now 2 is a common factor of a AND b — while Step 2 said a and b are co-prime, with NO common factor except 1. **Contradiction.**
   Why: The assumption has crashed. Every step was forced, so the only possible fault is the assumption itself.

9. Therefore the assumption is false: **√2 is irrational.** ∎
   Why: This completes the proof. Notice the rhythm: Assume → Square → p divides a → Substitute → p divides b → Contradiction.

TOPPER TIP: Learn the RHYTHM, not the lines: with 5 in place of 2, the same rhythm proves √5 irrational (Exercise 1.2 Q1, Section 07).

PIN POINT: Assume → Square → p | a → p | b → Contradiction: the five beats.

The Lemma — and the Pattern for √3 and √5

Lemma: if prime p divides a^2, then p divides a — why?

Step 1: Write a as a product of primes, say a = 2 × 3 × 3 × 7 (the argument works for any factorisation).
Step 2: Then $a^2 = a \times a$ has every prime of a appearing **twice as many times**: $a^2 = 2^2 \times 3^4 \times 7^2$. Squaring only DOUBLES exponents — it can never create a brand-new prime.
Step 3: So if a prime p divides a^2, p must be one of the primes inside a^2's factorisation — hence one of the primes inside a itself.
Step 4: Therefore p divides a. ∎ (This uses uniqueness of prime factorisation — the Fundamental Theorem again.)

The 5-beat pattern — one proof, many numbers

To prove √p irrational for any prime p, repeat Derivation B with p in place of 2:
Beat 1: Assume $\sqrt{p} = a/b$, co-prime. **Beat 2:** Square: $a^2 = p \cdot b^2$, so $p \mid a^2$.
Beat 3: Lemma: $p \mid a$, so $a = p \cdot c$. **Beat 4:** Substitute back: $b^2 = p \cdot c^2$, so $p \mid b$.
Beat 5: p divides both a and b — contradiction. Hence √p is irrational.

With p = 3 this proves **√3 irrational**; with p = 5 it proves **√5 irrational**. You will write the √5 proof in full in Section 07 (Ex 1.2 Q1) and the √3 proof in Section 12.

TOPPER TIP: *In the board exam, this proof is worth 3 marks and appears again and again. The marks are given beat by beat — which is why this book never compresses a proof into one paragraph.*

07 Detailed Solved Examples

8D • SOLVE — NCERT EXERCISES 1.1 & 1.2, EVERY STEP SHOWN

Here are the complete NCERT exercises, solved the KalviYogi way: **every step numbered, every step justified, nothing skipped**. The printed line under each solution shows a CBSE-style split of the marks — read it to see exactly where marks live. (The √5 proof in Exercise 1.2 follows the official CBSE marking pattern.)

EXERCISE 1.1 • QUESTION 1 (i)

Express 140 as a product of its prime factors.

THINK FIRST: Use the division ladder: divide by the smallest prime that works, again and again, until 1 is reached. Every division is one step.

1. 140 ÷ 2 = 70.
   Why: 140 is even, so start with the smallest prime, 2.
2. 70 ÷ 2 = 35.
   Why: 70 is still even.
3. 35 ÷ 5 = 7.
   Why: 35 is odd; it ends in 5, so 5 divides it.
4. 7 ÷ 7 = 1. The ladder is complete.
   Why: 7 is prime; dividing by itself gives 1, our stop signal.
5. Collect the divisors: 140 = 2 × 2 × 5 × 7 = **$2^2 \times 5 \times 7$**. Check: 4 × 5 × 7 = 140.
   Why: The check confirms no divisor was missed.

ANSWER: $140 = 2^2 \times 5 \times 7$

CBSE-STYLE MARKING SCHEME: 1 mark for the correct factorisation. The ladder working must be visible — a bare final answer earns nothing if one prime is wrong.

COMMON MISTAKE: Stopping at 140 = 2 × 70. But 70 is composite — the ladder must continue until 1 is reached.

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Exercise 1.1 Q1 (ii) — 156

EXERCISE 1.1 • QUESTION 1 (ii)

Express 156 as a product of its prime factors.

THINK FIRST: Same ladder, same rhythm. Watch the moment the divisor changes from 2 to 3 — that change is decided by a divisibility test, not by guessing.

1. 156 ÷ 2 = 78.
   Why: 156 is even.
2. 78 ÷ 2 = 39.
   Why: 78 is even once more.
3. 39 ÷ 3 = 13.
   Why: 39 is odd; digit sum 3 + 9 = 12, so 3 divides it.
4. 13 ÷ 13 = 1. The ladder is complete.
   Why: 13 is prime.
5. Collect: 156 = 2 × 2 × 3 × 13 = **$2^2 \times 3 \times 13$**. Check: 4 × 3 × 13 = 156.
   Why: Multiplying back confirms the ladder.

ANSWER: $156 = 2^2 \times 3 \times 13$

CBSE-STYLE MARKING SCHEME: 1 mark for the correct factorisation, with the ladder shown.

Ladder Clinic — one more number, fully worked: 176

Read the ladder downwards: each line divides the number above it by the smallest prime that works.

2	176
2	88
2	44
2	22
11	11
	1

Collect the left column: 176 = 2 × 2 × 2 × 2 × 11 = **$2^4 \times 11$**. Check: 16 × 11 = 176.

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Exercise 1.1 Q1 (iii) — 3825

EXERCISE 1.1 • QUESTION 1 (iii)

Express 3825 as a product of its prime factors.

THINK FIRST: Same ladder. For a big number, divisibility tests choose the prime at each step.

1. 3825 ÷ 3 = 1275.
   Why: Digit sum 3 + 8 + 2 + 5 = 18 is divisible by 3, so 3 divides first.
2. 1275 ÷ 3 = 425.
   Why: Digit sum of 1275 is 15, again divisible by 3.
3. 425 ÷ 5 = 85.
   Why: 425 is odd; it ends in 5, so move to 5.
4. 85 ÷ 5 = 17.
   Why: 85 ends in 5 once more.
5. 17 ÷ 17 = 1. The ladder is complete.
   Why: 17 is prime.
6. Collect the divisors: 3825 = 3 × 3 × 5 × 5 × 17 = $\mathbf{3^2 \times 5^2 \times 17}$.
   Why: Group equal primes and write them as powers.
7. Check: $3^2 \times 5^2 \times 17 = 9 \times 25 \times 17 = 225 \times 17 = 3825$.
   Why: Multiply back in chunks: 9 × 25 = 225 first.

ANSWER: 3825 = $3^2 \times 5^2 \times 17$

CBSE-STYLE MARKING SCHEME: 1 mark. Divisibility reasoning (digit-sum test) shown in the ladder is the method mark.

TOPPER TIP: *Numbers ending in 25 always have 5 at least twice in their factorisation — a handy prediction to check your ladder against.*

Exercise 1.1 Q1 (iv) and (v)

EXERCISE 1.1 • QUESTION 1 (iv) and (v)

Express each number as a product of its prime factors: (iv) 5005 (v) 7429

THINK FIRST: (iv) is friendly. (v) is the chapter's patience test: several primes fail before one works. Show every trial — the trials ARE the working.

1. **(iv) 5005:** 5005 ÷ 5 = 1001.
   Why: 5005 ends in 5, so 5 divides it.
2. 1001 ÷ 7 = 143.
   Why: 1001 is a famous number: 1001 = 7 × 11 × 13. Worth remembering!
3. 143 ÷ 11 = 13.
   Why: 143 = 11 × 13 — another pair worth recognising on sight.
4. 13 ÷ 13 = 1. Collect: 5005 = **5 × 7 × 11 × 13**. Check: 5 × 7 × 11 × 13 = 35 × 143 = 5005.
5. **(v) 7429:** Test 2, 3, 5 first: 7429 is odd (not divisible by 2); digit sum 7 + 4 + 2 + 9 = 22 (not divisible by 3); it does not end in 0 or 5 (not divisible by 5).
   Why: Rule out the small primes with quick tests before long division.
6. Test 7: 7 × 1061 = 7427 ≠ 7429. Test 11: 11 × 675 = 7425 ≠ 7429. Test 13: 13 × 571 = 7423 ≠ 7429. None works.
   Why: Multiply the prime by a trial quotient and compare — near misses prove the prime does NOT divide.
7. Test 17: 17 × 437 = 7429. Yes! So 7429 ÷ 17 = 437.
   Why: Patience pays — 17 is the first prime factor.
8. Now factorise 437: test 17 again (17 × 25 = 425 ≠ 437), then 19: 19 × 23 = 437. Yes! So 437 = 19 × 23, and 23 is prime.
   Why: 437 looks prime but is not — always keep testing until the quotient itself is prime or 1.
9. Collect: 7429 = **17 × 19 × 23**. Check: 17 × 19 = 323; 323 × 23 = 7429.

ANSWER: (iv) 5005 = 5 × 7 × 11 × 13 (v) 7429 = 17 × 19 × 23

CBSE-STYLE MARKING SCHEME: 1 mark each. In (v), visible trial divisions earn the method mark even though they take space.

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Exercise 1.1 Q2 (i) — HCF and LCM of 26 and 91

EXERCISE 1.1 • QUESTION 2 (i)

Find the LCM and HCF of 26 and 91, and verify that LCM × HCF = product of the two numbers.

THINK FIRST: Factorise both numbers completely first. Then HCF = common primes, smallest powers; LCM = all primes, greatest powers.

1. Factorise 26: 26 ÷ 2 = 13; 13 ÷ 13 = 1. So 26 = 2 × 13.
2. Factorise 91: 91 ÷ 7 = 13; 13 ÷ 13 = 1. So 91 = 7 × 13.
   Why: 91 is odd and doesn't end in 5; 7 works: 7 × 13 = 91. Many students wrongly call 91 prime!
3. The only common prime is 13, with power 1 in both. HCF = **13**.
   Why: HCF uses common primes only, at the smaller power.
4. LCM takes every prime at its greatest power: $2^1 \times 7^1 \times 13^1$ = **182**.
   Why: 2 appears only in 26 and 7 only in 91 — LCM still needs both.
5. Verify: LCM × HCF = 182 × 13 = 2366.
6. Product of the numbers: 26 × 91 = 2366.
7. 2366 = 2366. The identity is verified.
   Why: This verification is a required part of the question — never skip it.

ANSWER: HCF(26, 91) = 13 and LCM(26, 91) = 182; LCM × HCF = 2366 = 26 × 91. Verified.

CBSE-STYLE MARKING SCHEME: 3 marks: 1 for both factorisations, 1 for HCF and LCM, 1 for the verification.

COMMON MISTAKE: Calling 91 a prime number. 91 = 7 × 13 is the most famous 'fake prime' in this chapter — always test 7.

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Exercise 1.1 Q2 (ii) — HCF and LCM of 510 and 92

EXERCISE 1.1 • QUESTION 2 (ii)

Find the LCM and HCF of 510 and 92, and verify that LCM × HCF = product of the two numbers.

THINK FIRST: Same method. The LCM will be large — build it prime by prime and multiply in stages.

1. Factorise 510: 510 ÷ 2 = 255.
   Why: 510 is even.
2. 255 ÷ 3 = 85.
   Why: Digit sum 2 + 5 + 5 = 12, so 3 divides 255.
3. 85 ÷ 5 = 17.
   Why: 85 ends in 5.
4. 17 ÷ 17 = 1. So 510 = 2 × 3 × 5 × 17.
5. Factorise 92: 92 ÷ 2 = 46; 46 ÷ 2 = 23; 23 ÷ 23 = 1. So 92 = 2^2 × 23.
6. Common prime: only 2. Smaller power is 2^1. HCF = **2**.
   Why: 23, 3, 5, 17 are not common, so they cannot enter the HCF.
7. LCM takes every prime at its greatest power: 2^2 × 3 × 5 × 17 × 23.
8. Multiply in stages: 2^2 × 3 = 12; 12 × 5 = 60; 60 × 17 = 1020; 1020 × 23 = **23460**. So LCM = 23460.
   Why: Stage-wise multiplication prevents slips with big LCMs.
9. Verify: LCM × HCF = 23460 × 2 = 46920.
10. Product of the numbers: 510 × 92 = 46920.
    Why: 510 × 92 = 510 × 90 + 510 × 2 = 45900 + 1020 = 46920.
11. 46920 = 46920. The identity is verified.

ANSWER: HCF(510, 92) = 2 and LCM(510, 92) = 23460; LCM × HCF = 46920 = 510 × 92. Verified.

CBSE-STYLE MARKING SCHEME: 3 marks: factorisation (1), HCF and LCM (1), verification (1).

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Exercise 1.1 Q2 (iii) — HCF and LCM of 336 and 54

EXERCISE 1.1 • QUESTION 2 (iii)

Find the LCM and HCF of 336 and 54, and verify that LCM × HCF = product of the two numbers.

THINK FIRST: 336 has a long ladder of 2s. Take it one division at a time.

1. Factorise 336: 336 ÷ 2 = 168.
2. 168 ÷ 2 = 84.
3. 84 ÷ 2 = 42.
4. 42 ÷ 2 = 21.
   Why: Four divisions by 2 in a row — that becomes 2^4. Count carefully.
5. 21 ÷ 3 = 7.
   Why: 21 is odd; digit sum 3, so 3 divides it.
6. 7 ÷ 7 = 1. So 336 = $2^4 \times 3 \times 7$.
7. Factorise 54: 54 ÷ 2 = 27; 27 ÷ 3 = 9; 9 ÷ 3 = 3; 3 ÷ 3 = 1. So 54 = 2×3^3.
8. Common primes: 2 and 3. Smaller powers: 2^1 and 3^1. HCF = 2 × 3 = **6**.
9. LCM: every prime at its greatest power: $2^4 \times 3^3 \times 7$. Multiply in stages: 16 × 27 = 432; 432 × 7 = **3024**.
10. Verify: LCM × HCF = 3024 × 6 = 18144.
11. Product of the numbers: 336 × 54 = 18144.
    Why: 336 × 54 = 336 × 50 + 336 × 4 = 16800 + 1344 = 18144.
12. 18144 = 18144. The identity is verified.

ANSWER: HCF(336, 54) = 6 and LCM(336, 54) = 3024; LCM × HCF = 18144 = 336 × 54. Verified.

CBSE-STYLE MARKING SCHEME: 3 marks: factorisations (1), HCF and LCM (1), verification (1).

TOPPER TIP: *Tick off each repeated division with your pen as you count the power — losing one 2 is the commonest slip in this exercise.*

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Exercise 1.1 Q3 (i) — Three Numbers: 12, 15 and 21

EXERCISE 1.1 • QUESTION 3 (i)

Find the LCM and HCF of 12, 15 and 21 by applying the prime factorisation method.

THINK FIRST: Same rules, one extra care: for the HCF of three numbers, a prime must be present in ALL THREE to qualify.

1. Factorise each number: $12 = 2^2 \times 3$; $15 = 3 \times 5$; $21 = 3 \times 7$.
   Why: Short ladders: 12 = 2×2×3, 15 = 3×5, 21 = 3×7.
2. Look for a prime present in all three factorisations: only **3** appears in 12, 15 and 21.
   Why: 2 is missing from 15 and 21; 5 and 7 appear in only one number each.
3. HCF takes 3 at its smallest power anywhere, which is 3^1. HCF = **3**.
4. LCM takes every prime that appears anywhere, at its greatest power: 2^2 (from 12), 3^1, 5^1, 7^1.
5. Multiply in stages: 4 × 3 = 12; 12 × 5 = 60; 60 × 7 = **420**. So LCM = 420.
6. Sanity check: 420 ÷ 12 = 35, 420 ÷ 15 = 28, 420 ÷ 21 = 20 — all exact, so 420 is truly a common multiple.
   Why: Dividing the LCM back by each number is a fast honesty check.

ANSWER: HCF(12, 15, 21) = 3 and LCM(12, 15, 21) = 420.

CBSE-STYLE MARKING SCHEME: 2 marks: 1 for the three factorisations, 1 for HCF and LCM.

COMMON MISTAKE: Trying to verify with HCF × LCM = product here. That identity is for TWO numbers only (Section 06): 3 × 420 = 1260 ≠ 3780 — and nothing has gone wrong!

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Exercise 1.1 Q3 (ii) and (iii)

EXERCISE 1.1 • QUESTION 3 (ii) and (iii)

Find the LCM and HCF of the following integers by the prime factorisation method:
(ii) 17, 23 and 29 (iii) 8, 9 and 25

THINK FIRST: Both triples have NO prime common to all three. Watch what that forces the HCF to be.

1. **(ii)** 17, 23 and 29 are each prime themselves, so their factorisations are just 17, 23 and 29.
   Why: Test each: none of 17, 23, 29 has any divisor besides 1 and itself.
2. No prime is common to all three (in fact, no prime is shared by even two of them). So HCF = **1**.
   Why: When nothing is common, the HCF falls back to 1 — the numbers are pairwise co-prime.
3. LCM = 17 × 23 × 29. Multiply in stages: 17 × 23 = 391; 391 × 29 = **11339**.
   Why: With no shared primes, the LCM is simply the full product.
4. **(iii)** Factorise: $8 = 2^3$; $9 = 3^2$; $25 = 5^2$.
5. The three factorisations use completely different primes (2, 3 and 5). No prime is common to all three, so HCF = **1**.
   Why: The numbers need not be prime for HCF to be 1 — they only need to share no prime.
6. LCM takes every prime at its greatest power: $2^3 \times 3^2 \times 5^2 = 8 \times 9 \times 25$. Multiply: 8 × 9 = 72; 72 × 25 = **1800**.

ANSWER: (ii) HCF = 1, LCM = 11339 (iii) HCF = 1, LCM = 1800

CBSE-STYLE MARKING SCHEME: 2 marks for each part: factorisations + correct HCF and LCM.

TOPPER TIP: *HCF = 1 does NOT mean the numbers are prime. It means their prime factorisations share nothing — compare 8, 9, 25: all composite, yet HCF = 1.*

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Exercise 1.1 Q4 and Q5

EXERCISE 1.1 • QUESTION 4

Given that HCF (306, 657) = 9, find LCM (306, 657).

THINK FIRST: This is Derivation A at work: for two numbers, HCF × LCM = product of the numbers. We are given the HCF, so the LCM is one division away.

1. Write the identity with the known values: 9 × LCM (306, 657) = 306 × 657.
   Why: The identity holds for any TWO positive integers — Section 06 proved it.
2. Compute the product: 306 × 657 = 201042.
   Why: 306 × 657 = 306 × 600 + 306 × 57 = 183600 + 17442 = 201042.
3. Divide both sides by 9: LCM = 201042 ÷ 9 = **22338**.
   Why: Isolating the unknown is just undoing the multiplication.
4. Honesty check: 22338 ÷ 306 = 73 and 22338 ÷ 657 = 34 — both exact, so 22338 really is a common multiple of 306 and 657.
   Why: And since HCF × LCM matched the product, it is the SMALLEST such multiple.

ANSWER: LCM (306, 657) = 22338.

CBSE-STYLE MARKING SCHEME: 2 marks: 1 for using the identity correctly, 1 for the arithmetic.

Your turn — the same trick, fully worked: HCF(144, 180) = 36, find the LCM

Step 1: Write the identity: 36 × LCM = 144 × 180 = 25920.
Step 2: Divide: LCM = 25920 ÷ 36 = **720**.
Step 3: Check by primes: $144 = 2^4 \times 3^2$ and $180 = 2^2 \times 3^2 \times 5$, so LCM = $2^4 \times 3^2 \times 5 = 720$. The two roads meet — as Section 08 promises.

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Exercise 1.1 Q5 — Can 6^n End with the Digit 0?

EXERCISE 1.1 • QUESTION 5

Check whether 6^n can end with the digit 0 for any natural number n.

THINK FIRST: Think about what 'ending in 0' forces. A number ends in 0 exactly when 10 divides it — and 10 = 2 × 5. So the prime 5 must appear in the number's factorisation.

1. Suppose, if possible, that 6^n ends with the digit 0 for some natural number n.
   Why: We test the claim by examining the prime factorisation — it cannot lie.
2. Then 10 divides 6^n, so both 2 and 5 are prime factors of 6^n.
   Why: 10 = 2 × 5, and a divisor's primes must all live inside the number (Fundamental Theorem).
3. But factorise 6^n: $6^n = (2 \times 3)^n = 2^n \times 3^n$. Its only prime factors are 2 and 3 — for every n.
   Why: Powers never create new primes, however large n is.
4. The prime 5 is missing, so 10 can never divide 6^n.
   Why: Contradiction with Step 2 — the supposition collapses.
5. Therefore 6^n can NEVER end with the digit 0, for any natural number n.
   Why: In fact 6^n always ends in 6: 6, 36, 216, 1296, ...

ANSWER: No — 6^n can never end with the digit 0 for any natural number n.

CBSE-STYLE MARKING SCHEME: 2 marks: 1 for linking 'ends in 0' to the prime factor 5, 1 for the factorisation argument.

Ending digits of powers — the whole picture

2^n ends in the cycle 2, 4, 8, 6 — never 0, because 2^n has no prime factor 5.
5^n always ends in 5 — and never 0, because 5^n has no prime factor 2.
10^n always ends in 0 — because 10 = 2 × 5 contains BOTH primes. Ending in 0 needs the pair (2, 5) together; one without the other is never enough.

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Exercise 1.1 Q6 — Proving Numbers Composite

EXERCISE 1.1 • QUESTION 6

Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

THINK FIRST: A composite number is one that has a factor besides 1 and itself. Strategy: do NOT multiply everything out. Instead, spot the common factor in the two terms and factor it out — the two resulting factors expose the number as composite.

1. **First number:** 7 × 11 × 13 + 13. Both terms contain the factor 13: the first term is 13 × (7 × 11) and the second term is 13 × 1.

   Why: Look for a factor shared by BOTH terms of the sum — here it is 13.

2. Factor 13 out: 13 × (7 × 11 + 1) = 13 × (77 + 1) = 13 × 78.

   Why: This is just the distributive law run backwards.

3. Now the number is written as 13 × 78, where BOTH factors are greater than 1 and smaller than the number itself (the number is 1014).

   Why: A number expressed as a product of two smaller factors greater than 1 is composite by definition.

4. Hence 7 × 11 × 13 + 13 is composite. (Its full factorisation is $2 \times 3 \times 13^2$, if ever needed.)

ANSWER: 7 × 11 × 13 + 13 = 13 × 78 — composite.

CBSE-STYLE MARKING SCHEME: 1 mark for this number — awarded for the factorisation, not for the final value.

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Exercise 1.1 Q6 (continued) — The Second Number

EXERCISE 1.1 • QUESTION 6 — SECOND NUMBER

Explain why 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 is a composite number.

1. Both terms contain the factor 5: the long product contains × 5 × inside it, and the second term is 5 × 1.
   Why: The shared factor this time is 5, hiding inside the long multiplication.
2. Factor 5 out: 5 × (7 × 6 × 4 × 3 × 2 × 1 + 1) = 5 × (1008 + 1) = 5 × 1009.
   Why: Inside the bracket, the × 5 × has been removed from the product: 7 × 6 × 4 × 3 × 2 × 1 = 1008.
3. Both factors, 5 and 1009, are greater than 1 and smaller than the number itself (which is 5045).
4. Hence 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 is also composite.

ANSWER: 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 = 5 × 1009 — composite. (Together with the first number, Q6 is complete: 1 mark was for each factorisation.)

COMMON MISTAKE: Multiplying everything out first (getting 1014 and 5045) and then hunting for factors. That is the slow road — and for bigger numbers it is a dead end. Factor out FIRST.

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Exercise 1.1 Q7 — Sonia and Ravi

EXERCISE 1.1 • QUESTION 7

There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

THINK FIRST: This is Story Problem 2 wearing sports clothes. They meet at the start when the elapsed time is a whole number of rounds for BOTH — a common multiple of 18 and 12 — and we want the first such time: the LCM.

1. Sonia is back at the start after every multiple of 18 minutes: 18, 36, 54, ...

   Why: Each of her rounds takes exactly 18 minutes.

2. Ravi is back at the start after every multiple of 12 minutes: 12, 24, 36, 48, ...

3. They are at the start TOGETHER at the times common to both lists: 36, 72, ... The first such time is 36 minutes.

   Why: Common multiples of 18 and 12 — and 'again' means the smallest one greater than zero.

4. Confirm by prime factorisation: $18 = 2 \times 3^2$ and $12 = 2^2 \times 3$, so LCM = $2^2 \times 3^2 = 4 \times 9 = 36$.

   Why: The prime method and the listing method agree, as they must.

5. Check the meaning: in 36 minutes Sonia completes 36 ÷ 18 = 2 rounds and Ravi completes 36 ÷ 12 = 3 rounds — both finish a whole number of rounds at the same instant.

   Why: Translating the answer back into the story is the final, often-forgotten step.

ANSWER: They will meet again at the starting point after 36 minutes.

CBSE-STYLE MARKING SCHEME: 3 marks: 1 for recognising LCM, 1 for computing it, 1 for the concluding statement in context.

COMMON MISTAKE: Using HCF here (answer 6) because the numbers 'feel' like a sharing problem. Ask instead: do I need a divisor (sharing/cutting) or a multiple (meeting/repeating)? Meeting = multiples = LCM.

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Exercise 1.2 Q1 — Proving √5 is Irrational

EXERCISE 1.2 • QUESTION 1

Prove that √5 is irrational.

THINK FIRST: Run the 5-beat rhythm of Derivation B with p = 5. Every beat is one step — write them all.

1. Assume, to the contrary, that √5 is rational.
   Why: Proof by contradiction begins by assuming the opposite of the claim.
2. Then √5 = a/b, where a and b are co-prime positive integers and b ≠ 0.
   Why: Reduce the fraction to lowest form — this co-prime condition is what will be contradicted.
3. Square both sides: $5 = a^2/b^2$, so $a^2 = 5b^2$.
   Why: Squaring clears the root.
4. So 5 divides a^2, because $a^2 = 5 \times b^2$.
5. By the lemma (prime p dividing a^2 must divide a), 5 divides a.
   Why: Section 06 proved this lemma from the Fundamental Theorem.
6. Write a = 5c for some integer c. Substitute into $a^2 = 5b^2$: $(5c)^2 = 5b^2$, so $25c^2 = 5b^2$, so $b^2 = 5c^2$.
   Why: Divide $25c^2 = 5b^2$ by 5 to get $b^2 = 5c^2$.
7. So 5 divides b^2, and by the lemma again, 5 divides b.
   Why: The engine fires a second time, now on b.
8. Thus 5 is a common factor of a and b — contradicting Step 2, where a and b were co-prime.
   Why: Every step was forced; the only weak point is the opening assumption.
9. The assumption is false. Therefore √5 is irrational. ∎

ANSWER: √5 is irrational.

CBSE MARKING SCHEME (OFFICIAL PATTERN FOR THIS QUESTION): 3 marks: ½ for the assumption with the co-prime condition, 1 for showing 5 | a, 1 for showing 5 | b, ½ for the contradiction and conclusion.

TOPPER TIP: *Examiners deduct when the co-prime condition is not stated in Step 2 — without it, there is nothing to contradict in Step 8.*

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Exercise 1.2 Q2 — Proving 3 + 2√5 is Irrational

EXERCISE 1.2 • QUESTION 2

Prove that 3 + 2√5 is irrational.

THINK FIRST: New shape, same weapon. Assume it is rational, then use ordinary algebra to drag the √5 out alone. If √5 is forced to equal a rational expression, we have a contradiction — because Q1 just proved √5 irrational.

1. Assume, to the contrary, that 3 + 2√5 is rational.
2. Then 3 + 2√5 = a/b for some integers a and b with b ≠ 0.
   Why: This is simply the definition of a rational number — no co-prime condition is needed for this style of proof.
3. Subtract 3 from both sides: 2√5 = a/b − 3.
   Why: Begin isolating the irrational part.
4. Write the right side as a single fraction: 2√5 = (a − 3b)/b.
5. Divide both sides by 2: √5 = (a − 3b)/(2b).
   Why: Now √5 stands completely alone on the left.
6. The right side, (a − 3b)/(2b), is a ratio of two integers with a non-zero denominator — so it is a rational number.
   Why: a, b integers ⇒ a − 3b and 2b are integers, and 2b ≠ 0 since b ≠ 0.
7. So √5 would be rational — but Exercise 1.2 Q1 proved √5 is irrational. Contradiction.
   Why: The new proof stands on the shoulders of the previous one — that is why the exercise order matters.
8. Therefore our assumption is false: 3 + 2√5 is irrational. ∎

ANSWER: 3 + 2√5 is irrational.

CBSE-STYLE MARKING SCHEME: 3 marks: assumption (1/2), isolating √5 by correct rearrangement (1½), contradiction citing Q1 (1).

TOPPER TIP: *The general law you just used: rational ± irrational = irrational, and (non-zero rational) × irrational = irrational. Section 05's definition card plus this proof pattern settle every question of this family.*

Exercise 1.2 Q3 — Three More Irrationals

EXERCISE 1.2 • QUESTION 3 (i)

Prove that 1/√2 is irrational.

THINK FIRST: Assume it is rational and turn it upside down: then √2 would be a tidy fraction too.

1. Assume, to the contrary, that 1/√2 is rational.
2. Then 1/√2 = a/b for integers a, b with a ≠ 0 and b ≠ 0.
   Why: a ≠ 0 because 1/√2 is not zero.
3. Take reciprocals of both sides: √2 = b/a.
   Why: Flipping a fraction is legal since neither side is zero.
4. b/a is a ratio of integers with non-zero denominator, so √2 would be rational.
5. But √2 is irrational (proved in Section 06, Derivation B). Contradiction.
6. Therefore 1/√2 is irrational. ∎

ANSWER: 1/√2 is irrational.

CBSE-STYLE MARKING SCHEME: 1 mark: the reciprocal step plus citing √2's irrationality.

TOPPER TIP: *The reciprocal trick works for 1/√p for EVERY prime p: if 1/√p were a tidy fraction, flipping it would make √p a tidy fraction too — impossible. One proof, infinitely many numbers.*

Exercise 1.2 Q3 (continued) — Two More Irrationals

EXERCISE 1.2 • QUESTION 3 (ii) and (iii)

Prove that the following are irrationals: (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$

THINK FIRST: Same isolation trick as Q2: rearrange until the root stands alone, then cite Q1 / Section 06.

1. **(ii)** Assume $7\sqrt{5}$ is rational, say $7\sqrt{5} = a/b$ with $b \neq 0$.
2. Divide both sides by 7: $\sqrt{5} = a/(7b)$.
   Why: Isolating the root is the whole game.
3. $a/(7b)$ is rational, so $\sqrt{5}$ would be rational — contradicting Q1. Hence $7\sqrt{5}$ is irrational. ∎
4. **(iii)** Assume $6 + \sqrt{2}$ is rational, say $6 + \sqrt{2} = a/b$ with $b \neq 0$.
5. Subtract 6 from both sides: $\sqrt{2} = a/b - 6 = (a - 6b)/b$.
6. $(a - 6b)/b$ is rational, so $\sqrt{2}$ would be rational — contradicting Section 06. Hence $6 + \sqrt{2}$ is irrational. ∎

ANSWER: (ii) $7\sqrt{5}$ is irrational (iii) $6 + \sqrt{2}$ is irrational

CBSE-STYLE MARKING SCHEME: 1 mark each: rearrangement isolating the root (1/2) and the contradiction citing the known irrational (1/2).

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08 Multiple Solution Methods

8D • CHALLENGE — ONE PROBLEM, TWO ROADS

A mathematical thinker owns more than one road to every answer. Take the pair **48 and 120** and find its HCF by two completely different methods.

Method 1 — Prime factorisation

1. Factorise 48: 48 ÷ 2 = 24; 24 ÷ 2 = 12; 12 ÷ 2 = 6; 6 ÷ 2 = 3; 3 ÷ 3 = 1. So $48 = 2^4 \times 3$.
2. Factorise 120: 120 ÷ 2 = 60; 60 ÷ 2 = 30; 30 ÷ 2 = 15; 15 ÷ 3 = 5; 5 ÷ 5 = 1. So $120 = 2^3 \times 3 \times 5$.
3. HCF: common primes are 2 and 3; take smaller powers: $2^3 \times 3 = 8 \times 3 = \mathbf{24}$.
4. Bonus — the same factorisations also give the LCM: $2^4 \times 3 \times 5 = \mathbf{240}$.
   Why: The prime road hands you HCF and LCM together.

Method 2 — Euclid's division

The second road needs no factorisation at all — only division with remainder, repeated until the remainder is 0.

1. Divide the bigger by the smaller: 120 = 48 × 2 + 24.
   Why: We only care about the remainder, 24.
2. Now divide the previous divisor by the remainder: 48 = 24 × 2 + 0.
   Why: Each step replaces the pair by (divisor, remainder) — the HCF never changes.
3. The remainder is 0, so the last divisor is the HCF: **24**.
   Why: Euclid's method stops exactly when the remainder hits 0.
4. Euclid gives only the HCF. Get the LCM from the identity: LCM = (48 × 120) ÷ 24 = 5760 ÷ 24 = **240**.
   Why: Derivation A (Section 06) is the bridge between the two methods.

TOPPER TIP: *Same HCF (24), same LCM (240) — by two completely different roads. In the exam, choose the road that fits the numbers in front of you.*

Multiple Methods — When to Use Which?

When to use which?

Prime factorisation — best when numbers are small or medium, and when you need HCF *and* LCM together. It is also the method NCERT Exercise 1.1 asks for by name.
Euclid's division — best for very large numbers whose factors are hard to spot (try factorising 7429 again!). It needs no factorisation at all — just division with remainder. Euclid works on two numbers at a time, and it extends to three or more numbers by **chaining**: HCF(a, b, c) = HCF(HCF(a, b), c).

Euclid chained — HCF of 12, 15 and 21

Step 1: First find HCF(12, 15): 15 = 12 × 1 + 3; then 12 = 3 × 4 + 0. Remainder 0, so HCF(12, 15) = 3.
Step 2: Now find HCF(3, 21): 21 = 3 × 7 + 0. Remainder 0 immediately, so HCF(3, 21) = 3.
Step 3: Therefore HCF(12, 15, 21) = **3** — matching the prime-factorisation answer of Exercise 1.1 Q3(i). Two roads, one destination.

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09 Interactive Simulations & Hands-on Activities

TRY IT YOURSELF — LEARN BY DOING

Put the pen in your own hand. Complete these three activities before checking the answers at the bottom of the next page.

Activity 1 — Factor-tree challenge

Draw the complete factor tree for each number and write the prime factorisation. The first branch is started for you.

Number	First split	Prime factorisation (you write it)
72	72 = 8 × 9	72 = ...
180	180 = 18 × 10	180 = ...
294	294 = 2 × 147	294 = ...

Activity 2 — Plot it on the number line

On a hand-drawn number line from 0 to 3, mark these five numbers as accurately as you can: 3/2, √2, 7/3, √5, 5/2. Use the approximations √2 ≈ 1.414 and √5 ≈ 2.236. Then answer: which is larger, √5 or 7/3? How does the picture prove it?

KALVIYOGI DIGITAL COMPANION — CHAPTER 1: REAL NUMBERS
Video Lessons: every section of this chapter, explained by a teacher. **Simulations:** Number-Line Explorer (drag √2 and √5 onto the line and watch their decimals grow) and the Factor-Tree Simulator. **Online Tests:** chapter tests with instant marking.
QR code and link activate with the KalviYogi app/website at print.

Activity 3 — The HCF × LCM verification lab

For each pair, find the HCF and LCM yourself, then complete the last two columns. The final column must match the product column — if it doesn't, your HCF or LCM is wrong. The lab catches its own errors!

Pair	HCF	LCM	HCF × LCM	Product of the pair	Match?
16 and 20				320	☐
25 and 40				1000	☐
14 and 35				490	☐

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Activity Answers — Check After Trying

Activity 1: $72 = 2^3 \times 3^2$ ($72 = 8 \times 9 = 2^3 \times 3^2$). $180 = 2^2 \times 3^2 \times 5$. $294 = 2 \times 3 \times 7^2$ ($294 = 2 \times 147 = 2 \times 3 \times 49$).

Activity 2: Order on the line: $\sqrt{2}$ (1.414) < 3/2 (1.5) < $\sqrt{5}$ (2.236) < 7/3 (2.333) < 5/2 (2.5). So **7/3 is larger than $\sqrt{5}$** — its point lies to the right.

Activity 3: 16 and 20: HCF 4, LCM 80, $4 \times 80 = 320$. 25 and 40: HCF 5, LCM 200, $5 \times 200 = 1000$. 14 and 35: HCF 7, LCM 70, $7 \times 70 = 490$. All three match.

TOPPER TIP: If your lab row did not match, the error is almost always a missed prime in one factorisation. Re-run the ladder for that number — do not just adjust the answer.

DIGITAL CHALLENGE: In the Factor-Tree Simulator (see the KalviYogi Digital Companion box in this section), try to find a factorisation of 360 DIFFERENT from $2^3 \times 3^2 \times 5$. You will fail — and that failure IS the Fundamental Theorem.

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10 Common Mistakes

LEARN FROM OTHER STUDENTS' ERRORS — BEFORE THEY BECOME YOURS

MISTAKE 1 — Using HCF × LCM = a × b for THREE numbers.

Wrong way: For 12, 15, 21, writing "HCF × LCM = 12 × 15 × 21, so if HCF = 3 then LCM = 1260."

Why wrong: The identity was derived (Section 06) from min + max = sum of TWO exponents. With three exponents, the middle value is lost.

Right way: The identity is for exactly two numbers. For three numbers, build the LCM directly from prime powers: $2^2 \times 3 \times 5 \times 7 = 420$.

MISTAKE 2 — Stopping the factorisation at a composite number.

Wrong way: 156 = 12 × 13, full stop.

Why wrong: 12 is composite ($12 = 2^2 \times 3$). The answer is not yet in primes, so HCF/LCM read from it will be wrong.

Right way: Continue until every factor is prime: $156 = 2^2 \times 3 \times 13$. If any factor can be split, split it.

MISTAKE 3 — Writing "a = 5c" from $a^2 = 5b^2$ without justification.

Wrong way: "$a^2 = 5b^2$, so a = 5c" — jumping straight to the conclusion.

Why wrong: The step uses TWO facts: 5 divides a^2 (from the equation), and the lemma that a prime dividing a^2 divides a. Skipping them loses the method mark — and hides the logic from yourself.

Right way: Write both links: "$5 \mid a^2 \Rightarrow 5 \mid a$ (lemma) $\Rightarrow a = 5c$."

MISTAKE 4 — Forgetting the co-prime condition in an irrationality proof.

Wrong way: "Let $\sqrt{5} = a/b$. Then $a^2 = 5b^2$ … so 5 divides a and b. Hence irrational."

Why wrong: "5 divides both" is only a contradiction if a and b were promised to share no common factor. Without the promise, there is no contradiction and the proof proves nothing.

Right way: Always open with: "$\sqrt{5} = a/b$ where a, b are **co-prime**." Then the final line lands: "… contradicting that a and b are co-prime."

Common Mistakes 5–6

MISTAKE 5 — Choosing LCM for a 'largest equal pieces' problem.
Wrong way: Ribbons of 96 cm and 120 cm are cut into equal pieces of the greatest possible length; a student computes LCM = 480 cm — longer than both ribbons!
Why wrong: A piece length must DIVIDE both ribbons. LCM is a multiple — it is bigger than the ribbons and cannot be cut out of them.
Right way: Cutting/sharing/packing into equal largest parts = HCF (here 24 cm). Meeting/repeating together = LCM. Let the story's action decide.

MISTAKE 6 — "√2 = 1.414, so it is rational because I can write its decimal."
Wrong way: Treating a rounded approximation as the number itself.
Why wrong: 1.414 is not √2; it is a terminating decimal that sits very close to √2. The true decimal of √2 never terminates and never repeats — that is exactly what makes it irrational.
Right way: Judge rationality by the DEFINITION (can it be written as p/q?), never by a calculator display. Section 06 settles √2 by proof, not by approximation.

TOPPER TIP: Before submitting any answer in this chapter, run the three-point check: (1) Is every factor prime? (2) Did I use HCF/LCM for the right story-signal? (3) In a proof, did I state the co-prime condition and reach an explicit contradiction?

PIN POINT: Most errors in this chapter are one of these six — run your finished answer past this list before you submit.

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11 Graded Practice Problems

FIVE LEVELS — CLIMB ONE LEVEL AT A TIME • ANSWERS IN THE ANSWER KEY

Rules of the climb: finish a level with at most one error before moving up. Write full steps for Level 2 onwards — answers alone earn nothing in the board exam.

LEVEL 1 — FOUNDATION (direct concepts)

L1.1 Express 225 as a product of prime factors.

L1.2 Express 360 as a product of prime factors.

L1.3 Find the HCF of 18 and 48.

L1.4 Find the LCM of 14 and 20.

L1.5 Find the HCF and the LCM of 9 and 25. What special thing do you notice about the HCF?

L1.6 Using prime factorisation, find the HCF of 24 and 36.

LEVEL 2 — SKILL BUILDER (textbook standard)

L2.1 Find the HCF and LCM of 90 and 144, and verify that HCF × LCM = product of the two numbers.

L2.2 Find the HCF of 96 and 404 by prime factorisation. Hence, find their LCM.

L2.3 Find the HCF and LCM of 8, 36 and 90 by the prime factorisation method.

L2.4 Find the HCF and LCM of 867 and 255.

L2.5 Check whether 12^n can end with the digit 0 or the digit 5 for any natural number n. Give reasons.

L2.6 Explain why 3 × 5 × 7 + 7 is a composite number.

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Graded Practice — Levels 3, 4 and 5

LEVEL 3 — APPLICATION (real-life problems)

L3.1 Two traffic lights at a crossing change every 48 seconds and every 72 seconds. They change together at 7:00 a.m. At what time will they next change together?

L3.2 A hall floor is 18 m long and 12 m wide. Square tiles of the largest possible size are to be laid on it without cutting any tile. Find the side length of each tile and the number of tiles needed.

L3.3 A school takes 156 Class-9 students and 234 Class-10 students on a field trip. Each bus carries students of only one class, every bus carries the same number of students, and that number is as large as possible. How many students ride in each bus, and how many buses are needed in all?

L3.4 Three bells toll at intervals of 9, 12 and 15 minutes. They toll together at 8:00 a.m. When will they next toll together?

LEVEL 4 — ADVANCED REASONING (multi-concept)

L4.1 Two numbers are in the ratio 3 : 4 and their LCM is 180. Find the two numbers and their HCF.

L4.2 Find the prime factorisation of $5 \times 7 \times 11 + 11$, and hence explain why the number is composite.

L4.3 Prove that the product of any three consecutive positive integers is always divisible by 6. (Hint: among three consecutive integers, one is a multiple of 3 and at least one is even.)

LEVEL 5 — MATHEMATICAL GENIUS (olympiad flavour)

L5.1 A sweet seller packs ALL his sweets in groups of 12, or all in groups of 16, or all in groups of 18 — and in every case exactly 5 sweets are left unpacked, with at least one full group formed each time. What is the smallest possible number of sweets he has?

L5.2 Find the smallest number that leaves a remainder of 1 when divided by 5, by 6 or by 7, and is exactly divisible by 11.

> **TOPPER TIP:** *Stuck at a level? The worked examples of Section 07 contain a twin of almost every question here. Find the twin, study its steps, close the book, and try again.*

12 Mathematical Reasoning

JUSTIFY AND PROVE — FULL SOLUTIONS

REASONING 1

Prove that √3 is irrational.

THINK FIRST: Compare this proof with Ex 1.2 Q1 (√5) line by line. Only the prime changed — the logic is one single machine.

1. Assume, to the contrary, that √3 is rational.
   Why: The 5-beat rhythm of Section 06, now with p = 3.
2. Then √3 = a/b, where a and b are co-prime positive integers and b ≠ 0.
3. Square both sides: $3 = a^2/b^2$, so $a^2 = 3b^2$.
4. So 3 divides a^2, since $a^2 = 3 \times b^2$.
5. By the lemma, 3 divides a. Write a = 3c for some integer c.
   Why: Prime dividing the square must divide the number itself.
6. Substitute: $(3c)^2 = 3b^2$, so $9c^2 = 3b^2$, so $b^2 = 3c^2$.
   Why: Divide $9c^2 = 3b^2$ by 3.
7. So 3 divides b^2, and by the lemma again, 3 divides b.
8. Now 3 divides both a and b, contradicting the co-prime condition of Step 2.
9. The assumption is false. Therefore √3 is irrational. ∎

ANSWER: √3 is irrational.

Reasoning 2 — A Divisibility Proof by Cases

REASONING 2

Prove that the product of any two consecutive positive integers is always divisible by 2.

THINK FIRST: When a statement says 'any' or 'always', examples can only support it — never prove it. Cases or algebra must do the work.

1. Let the two consecutive integers be n and n + 1, where n is any positive integer.
   Why: Naming the unknown is always Step 1 of a general proof — we must prove it for EVERY n at once.
2. Every positive integer is either even or odd, so exactly two cases are possible. We settle both.
   Why: A proof by cases is complete only when the cases cover everything and each case is finished.
3. **Case 1: n is even.** Then n = 2k for some integer k, so n(n + 1) = 2k(n + 1) — visibly twice an integer, hence divisible by 2.
   Why: The factor n itself supplies the 2.
4. **Case 2: n is odd.** Then n + 1 is even (odd + 1 = even), so n + 1 = 2k for some integer k, and n(n + 1) = n × 2k — again twice an integer, hence divisible by 2.
   Why: This time the OTHER factor supplies the 2. One of the two neighbours is always even.
5. In both cases n(n + 1) is divisible by 2, and there are no other cases.
6. Therefore the product of any two consecutive positive integers is always divisible by 2. ∎
   Why: Check with examples: 7 × 8 = 56, 12 × 13 = 156 — both even, as proved.

ANSWER: n(n + 1) is always divisible by 2.

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Reasoning 3 — A Stronger Divisibility Proof

REASONING 3

Prove that $n^3 - n$ is divisible by 6 for every positive integer n.

THINK FIRST: Notice the ladder of ideas: factorise → recognise consecutive integers → use remainders → combine co-prime divisors. Each rung is a tool you now own.

1. Factorise the expression: $n^3 - n = n(n^2 - 1) = n(n - 1)(n + 1)$.

   Why: Use the identity $a^2 - 1 = (a - 1)(a + 1)$ with $a = n$.

2. Reorder the factors: $n^3 - n = (n - 1) \times n \times (n + 1)$ — a product of THREE consecutive integers.

   Why: The factorisation has revealed hidden structure: three neighbours in a row.

3. Among any three consecutive integers, one is divisible by 3: their remainders on division by 3 are 0, 1, 2 in some order, and the one with remainder 0 is a multiple of 3. So 3 divides the product.

   Why: Remainders cycle 0, 1, 2, 0, 1, 2, … — any three consecutive integers must contain a full cycle.

4. Among any two consecutive integers, one is even (Reasoning 2), so among three consecutive integers at least one is even. Hence 2 divides the product.

   Why: We reuse the previous result — proofs build on proofs.

5. So the product is divisible by both 2 and 3. Since 2 and 3 are co-prime, the product is divisible by $2 \times 3 = 6$.

   Why: Divisibility by two co-prime numbers combines into divisibility by their product.

6. Therefore $n^3 - n$ is divisible by 6 for every positive integer n. ∎

   Why: Test: $n = 4$ gives $64 - 4 = 60 = 6 \times 10$; $n = 5$ gives $125 - 5 = 120 = 6 \times 20$.

ANSWER: $n^3 - n$ is always divisible by 6.

13 Real-Life Applications

8D • APPLY — THE CHAPTER AT WORK IN THE WORLD

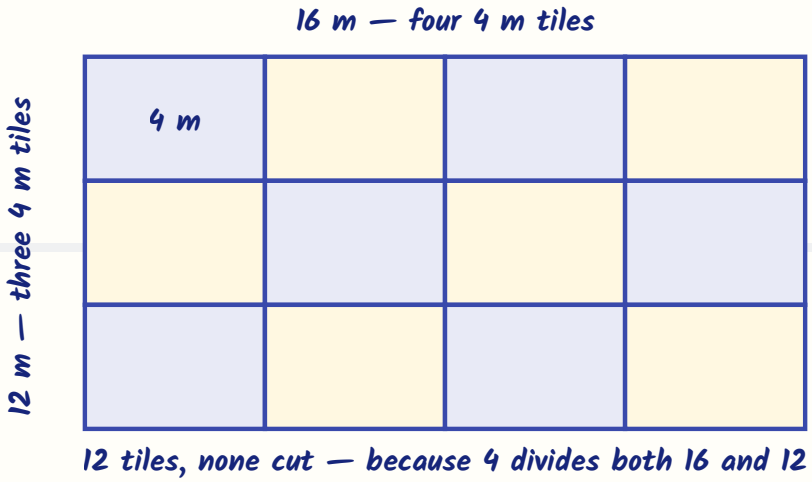
APPLICATION 1 — TILING A FLOOR (HCF)

A rectangular floor is 16 m long and 12 m wide. It is to be covered with square tiles of the largest possible size, without cutting any tile. Find the side of each tile and the number of tiles needed.

THINK FIRST: Cutting/packing/covering with the largest equal size → HCF. Always.

1. The tile side must divide BOTH 16 and 12 exactly — otherwise a tile would have to be cut at the edges.
   Why: No cutting allowed means the side is a common divisor of length and width.
2. 'Largest possible' tells us to take the greatest common divisor: the HCF.
   Why: Same signal as the gift boxes in the story.
3. Factorise: $16 = 2^4$ and $12 = 2^2 \times 3$. HCF = 2^2 = **4**. So each tile is 4 m × 4 m.
   Why: Common prime is 2; take the smaller power.
4. Tiles along the length: 16 ÷ 4 = 4. Tiles along the width: 12 ÷ 4 = 3.
5. Total tiles = 4 × 3 = **12**.
   Why: Multiply rows by columns — and note how the HCF made both divisions exact.

ANSWER: Each tile is 4 m square, and 12 tiles are needed.



Real-Life Application 2 — Buses Leaving Together

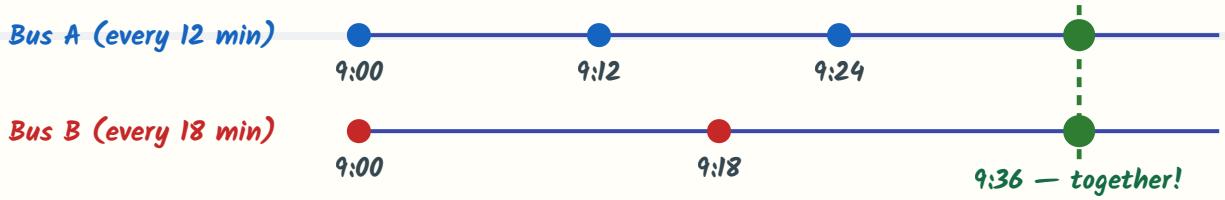
APPLICATION 2 — BUSES LEAVING TOGETHER (LCM)

From a bus stand, Bus A departs every 12 minutes and Bus B departs every 18 minutes. Both departed together at 9:00 a.m. At what time will they next depart together?

THINK FIRST: Repeating events meeting again → LCM. Always.

1. Bus A departs at multiples of 12 minutes after 9:00; Bus B at multiples of 18 minutes.
   Why: Departure times are multiples of the intervals.
2. They coincide at common multiples; the NEXT time together is the smallest one: the LCM.
   Why: Same signal as the suppliers in the story.
3. Factorise: $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$. LCM = $2^2 \times 3^2 = 36$.
   Why: Every prime at its greatest power.
4. 9:00 a.m. + 36 minutes = **9:36 a.m.**
   Why: Translate the number back into the story's language — a time, not just 36.

ANSWER: They will next depart together at 9:36 a.m.



See it: the two rows of dots march at different spacings, and the first place they line up again is 36 minutes — the LCM, drawn as a picture.

Real-Life Application 3 — Distributing Fruit

APPLICATION 3 — DISTRIBUTING FRUITS EQUALLY (HCF)

For a school celebration, 250 oranges and 350 apples are packed into fruit boxes. Each box contains only one kind of fruit, every box contains the same number of fruits, no fruit is left over, and the boxes are as large as possible. Find the number of fruits per box and the total number of boxes.

THINK FIRST: One method, many costumes: sweets, fruits, tiles, ribbons — the HCF does not care what the objects are.

1. The fruits-per-box number must divide both 250 and 350 exactly, and must be the largest such number: the HCF.

   Why: The story signal is identical to Application 1 — only the objects changed.

2. Factorise: $250 = 2 \times 5^3$ and $350 = 2 \times 5^2 \times 7$.

   Why: $250 = 2 \times 125$; $350 = 2 \times 175 = 2 \times 25 \times 7$.

3. Common primes: 2 and 5. Smaller powers: 2^1 and 5^2. HCF = $2 \times 25 = \mathbf{50}$ fruits per box.

4. Orange boxes: $250 \div 50 = 5$. Apple boxes: $350 \div 50 = 7$.

5. Total boxes = $5 + 7 = \mathbf{12}$.

ANSWER: 50 fruits in each box; 12 boxes in all (5 of oranges, 7 of apples).

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Real-Life Application 4 — Cutting Ribbons

APPLICATION 4 — CUTTING RIBBONS (HCF)

A red ribbon is 96 cm long and a gold ribbon is 120 cm long. Both are cut into pieces of equal length, the greatest possible length, with nothing wasted. Find the length of each piece and the total number of pieces.

THINK FIRST: If your answer for a cutting problem is LARGER than the ribbons themselves, you have computed an LCM by mistake — see Common Mistake 5.

1. Each piece length must divide both 96 and 120 exactly — a common divisor; 'greatest possible' makes it the HCF.
   Why: Cutting into equal pieces is the third costume of the same idea.
2. Factorise: $96 = 2^5 \times 3$ and $120 = 2^3 \times 3 \times 5$.
   Why: 96 = 32 × 3; 120 = 8 × 15.
3. Common primes: 2 and 3. Smaller powers: 2^3 and 3^1. HCF = 8 × 3 = **24** cm per piece.
4. Red pieces: 96 ÷ 24 = 4. Gold pieces: 120 ÷ 24 = 5.
5. Total pieces = 4 + 5 = **9**.

ANSWER: Each piece is 24 cm long; there are 9 pieces in all (4 red, 5 gold).

Section 13 in one breath

Four stories, two ideas. Tiling, distributing and cutting all asked for a **divisor** — the greatest one (HCF). Buses departing together asked for a **multiple** — the smallest common one (LCM). When you meet a new story, do not ask "which formula?" Ask: **"Is the answer a divisor of the given numbers, or a multiple of them?"** The story always tells you.

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14 Competency-Based Questions

CASE STUDY & ASSERTION-REASON — BOARD EXAM STYLE, FULLY SOLVED

Case Study — The School Garden Project. *Green Valley School is building a kitchen garden. The gardening club has received* ***126 rose saplings*** *and* ***210 marigold saplings****. The club decides that in each row, only one kind of sapling will be planted; every row must contain the* ***same number of saplings****; all saplings must be planted; and to keep the rows easy to maintain, the number of saplings in each row should be the* ***maximum possible****.*

CASE QUESTION (i)

What is the maximum number of saplings that can be planted in each row?

1. The row size must divide both 126 and 210 exactly, and must be maximum: the HCF of 126 and 210.

   Why: Recognise the story signal first — equal rows, nothing left, maximum size.

2. Factorise 126: 126 ÷ 2 = 63; 63 ÷ 3 = 21; 21 ÷ 3 = 7; 7 ÷ 7 = 1. So $126 = 2 \times 3^2 \times 7$.

3. Factorise 210: 210 ÷ 2 = 105; 105 ÷ 3 = 35; 35 ÷ 5 = 7; 7 ÷ 7 = 1. So $210 = 2 \times 3 \times 5 \times 7$.

4. Common primes: 2, 3 and 7. Smaller powers: 2^1, 3^1, 7^1. HCF = 2 × 3 × 7 = **42**.

ANSWER: 42 saplings in each row.

CASE QUESTION (ii)

How many rows of each kind will the garden have, and how many rows in total?

1. Rose rows = 126 ÷ 42 = **3**.

   Why: Divide each stock by the row size.

2. Marigold rows = 210 ÷ 42 = **5**.

3. Total rows = 3 + 5 = **8**.

ANSWER: 3 rows of roses and 5 rows of marigolds — 8 rows in all.

Competency — Case (continued) and Assertion–Reason

CASE QUESTION (iii)

The rose bed is watered every 6 days and the marigold bed every 8 days. If both beds were watered today, after how many days will both be watered on the same day again?

1. Watering days are multiples of 6 and of 8; the first common day is the LCM.
   Why: 'Together again' — the LCM signal.
2. $6 = 2 \times 3$ and $8 = 2^3$, so LCM = $2^3 \times 3 =$ **24**.

ANSWER: Both beds will be watered together again after 24 days.

CASE QUESTION (iv)

Verify the identity HCF × LCM = product of the numbers for 126 and 210.

1. From part (i), HCF = 42. Find the LCM from the factorisations: every prime at its greatest power: $2 \times 3^2 \times 5 \times 7 = 630$.
   Why: $126 = 2 \times 3^2 \times 7$ and $210 = 2 \times 3 \times 5 \times 7$ — collect the greatest powers.
2. HCF × LCM = 42 × 630 = 26460.
3. Product of the numbers: 126 × 210 = 26460.
4. 26460 = 26460 — the identity holds, as Derivation A guarantees for any two numbers.

ANSWER: HCF × LCM = 26460 = 126 × 210. Verified.

Assertion–Reason (choose: (a) both true, R explains A (b) both true, R does not explain A (c) A true, R false (d) A false, R true)

AR 1. Assertion (A): The HCF of 34 and 51 is 17. **Reason (R):** The HCF of two numbers is the product of the smallest powers of their common prime factors.
Solution: $34 = 2 \times 17$ and $51 = 3 \times 17$; the only common prime is 17, at power 1, so HCF = 17 — A is true. R states the exact rule used, and it is true. **Answer: (a)** — both true, and R is the correct explanation of A.

AR 2. Assertion (A): $\sqrt{2}$ is an irrational number. **Reason (R):** If a prime p divides a^2, then p divides a.
Solution: A is true (Derivation B, Section 06). R is true (the lemma, proved in Section 06) and

Assertion–Reason 3–6

Same choices: (a) both true, R explains A (b) both true, R does not explain A (c) A true, R false (d) A false, R true

AR 3. A: The LCM of 12, 15 and 21 is 420. **R:** The LCM of numbers is the product of the greatest powers of all primes occurring in the numbers.
Solution: $12 = 2^2 \times 3$, $15 = 3 \times 5$, $21 = 3 \times 7$. Greatest powers: $2^2 \times 3 \times 5 \times 7 = 420$ — A true, by exactly the rule R states. **Answer: (a).**

AR 4. A: 6^n cannot end with the digit 0 for any natural number n. **R:** $6^n = 2^n \times 3^n$, and a number ends in 0 only if its prime factorisation contains the prime 5.
Solution: Ending in 0 means divisible by $10 = 2 \times 5$, needing a factor 5; 6^n never has one — A true, and R is precisely why. **Answer: (a).**

AR 5. A: The HCF of 8, 9 and 25 is 1. **R:** 8, 9 and 25 are all prime numbers.
Solution: $8 = 2^3$, $9 = 3^2$, $25 = 5^2$ share no prime, so HCF = 1 — A is true. But none of 8, 9, 25 is prime — R is false. **Answer: (c).**

AR 6. A: $3 \times 5 \times 7 + 7$ is a composite number. **R:** $3 \times 5 \times 7 + 7 = 7 \times (15 + 1) = 7 \times 16$.
Solution: R shows the number as a product of two numbers both greater than 1, which is exactly what makes A true. **Answer: (a).**

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Assertion–Reason 7–10

Assertion–Reason (continued)

AR 7. A: If HCF(16, x) = 4 and LCM(16, x) = 48, then x = 12. **R:** For any two positive integers, HCF × LCM = product of the numbers.
Solution: By R: 4 × 48 = 16 × x, so x = 192 ÷ 16 = 12 — A true, and R is the tool that produces it. **Answer: (a).**

AR 8. A: $1/\sqrt{2}$ is an irrational number. **R:** The reciprocal of an irrational number is irrational.
Solution: $\sqrt{2}$ is irrational, and taking a reciprocal cannot create a terminating or repeating pattern (Section 07, Q3(i)) — A true, explained by R. **Answer: (a).**

AR 9. A: HCF(336, 54) × LCM(336, 54) = 336 × 54. **R:** The product of two numbers equals their HCF times their LCM.
Solution: HCF = 6 and LCM = 3024 (Exercise 1.1 Q2(iii)); 6 × 3024 = 18144 = 336 × 54 — A true by R. **Answer: (a).**

AR 10. A: 12^n can end with the digit 5 for some natural number n. **R:** $12^n = 2_2^n \times 3^n$.
Solution: R is true. But ending in 5 needs divisibility by 5, and 12^n has no prime factor 5 — its last digit is always even. A is false. **Answer: (d).**

TOPPER TIP: *In Assertion–Reason, mark the truth of A and R SEPARATELY first. Only if both are true, ask the third question: does R actually explain A?*

Case Study 2 — The Morning Bakery

Case Study 2. *Sunrise Bakery bakes* ***240 vanilla cupcakes*** *and* ***300 chocolate cupcakes*** *every morning. For delivery, each box must hold only one kind of cupcake; every box must hold the* ***same number*** *of cupcakes; no cupcake may be left over; and the bakery wants the* ***largest possible*** *box size, so that it buys as few boxes as possible.*

CASE 2 (i)

What is the largest number of cupcakes that each box can hold?

1. The box size must divide both 240 and 300 exactly, and be maximum: HCF(240, 300).
   Why: Equal boxes, nothing left, largest size — the HCF signal.
2. $240 = 2^4 \times 3 \times 5$ (240 ÷ 2 = 120, ÷ 2 = 60, ÷ 2 = 30, ÷ 2 = 15, ÷ 3 = 5, ÷ 5 = 1).
3. $300 = 2^2 \times 3 \times 5^2$ (300 ÷ 2 = 150, ÷ 2 = 75, ÷ 3 = 25, ÷ 5 = 5, ÷ 5 = 1).
4. Common primes at smallest powers: $2^2 \times 3 \times 5 =$ **60**.

ANSWER: 60 cupcakes in each box.

CASE 2 (ii)

How many boxes does the bakery need in all?

1. Vanilla boxes = 240 ÷ 60 = **4**.
2. Chocolate boxes = 300 ÷ 60 = **5**.
3. Total = 4 + 5 = **9 boxes**.

ANSWER: 9 boxes in all (4 vanilla + 5 chocolate).

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Case Study 2 (continued)

CASE 2 (iii)

Find the LCM of 240 and 300, and verify that HCF × LCM = 240 × 300.

1. LCM: every prime at its greatest power: $2^4 \times 3 \times 5^2 = 16 \times 3 \times 25 =$ **1200**.
2. HCF × LCM = 60 × 1200 = 72000.
3. 240 × 300 = 72000. Equal — verified.

ANSWER: LCM = 1200, and 60 × 1200 = 72000 = 240 × 300. Verified.

CASE 2 (iv)

The vanilla van visits the bakery every 6 days and the chocolate van every 8 days. Both visited today. After how many days will both vans visit on the same day again?

1. Joint visits happen on common multiples of 6 and 8; the first is the LCM.
   Why: 'Together again' – LCM.
2. $6 = 2 \times 3$, $8 = 2^3$, so LCM = $2^3 \times 3 =$ **24**.

ANSWER: Both vans will visit together again after 24 days.

Case Study 3 — Metro City Transport

Case Study 3. *At Central Station, a red bus leaves every* ***15 minutes*** *and a blue bus every* ***25 minutes****. Both buses left together at 7:00 a.m. Across town, a community hall measuring* ***15 m × 20 m*** *is to be tiled with the* ***largest possible square tiles****, with no cutting of tiles.*

CASE 3 (i)

After how many minutes will the two buses next leave together, and at what time?

1. Joint departures are common multiples of 15 and 25; the first is the LCM.
2. $15 = 3 \times 5$ and $25 = 5^2$, so LCM = $3 \times 5^2 = \mathbf{75}$.
3. 7:00 a.m. + 75 minutes = **8:15 a.m.**

ANSWER: The buses leave together again after 75 minutes, at 8:15 a.m.

CASE 3 (ii)

Find the HCF of 15 and 25, and verify HCF × LCM = 15 × 25.

1. Common prime: 5, at its smallest power 5^1. HCF = **5**.
2. HCF × LCM = 5 × 75 = 375.
3. 15 × 25 = 375. Equal — verified.

ANSWER: HCF = 5, and 5 × 75 = 375 = 15 × 25. Verified.

Case Study 3 (continued)

CASE 3 (iii)

What is the side of the largest square tile that tiles the hall exactly?

1. The tile side must divide both 15 m and 20 m exactly, and be maximum: HCF(15, 20).
   Why: Largest square, no cutting – HCF, exactly like Application 1.
2. $15 = 3 \times 5$ and $20 = 2^2 \times 5$. Common prime: 5. HCF = **5**.

ANSWER: The largest square tile has side 5 m.

CASE 3 (iv)

How many such tiles are needed to cover the hall?

1. Along the length: $20 \div 5 = 4$ tiles. Along the width: $15 \div 5 = 3$ tiles.
2. Total = 4×3 = **12 tiles**.

ANSWER: 12 tiles of side 5 m.

Case Study 4 — The School Library

Case Study 4. *A school library receives* ***432 mathematics books*** *and* ***576 science books****. The librarian stacks books of one subject in piles; every pile must have the* ***same number of books****, no book may be left over, and she wants the* ***tallest possible piles*** *so that the shelf count stays small.*

CASE 4 (i)

What is the greatest number of books in each pile?

1. The pile size divides both 432 and 576, maximum: HCF(432, 576).
2. $432 = 2^4 \times 3^3$ and $576 = 2^6 \times 3^2$.
   Why: Check: 16 × 27 = 432 and 64 × 9 = 576.
3. Common primes at smallest powers: $2^4 \times 3^2 = 16 \times 9 = \mathbf{144}$.

ANSWER: 144 books in each pile.

CASE 4 (ii)

How many piles of each subject are there?

1. Mathematics piles = 432 ÷ 144 = **3**.
2. Science piles = 576 ÷ 144 = **4**. Total piles = 7.

ANSWER: 3 piles of mathematics and 4 piles of science.

Case Study 4 (continued)

CASE 4 (iii)

Find LCM(432, 576) and verify the product identity.

1. Greatest powers: $2^6 \times 3^3 = 64 \times 27 =$ **1728**.
2. HCF × LCM = 144 × 1728 = 248832.
3. 432 × 576 = 248832. Equal — verified.

ANSWER: LCM = 1728, and the identity holds.

CASE 4 (iv)

A donation adds 48 more science books, making 624. What is the new greatest pile size, and how many piles are there now?

1. New pair: 432 and 624 = $2^4 \times 3 \times 13$.
   Why: 624 = 16 × 39 = $2^4 \times 3 \times 13$.
2. Common primes at smallest powers: $2^4 \times 3 =$ **48**.
3. Piles: 432 ÷ 48 = 9 mathematics piles; 624 ÷ 48 = 13 science piles. Total = **22 piles**.

ANSWER: New pile size 48 books; 9 + 13 = 22 piles.

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Case Study 5 — The Festival of Lights

Case Study 5. *For the town festival, three strings of lights blink every* ***20 seconds, 30 seconds and 45 seconds****. All three flashed together at the switch-on moment. In the food court, a stall packs* ***150 laddus*** *and* ***225 jalebis*** *into gift packs: each pack holds one sweet only, every pack holds the* ***same number*** *of sweets, nothing is left over, and packs are as* ***large as possible****.*

CASE 5 (i)

After how many seconds will all three light strings flash together again?

1. Joint flashes are common multiples of 20, 30, 45; the first is the LCM.
2. $20 = 2^2 \times 5$, $30 = 2 \times 3 \times 5$, $45 = 3^2 \times 5$. Greatest powers: $2^2 \times 3^2 \times 5 =$ **180**.

ANSWER: All three flash together again after 180 seconds (3 minutes).

CASE 5 (ii)

In the 30 minutes after switch-on, how many times do all three flash together (not counting the switch-on flash itself)?

1. 30 minutes = 1800 seconds.
2. Joint flashes occur every 180 seconds: 1800 ÷ 180 = **10**.

ANSWER: 10 joint flashes in 30 minutes.

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Case Study 5 (continued)

CASE 5 (iii)

What is the largest number of sweets in each gift pack, and how many packs of each sweet are made?

1. Pack size = HCF(150, 225): $150 = 2 \times 3 \times 5^2$, $225 = 3^2 \times 5^2$. Common at smallest powers: $3 \times 5^2 = \mathbf{75}$.
2. Laddu packs = $150 \div 75 = \mathbf{2}$; jalebi packs = $225 \div 75 = \mathbf{3}$.

ANSWER: 75 sweets per pack; 2 laddu packs and 3 jalebi packs.

CASE 5 (iv)

Verify the product identity for 150 and 225.

1. LCM: greatest powers $2 \times 3^2 \times 5^2 = \mathbf{450}$.
2. HCF × LCM = 75 × 450 = 33750.
3. 150 × 225 = 33750. Equal — verified.

ANSWER: 75 × 450 = 33750 = 150 × 225. Verified.

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15 Olympiad & Foundation Challenges

8D • CHALLENGE — FOR STUDENTS WHO FINISH EARLY AND THINK DEEP

CHALLENGE 1

Find the smallest number that leaves a remainder of 1 when divided by 5, by 6 or by 7, and is exactly divisible by 11.

THINK FIRST: Convert a remainder condition into a divisibility condition (subtract the remainder), then let the LCM generate all candidates at once.

1. A number leaving remainder 1 on division by 5, 6 and 7 must be exactly 1 more than a common multiple of 5, 6 and 7.

   Why: Remainder 1 means: subtract 1 and the number becomes divisible by all three.

2. The common multiples of 5, 6, 7 are the multiples of their LCM. Compute it: 5 = 5; 6 = 2 × 3; 7 = 7, so LCM = 2 × 3 × 5 × 7 = 210.

   Why: No prime is shared, so the LCM is the full product.

3. So our number has the form 210k + 1 for some positive integer k: 211, 421, 631, ...

4. We also need 11 to divide 210k + 1. Split 210 using 11: 210 = 11 × 19 + 1, so 210k + 1 = 11 × 19k + (k + 1).

   Why: The first part is automatically divisible by 11 — so everything depends on the small remainder k + 1.

5. Thus 11 must divide k + 1. The smallest possibility is k + 1 = 11, i.e. k = 10.

   Why: Choosing the smallest k gives the smallest number, as asked.

6. The number is 210 × 10 + 1 = **2101**.

7. Full check: 2101 = 11 × 191 exactly; 2101 ÷ 5 = 420 remainder 1; 2101 ÷ 6 = 350 remainder 1; 2101 ÷ 7 = 300 remainder 1. All four conditions hold.

   Why: A challenge answer is not finished until it is checked against EVERY condition.

ANSWER: The smallest such number is 2101.

Olympiad Challenge 2

CHALLENGE 2

Find the greatest number that divides 245 and 2053, leaving a remainder of 5 in each case.

THINK FIRST: Remainder problems: first remove the remainder, then the problem becomes a familiar HCF/LCM question in disguise.

1. Call the required number d. Dividing 245 by d leaves remainder 5, so d divides 245 − 5 = 240 exactly.
   Why: Subtracting the remainder converts the problem into an exact-division problem.
2. Similarly, d divides 2053 − 5 = 2048 exactly.
3. So d is a common divisor of 240 and 2048; the GREATEST such number is their HCF.
   Why: 'Greatest' + 'common divisor' = HCF, as always.
4. Factorise: $240 = 2^4 \times 3 \times 5$. And $2048 = 2^{11}$ (keep doubling: 2, 4, 8, ..., 1024, 2048 — eleven 2s).
   Why: 2048 is a pure power of 2 — recognising powers of 2 up to $2^{11} = 2048$ is a handy olympiad skill.
5. The only common prime is 2; take the smaller power: HCF = 2^4 = **16**.
6. Verify: $245 = 16 \times 15 + 5$ and $2053 = 16 \times 128 + 5$. Both leave remainder 5, and no larger number can work, because any such number must divide both 240 and 2048.
   Why: The verification uses the ORIGINAL numbers, not the reduced ones.

ANSWER: The greatest such number is 16.

Olympiad Challenge 3 — The Grand Finale

CHALLENGE 3

Prove that √2 + √3 is irrational. (You may first prove that √6 is irrational.)

THINK FIRST: Big proofs are built from small proved blocks. When a problem looks too hard, ask: what smaller fact would make this easy? Prove THAT first.

1. **Part 1 — a lemma about 6:** if 6 divides a^2, then 6 divides a. Proof: $6 \mid a^2$ means $a^2 = 6 \times$ (an integer), so $2 \mid a^2$ AND $3 \mid a^2$.
   Why: Careful: the Section 06 lemma is about a PRIME dividing a^2. 6 is not prime, so split it into primes 2 and 3 and use the lemma on each.
2. By the prime lemma, $2 \mid a^2 \Rightarrow 2 \mid a$, and $3 \mid a^2 \Rightarrow 3 \mid a$.
   Why: The prime lemma fires twice — once for each prime factor of 6.
3. A number divisible by both 2 and 3 is divisible by 6, because 2 and 3 are co-prime. Hence $6 \mid a$. **Lemma proved.**
4. **Now prove √6 is irrational.** Assume $\sqrt{6} = a/b$ with a, b co-prime. Squaring: $a^2 = 6b^2$, so 6 divides a^2.
   Why: We have built the tool; now use it.
5. By the lemma, 6 divides a. Write $a = 6c$. Substitute: $36c^2 = 6b^2$, so $b^2 = 6c^2$.
6. So 6 divides b^2, and by the lemma again, 6 divides b.
7. Now 6 divides both a and b — contradicting co-prime. So √6 is irrational. ∎
8. **Part 2 — the main proof.** Assume, to the contrary, that √2 + √3 is rational; call it r.
9. Square both sides: $r^2 = (\sqrt{2} + \sqrt{3})^2 = 2 + 2\sqrt{6} + 3 = 5 + 2\sqrt{6}$.
   Why: Use $(x + y)^2 = x^2 + 2xy + y^2$ with $xy = \sqrt{2} \times \sqrt{3} = \sqrt{6}$.
10. Rearrange: $2\sqrt{6} = r^2 - 5$, so $\sqrt{6} = (r^2 - 5)/2$.
    Why: Isolate the root — the same finishing move as Ex 1.2 Q2 and Q3.
11. Since r is rational, r^2 is rational, so $(r^2 - 5)/2$ is rational. That would make √6 rational.
12. But Part 1 proved √6 is irrational. Contradiction.
13. Therefore √2 + √3 is irrational. ∎

Olympiad Challenges 4–6

CHALLENGE 4

Two positive integers have HCF 12 and product 2592. Find their LCM, and find ALL possible pairs of such numbers.

1. LCM = product ÷ HCF = 2592 ÷ 12 = **216**.
   Why: The identity, used in reverse.
2. Write the numbers as 12x and 12y with x, y co-prime (the 12 carries the whole HCF).
   Why: Any common factor inside x and y would enlarge the HCF beyond 12.
3. Product: 12x × 12y = 144xy = 2592, so xy = 18.
4. Co-prime factor pairs of 18: (1, 18) and (2, 9). The pair (3, 6) is rejected — 3 and 6 are not co-prime.
   Why: Check pairs against the co-prime condition, not just the product.
5. Pairs: (12 × 1, 12 × 18) = (12, 216) and (12 × 2, 12 × 9) = (24, 108). Check: HCF(24, 108) = 12 and 24 × 108 = 2592. Both work.

ANSWER: LCM = 216. The pairs are (12, 216) and (24, 108).

CHALLENGE 5

Find the largest number which divides 62, 132 and 237 leaving the SAME remainder in each case, and find that remainder.

1. Let the number be d and the common remainder r. Then 62 − r, 132 − r and 237 − r are all divisible by d.
   Why: Subtract the unknown remainder away.
2. Subtract in pairs to eliminate r: 132 − 62 = 70, 237 − 132 = 105, 237 − 62 = 175. So d divides 70, 105 and 175.
   Why: Pairwise differences are divisible by d — the remainder cancels.
3. d is the largest such number: d = HCF(70, 105, 175). 70 = 2 × 5 × 7, 105 = 3 × 5 × 7, 175 = 5^2 × 7. Common: 5 × 7 = **35**.
4. Remainder: 62 ÷ 35 leaves remainder **27**. Check: 132 = 35 × 3 + 27 and 237 = 35 × 6 + 27. Same remainder, as required.

ANSWER: The largest number is 35, leaving remainder 27 each time.

CHALLENGE 6

Prove that 5 − √3 is irrational. (You may use the fact that √3 is irrational.)

1. Assume, to the contrary, that 5 − √3 is rational; call it r.

   Why: The isolation method of Exercise 1.2 Q2, with a minus sign.

2. Then √3 = 5 − r.

   Why: Isolate the root on one side.

3. Since 5 and r are both rational, 5 − r is rational — so √3 would be rational.

4. But √3 is irrational. Contradiction.

5. Therefore 5 − √3 is irrational. ∎

ANSWER: 5 − √3 is irrational.

Olympiad Challenges 7–8

CHALLENGE 7

Find the smallest number which, when divided by 8, 9 and 10, leaves remainders 5, 6 and 7 respectively.

1. Call the number N. The remainders fall short of the divisors by the same amount: 8 − 5 = 3, 9 − 6 = 3, 10 − 7 = 3.

   Why: Compare remainder with divisor FIRST — a constant gap is the key pattern.

2. So N + 3 is divisible by 8, by 9 and by 10: N + 3 is a common multiple of 8, 9, 10.

   Why: Adding the gap completes every division exactly.

3. Smallest such: N + 3 = LCM(8, 9, 10) = $2^3 \times 3^2 \times 5$ = **360**.

4. Therefore N = 360 − 3 = **357**. Check: 357 = 8 × 44 + 5 = 9 × 39 + 6 = 10 × 35 + 7. All three remainders correct.

ANSWER: The smallest such number is 357.

CHALLENGE 8

Find the smallest positive number which leaves remainder 7 when divided by 12 and remainder 10 when divided by 15.

1. Call the number x. Then x − 7 is a multiple of 12, and x − 10 is a multiple of 15.

2. Notice the gaps: 12 − 7 = 5 and 15 − 10 = 5. So x + 5 is a multiple of 12 AND of 15.

   Why: Same trick as Challenge 7 — a constant gap of 5.

3. Smallest: x + 5 = LCM(12, 15) = $2^2 \times 3 \times 5$ = **60**.

4. So x = 60 − 5 = **55**. Check: 55 = 12 × 4 + 7 and 55 = 15 × 3 + 10. Both remainders correct.

ANSWER: The smallest such number is 55.

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Olympiad Challenges 9–10

CHALLENGE 9

Prove that the product of any four consecutive positive integers is divisible by 24.

1. Let the numbers be n, n + 1, n + 2, n + 3. Since 24 = 8 × 3 with 8 and 3 co-prime, it is enough to prove divisibility by 8 and by 3 separately.

   Why: Split the target into co-prime parts – then conquer each part.

2. Divisibility by 3: among any three consecutive integers, one is a multiple of 3 (the residues mod 3 cycle 0, 1, 2). Our four numbers contain three consecutive ones, so 3 divides the product.

3. Divisibility by 8: among the four numbers, two are even — call them 2a and 2a + 2. One of any two consecutive even numbers is a multiple of 4. So the product contains factors 2 × 4 = 8.

   Why: In every pair of consecutive even numbers, exactly one is divisible by 4.

4. The product is divisible by 8 and by 3, and gcd(8, 3) = 1, so it is divisible by 8 × 3 = 24. ∎

ANSWER: The product of four consecutive positive integers is always divisible by 24.

CHALLENGE 10

Find the smallest 4-digit number that is divisible by 12, 15 and 18.

1. The number must be a common multiple of 12, 15 and 18.

2. LCM(12, 15, 18): $12 = 2^2 \times 3$, $15 = 3 \times 5$, $18 = 2 \times 3^2$. Greatest powers: $2^2 \times 3^2 \times 5 = \mathbf{180}$.

3. Multiples of 180: 180, 360, 540, 720, 900, 1080, … The first one with four digits is 180 × 6 = **1080** (since 1000 ÷ 180 = 5.55…, round UP to 6).

   Why: The smallest multiple that reaches 1000 – divide and round up, never down.

4. Check: 1080 ÷ 12 = 90, 1080 ÷ 15 = 72, 1080 ÷ 18 = 60. All exact.

ANSWER: The smallest 4-digit number divisible by 12, 15 and 18 is 1080.

TOPPER TIP: *Notice how many Olympiad problems are just this chapter in disguise: remainders become LCM-minus-a-gap, and 'largest divisor with equal remainders' becomes HCF of differences.*

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16

Chapter Mind Map

THE WHOLE CHAPTER ON ONE PAGE — REVISE FROM HERE

REAL NUMBERS

Heart of the chapter: **Fundamental Theorem of Arithmetic** — every integer greater than 1 is a product of primes in exactly ONE way (apart from order).

Prime Factorisation

Factor tree or division ladder. Divide by the smallest prime; stop at 1. Multiply back to check.

→

HCF

Common primes only, SMALLEST powers. Story signal: largest, equal, cut, pack, no leftovers.

LCM

All primes, GREATEST powers. Story signal: meet again, together, repeat, smallest that fits all.

The Golden Identity

HCF × LCM = a × b — TWO numbers only. Derived from min + max = sum of exponents.

Ending-Digit Proofs

Ending in 0 needs primes 2 AND 5. If 5 is missing from the factorisation, the number can never end in 0. (6^n, 12^n, 8^n)

Composite Proofs

Do NOT multiply out. Factor out the common factor: 7×11×13 + 13 = 13 × 78.

Irrational Numbers — Proof by Contradiction

5 beats: Assume rational a/b, co-prime → Square → p | a^2 so p | a → Substitute, get p | b → Contradiction. Engine: the lemma "p | a^2 ⇒ p | a".

→

The Family

√2, √3, √5 irrational → also 3 + 2√5, 1/√2, 7√5, 6 + √2 — by isolating the root.

Key Results to Quote

HCF(336, 54) = 6, LCM = 3024 • LCM(306, 657) = 22338 from HCF 9 • Sonia–Ravi meet after 36 min • 3825 = $3^2 \times 5^2 \times 17$ • 7429 = 17 × 19 × 23

TOPPER TIP: *Revise actively: cover a box, speak its contents aloud from memory, then uncover and check. A mind map you can REPRODUCE from memory is worth ten readings.*

17 Pin-Point Flash Cards

THE WHOLE CHAPTER IN YOUR POCKET — CUT THEM OUT, CARRY THEM, MASTER THEM

How to use these cards

Cut along the dashed borders and keep the 16 cards in your pocket or a small box. On Smart Memory **Day 3, Day 7 and Day 21**, shuffle the cards and test yourself: read the title, say the pin-point aloud from memory, then turn the card over (flip it in your hand) to check. Any card you miss goes back into the box for tomorrow. Two minutes a day — that is all it takes to keep this chapter alive all year.

CARD 1 — Fundamental Theorem of Arithmetic

Every integer greater than 1 is a product of prime numbers in exactly ONE way (apart from order).

CARD 2 — HCF by primes

Take the COMMON primes, each at its SMALLEST power. HCF = product of the smallest powers.

CARD 3 — LCM by primes

Take ALL primes that appear in either number, each at its GREATEST power.

CARD 4 — HCF × LCM

HCF × LCM = a × b — for TWO numbers only. Never for three.

Flash Cards 5–10

CARD 5 — Three numbers

HCF(a, b, c) = HCF(HCF(a, b), c). Chain it two at a time — Euclid chains the same way.

CARD 6 — The Lemma

If p is PRIME and p divides a^2, then p divides a. The engine of every irrationality proof.

CARD 7 — The famous irrationals

√2, √3 and √5 are IRRATIONAL — proved by contradiction, all with the same 5-beat rhythm.

CARD 8 — Rational + Irrational

A rational number ± an irrational number is ALWAYS irrational. Isolate the root to prove it.

CARD 9 — Products and reciprocals

A non-zero rational × an irrational is irrational. And 1 ÷ (irrational) is irrational — so 1/√2 is irrational.

CARD 10 — Ending in zero

A power ends in 0 only if its primes include 2 AND 5. 6^n has only 2 and 3 — it NEVER ends in 0 (it always ends in 6).

TOPPER TIP: *Say each card aloud once before you test yourself on it — hearing your own voice say the fact makes the memory stick twice as fast.*

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Flash Cards 11–16

CARD 11 — Composite test

If n = a × b with a, b both greater than 1, then n is COMPOSITE. Factor a piece out — never multiply the whole thing out.

CARD 12 — Euclid's division

a = bq + r, with 0 ≤ r < b. Repeat with (b, r) until r = 0 — the last divisor is the HCF.

CARD 13 — Co-prime

HCF(a, b) = 1 means a and b are co-prime. They need NOT be prime themselves: 8 and 9 are co-prime!

CARD 14 — The number 1

1 is NEITHER prime NOR composite. It is the only number with exactly one factor.

CARD 15 — Smallest of all

Smallest prime = 2 (the only even prime). Smallest composite = 4. Smallest odd prime = 3.

CARD 16 — Story signals

Largest / equal groups / cut / pack / distribute → HCF. Meet again / together / repeat / bells → LCM.

MAKE YOUR OWN CARD

Write one pin-point YOU keep forgetting, in your own words. A card you write yourself is the card you remember best.

_____

_____

MAKE YOUR OWN CARD

One more: the mistake you made most often in this chapter, and the one-line rule that fixes it.

_____

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18 Active Recall

8D • RECALL — CLOSE THE BOOK, THEN ANSWER

How this works: close every other page. Answer all ten questions on rough paper, from memory alone. Only then open the Answer Key. Anything you could not recall is exactly what your Day-3 revision must target — write it in the Error Tracker (Section 19).

Ten rapid questions

R1. State the Fundamental Theorem of Arithmetic.

R2. What is the HCF of 12 and 18?

R3. What is the LCM of 12 and 18?

R4. Write 100 as a product of prime factors.

R5. Find the HCF and the LCM of 20 and 28.

R6. Can 8^n end with the digit 0 for any natural number n? Why, or why not?

R7. Is $1/\sqrt{2}$ rational or irrational?

R8. Find the HCF and the LCM of 7, 11 and 13.

R9. If HCF(16, x) = 4 and LCM(16, x) = 48, find x.

R10. Is $7 \times 11 \times 13 + 13$ prime or composite? Justify in one line.

Five recall prompts — say each answer aloud, in a full sentence

P1. Define HCF in one sentence.

P2. Define LCM in one sentence.

P3. State the identity connecting HCF, LCM and two numbers — and state its limit.

P4. Define an irrational number.

P5. State the lemma that powers every irrationality proof in this chapter.

> **TOPPER TIP:** *Scoring: 13–15 correct — chapter mastered, move on. 9–12 — revise the missed topics, retake in 3 days. Below 9 — re-study Sections 05–07, then retake. Record your score in the Mastery Tracker.*

19 Self-Assessment & Mastery Tracker

KALVIYOGI SMART MEMORY SYSTEM — LEARN IT, THEN KEEP IT

Memory fades on a predictable curve — unless you revisit at the right moments. This schedule is your revisit plan for THIS chapter. Tick each day off when done.

Day	Your activity for Chapter 1	Done
Day 0	Pre-learning assessment: attempt the Quick Check (Section 02) and Rapid Questions R1–R10 (Section 18) BEFORE studying. Note your score.	☐
Day 1	Learn the chapter fully (Sections 01–10) and complete Level 1–3 practice.	☐
Day 3	Without-book recall: reproduce the Mind Map (Section 16) on a blank page; redo Rapid Questions; finish Level 4.	☐
Day 7	Independent problem-solving: Level 5 + Olympiad Challenges, timed, book closed. Then take the **Day 7 Mini-Test** (Practice Plus) and run the Flash Cards (Section 17).	☐
Day 14	Mixed practice: re-attempt every question in your Error Tracker; solve Ex 1.1 Q2 and Q3 fresh.	☐
Day 21	Long-term recall test: write the √5 proof and the HCF×LCM derivation from memory, then verify line by line. Then take the **Day 21 Mini-Test** (Practice Plus).	☐
Day 30	Chapter mastery assessment: take the **Day 30 Mastery Test** (Practice Plus) under exam conditions; update your Concept Mastery Score below.	☐

Formula Recall Tracker — tick when you can say it with the book closed

Formula / Statement	Day 1	Day 3	Day 7	Day 21
Fundamental Theorem of Arithmetic	☐	☐	☐	☐
HCF rule (common primes, smallest powers)	☐	☐	☐	☐
LCM rule (all primes, greatest powers)	☐	☐	☐	☐
HCF × LCM = a × b (two numbers only)	☐	☐	☐	☐
Lemma: $p \mid a^2 \Rightarrow p \mid a$	☐	☐	☐	☐
The 5-beat contradiction pattern	☐	☐	☐	☐

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Mastery Tracker — Errors and Concept Scores

Error Tracker — every mistake becomes a lesson

Date	Question	What went wrong	The correct idea

Concept Mastery Score — rate yourself honestly, out of 5

Concept	Score / 5	Status
Prime factorisation (tree and ladder)	___ / 5	☐ Weak ☐ Okay ☐ Mastered
HCF & LCM — two numbers	___ / 5	☐ Weak ☐ Okay ☐ Mastered
HCF & LCM — three numbers	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Ending-digit & composite-number proofs	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Irrationality proofs	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Word problems (choosing HCF vs LCM)	___ / 5	☐ Weak ☐ Okay ☐ Mastered

Learning Time Tracker

Day / Date	Minutes studied	What I studied

Mastery rule: a concept counts as MASTERED only when you score 4 or 5 AND can reproduce it on Day 21 without the book. Anything less — revisit, don't just re-read.

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Practice Plus — 25 Concept MCQs (1–13)

One mark each. Cover the Answer Key and choose the single best option — then check the key, where the tricky ones carry a one-line explanation.

Q1. The HCF of 12 and 18 is
(a) 3 (b) 6 (c) 12 (d) 36

Q2. The LCM of 12 and 18 is
(a) 18 (b) 24 (c) 36 (d) 216

Q3. 140 expressed as a product of primes is
(a) $2 \times 5 \times 14$ (b) $2^2 \times 5 \times 7$ (c) $2^2 \times 35$ (d) $4 \times 5 \times 7$

Q4. The HCF of 7, 11 and 13 is
(a) 0 (b) 1 (c) 7 (d) 1001

Q5. The LCM of 7, 11 and 13 is
(a) 1 (b) 31 (c) 91 (d) 1001

Q6. Which of the following is irrational?
(a) $\sqrt{4}$ (b) 22/7 (c) $\sqrt{9}$ (d) $\sqrt{5}$

Q7. For every natural number n, 6^n always ends with the digit
(a) 0 (b) 2 (c) 6 (d) 5

Q8. If HCF(306, 657) = 9, then LCM(306, 657) is
(a) 2238 (b) 5913 (c) 22338 (d) 657

Q9. Two numbers have product 2366 and HCF 13. Their LCM is
(a) 13 (b) 169 (c) 182 (d) 2366

Q10. Which of the following is a prime number?
(a) 87 (b) 91 (c) 93 (d) 97

Q11. The HCF of $2^3 \times 3^2$ and $2^2 \times 3^3$ is
(a) 6 (b) $2^2 \times 3^2$ (c) $2^3 \times 3^3$ (d) 2×3

Q12. The LCM of 5, 10 and 15 is
(a) 15 (b) 30 (c) 150 (d) 750

Q13. If p is a prime and p divides a^2, then
(a) a divides p (b) p^2 divides a (c) p divides a (d) p divides a + 1

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25 Concept MCQs (14–25)

Q14. The number $7 \times 11 \times 13 + 13$ is
(a) prime (b) composite (c) neither prime nor composite (d) irrational

Q15. The smallest composite number is
(a) 1 (b) 2 (c) 4 (d) 6

Q16. The smallest prime number is
(a) 0 (b) 1 (c) 2 (d) 3

Q17. If three numbers are pairwise co-prime, their HCF is
(a) 0 (b) 1 (c) their product (d) the smallest of them

Q18. The number $3 + \sqrt{5}$ is
(a) rational (b) an integer (c) a natural number (d) irrational

Q19. The HCF of 100 and 125 is
(a) 5 (b) 20 (c) 25 (d) 125

Q20. The LCM of 100 and 125 is
(a) 125 (b) 250 (c) 500 (d) 12500

Q21. Two bells toll every 9 minutes and every 12 minutes. If they toll together now, they will next toll together after
(a) 3 minutes (b) 21 minutes (c) 36 minutes (d) 108 minutes

Q22. The number 1 is
(a) prime (b) composite (c) both prime and composite (d) neither prime nor composite

Q23. Which of the following powers can end with the digit 0?
(a) 4^n (b) 6^n (c) 10^n (d) 12^n

Q24. The HCF of the smallest prime number and the smallest composite number is
(a) 1 (b) 2 (c) 4 (d) 8

Q25. If HCF(a, b) = 1, then LCM(a, b) equals
(a) a + b (b) 1 (c) a × b (d) (a × b)/2

TOPPER TIP: *Scored below 20? Do not just re-read the key — find WHICH pin-point card covers each missed question and revise that card first.*

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Find the Error — Questions 1–5

Each piece of working below contains exactly one planted mistake — the kind real students make in board exams. Spot it before you check the key.

ERROR 1

"HCF(12, 18): $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$. For the HCF, take the GREATEST powers: $2^2 \times 3^2 = 36$. So HCF = 36."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 2

"91 is a prime number, because it is not divisible by 2, 3 or 5."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 3

"6^n can end with the digit 0. Proof: $6 \times 10 = 60$, which ends in 0."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 4

"LCM(4, 6) = $4 \times 6 = 24$, because LCM is always the product of the two numbers."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 5

"Proof that $\sqrt{5}$ is irrational: Let $\sqrt{5} = a/b$. Then $a^2 = 5b^2$, so 5 divides a; write $a = 5c$. Then $b^2 = 5c^2$, so 5 divides b. Hence $\sqrt{5}$ is irrational."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

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Find the Error — Questions 6–10

ERROR 6

"HCF(12, 15, 21) × LCM(12, 15, 21) = 12 × 15 × 21 = 3780. Since LCM = 420, HCF = 3780 ÷ 420 = 9."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 7

"3825 = 3 × 5^2 × 17."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 8

"1/√2 = √2/2. Since √2/2 is written as a fraction, 1/√2 is a rational number."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 9

"Sonia takes 18 minutes and Ravi 12 minutes for one round. They meet at the starting point after HCF(18, 12) = 6 minutes."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 10

"√2 + √3 is irrational, because √2 is irrational, √3 is irrational, and the sum of two irrational numbers is always irrational."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

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Previous Year Questions — Real CBSE Board Questions (1)

These are genuine CBSE board questions, with the year and paper verified. Solve them exactly as you would in the exam hall — full steps earn full marks.

CBSE 2018 (30/1) — 1 MARK

What is the HCF of the smallest prime number and the smallest composite number?

1. The smallest prime number is 2, and the smallest composite number is 4.
   Why: Two facts straight off Flash Cards 14 and 15.
2. HCF(2, 4) = 2, since 2 divides 4.

ANSWER: HCF = 2.

CBSE 2018 (30/1) — 3 MARKS

Find the HCF and LCM of 404 and 96, and verify that HCF × LCM = product of the two numbers.

1. Factorise 404: 404 ÷ 2 = 202; 202 ÷ 2 = 101; 101 is prime. So $404 = 2^2 \times 101$.
2. Factorise 96: divide by 2 five times (96, 48, 24, 12, 6, 3), then by 3. So $96 = 2^5 \times 3$.
3. HCF: the only common prime is 2, at its smallest power 2^2 = **4**.
4. LCM: greatest powers $2^5 \times 3 \times 101 = 32 \times 303$ = **9696**.
5. Verify: HCF × LCM = 4 × 9696 = 38784. Product: 404 × 96 = 38784. Equal — verified.

ANSWER: HCF = 4, LCM = 9696, and 4 × 9696 = 38784 = 404 × 96.

CBSE 2020 (30/1/1) — MCQ

The HCF and LCM of 12, 21 and 15 respectively are: (a) 3, 140 (b) 12, 420 (c) 3, 420 (d) 420, 3

1. $12 = 2^2 \times 3$, $21 = 3 \times 7$, $15 = 3 \times 5$. The only prime common to all three is 3: HCF = **3**.
2. LCM: greatest powers $2^2 \times 3 \times 5 \times 7$ = **420**.
3. HCF 3 and LCM 420 — option **(c)**.

ANSWER: Option (c): HCF = 3, LCM = 420.

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Previous Year Questions (2)

CBSE 2024 — 3 MARKS

Prove that √5 is irrational. Hence, prove that 3 + 2√5 is irrational.

1. Part 1 is Exercise 1.2 Q1, solved step by step in Section 07: assume √5 = a/b co-prime, reach 5 dividing both a and b — contradiction. So √5 is irrational.
   Why: In the exam, write the full proof — do not write 'as proved earlier'. The beats: assume, square, 5 | a, substitute, 5 | b, contradict.
2. For the second part, assume 3 + 2√5 is rational; call it r.
3. Then 2√5 = r − 3, so √5 = (r − 3)/2.
   Why: Isolate the root.
4. The right side is rational, so √5 would be rational — contradicting Part 1.
5. Therefore 3 + 2√5 is irrational. ∎

ANSWER: √5 is irrational, and hence 3 + 2√5 is irrational.

CBSE 2025 — MCQ

The greatest number which divides 70 and 125, leaving remainders 5 and 8 respectively, is: (a) 13 (b) 65 (c) 875 (d) 1750

1. Remove the remainders first: 70 − 5 = 65 and 125 − 8 = 117. The required number divides both 65 and 117 exactly, and is the greatest such: HCF(65, 117).
   Why: The Challenge 5 idea — subtract the remainder away.
2. 65 = 5 × 13 and 117 = 9 × 13 = 3^2 × 13. Common prime: 13. HCF = **13**.
3. Option **(a)**.

ANSWER: Option (a): 13.

Previous Year Questions (3)

CBSE 2025 (30/1/1) — MCQ

If HCF(98, 28) = m and LCM(98, 28) = n, then the value of n − 7m is: (a) 0 (b) 28 (c) 98 (d) 198

1. $98 = 2 \times 7^2$ and $28 = 2^2 \times 7$. HCF: smallest powers 2×7 = **14**, so m = 14.
2. LCM: greatest powers $2^2 \times 7^2$ = **196**, so n = 196. (Check: 14 × 196 = 2744 = 98 × 28.)
3. n − 7m = 196 − 7 × 14 = 196 − 98 = **98**. Option **(c)**.

ANSWER: Option (c): 98.

TOPPER TIP: *Board MCQ tactic: compute the HCF first, then get the LCM free from the identity HCF × LCM = product of the numbers, instead of factorising twice. It halves your time — and your error rate.*

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Chapter Model Test A — Basics (20 Marks)

Time: 40 minutes. Attempt all questions. Show every step — the marking key in the Answer Key shows where each mark lives.

Q1. Express 252 as a product of prime factors. **[2 marks]**

Q2. Find the HCF and LCM of 15 and 20. **[3 marks]**

Q3. Find the HCF and LCM of 84 and 90. **[3 marks]**

Q4. Check whether 4^n can end with the digit 0 for any natural number n. Give a reason. **[3 marks]**

Q5. Find the HCF and LCM of 12, 18 and 30. **[4 marks]**

Q6. State the Fundamental Theorem of Arithmetic. Given that HCF(120, 150) = 30, find LCM(120, 150). **[5 marks]**

Chapter Model Test B — Standard (20 Marks)

Time: 40 minutes. Board-pattern questions at full Class 10 level.

Q1. Find the HCF and LCM of 336 and 54, and verify that HCF × LCM = product of the two numbers. **[4 marks]**

Q2. Prove that $3 + 2\sqrt{5}$ is irrational, given that $\sqrt{5}$ is irrational. **[4 marks]**

Q3. Two bells toll at intervals of 12 minutes and 18 minutes. If they toll together at 10:00 a.m., at what time will they toll together again? **[3 marks]**

Q4. Explain why $7 \times 11 \times 13 + 13$ is a composite number. **[3 marks]**

Q5. Find the HCF and LCM of 8, 9 and 25. **[3 marks]**

Q6. Given that HCF(306, 657) = 9, find LCM(306, 657). **[3 marks]**

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Chapter Model Test C — Advanced (20 Marks)

Time: 45 minutes. Proof-heavy, HOTS-level — this is the paper that separates 95 from 100.

Q1. Prove that √5 is irrational. **[5 marks]**

Q2. Find the greatest number that divides 245 and 2053, leaving remainder 5 in each case. **[4 marks]**

Q3. Two numbers are in the ratio 3 : 4 and their LCM is 180. Find the numbers. **[4 marks]**

Q4. Prove that $n^3 - n$ is divisible by 6 for every positive integer n. **[4 marks]**

Q5. Show that 5 × 7 × 11 + 11 is a composite number, and write its prime factorisation. **[3 marks]**

> ***TOPPER TIP:*** *Take these three tests on three different days — Test A after Section 11, Test B after Section 14, Test C in the final revision week. Record every score in your Mastery Tracker.*

My Test Record — Chapter 1

Test	Date	Score	Weakest topic — revise this first
Model Test A — Basics		/ 20	
Model Test B — Standard		/ 20	
Model Test C — Advanced		/ 20	
Day 7 Mini-Test		/ 10	
Day 21 Mini-Test		/ 10	
Day 30 Mastery Test		/ 10	

A rising line of scores is the real certificate of this chapter — not any single day's mark. If a score dips, the 'weakest topic' column tells you exactly which section to revisit before moving on.

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Smart Memory Mini-Tests — Day 7 and Day 21

Ten quick questions each, closed book, five minutes. Answers are in the Answer Key. These are the Day 7 and Day 21 stops of your Smart Memory schedule (Section 19).

DAY 7 TEST

1. Express 144 as a product of prime factors.

2. Find HCF(28, 42).

3. Find LCM(28, 42).

4. For the pair (9, 12), write the value of HCF × LCM.

5. Can 10^n end with the digit 0? Answer yes or no, with the reason in one line.

6. Is 51 prime or composite? Give its factors.

7. Find LCM(5, 7).

8. Find HCF(100, 75).

9. State, in one line, why $6 + \sqrt{2}$ is irrational.

10. Given HCF(18, 24) = 6, find LCM(18, 24).

DAY 21 TEST

1. Express 500 as a product of prime factors.

2. Find HCF(45, 60).

3. Find LCM(45, 60).

4. Can 7^n end with the digit 0? Why?

5. Find HCF(13, 17).

6. Find LCM(13, 17).

7. Is $7 \times 11 \times 13 + 13$ prime or composite? Why?

8. If HCF(20, x) = 5 and LCM(20, x) = 100, find x.

9. State whether $\sqrt{3}$ is rational or irrational, and name the method used to prove it.

10. Find HCF(8, 12, 20) and LCM(8, 12, 20).

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Smart Memory Mini-Test — Day 30 Mastery Test

The final checkpoint. Ten questions, closed book — if you score 9 or 10, this chapter is yours for the whole year.

DAY 30 MASTERY TEST

1. Express 7429 as a product of prime factors.

2. Find HCF(336, 54).

3. Find LCM(306, 657), given HCF(306, 657) = 9.

4. Write the five beats of the contradiction proof that √p is irrational.

5. Sonia (18 min) and Ravi (12 min) start a round together. After how long do they meet at the start again?

6. Find HCF(12, 15, 21) and LCM(12, 15, 21).

7. In one line: why is 1/√2 irrational?

8. Find the smallest number that leaves remainder 5 when divided by 12, 16 and 18, forming at least one full group each time.

9. Find HCF(96, 120).

10. State the lemma used in every irrationality proof of this chapter.

> **TOPPER TIP:** *Missed any Day 30 question? That question's Flash Card goes back into your pocket box for one more week.*

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Answer Key — Graded Practice, Levels 1 and 2

Level 1 — Foundation

L1.1 $225 = 3^2 \times 5^2$. **L1.2** $360 = 2^3 \times 3^2 \times 5$. **L1.3** HCF(18, 48) = 6. **L1.4** LCM(14, 20) = 140. **L1.5** HCF(9, 25) = 1 and LCM(9, 25) = 225; the special point: 9 and 25 are co-prime — their factorisations (3^2 and 5^2) share no prime. **L1.6** HCF(24, 36) = 12.

Level 2 — Skill Builder

L2.1 $90 = 2 \times 3^2 \times 5$; $144 = 2^4 \times 3^2$. HCF = $2 \times 3^2 = 18$; LCM = $2^4 \times 3^2 \times 5 = 720$. Check: $18 \times 720 = 12960 = 90 \times 144$.

L2.2 $96 = 2^5 \times 3$; $404 = 2^2 \times 101$. HCF = $2^2 = 4$. LCM = $(96 \times 404) \div 4 = 38784 \div 4 = 9696$.

L2.3 $8 = 2^3$; $36 = 2^2 \times 3^2$; $90 = 2 \times 3^2 \times 5$. HCF = $2^1 = 2$ (only prime in all three is 2 — 3 is missing from 8). LCM = $2^3 \times 3^2 \times 5 = 360$.

L2.4 $867 = 3 \times 17^2$; $255 = 3 \times 5 \times 17$. HCF = $3 \times 17 = 51$. LCM = $(867 \times 255) \div 51 = 867 \times 5 = 4335$.

L2.5 No. $12^n = 2^{2n} \times 3^n$ — its only primes are 2 and 3. Ending in 0 requires the prime 5 ($10 = 2 \times 5$), and ending in 5 also requires the factor 5 (the number would be a multiple of 5). Since 5 never appears, neither ending is possible.

L2.6 $3 \times 5 \times 7 + 7 = 7 \times (3 \times 5 + 1) = 7 \times 16 = 112$. It is a product of two integers both greater than 1, so it is composite.

TOPPER TIP: *Marked yourself wrong at Level 2? Compare your working with the twin solution in Section 07 step by step — find the FIRST step where you differ. That step is the lesson.*

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Answer Key — Levels 3, 4, 5 and Active Recall

Level 3 — Application

L3.1 LCM(48, 72) = 144 seconds = 2 min 24 s, so they change together again at **7:02:24 a.m.**

L3.2 Tile side = HCF(18, 12) = 6 m. Tiles = (18 ÷ 6) × (12 ÷ 6) = 3 × 2 = **6 tiles**.

L3.3 Students per bus = HCF(156, 234) = 78. Buses: 156 ÷ 78 = 2 and 234 ÷ 78 = 3, so **5 buses**.

L3.4 LCM(9, 12, 15) = 180 minutes = 3 hours, so they toll together again at **11:00 a.m.**

Level 4 — Advanced Reasoning

L4.1 Write the numbers as 3k and 4k. Since 3 and 4 are co-prime, LCM = 3 × 4 × k = 12k = 180, so k = 15. The numbers are **45 and 60**, and HCF = k = **15**. (Check: HCF × LCM = 15 × 180 = 2700 = 45 × 60.)

L4.2 $5 \times 7 \times 11 + 11 = 11 \times (5 \times 7 + 1) = 11 \times 36 = 2^2 \times 3^2 \times 11$. It has factors other than 1 and itself (for example 11 and 36), so it is composite.

L4.3 Let the integers be n, n + 1, n + 2. Among any three consecutive integers, the remainders on division by 3 are 0, 1, 2 in some order, so one of them is divisible by 3. Among the first two, one is even, so the product is divisible by 2 as well. A number divisible by both 2 and 3 (co-prime) is divisible by 6. ∎

Level 5 — Mathematical Genius

L5.1 The number of sweets is 5 more than a common multiple of 12, 16, 18. LCM(12, 16, 18) = $2^4 \times 3^2 = 144$. Smallest such number = 144 + 5 = **149 sweets**. (The 'at least one full group' condition matters: without it, 5 sweets alone would leave 5 unpacked with zero groups formed.)

L5.2 2101. (Candidates are 210k + 1, since LCM(5, 6, 7) = 210. Because 210 = 11 × 19 + 1, we need 11 | (k + 1); smallest k = 10 gives 2101 = 11 × 191. Full solution: Challenge 1, Section 15.)

Active Recall (Section 18)

R1 Every integer greater than 1 can be expressed as a product of prime numbers in a unique way, apart from the order of the factors. **R2** 6. **R3** 36. **R4** 100 = $2^2 \times 5^2$. **R5** HCF = 4, LCM = 140. **R6** No — $8^n = 2^{3n}$ has no prime factor 5, and ending in 0 requires the factor 5. **R7** Irrational. **R8** HCF = 1, LCM = 1001. **R9** x = (4 × 48) ÷ 16 = 12. **R10** Composite — it equals 13 × 78.

P1 The HCF is the largest number that divides each of the given numbers exactly. **P2** The LCM is the smallest number that is a multiple of each of the given numbers. **P3** HCF × LCM = a × b — valid for two numbers only. **P4** A real number that cannot be written as p/q with p, q integers, $q \neq 0$. **P5** If a prime p divides a^2, then p divides a.

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Answer Key — Practice Plus

25 Concept MCQs

1 (b) 6 **2** (c) 36 **3** (b) $2^2 \times 5 \times 7$ **4** (b) 1 **5** (d) 1001 **6** (d) $\sqrt{5}$ **7** (c) 6 **8** (c) 22338 **9** (c) 182 **10** (d) 97 **11** (b) $2^2 \times 3^2$ **12** (b) 30 **13** (c) p divides a

14 (b) composite **15** (c) 4 **16** (c) 2 **17** (b) 1 **18** (d) irrational **19** (c) 25 **20** (c) 500 **21** (c) 36 minutes **22** (d) neither **23** (c) 10^n **24** (b) 2 **25** (c) a × b

Explanations for the tricky ones. Q6: $\sqrt{4} = 2$ and $\sqrt{9} = 3$ are integers, and 22/7 is a ratio of integers — all rational; only $\sqrt{5}$ is irrational. Q10: 87 = 3 × 29, 91 = 7 × 13, 93 = 3 × 31 — only 97 has no factor. Q11: HCF takes SMALLEST powers: $2^2 \times 3^2 = 36$. Q14: 7 × 11 × 13 + 13 = 13 × (77 + 1) = 13 × 78, a product of numbers above 1. Q23: only 10^n contains the prime 5 (10 = 2 × 5), so only it can end in 0. Q24: smallest prime 2, smallest composite 4, and HCF(2, 4) = 2.

Find the Error — Corrections

E1 HCF takes the SMALLEST powers of common primes, not the greatest (that is the LCM rule). HCF(12, 18) = 2 × 3 = **6**. (36 is the LCM.)

E2 91 = 7 × 13 — the check stopped too early. A prime test must try every prime up to $\sqrt{91} \approx 9.5$, including 7.

E3 6 × 10 is not a power of 6. Powers are 6, 36, 216, … = $2^n \times 3^n$ — no prime factor 5, so 6^n can never end in 0.

E4 LCM = product only when the numbers are co-prime. 4 and 6 share the factor 2: LCM(4, 6) = $2^2 \times 3$ = **12**.

E5 The proof never assumed a and b are **co-prime** — so '5 divides both' contradicts nothing, and the proof proves nothing. The co-prime condition in Step 1 is what Step 8 contradicts.

E6 HCF × LCM = product holds for TWO numbers only. HCF(12, 15, 21) = **3** (and indeed 3 × 420 = 1260 ≠ 3780).

E7 Multiply back: 3 × 25 × 17 = 1275 ≠ 3825. A power was dropped: $3825 = 3^2 \times 5^2 \times 17$. Always multiply back to check.

E8 A fraction is rational only when numerator and denominator are INTEGERS. $\sqrt{2}/2$ still contains $\sqrt{2}$ — $1/\sqrt{2}$ is irrational.

E9 Meeting again is a 'together' event — LCM, not HCF. LCM(18, 12) = **36 minutes**.

E10 'Irrational + irrational is irrational' is FALSE: $(2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$, rational. The real proof squares: if $\sqrt{2} + \sqrt{3} = r$, then $\sqrt{6} = (r^2 - 5)/2$ would be rational — contradiction (Challenge 3).

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Answer Key — Model Tests and Day Tests

Model Test A — Basics (20)

Q1 $252 = 2^2 \times 3^2 \times 7$ [ladder 1, collecting primes 1]. **Q2** HCF = 5 [1], LCM = 60 [2]. **Q3** HCF = 6 [1], LCM = 1260 [2]. **Q4** No [1]: $4^n = 2_2{}^n$ has no prime factor 5 [2]. **Q5** HCF = 6 [2], LCM = 180 [2]. **Q6** FTA statement [2]; LCM = 120 × 150 ÷ 30 = 600 [3].

Model Test B — Standard (20)

Q1 HCF = 6 [1], LCM = 3024 [2], verification 6 × 3024 = 18144 = 336 × 54 [1]. **Q2** Assume rational r [1]; isolate $\sqrt{5} = (r - 3)/2$ [2]; contradiction, hence irrational [1]. **Q3** LCM(12, 18) = 36 minutes [2]; together again at **10:36 a.m.** [1]. **Q4** 7 × 11 × 13 + 13 = 13 × (77 + 1) = 13 × 78 [2]; a product of two numbers above 1, hence composite [1]. **Q5** HCF = 1 [1], LCM = 1800 [2]. **Q6** LCM = 306 × 657 ÷ 9 = **22338** [3].

Model Test C — Advanced (20)

Q1 Assumption with co-prime [½]; squaring $a^2 = 5b^2$ [1]; 5 | a [½]; substitute a = 5c, get $b^2 = 5c^2$ [1]; 5 | b [½]; contradiction and conclusion [1½]. **Q2** Subtract remainders: HCF(240, 2048) = **16** [differences 2, HCF 2]. **Q3** Numbers 3k, 4k with LCM 12k = 180 [2], so k = 15: the numbers are **45 and 60** [2]. **Q4** $n^3 - n = (n - 1)n(n + 1)$ [1]; among three consecutive integers one is divisible by 3 [1] and at least one is even [1]; 2 and 3 co-prime, so divisible by 6 [1]. **Q5** 11 × (35 + 1) = 11 × 36 [1]; composite [1]; factorisation $2^2 \times 3^2 \times 11$ [1].

Day 7 Test

1 $144 = 2^4 \times 3^2$ **2** 14 **3** 84 **4** 108 (= 9 × 12) **5** Yes — $10^n = 2^n \times 5^n$ contains both 2 and 5 **6** Composite: 51 = 3 × 17 **7** 35 **8** 25 **9** Rational + irrational is always irrational **10** 72 (= 18 × 24 ÷ 6)

Day 21 Test

1 $500 = 2^2 \times 5^3$ **2** 15 **3** 180 **4** No — 7^n has no prime factor 5 **5** 1 **6** 221 **7** Composite: 13 × 78 **8** x = 5 × 100 ÷ 20 = 25 **9** Irrational; proof by contradiction (the lemma) **10** HCF = 4, LCM = 120

Day 30 Mastery Test

1 7429 = 17 × 19 × 23 **2** 6 **3** 22338 **4** Assume → Square → p divides a → p divides b → Contradiction **5** 36 minutes (LCM of 18 and 12) **6** HCF = 3, LCM = 420 **7** The reciprocal of an irrational number is irrational **8** 149 (= LCM(12, 16, 18) + 5 = 144 + 5) **9** 24 **10** If a prime p divides a^2, then p divides a

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Revision Card — Chapter 1 on One Page

The one theorem

Fundamental Theorem of Arithmetic: every integer greater than 1 = a product of prime numbers, in exactly one way (apart from order).

The two rules

HCF = common primes, smallest powers. **LCM** = all primes, greatest powers. **Identity:** HCF × LCM = a × b (two numbers only — derived from min + max = sum).

The proof machine (irrationality)

Assume rational a/b, co-prime → square: $a^2 = p \cdot b^2$ → $p \mid a^2 \Rightarrow p \mid a$ (lemma) → a = pc, substitute → $p \mid b$ → contradiction. Works for √p for every PRIME p (√2, √3, √5). For √6, split 6 into primes first: $6 \mid a^2 \Rightarrow 6 \mid a$ (Challenge 3, Section 15). For sums/products (3 + 2√5, 7√5, 6 + √2, 1/√2): isolate the root and cite the known irrational.

Story signals

HCF: largest / greatest / equal pieces / cut / pack / no leftovers. **LCM:** meet again / together again / repeat / smallest that fits all. **Ending in 0:** needs prime 5 in the factorisation. **Composite proof:** factor out the common factor — never multiply out.

Exam-eve checklist

☐ Ladder shown for every factorisation ☐ Verification line written in Q2-type questions ☐ Co-prime condition stated in every proof ☐ Final answer translated back into the story's words ☐ 91 = 7 × 13 — not prime!

Chapter 1 complete. You did not memorise Real Numbers — you discovered, visualised, derived and mastered them.

Next: Chapter 2 — Polynomials, in the same 8D format. • KalviYogi • Mathematics – Beyond the Formula • 369 Tesla × KalviYogi • © Edunivra Pvt Ltd

CBSE • CLASS 10 • MATHEMATICS

MATHEMATICS BEYOND THE FORMULA

CHAPTER 2 • POLYNOMIALS

Don't Memorize Mathematics. Understand, Visualise and Discover It.

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01 Chapter Introduction

8D • DISCOVER — WHY SHOULD I LEARN THIS?

Throw a ball into the air. It rises, pauses for a heartbeat, and falls back to the ground. That graceful curve it draws in the sky is not just physics — it is a **polynomial** drawing itself. The arch of a bridge, the path of water from a fountain, the profit curve of a small business — nature and life keep writing the same kind of mathematics: **polynomials**.

In this chapter you will learn to **see** a polynomial before you calculate with it. Its graph will tell you its secrets at a glance: where it touches the ground (its **zeroes**), how many such touching points are possible, and how the zeroes are secretly wired to the coefficients — a bridge so reliable that the board examiner tests it every single year.

What you will master in this chapter

- What a polynomial is: degree, types (linear, quadratic, cubic) and the value p(a).
- The **geometrical meaning of zeroes** — zeroes are the x-coordinates where the graph meets the x-axis (NCERT Exercise 2.1).
- Finding zeroes by **splitting the middle term**, with every split justified.
- The zeroes–coefficients bridge for quadratics: $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$ — **derived** in Section 06, then verified on every NCERT question.
- The same bridge for **cubic** polynomials: three relations, one of them with a famous minus sign.
- **Forming** a quadratic polynomial when the sum and product of its zeroes are given.
- The **division algorithm**: $p(x) = g(x) \times q(x) + r(x)$, worked by full long division.

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Your Chapter Map — 18 Stops, One Journey

Tick each stop as you finish it. The Plus blocks are your exam-fitness training — do not skip them.

Discover & Visualise	Understand & Derive	Solve, Apply, Recall
☐ 01 Chapter Introduction ☐ 02 Prerequisites ☐ 03 Real-Life Story ☐ 04 Visual Concepts (CPA)	☐ 05 Mathematical Foundations ☐ 06 Formula Derivations ☐ 08 Multiple Methods ☐ 10 Common Mistakes	☐ 07 Detailed Solved Examples ☐ 09 Hands-on Activities ☐ 11 Graded Practice (5 Levels) ☐ 12 Mathematical Reasoning
☐ 13 Real-Life Applications ☐ 14 Competency-Based Questions ☐ 15 Olympiad Challenges	☐ PLUS: Concept MCQs (25) ☐ PLUS: Find the Error (10) ☐ PLUS: Board PYQs (solved)	☐ 16 Chapter Mind Map ☐ Pin-Point Flash Cards ☐ 17 Active Recall ☐ 18 Mastery Tracker ☐ PLUS: 3 Model Tests

How to use this chapter (read this first!)

1. **Never read a solution passively.** Cover the steps, try yourself, then uncover one step at a time.
2. Every solution shows **Step 1, Step 2, Step 3…** with a small *Why* note in purple handwriting — the reason behind the step. Examiners award marks for these steps, not only for the final answer.
3. After Section 07, test yourself in Sections 11–15 and the Plus blocks **before** looking at the Answer Key.
4. Finish with the Mind Map (16), the Flash Cards, the closed-book Recall (17) and your Mastery Tracker (18). Come back on Day 3, 7, 14, 21 and 30 — the Smart Memory schedule, with its Day 7 / 21 / 30 mini-tests, tells you exactly what to do each day.

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02 Prerequisites

EARLIER CLASSES RECAP — THE TOOLS YOU ALREADY OWN

Polynomials grow out of four ideas you already know: algebraic expressions, equations, factorisation and graphs. Refresh them here.

1. Expressions, terms and substitution

An **algebraic expression** is built from variables and constants using +, −, ×, ÷. In $3x^2 - 2x + 1$, the terms are $3x^2$, $-2x$ and 1. **Substituting** a value means replacing the variable everywhere: for $p(x) = 3x^2 - 2x + 1$, $p(1) = 3 - 2 + 1 = 2$ and $p(0) = 1$. This chapter calls such a result the **value of the polynomial**.

2. Solving a linear equation

To solve $2x - 6 = 0$, you undo operations in reverse: $2x = 6$, so $x = 3$. Every "find the zeroes" step in this chapter ends with exactly this move, applied to each factor separately: if $(x - 4)(x + 2) = 0$, then $x - 4 = 0$ OR $x + 2 = 0$. A product is zero only when at least one factor is zero — this is called the **zero-product rule**.

3. Factorisation by splitting the middle term

To factorise $x^2 - 3x - 4$, find two numbers whose **sum** is −3 and whose **product** is −4: they are −4 and 1. Split: $x^2 - 4x + x - 4 = x(x - 4) + 1(x - 4) = (x - 4)(x + 1)$. For $ax^2 + bx + c$ with $a \neq 1$, the two numbers must multiply to **a × c** — Section 07 uses this in every solution.

4. Coordinates and graphs

A point (x, y) sits x units across and y units up. The graph of $y = p(x)$ is the picture of all points whose y-value equals $p(x)$. Where this picture meets the **x-axis**, the y-value is 0 — and those x-values are about to become the heroes of this chapter: the **zeroes**.

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Quick Check — Are You Ready? (5 Questions)

Try all five without looking back. Answers are given right below — cover them with your hand first!

Q1. Find the value of $p(x) = 2x^2 + 5x - 3$ at $x = 1$ and at $x = 0$.

Q2. Solve: (a) $3x - 9 = 0$ (b) $2x + 7 = 0$.

Q3. Factorise $x^2 + 5x + 6$.

Q4. Factorise $2x^2 + 5x + 3$. (Careful: here $a = 2$, so the split numbers must multiply to $a \times c = 6$.)

Q5. The graph of $y = x - 3$ meets the x-axis at which point? What is the y-value there?

Answers — check only after trying

Q1. $p(1) = 2 + 5 - 3 = 4$. $p(0) = -3$ (only the constant term survives).

Q2. (a) $3x = 9$, so $x = 3$. (b) $2x = -7$, so $x = -7/2$.

Q3. Split numbers: 2 and 3 (sum 5, product 6). $x^2 + 2x + 3x + 6 = x(x + 2) + 3(x + 2) = (x + 2)(x + 3)$.

Q4. Split numbers: 2 and 3 (sum 5, product $6 = a \times c$). $2x^2 + 2x + 3x + 3 = 2x(x + 1) + 3(x + 1) = (x + 1)(2x + 3)$.

Q5. It meets the x-axis at (3, 0); the y-value on the x-axis is always 0. Setting $y = 0$ gives $x = 3$ — you have just found your first zero of a polynomial!

TOPPER TIP: *Got 4 or 5 right? You are fully ready. Got fewer? Re-read the four cards on the previous page once, then come back — do not skip this step.*

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03 Real-Life Mathematical Story

DISCOVER • VISUALISE — THE ARCH THAT TOUCHED THE GROUND

Before any formula, live the problem. Read this story slowly — the mathematics will walk out of the story by itself.

*The town school is getting a new entrance, and the young architect **Anitha** has drawn something beautiful: a smooth, curved **arch** over the main gate, shaped like an upside-down U.*

*At the site, the head mason scratches his head. "Madam, my workers build with measurements, not with curves. At every point, I must know **how high** the arch is. And first of all — **where exactly do the two feet of the arch stand?** How wide is the opening?"*

Anitha smiles and opens her notebook. She draws a ground line and marks the left foot of the arch as her starting point. "Measure x metres from the left foot along the ground. Then the height of the arch above that point, in metres, is given by one small rule:

$$\textbf{height} = -x^2 + 4x$$

Try it, she says. At x = 1, the height is −1 + 4 = 3 metres. At x = 2, the height is −4 + 8 = 4 metres — the top of the arch. At x = 3, the height is −9 + 12 = 3 metres again. The arch rises, peaks, and comes down — perfectly symmetric.

*"But madam," the mason asks, "where does it come back to the **ground**? Where is the second foot?"*

*That question — where does the height become zero? — is the question of this whole chapter. In mathematics, the rule $-x^2 + 4x$ is called a **polynomial**, and the values of x where it becomes 0 are called its **zeroes**.*

Anitha has a second puzzle in her notebook too. Behind the school, a rectangular garden plot is being planned. Its length is 3 metres more than a chosen width number x, and its breadth is 4 metres more than x — so its area works out to $x^2 + 7x + 12$ square metres. The gardener wants the two side-lengths written separately, not hidden inside one formula. Both puzzles are solved on the next two pages — one step at a time.

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Solving the Story — Problem 1: Where Does the Arch Land?

STORY PROBLEM 1

The height of the arch at distance x metres from its left foot is $h(x) = -x^2 + 4x$ metres. Find where the arch meets the ground again, and hence the width of the opening.

THINK FIRST: Whenever a story asks 'where does it touch / land / become zero', it is asking for the ZEROES of a polynomial.

1. "Meets the ground" means the height is 0, so we must solve $h(x) = 0$, that is, $-x^2 + 4x = 0$.
   Why: A zero of a polynomial is exactly an input that makes the output zero.

2. Take out the common factor x: $-x^2 + 4x = x(-x + 4) = 0$. Factorising the same expression another way: $x(4 - x) = 0$.
   Why: Both terms contain x, so x comes out as a common factor.

3. By the zero-product rule, either $x = 0$ or $4 - x = 0$.
   Why: A product can be zero only if one of its factors is zero.

4. From $4 - x = 0$, we get $x = 4$.

5. So $h(x)$ has two zeroes: $x = 0$ and $x = 4$. The first is the left foot (where we started measuring); the second is the right foot.
   Why: Zeroes are positions here — the two feet of the arch.

6. Width of the opening = distance between the feet = $4 - 0$ = **4 metres**.

ANSWER: The arch meets the ground at x = 0 m (left foot) and x = 4 m (right foot). The opening is 4 metres wide.

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Solving the Story — Problem 2: The Garden's Two Sides

STORY PROBLEM 2

The area of the rectangular garden is $A(x) = x^2 + 7x + 12$ m^2. Write the area as a product of the two side-lengths. If x = 2, find the actual length, breadth and area.

THINK FIRST: An area written as one polynomial often hides two side-lengths. Factorising is how you set them free.

1. We need two numbers whose sum is 7 (the coefficient of x) and whose product is 12 (the constant term).
   Why: This is the splitting-the-middle-term hunt from Section 02.

2. Try pairs multiplying to 12: 1 × 12 (sum 13), 2 × 6 (sum 8), 3 × 4 (sum 7). The pair 3 and 4 fits both conditions.
   Why: Check BOTH conditions every time — sum and product.

3. Split the middle term: $x^2 + 7x + 12 = x^2 + 3x + 4x + 12$.

4. Group and factorise: $x(x + 3) + 4(x + 3) = (x + 3)(x + 4)$.
   Why: The common bracket (x + 3) is the signal that the grouping worked.

5. So the two sides are (x + 3) m and (x + 4) m. Check by expanding: $(x + 3)(x + 4) = x^2 + 4x + 3x + 12 = x^2 + 7x + 12$. Correct.
   Why: Always expand back to verify a factorisation.

6. Put x = 2: length = 6 m, breadth = 5 m, area = 30 m^2. Cross-check: $A(2) = 4 + 14 + 12 = 30$ m^2. Same.
   Why: Two routes, one answer — real verification.

ANSWER: Area = (x + 3)(x + 4) m^2, so the sides are (x + 3) m and (x + 4) m. For x = 2: 6 m by 5 m, area 30 m^2.

The two story signals of this chapter — pin these in memory

Touches the ground / lands / height becomes zero / break-even → set the polynomial equal to 0 and find its **zeroes**.
Area given as one expression, sides wanted → factorise the polynomial; the factors are the sides. You will meet both signals again in Sections 13 and 14 — and in the board exam.

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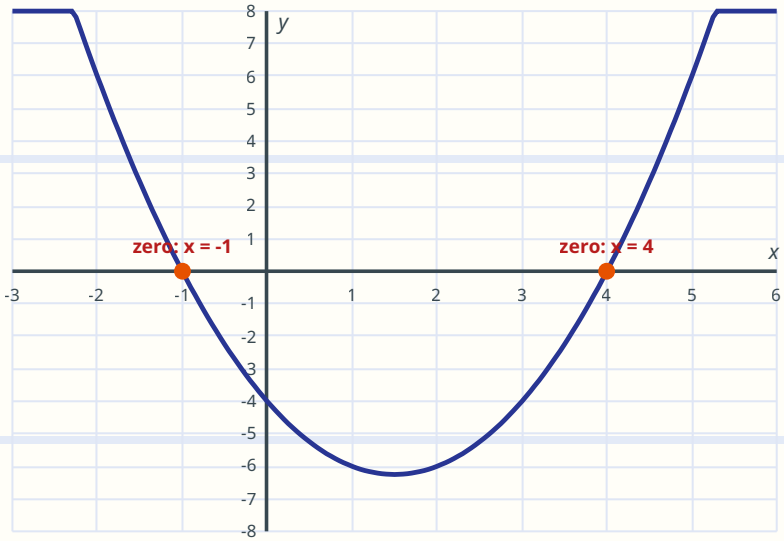
04 Visual Concept Explanation

SINGAPORE CPA — CONCRETE → PICTORIAL → ABSTRACT

Concrete: the arch and the garden of Section 03 were real objects. **Pictorial:** now we draw the polynomial as a picture — a graph. **Abstract:** only after the picture is clear do we touch formulas. Here is the hero picture of this chapter: the graph of $y = x^2 - 3x - 4$, built point by point from this table:

x	−2	−1	0	1	2	3	4	5
$y = x^2 - 3x - 4$	6	**0**	−4	−6	−6	−4	**0**	6



Look at the two orange dots. The curve crosses the x-axis at **x = −1** and **x = 4** — exactly the two table entries where y became 0. That is the whole secret, visible to the eye:

PIN POINT

The ZEROES of a polynomial p(x) are the x-coordinates of the points where the graph of y = p(x) meets the x-axis. Count the meetings, and you have counted the zeroes.

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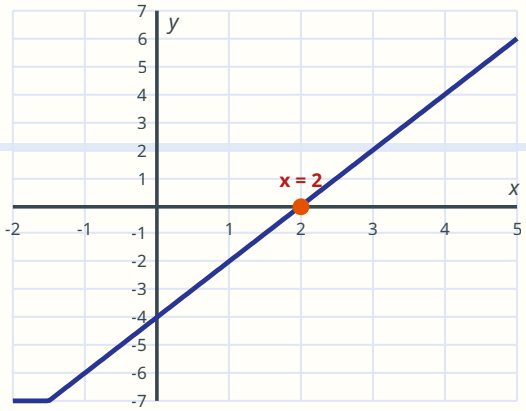
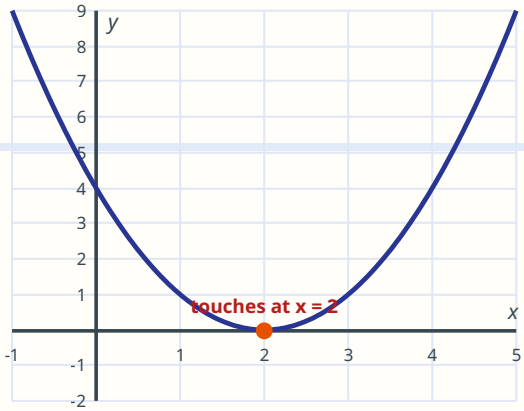
Four Pictures You Must Recognise Instantly

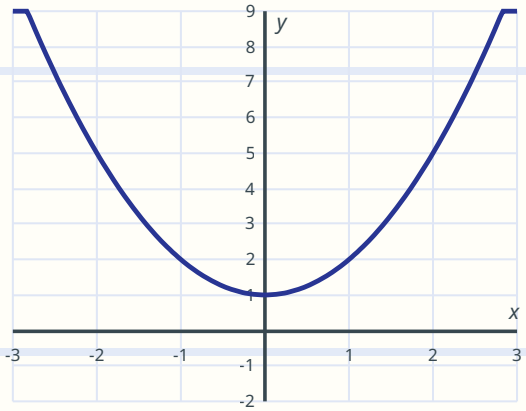
Every polynomial graph you will ever meet in Class 10 is one of these shapes. Read each picture the same way: **where does it meet the x-axis?**



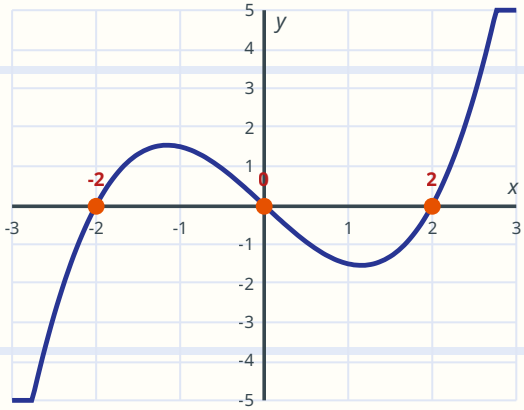
Linear $y = 2x - 4$: a straight line. Crosses once → **1 zero** ($x = 2$).



Quadratic $y = (x - 2)^2$: a parabola that **touches** the axis at one point → **1 zero**, counted twice ($x = 2, 2$).



Quadratic $y = x^2 + 1$: floats above the axis, never meets it → **0 zeroes** (no real zeroes).



Cubic $y = (x^3 - 4x)/2$: an S-curve crossing three times → **3 zeroes** ($x = -2, 0, 2$).

The Degree Rule — How Many Zeroes Can There Be?

Compare the four pictures with the degrees of their polynomials and a law appears:

Polynomial type	Degree	Graph shape	Zeroes possible
Linear, e.g. $2x - 4$	1	Straight line	Exactly 1
Quadratic, e.g. $x^2 - 3x - 4$	2	Parabola (U or ∩)	0, 1 (touching) or 2
Cubic, e.g. $x^3 - 4x$	3	S-shaped curve	1, 2 (one touching) or 3
Degree n (general)	n	—	**At most n**

Why "at most"? Each zero a gives a factor $(x - a)$ of the polynomial. A degree-n polynomial has room for at most n such linear factors — so it can have at most n zeroes. Section 12 proves this properly. For the exam, the working form is:

PIN POINT

A polynomial of degree n has AT MOST n zeroes. Quadratic → at most 2. Cubic → at most 3. If your answer ever shows more zeroes than the degree, your answer is certainly wrong — recheck.

One more reading skill: the parabola of a quadratic opens **upwards** when the coefficient of x^2 is positive (like $x^2 - 3x - 4$) and **downwards** when it is negative (like the arch, $-x^2 + 4x$). The lowest (or highest) point of the parabola is called its **vertex** — for the arch, the vertex (2, 4) was the top of the gate, 4 metres high.

TOPPER TIP: In Exercise 2.1 the board gives you ONLY pictures. Practise the reflex now: picture → count x-axis meetings → that count is your answer. Nothing else is asked.

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05 Mathematical Foundations

UNDERSTAND — THE PRECISE IDEAS, WORD BY WORD

1. What is a polynomial?

A **polynomial** in x is an expression in which x appears only with **whole-number powers** (0, 1, 2, 3, ...) and the coefficients are real numbers. Examples: $4x + 5$; $x^2 - 3x - 4$; $2x^3 - 7x + 1$; 9 (a constant is a polynomial too).

Not polynomials: $x^{-1} + 2$ (negative power), $\sqrt{x} + 3$ (fractional power), $1/x + x$ (x in a denominator). The powers must never be negative or fractional.

2. Degree — the polynomial's rank

The **degree** is the highest power of x in the polynomial. $5x^3 - 4x^2 + 7$ has degree 3. A non-zero constant (like 9) has degree 0. The **zero polynomial** 0 is special: its degree is **not defined** — a favourite one-mark trap.

Degree	Name	General form	Example
1	Linear	$ax + b$, $a \neq 0$	$2x - 4$
2	Quadratic	$ax^2 + bx + c$, $a \neq 0$	$x^2 - 3x - 4$
3	Cubic	$ax^3 + bx^2 + cx + d$, $a \neq 0$	$x^3 - 4x$

By number of terms: one term = **monomial** ($7x^2$), two terms = **binomial** ($x + 5$), three terms = **trinomial** ($x^2 - 3x - 4$).

3. Value of a polynomial — p(a)

p(a) means: replace x by a everywhere and calculate. For $p(x) = x^2 - 3x - 4$: $p(4) = 16 - 12 - 4 = 0$; $p(0) = -4$; $p(-1) = 1 + 3 - 4 = 0$. Keep $p(4) = 0$ and $p(-1) = 0$ in mind — they are about to earn a name.

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Zero of a Polynomial — The Key Definition

4. Zero of a polynomial

A real number **a** is called a **zero** of the polynomial p(x) if **p(a) = 0**. So 4 and −1 are zeroes of $x^2 - 3x - 4$, because p(4) = 0 and p(−1) = 0. Read it both ways: "a is a zero" and "p(a) = 0" are the same statement wearing two coats.

PIN POINT

a is a ZERO of p(x) ⇔ p(a) = 0 ⇔ the graph of y = p(x) meets the x-axis at x = a ⇔ (x − a) is a factor of p(x). Four faces of one fact — the exam tests each face separately.

5. The zeroes–coefficients bridge (statements — derived in Section 06)

If α and β are the zeroes of the quadratic $ax^2 + bx + c$ ($a \neq 0$), then
α + β = −b/a and **αβ = c/a**.

If α, β, γ are the zeroes of the cubic $ax^3 + bx^2 + cx + d$ ($a \neq 0$), then
α + β + γ = −b/a, **αβ + βγ + γα = c/a**, **αβγ = −d/a**.

Notice the pattern of signs: minus, plus, minus — they alternate, starting with minus. Section 06 shows you WHY, so you will never need to memorise blindly.

6. The division algorithm (foundation for Section 07)

Just as 17 ÷ 5 gives quotient 3 and remainder 2 with 17 = 5 × 3 + 2, polynomials divide the same way. For any two polynomials p(x) and g(x) (g(x) ≠ 0), there exist q(x) and r(x) with
p(x) = g(x) × q(x) + r(x), where r(x) = 0 or **degree of r(x) < degree of g(x)**.

You divide until the remainder's degree is smaller than the divisor's degree — then you must stop. Two fully worked long divisions appear in Section 07.

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06 Formula Derivations

8D • DERIVE — NEVER MEMORISE WHAT YOU CAN DERIVE

Derivation 1 — The quadratic bridge: α + β = −b/a, αβ = c/a

Let α and β be the zeroes of $p(x) = ax^2 + bx + c$, with $a \neq 0$. We build the bridge in five steps.

1. Since α and β are the zeroes, $(x - \alpha)$ and $(x - \beta)$ are factors of p(x). So $p(x) = a(x - \alpha)(x - \beta)$ for some constant multiple — and the multiple must be a, to match the x^2 coefficient.
   Why: A zero a always brings its factor (x − a) with it (Section 05, Pin Point).

2. First expand the brackets: $(x - \alpha)(x - \beta) = x^2 - \beta x - \alpha x + \alpha\beta = x^2 - (\alpha + \beta)x + \alpha\beta$.
   Why: Multiply term by term; collect the two middle terms.

3. Multiply by a: $p(x) = ax^2 - a(\alpha + \beta)x + a\alpha\beta$.

4. But p(x) is also $ax^2 + bx + c$. Two writings of one polynomial must match coefficient by coefficient: coefficient of x gives $b = -a(\alpha + \beta)$, and the constant term gives $c = a\alpha\beta$.
   Why: Equal polynomials have equal coefficients — this comparison is the engine of the derivation.

5. Divide: from $b = -a(\alpha + \beta)$, we get **$\alpha + \beta = -b/a$**. From $c = a\alpha\beta$, we get **$\alpha\beta = c/a$**. The bridge is built.
   Why: Notice where the minus sign is born: it comes from the −(α + β) in the expansion. That is why the sum formula carries a minus and the product formula does not.

Instant check on an old friend, $x^2 - 3x - 4$ (zeroes −1 and 4): sum of zeroes = −1 + 4 = 3, and $-b/a = -(-3)/1 = 3$. Product = −1 × 4 = −4, and $c/a = -4/1 = -4$. Both match — the bridge holds.

PIN POINT

SUM of zeroes = −(coefficient of x) / (coefficient of x^2). PRODUCT of zeroes = (constant term) / (coefficient of x^2). The minus lives ONLY in the sum.

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Derivation 2 — The Cubic Bridge (Same Logic, One More Zero)

Let α, β, γ be the zeroes of $p(x) = ax^3 + bx^2 + cx + d$. Then $p(x) = a(x - \alpha)(x - \beta)(x - \gamma)$. We expand in two stages.

1. Stage 1: $(x - \alpha)(x - \beta) = x^2 - (\alpha + \beta)x + \alpha\beta$ — exactly as in Derivation 1.

2. Stage 2: multiply the result by $(x - \gamma)$. The x^3 term: $x^2 \times x = x^3$. The x^2 terms: $-(\alpha + \beta)x \times x$ and $x^2 \times (-\gamma)$ give $-(\alpha + \beta + \gamma)x^2$.
Why: Track one power at a time — that keeps a long expansion error-free.

3. The x terms: $\alpha\beta \times x$ and $-(\alpha + \beta)x \times (-\gamma)$ give $(\alpha\beta + \beta\gamma + \gamma\alpha)x$. The constant term: $\alpha\beta \times (-\gamma) = -\alpha\beta\gamma$.
Why: Each x-term pairs two zeroes; the constant pairs all three — and collects a minus sign from the single (−γ).

4. So $(x - \alpha)(x - \beta)(x - \gamma) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$. Multiply by a and compare with $ax^3 + bx^2 + cx + d$.

5. Coefficient of x^2: $b = -a(\alpha + \beta + \gamma) \Rightarrow$ **$\alpha + \beta + \gamma = -b/a$**. Coefficient of x: $c = a(\alpha\beta + \beta\gamma + \gamma\alpha) \Rightarrow$ **$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$**. Constant: $d = -a\alpha\beta\gamma \Rightarrow$ **$\alpha\beta\gamma = -d/a$**.
Why: Signs alternate −, +, − — one minus for each bracket you multiply by. Now the pattern explains itself.

Instant check on $x^3 - 6x^2 + 11x - 6$, whose zeroes are 1, 2, 3: sum = 6 and $-b/a = 6/1 = 6$. Pairs: $1\cdot2 + 2\cdot3 + 3\cdot1 = 2 + 6 + 3 = 11$ and $c/a = 11$. Product: $1 \cdot 2 \cdot 3 = 6$ and $-d/a = -(-6)/1 = 6$. All three match.

PIN POINT

CUBIC: $\alpha + \beta + \gamma = -b/a$; $\alpha\beta + \beta\gamma + \gamma\alpha = c/a$; $\alpha\beta\gamma = -d/a$. The product of three zeroes carries a MINUS — the most-lost mark in this chapter.

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Derivation 3 — Building a Polynomial from Sum and Product

The bridge also works in reverse. Suppose a quadratic has zeroes α, β with **sum S = α + β** and **product P = αβ**. What is the polynomial?

1. From Derivation 1, the polynomial is $a(x - \alpha)(x - \beta) = a[x^2 - (\alpha + \beta)x + \alpha\beta]$.
   Why: We reuse the expansion — no new work needed.

2. Replace α + β by S and αβ by P: $p(x) = a[x^2 - Sx + P]$, where a can be ANY non-zero number.
   Why: That is why Exercise 2.2 Q2 says 'find A quadratic polynomial' — there are infinitely many, all multiples of one another.

3. Choose a to clear fractions: if S and P are whole numbers, a = 1 is fine. If S = 1/4 and P = −1, choose a = 4 to get $4x^2 - x - 4$ — no fractions anywhere.
   Why: Multiplying a polynomial by a non-zero constant never changes its zeroes.

Mini-example: sum = 5, product = 6. Then $p(x) = x^2 - 5x + 6$. Check: its zeroes are 2 and 3 (factorise: (x − 2)(x − 3)), and indeed 2 + 3 = 5, 2 × 3 = 6.

PIN POINT

FORMING A QUADRATIC: $p(x) = k[x^2 - (\text{sum of zeroes})x + (\text{product of zeroes})]$, $k \neq 0$. Minus before the sum, plus before the product — every time.

TOPPER TIP: Derivations 1–3 are one single idea viewed from three sides: expand $a(x - \alpha)(x - \beta)$ and compare. If you can redo that expansion, you own every formula in this chapter.

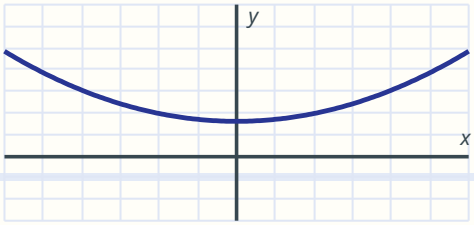
07 Detailed Solved Examples

8D • SOLVE — NCERT EXERCISES 2.1 & 2.2, EVERY STEP SHOWN

Exercise 2.1 has just one question with six graphs: *"The graphs of y = p(x) are given below, for some polynomials p(x). Find the number of zeroes of p(x), in each case."* The method never changes: **count the x-axis meetings**. We still show the steps — because in the exam hall, a calm method beats a quick guess.

EXERCISE 2.1 • GRAPH (i)

Find the number of zeroes of p(x) from the graph of y = p(x).



1 Look only at the x-axis and count the meetings. The curve stays completely above the x-axis. It neither crosses nor touches it — the count of meetings is 0.

Why: Zeroes live on the x-axis and nowhere else (Section 04).

ANSWER: The number of zeroes of p(x) is 0.

CBSE MARKING SCHEME: 1 mark — correct count of x-axis intersections.

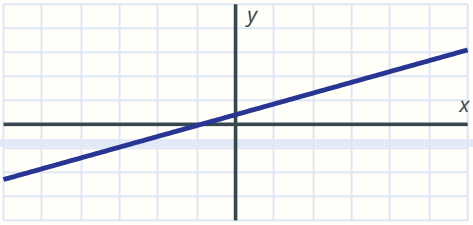
Before you turn the page — predict!

For each of the five graphs ahead, cover the answer and say the count out loud *before* reading the step. Predicting first — even when you are wrong — makes the correction stick. That is the Discover habit of the 8D system, and it costs you ten seconds a graph.

Exercise 2.1 — Graphs (ii) and (iii)

EXERCISE 2.1 • GRAPH (ii)

Find the number of zeroes of p(x) from the graph of y = p(x).



1 Look only at the x-axis and count the meetings. The straight line cuts the x-axis at exactly one point — the count of meetings is 1.

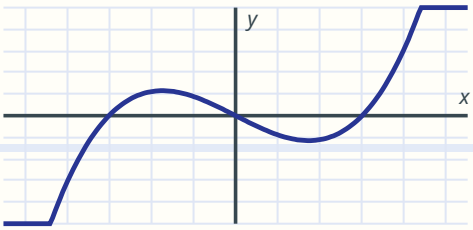
Why: Zeroes live on the x-axis and nowhere else (Section 04).

ANSWER: The number of zeroes of p(x) is 1.

CBSE MARKING SCHEME: 1 mark — correct count of x-axis intersections.

EXERCISE 2.1 • GRAPH (iii)

Find the number of zeroes of p(x) from the graph of y = p(x).



1 Look only at the x-axis and count the meetings. The curve weaves across the axis: it crosses once on the left, once near the middle and once on the right — the count of meetings is 3.

Why: Zeroes live on the x-axis and nowhere else (Section 04).

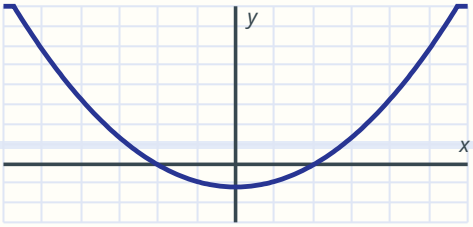
ANSWER: The number of zeroes of p(x) is 3.

CBSE MARKING SCHEME: 1 mark — correct count of x-axis intersections.

Exercise 2.1 — Graphs (iv) and (v)

EXERCISE 2.1 • GRAPH (iv)

Find the number of zeroes of p(x) from the graph of y = p(x).



1 Look only at the x-axis and count the meetings. The parabola dips below the axis and comes back up, crossing it at two points — the count of meetings is 2.

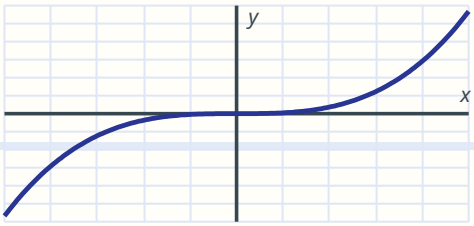
Why: Zeroes live on the x-axis and nowhere else (Section 04).

ANSWER: The number of zeroes of p(x) is 2.

CBSE MARKING SCHEME: 1 mark — correct count of x-axis intersections.

EXERCISE 2.1 • GRAPH (v)

Find the number of zeroes of p(x) from the graph of y = p(x).



1 Look only at the x-axis and count the meetings. The curve flattens as it passes through the origin but meets the axis only there — the count of meetings is 1.

Why: Zeroes live on the x-axis and nowhere else (Section 04).

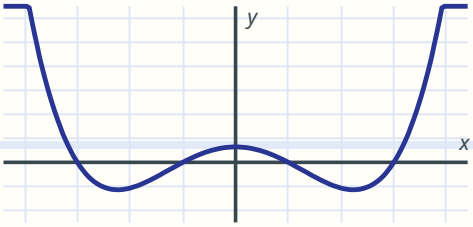
ANSWER: The number of zeroes of p(x) is 1.

CBSE MARKING SCHEME: 1 mark — correct count of x-axis intersections.

Exercise 2.1 — Graph (vi) and the Summary

EXERCISE 2.1 • GRAPH (vi)

Find the number of zeroes of p(x) from the graph of y = p(x).



1 Look only at the x-axis and count the meetings. The wavy curve crosses the axis four times — twice on the left of the origin and twice on the right — the count of meetings is 4.

Why: Zeroes live on the x-axis and nowhere else (Section 04).

ANSWER: The number of zeroes of p(x) is 4.

CBSE MARKING SCHEME: 1 mark — correct count of x-axis intersections.

Exercise 2.1 — all six answers at a glance

(i) 0 • (ii) 1 • (iii) 3 • (iv) 2 • (v) 1 • (vi) 4. Graph (vi)'s four zeroes force degree at least 4 — the degree rule in reverse. Exam trap: never count the Y-axis meeting point — that is p(0), not a zero.

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07B Exercise 2.2 Q1 — Find Zeroes and Verify

THE BOARD'S FAVOURITE QUESTION TYPE • 3 MARKS EACH

NCERT Exercise 2.2, Q1: *"Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients."* Every part follows the same five-beat rhythm: **read a, b, c → split the middle term → factorise → zeroes → verify sum → verify product**. Learn the rhythm once; play it six times.

EX 2.2 Q1 (i)

Find the zeroes of $x^2 - 2x - 8$ and verify the relationship between the zeroes and the coefficients.

THINK FIRST: Split, factorise, then verify. The split pair must multiply to a × c = −8 and add to b = −2.

1. Read the coefficients: a = 1, b = −2, c = −8. So a × c = −8.
   Why: Always begin by naming a, b, c — with their signs.
2. Find the split pair: two numbers with product −8 and sum −2. They are −4 and +2 (since −4 × 2 = −8 and −4 + 2 = −2).
   Why: For a × c negative, the two numbers have opposite signs.
3. Split and group: $x^2 - 4x + 2x - 8 = x(x - 4) + 2(x - 4) = (x - 4)(x + 2)$.
4. Zero-product rule: x − 4 = 0 or x + 2 = 0, so the zeroes are α = 4 and β = −2.
5. Verify the sum: α + β = 4 + (−2) = 2. And −b/a = −(−2)/1 = 2. Equal.
   Why: Write both sides explicitly — the verification IS the second half of the question.
6. Verify the product: αβ = 4 × (−2) = −8. And c/a = −8/1 = −8. Equal.

ANSWER: Zeroes: 4 and −2. Sum = 2 = −b/a and product = −8 = c/a — relationship verified.

CBSE MARKING SCHEME: 1 mark for correct factorisation and zeroes • 1 mark for verifying the sum • 1 mark for verifying the product.

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Exercise 2.2 Q1 (ii) — The Equal-Zeroes Case

EX 2.2 Q1 (ii)

Find the zeroes of $4s^2 - 4s + 1$ and verify the relationship between the zeroes and the coefficients. (Variable is s.)

THINK FIRST: Here a × c = 4. The split pair must multiply to +4 and add to −4 — suspect a repeated pair.

1. Read the coefficients: a = 4, b = −4, c = 1. So a × c = 4.
2. Find the split pair: −2 and −2 (product +4, sum −4).
   Why: A repeated split pair is the fingerprint of equal zeroes.
3. Split and group: $4s^2 - 2s - 2s + 1 = 2s(2s - 1) - 1(2s - 1) = (2s - 1)(2s - 1) = (2s - 1)^2$.
4. Zero-product rule: 2s − 1 = 0 (twice), so s = 1/2. The zeroes are α = 1/2 and β = 1/2 — equal zeroes.
   Why: Equal zeroes mean the graph TOUCHES the x-axis at s = 1/2, like picture (b) of Section 04.
5. Verify the sum: α + β = 1/2 + 1/2 = 1. And −b/a = −(−4)/4 = 1. Equal.
6. Verify the product: αβ = 1/2 × 1/2 = 1/4. And c/a = 1/4. Equal.

ANSWER: Zeroes: 1/2 and 1/2 (equal). Sum = 1 = −b/a and product = 1/4 = c/a — relationship verified.

CBSE MARKING SCHEME: 1 mark for correct factorisation and zeroes • 1 mark for verifying the sum • 1 mark for verifying the product.

Exercise 2.2 Q1 (iii) — First, Arrange in Standard Form

EX 2.2 Q1 (iii)

Find the zeroes of $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients.

THINK FIRST: NCERT has shuffled the terms on purpose. Your first step is not splitting — it is housekeeping: write the polynomial in the standard order $ax^2 + bx + c$.

1. Rewrite in standard form: $6x^2 - 7x - 3$. Now read: $a = 6$, $b = -7$, $c = -3$, so $a \times c = -18$.
   Why: Reading b = 7 from the shuffled form is the classic error here (see Find-the-Error E7). The coefficient of x is −7.
2. Find the split pair: product -18, sum -7. They are -9 and $+2$.
3. Split and group: $6x^2 - 9x + 2x - 3 = 3x(2x - 3) + 1(2x - 3) = (2x - 3)(3x + 1)$.
4. Zero-product rule: $2x - 3 = 0$ gives $x = 3/2$; $3x + 1 = 0$ gives $x = -1/3$. Zeroes: $\alpha = 3/2$, $\beta = -1/3$.
5. Verify the sum: $3/2 + (-1/3) = 9/6 - 2/6 = 7/6$. And $-b/a = -(-7)/6 = 7/6$. Equal.
   Why: Fraction addition: use LCM 6 — show it, do not jump.
6. Verify the product: $(3/2) \times (-1/3) = -1/2$. And $c/a = -3/6 = -1/2$. Equal.

ANSWER: Zeroes: 3/2 and −1/3. Sum = 7/6 = −b/a and product = −1/2 = c/a — relationship verified.

CBSE MARKING SCHEME: 1 mark for correct factorisation and zeroes • 1 mark for verifying the sum • 1 mark for verifying the product.

Exercise 2.2 Q1 (iv) — When the Constant Term Is Zero

EX 2.2 Q1 (iv)

Find the zeroes of $4u^2 + 8u$ and verify the relationship between the zeroes and the coefficients. (Variable is u.)

THINK FIRST: No constant term — so no splitting hunt is needed. A common factor is staring at you.

1. Read the coefficients: a = 4, b = 8, c = 0.
   Why: c = 0 is information, not a gap: it promises that one zero will be 0.
2. Take out the common factor: $4u^2 + 8u = 4u(u + 2)$.
   Why: Both terms contain 4u.
3. Zero-product rule: 4u = 0 gives u = 0; u + 2 = 0 gives u = −2. Zeroes: α = 0, β = −2.
4. Verify the sum: 0 + (−2) = −2. And −b/a = −8/4 = −2. Equal.
5. Verify the product: 0 × (−2) = 0. And c/a = 0/4 = 0. Equal.
   Why: Product zero ⇔ at least one zero is zero. The bridge agrees with common sense.

ANSWER: Zeroes: 0 and −2. Sum = −2 = −b/a and product = 0 = c/a — relationship verified.

CBSE MARKING SCHEME: 1 mark for correct factorisation and zeroes • 1 mark for verifying the sum • 1 mark for verifying the product.

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Exercise 2.2 Q1 (v) — The Difference of Squares

EX 2.2 Q1 (v)

Find the zeroes of $t^2 - 15$ and verify the relationship between the zeroes and the coefficients. (Variable is t.)

THINK FIRST: There is no middle term (b = 0), so splitting has nothing to split. Recognise the shape: a square minus a number — the difference-of-squares pattern.

1. Read the coefficients: a = 1, b = 0, c = −15.
2. Write 15 as a square: $15 = (\sqrt{15})^2$. So $t^2 - 15 = t^2 - (\sqrt{15})^2$.
   Why: Any positive number is the square of its square root — this trick unlocks every 'x^2 − number' polynomial.
3. Apply $a^2 - b^2 = (a - b)(a + b)$: $t^2 - 15 = (t - \sqrt{15})(t + \sqrt{15})$.
4. Zero-product rule: $t = \sqrt{15}$ or $t = -\sqrt{15}$. Zeroes: $\alpha = \sqrt{15}$, $\beta = -\sqrt{15}$.
   Why: Never report only the positive root — see Find-the-Error ES. A quadratic with b = 0 has zeroes in ± pairs.
5. Verify the sum: $\sqrt{15} + (-\sqrt{15}) = 0$. And $-b/a = -0/1 = 0$. Equal.
6. Verify the product: $\sqrt{15} \times (-\sqrt{15}) = -15$. And $c/a = -15/1 = -15$. Equal.
   Why: √15 × √15 = 15 — a root times itself returns the number.

ANSWER: Zeroes: $\sqrt{15}$ and $-\sqrt{15}$. Sum = 0 = −b/a and product = −15 = c/a — relationship verified.

CBSE MARKING SCHEME: 1 mark for correct factorisation and zeroes • 1 mark for verifying the sum • 1 mark for verifying the product.

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Exercise 2.2 Q1 (vi) — The Full Rhythm, One Last Time

EX 2.2 Q1 (vi)

Find the zeroes of $3x^2 - x - 4$ and verify the relationship between the zeroes and the coefficients.

THINK FIRST: $a \times c = -12$ and $b = -1$. Hunt for the pair calmly.

1. Read the coefficients: $a = 3$, $b = -1$, $c = -4$. So $a \times c = -12$.
2. Find the split pair: product -12, sum -1. They are -4 and $+3$.
3. Split and group: $3x^2 - 4x + 3x - 4 = x(3x - 4) + 1(3x - 4) = (3x - 4)(x + 1)$.
4. Zero-product rule: $3x - 4 = 0$ gives $x = 4/3$; $x + 1 = 0$ gives $x = -1$. Zeroes: $\alpha = 4/3$, $\beta = -1$.
5. Verify the sum: $4/3 + (-1) = 1/3$. And $-b/a = -(-1)/3 = 1/3$. Equal.
6. Verify the product: $(4/3) \times (-1) = -4/3$. And $c/a = -4/3$. Equal.

ANSWER: Zeroes: 4/3 and −1. Sum = 1/3 = −b/a and product = −4/3 = c/a — relationship verified.

CBSE MARKING SCHEME: 1 mark for correct factorisation and zeroes • 1 mark for verifying the sum • 1 mark for verifying the product.

TOPPER TIP: *Six questions, one rhythm. In the board exam this exact question type appeared in 2023 AND 2024 papers (see the PYQ section). Master the rhythm here and those marks are already yours.*

07C Exercise 2.2 Q2 — Form a Polynomial from Sum and Product

THE BRIDGE, CROSSED BACKWARDS

NCERT Exercise 2.2, Q2: *"Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively."* Six parts, one formula. Watch the signs: **minus** the sum, **plus** the product.

EX 2.2 Q2 (i)

Find a quadratic polynomial whose sum of zeroes is 1/4 and product of zeroes is −1.

THINK FIRST: Use the forming formula from Derivation 3: $p(x) = k[x^2 - Sx + P]$. Then choose k to clear fractions.

1. Write S = 1/4 and P = −1. The monic form is $x^2 - (1/4)x + (-1) = x^2 - (1/4)x - 1$.
   Why: The product P = −1 enters with its own sign: + (−1) = −1.

2. Clear the fraction by choosing k = 4: multiply every term by 4: $4x^2 - x - 4$.
   Why: Multiplying by 4 changes the polynomial's size, never its zeroes.

3. Self-check: for $4x^2 - x - 4$, sum of zeroes = −(−1)/4 = 1/4 and product = −4/4 = −1. Both as given.
   Why: Ten seconds of checking protects two full marks.

ANSWER: $4x^2 - x - 4$ (any non-zero multiple is equally correct).

CBSE MARKING SCHEME: 1 mark for writing $x^2 - Sx + P$ with correct signs • 1 mark for the final polynomial with fractions cleared.

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Exercise 2.2 Q2 — Parts (ii) and (iii)

EX 2.2 Q2 (ii)

Find a quadratic polynomial whose sum of zeroes is √2 and product of zeroes is 1/3.

THINK FIRST: Use the forming formula from Derivation 3: $p(x) = k[x^2 - Sx + P]$. Then choose k to clear fractions.

1. Write $S = \sqrt{2}$ and $P = 1/3$. The monic form is $x^2 - \sqrt{2}\,x + 1/3$.
2. Clear the fraction by choosing $k = 3$: $3x^2 - 3\sqrt{2}\,x + 1$.
   Why: Only the 1/3 needed clearing; √2 is a perfectly respectable coefficient.
3. Self-check: sum $= -(-3\sqrt{2})/3 = \sqrt{2}$; product $= 1/3$. Both as given.

ANSWER: $3x^2 - 3\sqrt{2}\,x + 1$.

CBSE MARKING SCHEME: 1 mark for writing $x^2 - Sx + P$ with correct signs • 1 mark for the final polynomial with fractions cleared.

EX 2.2 Q2 (iii)

Find a quadratic polynomial whose sum of zeroes is 0 and product of zeroes is √5.

THINK FIRST: Use the forming formula from Derivation 3: $p(x) = k[x^2 - Sx + P]$. Then choose k to clear fractions.

1. Write $S = 0$ and $P = \sqrt{5}$. The monic form is $x^2 - 0 \cdot x + \sqrt{5} = x^2 + \sqrt{5}$.
   Why: Sum zero means the x-term vanishes — the zeroes will be opposites of each other, like ± pairs.
2. No fractions to clear, so $k = 1$: the polynomial is $x^2 + \sqrt{5}$.
3. Self-check: sum $= -0/1 = 0$; product $= \sqrt{5}/1 = \sqrt{5}$. Both as given.

ANSWER: $x^2 + \sqrt{5}$.

CBSE MARKING SCHEME: 1 mark for writing $x^2 - Sx + P$ with correct signs • 1 mark for the final polynomial with fractions cleared.

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Exercise 2.2 Q2 — Parts (iv) and (v)

EX 2.2 Q2 (iv)

Find a quadratic polynomial whose sum of zeroes is 1 and product of zeroes is 1.

THINK FIRST: Use the forming formula from Derivation 3: $p(x) = k[x^2 - Sx + P]$. Then choose k to clear fractions.

1. Write S = 1 and P = 1. The monic form is $x^2 - 1 \cdot x + 1 = x^2 - x + 1$.
2. No fractions, k = 1: the polynomial is $x^2 - x + 1$.
3. Self-check: sum = $-(-1)/1 = 1$; product = $1/1 = 1$. Both as given.
   Why: Note: this polynomial's graph never touches the x-axis — its 'zeroes' are not real numbers. The forming formula still works; Class 11 will explain why.

ANSWER: $x^2 - x + 1$.

CBSE MARKING SCHEME: 1 mark for writing $x^2 - Sx + P$ with correct signs • 1 mark for the final polynomial with fractions cleared.

EX 2.2 Q2 (v)

Find a quadratic polynomial whose sum of zeroes is −1/4 and product of zeroes is 1/4.

THINK FIRST: Use the forming formula from Derivation 3: $p(x) = k[x^2 - Sx + P]$. Then choose k to clear fractions.

1. Write S = −1/4 and P = 1/4. The monic form is $x^2 - (-1/4)x + 1/4 = x^2 + (1/4)x + 1/4$.
   Why: Minus a negative is plus — this is THE sign trap of this exercise. Slow down here.
2. Clear fractions with k = 4: $4x^2 + x + 1$.
3. Self-check: sum = −1/4; product = 1/4. Both as given.

ANSWER: $4x^2 + x + 1$.

CBSE MARKING SCHEME: 1 mark for writing $x^2 - Sx + P$ with correct signs • 1 mark for the final polynomial with fractions cleared.

Exercise 2.2 Q2 — Part (vi) and the Method

EX 2.2 Q2 (vi)

Find a quadratic polynomial whose sum of zeroes is 4 and product of zeroes is 1.

THINK FIRST: Use the forming formula from Derivation 3: $p(x) = k[x^2 - Sx + P]$. Then choose k to clear fractions.

1. Write S = 4 and P = 1. The monic form is $x^2 - 4x + 1$.
2. No fractions, k = 1: the polynomial is $x^2 - 4x + 1$.
3. Self-check: sum = $-(-4)/1 = 4$; product = $1/1 = 1$. Both as given.

ANSWER: $x^2 - 4x + 1$.

CBSE MARKING SCHEME: 1 mark for writing $x^2 - Sx + P$ with correct signs • 1 mark for the final polynomial with fractions cleared.

TOPPER TIP: *Finished all six? Notice the pattern of your own work: read S and P → write $x^2 - Sx + P$ → clear fractions → self-check. Four moves, every time. That is a method, and methods are what toppers actually memorise.*

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Beyond the Exercise — The Division Algorithm at Work

ENRICHMENT • FULLY WORKED

The rationalised NCERT exercise list no longer asks for polynomial long division, but the **division algorithm** (Section 05, idea 6) remains in the chapter's foundations — and it returns in Class 10's later chapters and in Class 11. Learn it here, completely, with two examples. In the working below, x^2 means x^2 and x^3 means x^3.

DIVISION EXAMPLE 1

Divide $2x^2 + 3x + 1$ by $x + 2$. Find the quotient and the remainder.

THINK FIRST: Treat it exactly like school long division of numbers: divide, multiply, subtract, bring down — but with degrees instead of digits.

1. Ask: what must multiply $(x + 2)$ to give the first term $2x^2$? Answer: $2x$. Write $2x$ as the first term of the quotient.
   Why: Divide leading term by leading term: $2x^2 \div x = 2x$.
2. Multiply back: $2x \times (x + 2) = 2x^2 + 4x$. Subtract from the dividend: $(2x^2 + 3x) - (2x^2 + 4x) = -x$. Bring down the +1: new dividend $-x + 1$.
   Why: Subtraction kills the x^2 term — that is the point of the step.
3. Repeat: $-x \div x = -1$, the next quotient term. Multiply back: $-1 \times (x + 2) = -x - 2$. Subtract: $(-x + 1) - (-x - 2) = 3$.
4. The remainder 3 has degree 0, which is less than the divisor's degree 1 — so we STOP. Quotient $q(x) = 2x - 1$, remainder $r(x) = 3$.
   Why: The stopping rule is about DEGREES, not about the remainder being zero.
5. Verify with the algorithm: $g(x)q(x) + r(x) = (x + 2)(2x - 1) + 3 = (2x^2 + 3x - 2) + 3 = 2x^2 + 3x + 1 = p(x)$. Correct.
   Why: Never trust a division you have not multiplied back.

ANSWER: Quotient = $2x - 1$, remainder = 3.

The Written Layout — How a Long Division Looks on Paper

Here is Division Example 1 written out in the traditional layout. Read it with the five steps of the previous page: each "divide – multiply – subtract" cycle produces one line of the quotient and one subtraction block.

```
           2x - 1            <- quotient
         ____________
x + 2  ) 2x^2 + 3x + 1
         2x^2 + 4x
         ----------
               -x + 1
               -x - 2
               ------
                    3        <- remainder
```

Reading the layout — three habits

1. **Align by degree:** x^2 terms above x^2 terms, x terms above x terms — misalignment is the number-one cause of subtraction errors.
2. **Subtract the WHOLE row:** when you subtract $2x^2 + 4x$, the sign of every term flips — this is where the $-x$ in the working comes from $(3x - 4x)$.
3. **The remainder sits alone at the end** only when its degree has dropped below the divisor's. Here 3 (degree 0) < x + 2 (degree 1) — stop.

TOPPER TIP: *On rough paper, rule faint vertical guide-lines under each degree column before you begin. Thirty seconds of drawing saves three minutes of error-hunting.*

Division Example 2 — A Cubic Divided by a Quadratic

DIVISION EXAMPLE 2

Divide $x^3 - 3x^2 + 5x - 3$ by $x^2 - 2$. Find the quotient and the remainder.

THINK FIRST: The divisor has degree 2, so the quotient will have degree $3 - 2 = 1$, and the remainder's degree must drop below 2.

1. Write the dividend with all powers present: $x^3 - 3x^2 + 5x - 3$ (no term is missing here). Divide leading terms: $x^3 \div x^2 = x$. First quotient term: x.

2. Multiply back: $x \times (x^2 - 2) = x^3 - 2x$. Subtract: $(x^3 - 3x^2 + 5x) - (x^3 - 2x) = -3x^2 + 7x$. Bring down -3: new dividend $-3x^2 + 7x - 3$.
   Why: Watch the x-term: 5x − (−2x) = 7x. Sign slips live here.

3. Repeat: $-3x^2 \div x^2 = -3$. Multiply back: $-3 \times (x^2 - 2) = -3x^2 + 6$. Subtract: $(-3x^2 + 7x - 3) - (-3x^2 + 6) = 7x - 9$.

4. The remainder $7x - 9$ has degree $1 < 2$ (the divisor's degree) — STOP. Quotient $q(x) = x - 3$, remainder $r(x) = 7x - 9$.

5. Verify: $(x^2 - 2)(x - 3) + (7x - 9) = (x^3 - 3x^2 - 2x + 6) + 7x - 9 = x^3 - 3x^2 + 5x - 3 = p(x)$. Correct.

ANSWER: Quotient = x − 3, remainder = 7x − 9.

```
          x - 3            <- quotient
        _________________
x^2 - 2 ) x^3 - 3x^2 + 5x - 3
          x^3       - 2x
          -----------------
               -3x^2 + 7x - 3
               -3x^2      + 6
               --------------
                        7x - 9   <- remainder
```

PIN POINT

DIVISION ALGORITHM: p(x) = g(x) × q(x) + r(x), where r(x) = 0 OR deg r(x) < deg g(x). If r(x) = 0, then g(x) is a FACTOR of p(x) — and every zero of g(x) is a zero of p(x).

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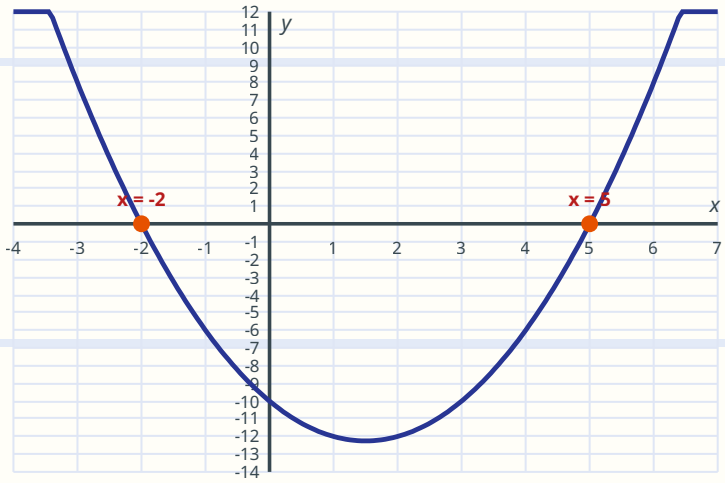
08 Multiple Methods

8D • CHALLENGE — ONE PROBLEM, THREE ROADS

The problem: find the zeroes of $p(x) = x^2 - 3x - 10$. A one-method student is stuck when that method fails. A three-method student simply changes roads. Learn all three.

Method A — Read the graph (the Visual road)



The parabola crosses the x-axis at $x = -2$ and $x = 5$, so the zeroes are **-2 and 5**. Strength: instant understanding. Weakness: needs an accurate graph — useless for zeroes like 4/3.

Method B — Factorise by splitting (the Algebra road)

1. $a \times c = -10$, $b = -3$. Split pair: -5 and $+2$.
2. $x^2 - 5x + 2x - 10 = x(x - 5) + 2(x - 5) = (x - 5)(x + 2)$.
3. Zero-product rule: $x = 5$ or $x = -2$.

Strength: exact, works in the exam hall with no graph paper. This is the board's expected road.

Method C — and Choosing Your Road

Method C — The sum-product detective (the Bridge road)

1. From the coefficients, the zeroes (call them α, β) must satisfy α + β = −b/a = 3 and αβ = c/a = −10.
   Why: The bridge tells us the zeroes' fingerprints before we know the zeroes.
2. Hunt two numbers with sum 3 and product −10: factor pairs of −10 are (1, −10), (−1, 10), (2, −5), (−2, 5). Only −2 and 5 add to 3.
3. So the zeroes are −2 and 5 — agreeing with Methods A and B.
   Why: Three roads, one destination. When two roads agree, you can walk into the answer box with confidence.

Forming a polynomial — two roads

Problem: form a quadratic whose zeroes are 3 and −2.

Road 1 (factors): $(x - 3)(x + 2) = x^2 - x - 6$.

Road 2 (formula): $S = 3 + (-2) = 1$, $P = 3 \times (-2) = -6$, so $p(x) = x^2 - Sx + P = x^2 - x - 6$. Same road-end — Road 2 is faster when only S and P are given; Road 1 is faster when the zeroes themselves are given.

Situation	Best road
Graph is drawn / picture question	Method A — read the x-axis meetings
Exam question: "find the zeroes"	Method B — split and factorise
Only sum/product conditions given	Method C — detective work on the bridge
Checking any answer	Use a DIFFERENT road from the one that produced it

> **TOPPER TIP:** *The golden rule of Method-choosing: solve by one road, verify by another. A graph check takes twenty seconds and has saved more board marks than any other habit in this chapter.*

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09 Interactive Simulations & Hands-on Activities

TRY IT YOURSELF — LEARN BY DOING

Graph paper ready? In this chapter, your hand must learn what your eye saw in Section 04. Complete all three activities before opening the answers.

Activity 1 — The graph-plotting lab: $y = x^2 - 3x - 4$

Complete the table, plot the eight points on graph paper, join them with a smooth curve, and read off the zeroes. (This is the same polynomial as Section 04 — now YOU draw it.)

x	−2	−1	0	1	2	3	4	5
y	...	...	−4	...	−6	...	...	...

Questions: (a) At which two x-values is y = 0? (b) What is the lowest point (vertex) of your curve? (c) The curve is symmetric about a vertical line — which line?

Activity 2 — Predict first, plot second

For each polynomial, FIRST write your prediction ("it will cross / touch / miss the x-axis because..."), THEN compute the table and plot.

Polynomial	Table (x, y)	My prediction
$y = x^2 - 4x + 4$	x = 0, 1, 2, 3, 4 → y = 4, ..., ..., ..., 4	
$y = x^2 + 1$	x = −2, −1, 0, 1, 2 → y = 5, ..., ..., ..., 5	

Activity 3 — The zero hunt

A number is a zero only if substituting it makes the polynomial exactly 0. Test each candidate and mark Zero / Not a zero.

Polynomial	Candidate	Value on substituting	Zero?
$p(x) = x^2 - 5x + 6$	x = 1, 2, 3, 4		
$q(x) = 2x^2 + 3x - 2$	x = 1/2, −2, 1		
$r(x) = x^2 - 2x - 3$	x = −1, 2, 3		

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Activity Answers — Check After Trying

Activity 1: y-values: 6, 0, −4, −6, −6, −4, 0, 6. (a) y = 0 at x = −1 and x = 4 — the zeroes. (b) Vertex: the lowest value is y = −6, reached at x = 1 and x = 2 — the true vertex sits between them at x = 3/2, y = −25/4 (the midpoint of the zeroes −1 and 4 is 3/2 — Section 12 explains why). (c) The line x = 3/2.

Activity 2: $y = x^2 - 4x + 4$: values 4, 1, 0, 1, 4 — the parabola TOUCHES the axis at x = 2 (one zero, counted twice: it is $(x - 2)^2$). $y = x^2 + 1$: values 5, 2, 1, 2, 5 — the smallest value is 1, never 0, so the curve MISSES the axis: no real zeroes. If your prediction used the words “the values never reach 0”, you were thinking like a mathematician.

Activity 3: p(x): p(1) = 2 (not), p(2) = 0 (ZERO), p(3) = 0 (ZERO), p(4) = 2 (not). q(x): q(1/2) = 1/2 + 3/2 − 2 = 0 (ZERO), q(−2) = 8 − 6 − 2 = 0 (ZERO), q(1) = 3 (not). r(x): r(−1) = 0 (ZERO), r(2) = −3 (not), r(3) = 0 (ZERO).

TOPPER TIP: *Notice Activity 3's hidden lesson: p(1) = 2 and p(4) = 2 — equal values, neither zero. Only the exact value 0 earns the title ‘zero of the polynomial’. Close is not zero.*

THE KALVIYOGI DIGITAL COMPANION — this chapter continues in the KalviYogi app/website:
- **Video lesson:** “Zeroes You Can See” — the graph method, animated.
- **Simulation:** Graph Explorer — drag a parabola up and down and watch its zeroes appear, merge and vanish.
- **Simulation:** Split-the-Middle-Term Trainer — endless factorisation practice with instant feedback.
- **Online test:** Chapter 2 test — 20 questions, marked instantly, errors explained.
- **Podcast:** “The Arch That Touched the Ground” — the Section 03 story as audio revision.
- **Active Recall deck:** 15 flashcards built from this chapter's Pin Points.

QR code and link activate with the KalviYogi app/website at print.

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10 Common Mistakes

LEARN FROM OTHER STUDENTS' ERRORS — BEFORE THEY BECOME YOURS

MISTAKE 1 — Confusing 'zero of the polynomial' with 'the value at zero'.

Wrong way: "The zero of $x^2 - 3x - 4$ is $p(0) = -4$."

Why wrong: $p(0)$ is the value of the polynomial AT $x = 0$ — here it is the constant term, -4. A **zero** is an input that makes the value zero.

Right way: Solve $p(x) = 0$: zeroes are -1 and 4, because $p(-1) = 0$ and $p(4) = 0$. Keep the two phrases in separate boxes in your mind: 'value at 0' vs 'zero of $p(x)$'.

MISTAKE 2 — Losing the minus sign in the sum of zeroes.

Wrong way: For $2x^2 - 7x + 3$: "sum of zeroes = $b/a = -7/2$."

Why wrong: The formula is $\alpha + \beta = -b/a$. Here $b = -7$, so $-b/a = -(-7)/2 = +7/2$. Check against the actual zeroes 3 and 1/2: their sum is 7/2, positive.

Right way: Say the formula aloud with its minus: "sum = minus-b-by-a". Then substitute b WITH its sign.

MISTAKE 3 — Reading b from a shuffled polynomial.

Wrong way: For $6x^2 - 3 - 7x$, taking $b = 7$ (or $c = -7$) straight from the printed order.

Why wrong: The letters a, b, c are defined by POSITION in the standard form $ax^2 + bx + c$ — a with x^2, b with x, c constant. In the shuffled polynomial the x-term is $-7x$.

Right way: First rewrite: $6x^2 - 7x - 3$. Only then read $a = 6$, $b = -7$, $c = -3$. (NCERT Q1(iii) is designed to catch exactly this.)

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Common Mistakes 4–6

MISTAKE 4 — Counting the wrong meetings in graph questions.

Wrong way: Counting where the curve crosses the y-axis, or counting its turning points, as zeroes.

Why wrong: A zero needs y = 0 — the point must lie ON the x-axis. A y-axis crossing has x = 0, not y = 0.

Right way: Run your finger along the X-AXIS only, and count touches/crossings there. Nothing else counts.

MISTAKE 5 — Sign slip when forming a polynomial.

Wrong way: Sum = −1/4, product = 1/4 → student writes $x^2 - (1/4)x + 1/4$.

Why wrong: The formula subtracts the sum AS IT IS: $x^2 - Sx + P$ with S = −1/4 gives $x^2 - (-1/4)x + 1/4 = x^2 + (1/4)x + 1/4$. The student subtracted +1/4 instead of −1/4.

Right way: Write S in brackets first — $x^2 - (-1/4)x + (1/4)$ — then simplify the signs. (This is NCERT Q2(v), and it appears in Find-the-Error E3's family.)

MISTAKE 6 — A split pair that satisfies only ONE condition.

Wrong way: For $3x^2 - x - 4$, splitting with −3 and +2 because −3 + 2 = −1 = b.

Why wrong: The pair must ALSO multiply to a × c = −12. But −3 × 2 = −6. The grouping then fails — no common bracket appears — and students force a wrong factorisation.

Right way: The pair is −4 and +3: sum −1, product −12. Check BOTH conditions before writing a single term. If the grouping produces no common bracket, your pair was wrong — go back, do not force it.

TOPPER TIP: *Three of these six mistakes are pure SIGN mistakes. In this chapter, marks are rarely lost on ideas — they are lost on minus signs. Slow down at every −b/a, every −(−...), every split.*

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11 Graded Practice Problems

FIVE LEVELS — CLIMB ONE LEVEL AT A TIME • ANSWERS IN THE ANSWER KEY

Rules of the climb: finish a level with at most one error before moving up. Write full steps for Level 2 onwards — answers alone earn nothing in the board exam.

LEVEL 1 — FOUNDATION (direct concepts)

L1.1 For $p(x) = 2x^2 - 3x + 5$, find $p(1)$.

L1.2 Is $x = 2$ a zero of $x^2 - 5x + 6$? Justify in one line.

L1.3 Find the zeroes of $x^2 - 9$.

L1.4 For $5x^2 - 3x + 2$, write down the sum of zeroes and the product of zeroes. (Do not find the zeroes.)

L1.5 Find the zeroes of $x^2 - 7x + 12$.

L1.6 Write a quadratic polynomial whose zeroes are 2 and 3.

LEVEL 2 — SKILL BUILDER (textbook standard)

L2.1 Find the zeroes of $2x^2 - 7x + 3$ and verify the relationship between the zeroes and the coefficients.

L2.2 Find the zeroes of $4x^2 + 4x - 3$ and verify the relationship between the zeroes and the coefficients.

L2.3 Find a quadratic polynomial whose sum of zeroes is -3 and product of zeroes is 2.

L2.4 Find a quadratic polynomial whose sum of zeroes is 0 and product of zeroes is -4.

L2.5 Find the zeroes of $x^2 + 7x + 10$ and verify the relationship between the zeroes and the coefficients.

L2.6 Find the zeroes of $3x^2 + 5x - 2$ and verify the relationship between the zeroes and the coefficients.

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Graded Practice — Levels 3, 4 and 5

LEVEL 3 — APPLICATION (real-life problems)

L3.1 A ball's height after t seconds is $h(t) = -5t^2 + 30t$ metres. After how many seconds does it hit the ground? (Ignore t = 0, the moment of throwing.)

L3.2 A rectangular lawn has area $A(x) = x^2 + 11x + 30$ m^2. Express the area as a product of the two side-lengths.

L3.3 An arch has height $h(x) = -x^2 + 6x - 5$ metres at distance x from a fixed pillar. Find where the arch meets the ground, and hence the width of the opening.

L3.4 Two numbers have sum 11 and product 30. Form a quadratic polynomial with these as sum and product of zeroes, and hence find the two numbers.

LEVEL 4 — ADVANCED REASONING (multi-concept)

L4.1 If α and β are the zeroes of $x^2 - 5x + 6$, find $\alpha^2 + \beta^2$ — without finding α and β separately.

L4.2 One zero of $3x^2 - 8x + (2k + 1)$ is seven times the other. Find the value of k.

L4.3 If α, β are the zeroes of $ax^2 + bx + c$ and $\alpha + \beta = \alpha\beta$, show that $b + c = 0$.

LEVEL 5 — MATHEMATICAL GENIUS (olympiad flavour)

L5.1 Find all values of k for which $x^2 + kx + 9$ has equal zeroes.

L5.2 If α, β are the zeroes of $x^2 - 6x + 5$, form a quadratic polynomial whose zeroes are α + 1 and β + 1.

> **TOPPER TIP:** *Stuck at a level? Section 07 contains a worked twin of almost every question here. Find the twin, study its steps, close the book, and try again.*

Practice Plus — Concept MCQs (1 of 3)

25 one-mark questions in true board style. Choose the ONE correct option. The Answer Key gives the key AND a one-line explanation for every question — read the explanation even when you are right.

M1. The degree of the polynomial $5x^3 - 4x^2 + 7x - 2$ is

(A) 2 (B) 4 (C) 3 (D) 1

M2. The zeroes of $x^2 - 3x - 4$ are

(A) −4 and 1 (B) 4 and −1 (C) 4 and 1 (D) −4 and −1

M3. Which of the following is a zero of $x^2 - 7x + 10$?

(A) 3 (B) 2 (C) 4 (D) −2

M4. The sum of the zeroes of $3x^2 - 5x + 2$ is

(A) 5/3 (B) −5/3 (C) 2/3 (D) −2/3

M5. The product of the zeroes of $2x^2 + 3x - 5$ is

(A) 5/2 (B) −3/2 (C) −5/2 (D) 3/5

M6. A quadratic polynomial whose zeroes are −3 and 4 is

(A) $x^2 + x - 12$ (B) $x^2 - x + 12$ (C) $x^2 + x + 12$ (D) $x^2 - x - 12$

M7. The maximum number of zeroes a cubic polynomial can have is

(A) 2 (B) 3 (C) 4 (D) 1

M8. The graph of $y = p(x)$ is a parabola touching the x-axis at exactly one point. The number of distinct zeroes of $p(x)$ is

(A) 1 (B) 2 (C) 0 (D) 3

M9. If $\alpha + \beta = 0$ and one zero of a quadratic is 3, the other zero is

(A) 0 (B) 3 (C) −3 (D) 6

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Practice Plus — Concept MCQs (2 of 3)

M10. The number of real zeroes of $x^2 + 1$ is

(A) 0 (B) 1 (C) 2 (D) 4

M11. If $x = 1$ is a zero of $x^2 + kx - 6$, then k equals

(A) -5 (B) 6 (C) -6 (D) 5

M12. The sum of the zeroes of $-x^2 + 5x - 6$ is

(A) -5 (B) 5 (C) 6 (D) -6

M13. The product of the zeroes of $4x^2 - 9$ is

(A) 9/4 (B) $-4/9$ (C) $-9/4$ (D) 0

M14. The zeroes of $6x^2 - 7x - 3$ are

(A) 3/2 and $-1/3$ (B) $-3/2$ and 1/3 (C) 3 and $-1/3$ (D) 3/2 and 1/3

M15. A linear polynomial has at most how many zeroes?

(A) 0 (B) 1 (C) 2 (D) 3

M16. If α, β are the zeroes of $x^2 - 6x + 8$, then $\alpha + \beta + \alpha\beta$ equals

(A) 6 (B) 8 (C) 2 (D) 14

M17. A polynomial of degree 4 has at most how many zeroes?

(A) 1 (B) 2 (C) 4 (D) 5

M18. For $p(x) = x^2 - 5x + 6$, $p(x) = 0$ when x equals

(A) 1 (B) 3 (C) 4 (D) 6

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Practice Plus — Concept MCQs (3 of 3)

M19. The zeroes of $x^2 - 2x$ are

(A) 0 and 2 (B) 0 and −2 (C) 2 and −2 (D) only 2

M20. If the zeroes of $x^2 + px + q$ are 2 and 3, then q equals

(A) 5 (B) −5 (C) 6 (D) −6

M21. A parabola crosses the x-axis at two distinct points. The number of zeroes of the corresponding polynomial is

(A) 0 (B) 1 (C) 2 (D) 3

M22. If α and β are the zeroes of a quadratic and αβ = 0, then

(A) at least one zero is 0 (B) both zeroes are 0 (C) α + β = 0 (D) the polynomial has no zeroes

M23. A quadratic polynomial with sum of zeroes −1/2 and product of zeroes −3 is

(A) $2x^2 - x - 6$ (B) $x^2 - (1/2)x + 3$ (C) $2x^2 + x + 6$ (D) $2x^2 + x - 6$

M24. The zero of the polynomial $3x - 6$ is

(A) 6 (B) 2 (C) −2 (D) 3

M25. If α, β are the zeroes of $x^2 - x - 6$, then $1/\alpha + 1/\beta$ equals

(A) 1/6 (B) −5/6 (C) −1/6 (D) 5/6

TOPPER TIP: *Scoring guide: 22–25 — board-ready on concepts. 16–21 — revise the sections named in the key's explanations. Below 16 — redo Sections 04–06, then retake on Day 3.*

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12 Mathematical Reasoning

JUSTIFY AND PROVE — FULL SOLUTIONS

REASONING 1

If α and β are the zeroes of $ax^2 + bx + c$ ($a \neq 0$, $c \neq 0$), prove that $1/\alpha + 1/\beta = -b/c$.

THINK FIRST: Almost every 'prove' question in this chapter is the bridge wearing a disguise. Undress it: rewrite the target using α + β and αβ, then substitute.

1. Combine the fractions on the left: $1/\alpha + 1/\beta = (\beta + \alpha) / (\alpha\beta) = (\alpha + \beta) / (\alpha\beta)$.
   Why: Add fractions by the common denominator αβ — and note the numerator becomes the SUM of zeroes.
2. Substitute the bridge values: $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$. So $1/\alpha + 1/\beta = (-b/a) / (c/a)$.
3. Dividing by a fraction multiplies by its reciprocal: $(-b/a) \times (a/c) = -b/c$. The a cancels.
   Why: That cancellation is why the answer is so clean — a disappears completely.
4. Hence $1/\alpha + 1/\beta = -b/c$. Proved. ∎

ANSWER: $1/\alpha + 1/\beta = (\alpha + \beta)/(\alpha\beta) = -b/c$. ∎

REASONING 2

Prove that a quadratic polynomial cannot have more than two zeroes.

1. Suppose α is one zero of the quadratic p(x). Then (x − α) is a factor of p(x), so p(x) = (x − α) × (a linear polynomial), say p(x) = (x − α)(mx + n).
   Why: Dividing a degree-2 polynomial by a degree-1 factor leaves a degree-1 quotient (division algorithm, Section 05).
2. Let β be any other zero, $\beta \neq \alpha$. Then p(β) = 0 gives (β − α)(mβ + n) = 0.
3. Since $\beta \neq \alpha$, the first factor β − α is not zero. So the second factor must be: mβ + n = 0, giving $\beta = -n/m$.
   Why: The zero-product rule again — one factor MUST be zero, and we have ruled out the first.
4. −n/m is a single fixed number. So besides α, there is room for only ONE more zero. Total: at most 2 zeroes. Proved. ∎
   Why: The same argument, repeated, shows a degree-n polynomial has at most n zeroes — the degree rule of Section 04 is a theorem, not a coincidence.

ANSWER: A quadratic polynomial has at most two zeroes. ∎

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Reasoning 3 — The Parabola's Mirror Line

REASONING 3

Show that the axis of symmetry of the parabola $y = ax^2 + bx + c$ is the line $x = -b/(2a)$. Verify for $y = x^2 - 3x - 4$.

1. Let the zeroes of the polynomial be α and β (assume two real zeroes). The graph meets the x-axis at (α, 0) and (β, 0).

2. A parabola is symmetric: its axis is the vertical line passing midway between any two points at the same height — in particular, midway between the two x-axis meetings. So the axis is $x = (\alpha + \beta)/2$.
Why: Symmetry means the two zeroes stand at equal distances from the mirror line.

3. Substitute the bridge: $\alpha + \beta = -b/a$, so the axis is $x = (-b/a)/2 = -b/(2a)$. Proved. ∎

4. Verify on $x^2 - 3x - 4$: zeroes are -1 and 4, midpoint $= (-1 + 4)/2 = 3/2$. And $-b/(2a) = 3/2$. Match.
Why: This is exactly the symmetry line you found by drawing in Activity 1 — the picture and the algebra agree.

5. Bonus: the vertex sits on this line, so its x-coordinate is $-b/(2a)$. For our parabola the vertex is $(3/2, -25/4)$.

ANSWER: The axis of symmetry is $x = -b/(2a)$; for $x^2 - 3x - 4$ it is $x = 3/2$. ∎

TOPPER TIP: *Reasoning questions reward structure: state what you use, substitute, conclude. Write 'Proved' (or the symbol ∎) at the end — it tells the examiner the argument is complete.*

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13 Real-Life Applications

8D • APPLY — THE CHAPTER AT WORK IN THE WORLD

APPLICATION 1

A cricketer hits a ball. Its height after t seconds is $h(t) = -5t^2 + 20t$ metres. (a) After how long does the ball land? (b) How high is it after 1 second?

THINK FIRST: One polynomial, two question types: 'when is it zero' (solve) and 'what is it at time t' (substitute). Read the question to know which.

1. (a) Landing means height = 0: solve $-5t^2 + 20t = 0$.
   Why: The story signal 'lands' = zeroes, as in Section 03.
2. Factor out $-5t$: $-5t(t - 4) = 0$.
3. Zero-product rule: $t = 0$ or $t = 4$. $t = 0$ is the moment of the hit, so the ball lands at $t = 4$ seconds.
   Why: Both zeroes are real events — launch and landing. The story chooses which one answers the question.
4. (b) Substitute $t = 1$: $h(1) = -5 + 20 = 15$ metres.

ANSWER: (a) The ball lands after 4 seconds. (b) After 1 second it is 15 m high.

APPLICATION 2

A rectangular flower bed has area $A(x) = x^2 + 8x + 15$ m^2. (a) Write the area as a product of the two side-lengths. (b) If $x = 4$, find the sides and verify the area.

1. (a) Split hunt: two numbers with sum 8 and product 15 — they are 3 and 5.
2. $x^2 + 8x + 15 = x^2 + 3x + 5x + 15 = x(x + 3) + 5(x + 3) = (x + 3)(x + 5)$. So the sides are $(x + 3)$ m and $(x + 5)$ m.
3. (b) Put $x = 4$: sides are 7 m and 9 m. Area $= 7 \times 9 = 63$ m^2.
4. Verify from the formula: $A(4) = 16 + 32 + 15 = 63$ m^2. Same.

ANSWER: (a) Sides: $(x + 3)$ m and $(x + 5)$ m. (b) For $x = 4$: 7 m by 9 m, area 63 m^2.

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Application 3 — The Arch Again

APPLICATION 3

Remember Anitha's arch from Section 03, with height $h(x) = -x^2 + 4x$. A delivery lorry 3.75 m tall must pass through the centre of the arch. Can it pass, and with how much clearance?

THINK FIRST: 'Centre of the arch' is a midpoint-of-zeroes question; 'can it pass' is a comparison. Two small ideas, one real decision.

1. The arch's feet (zeroes) are at x = 0 and x = 4, so its centre is at x = (0 + 4)/2 = 2.
   Why: The centre of a symmetric arch is the midpoint of its zeroes — Reasoning 3 in disguise.
2. Height at the centre: h(2) = −4 + 8 = 4 metres.
3. Compare: arch height 4 m > lorry height 3.75 m. Clearance = 4 − 3.75 = 0.25 m.

ANSWER: Yes, the lorry passes, with 0.25 m (25 cm) of clearance at the centre.

The story signals of this chapter — collected

Lands / touches down / returns to the ground → zeroes of the height polynomial.
Area as one expression, sides wanted → factorise.
Centre / topmost point → midpoint of the zeroes, then substitute.
Break-even / no profit no loss → zeroes of the profit polynomial (next page).
Every application in this chapter — and nearly every one in the board exam — is one of these four signals in a costume.

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Application 4 — A Business Breaks Even

APPLICATION 4

A small business models its daily profit (in rupees) on selling x items as $P(x) = -x^2 + 40x - 300$. Find the break-even points — the sales levels at which profit is exactly zero.

THINK FIRST: Break-even = zeroes of the profit polynomial. Whenever a story says 'no profit, no loss', set the polynomial to zero.

1. Break-even means $P(x) = 0$: solve $-x^2 + 40x - 300 = 0$. Multiply by -1: $x^2 - 40x + 300 = 0$.
   Why: Multiplying an equation by −1 keeps its zeroes — and makes the splitting hunt friendlier.
2. Split hunt: two numbers with sum -40 and product 300 — they are -30 and -10.
3. $x^2 - 30x - 10x + 300 = x(x - 30) - 10(x - 30) = (x - 30)(x - 10) = 0$.
4. Zero-product rule: $x = 30$ or $x = 10$.
5. Interpret: below 10 items the business loses money (setting-up costs dominate); between 10 and 30 it profits; beyond 30, discounting and overtime pull profit back to zero at 30.
   Why: A downward parabola is positive BETWEEN its zeroes — the graph from Section 04 is doing business analysis.

ANSWER: Break-even at 10 items and at 30 items. Profit is positive for sales strictly between 10 and 30 items.

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14 Competency-Based Questions

FIVE CASE STUDIES • ASSERTION–REASON • BOARD EXAM STYLE, FULLY SOLVED

Case Study 1 — The Bridge Arch

A river bridge has a parabolic arch. Taking the left bank at x = 0, the height of the arch above water at distance x metres is $p(x) = -x^2 + 8x - 12$ metres. Answer the four questions.

(i) Find the zeroes of p(x).

1. Set $p(x) = 0$ and multiply by -1: $x^2 - 8x + 12 = 0$.
2. Split: -2 and -6 (sum -8, product 12): $(x - 2)(x - 6) = 0$.
3. Zeroes: $x = 2$ and $x = 6$.

ANSWER: 2 and 6.

(ii) What is the width of the arch at water level?

1. The arch meets the water at its zeroes, $x = 2$ and $x = 6$.
   Why: Water level is height 0 — the zeroes ARE the meeting points.
2. Width $= 6 - 2 = 4$ metres.

ANSWER: 4 metres.

Case Study 1 (continued) and Case Study 2

(iii) Verify that the sum and product of the zeroes match the coefficients of p(x).

1. Sum of zeroes = 2 + 6 = 8. From coefficients (a = −1, b = 8): −b/a = −8/−1 = 8. Match.
   Why: Careful: a is −1 here, not 1 — read coefficients from p(x) as written.
2. Product of zeroes = 2 × 6 = 12. And c/a = −12/−1 = 12. Match.

ANSWER: Sum = 8 = −b/a; product = 12 = c/a. Verified.

(iv) Find the greatest height of the arch above the water.

1. The vertex stands midway between the zeroes: x = (2 + 6)/2 = 4.
   Why: Reasoning 3: the axis of symmetry passes through the midpoint.
2. p(4) = −16 + 32 − 12 = 4 metres.

ANSWER: 4 metres, at the centre x = 4.

Case Study 2 — The Juggler's Ball

A juggler throws a ball straight up. Its height after t seconds is $h(t) = -5t^2 + 25t$ metres.

(i) Find the zeroes of h(t).

1. Set h(t) = 0: $-5t^2 + 25t = 0$. Factor out −5t: $-5t(t - 5) = 0$.
2. Zeroes: t = 0 and t = 5.

ANSWER: 0 and 5.

(ii) For how long is the ball in the air?

1. It leaves the hand at t = 0 and returns to hand height at the next zero, t = 5.

ANSWER: 5 seconds.

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Case Studies 2 (continued) and 3

(iii) How high is the ball after 2 seconds?

1. Substitute t = 2: h(2) = −5(4) + 25(2) = −20 + 50 = 30 metres.

ANSWER: 30 metres.

(iv) Use the coefficients of h(t) to predict the sum of its zeroes, and check against part (i).

1. a = −5, b = 25, so −b/a = −25/−5 = 5.
2. From part (i): 0 + 5 = 5. Match.

ANSWER: Predicted sum 5; actual sum 5. Verified.

Case Study 3 — The School Garden Plot

A rectangular garden plot has area $A(x) = x^2 + 9x + 20$ m^2, where x is a design number chosen by the gardener.

(i) Express A(x) as a product of the two side-lengths.

1. Split hunt: sum 9, product 20 — the pair is 4 and 5.
2. $x^2 + 9x + 20 = (x + 4)(x + 5)$. Sides: (x + 4) m and (x + 5) m.

ANSWER: (x + 4) m and (x + 5) m.

(ii) If the gardener chooses x = 3, find the two sides.

1. Substitute: 3 + 4 = 7 m and 3 + 5 = 8 m.

ANSWER: 7 m and 8 m.

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Case Studies 3 (continued) and 4

(iii) For x = 3, verify the area in two ways.

1. From the sides: $7 \times 8 = 56$ m^2.
2. From the formula: $A(3) = 9 + 27 + 20 = 56$ m^2. Same.

ANSWER: 56 m^2 by both routes.

(iv) The zeroes of A(x) are −4 and −5. Why can the gardener not choose x = −4, even though it is a zero?

1. At x = −4, one side becomes x + 4 = 0 and the other is −1 — a plot with a zero or negative side does not exist.
   Why: Zeroes belong to algebra; the STORY decides which numbers are allowed. Lengths must be positive.
2. So x must be positive in this context (in fact x > 0).

ANSWER: Because side-lengths must be positive; at x = −4 the sides would be 0 m and −1 m, which is impossible for a real plot.

Case Study 4 — The Snack Stall

A school-fair snack stall models its profit (in rupees) on selling x items as $P(x) = -2x^2 + 32x - 96$.

(i) Find the break-even sales levels (profit exactly zero).

1. Set P(x) = 0 and divide by −2: $x^2 - 16x + 48 = 0$.
2. Split: −4 and −12 (sum −16, product 48): $(x - 4)(x - 12) = 0$.
3. Break-even at x = 4 and x = 12 items.

ANSWER: 4 items and 12 items.

(ii) What is the profit on selling 8 items?

1. $P(8) = -2(64) + 32(8) - 96 = -128 + 256 - 96 = 32$.

ANSWER: Rs. 32 profit.

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Case Studies 4 (continued) and 5

(iii) Predict the sum of the zeroes from the coefficients of P(x), and check.

1. $-b/a = -32/-2 = 16$.
2. Actual: 4 + 12 = 16. Match.

ANSWER: 16, matching 4 + 12.

(iv) Between which sales levels does the stall make a profit?

1. P(x) is a downward parabola, positive between its zeroes 4 and 12.
   Why: Section 04's picture, doing business: between the zeroes the curve is above the axis.
2. So profit is positive for $4 < x < 12$ items.

ANSWER: Strictly between 4 and 12 items.

Case Study 5 — The Fountain Jet

Water leaves a fountain nozzle at ground level. At horizontal distance x metres, the jet's height is $y = -x^2 + 6x$ metres.

(i) How far from the nozzle does the water land?

1. Landing means $y = 0$: $-x^2 + 6x = 0$, i.e. $x(6 - x) = 0$ (after taking out $-x$, signs arranged).
2. Zeroes: $x = 0$ (the nozzle) and $x = 6$. The water lands 6 m away.

ANSWER: 6 metres.

Case Study 5 (continued)

(ii) Find the product of the zeroes from the coefficients, and check it directly.

1 c/a = 0/−1 = 0. Directly: 0 × 6 = 0. Match.

Why: c = 0 always forces a zero at x = 0 — here, the nozzle itself.

ANSWER: 0, matching 0 × 6.

(iii) What is the greatest height of the jet, and where?

1 Midpoint of zeroes: x = (0 + 6)/2 = 3. Height there: y = −9 + 18 = 9 metres.

ANSWER: 9 metres, at x = 3 m from the nozzle.

TOPPER TIP: *Five case studies, one skeleton: read the story → name the polynomial → zeroes or substitute → answer in the story's words. CBSE's competency questions reward exactly this skeleton — the context changes, the skeleton does not.*

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Assertion–Reason Questions (1 of 2)

Options: (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is NOT the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.

AR1. Assertion (A): The graph of $y = x^2 - 4$ crosses the x-axis at two points.
Reason (R): The number of zeroes of a polynomial equals the number of points where its graph meets the x-axis.

AR2. Assertion (A): The sum of the zeroes of $3x^2 - 6x + 2$ is 2.
Reason (R): The sum of the zeroes of $ax^2 + bx + c$ is c/a.

AR3. Assertion (A): $x = 2$ is a zero of $x^2 - 5x + 6$.
Reason (R): A number a is a zero of p(x) if and only if p(a) = 0, and here p(2) = 0.

AR4. Assertion (A): A quadratic polynomial can have three distinct zeroes.
Reason (R): A polynomial of degree n has at most n zeroes.

AR5. Assertion (A): The product of the zeroes of $2x^2 - 3x + 7$ is 7/2.
Reason (R): The sum of the zeroes of $ax^2 + bx + c$ is $-b/a$.

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Assertion–Reason Questions (2 of 2)

Options: (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is NOT the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.

AR6. Assertion (A): If $\alpha + \beta = 5$ and $\alpha\beta = 6$, then a quadratic polynomial with zeroes α, β is $x^2 - 5x + 6$.

Reason (R): A quadratic polynomial with sum of zeroes S and product P is $k[x^2 - Sx + P]$.

AR7. Assertion (A): The zeroes of $4x^2 - 4x + 1$ are 1/2 and 1/2.

Reason (R): $4x^2 - 4x + 1 = (2x - 1)^2$.

AR8. Assertion (A): $p(x) = x^2 + 4$ has two real zeroes.

Reason (R): The equation $x^2 = -4$ has no real solution.

AR9. Assertion (A): The sum of the zeroes of $x^2 - 2x - 8$ is 2.

Reason (R): The sum of the zeroes of a quadratic polynomial equals b/a.

AR10. Assertion (A): The zeroes of $x^3 - 6x^2 + 11x - 6$ are 1, 2 and 3.

Reason (R): The sum of the zeroes of this cubic is $-b/a = 6$, and $1 + 2 + 3 = 6$.

> ***TOPPER TIP:*** *Answers with one-line justifications are in the Answer Key. Remember: option (a) needs R to EXPLAIN A — 'both true' alone is only option (b). AR10 is the classic trap: a matching sum is necessary, but it does not by itself prove those are the zeroes.*

15 Olympiad & Foundation Challenges

8D • CHALLENGE — TEN PROBLEMS THAT STRETCH THE BRIDGE

CHALLENGE 1

If α, β are the zeroes of $x^2 + px + q$, express $\alpha^3 + \beta^3$ in terms of p and q.

THINK FIRST: Olympiad rule of this chapter: never find α, β themselves. Work with their sum and product as two single numbers.

1. Read the bridge: $\alpha + \beta = -p$ and $\alpha\beta = q$.
2. Use the identity $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$.
   Why: Cubes of a sum split into a cube of the sum minus a correction — the correction is where αβ enters.
3. Substitute: $(-p)^3 - 3(q)(-p) = -p^3 + 3pq$.

ANSWER: $\alpha^3 + \beta^3 = 3pq - p^3$.

CHALLENGE 2

The zeroes of $x^2 - kx + 6$ are in the ratio 2 : 3. Find k.

1. Let the zeroes be 2m and 3m for some number m.
   Why: Ratio problems: give the parts a common unit m.
2. Product: $(2m)(3m) = 6m^2 = c/a = 6$, so $m^2 = 1$ and $m = \pm 1$.
3. Sum: $2m + 3m = 5m = -b/a = k$. So $k = 5$ (when $m = 1$) or $k = -5$ (when $m = -1$).
   Why: Both signs are genuine: zeroes (2, 3) give k = 5; zeroes (−2, −3) give k = −5.

ANSWER: k = 5 or k = −5.

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Olympiad Challenges 3 and 4

CHALLENGE 3

Show that 1, 2, 3 are the zeroes of $x^3 - 6x^2 + 11x - 6$, and verify all three cubic relations.

1. Test $x = 1$: $p(1) = 1 - 6 + 11 - 6 = 0$. So $(x - 1)$ is a factor.
   Why: Always test $x = 1$ and $x = -1$ first in a cubic — they are the cheapest tests in algebra.
2. Divide $p(x)$ by $(x - 1)$ (division algorithm): quotient $x^2 - 5x + 6$, remainder 0.
3. Factorise the quotient: $x^2 - 5x + 6 = (x - 2)(x - 3)$. So $p(x) = (x - 1)(x - 2)(x - 3)$ and the zeroes are 1, 2, 3.
4. Verify: sum $1 + 2 + 3 = 6 = -b/a = 6$. Pairs: $2 + 6 + 3 = 11 = c/a$. Product: $6 = -d/a = 6$. All three relations hold.

ANSWER: Zeroes 1, 2, 3; sum = 6 = −b/a, pairwise sum = 11 = c/a, product = 6 = −d/a. All verified.

CHALLENGE 4

If α, β are the zeroes of $x^2 - 5x + 6$, find $\alpha^4 + \beta^4$.

1. Bridge: $\alpha + \beta = 5$, $\alpha\beta = 6$.
2. First squares: $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 25 - 12 = 13$.
3. Now treat α^2, β^2 as a new pair with sum 13 and product $(\alpha\beta)^2 = 36$: $\alpha^4 + \beta^4 = 13^2 - 2(36) = 169 - 72 = 97$.
   Why: The same identity, climbed twice — like stairs.
4. Sanity check with the actual zeroes 2 and 3: $16 + 81 = 97$. Match.

ANSWER: $\alpha^4 + \beta^4 = 97$.

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Olympiad Challenges 5 and 6

CHALLENGE 5

The zeroes of $x^2 - 7x + k$ differ by 3. Find k.

1. Bridge: $\alpha + \beta = 7$ and $\alpha\beta = k$.
2. Use $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$: we get $3^2 = 49 - 4k$.
   Why: 'Differ by' questions always enter through this identity.
3. So $4k = 49 - 9 = 40$, and $k = 10$.
4. Check: $x^2 - 7x + 10$ has zeroes 2 and 5, which indeed differ by 3.

ANSWER: k = 10.

CHALLENGE 6

If α, β are the zeroes of $x^2 - 8x + k$ and $\alpha + \beta = \alpha\beta$, find k.

1. Bridge: $\alpha + \beta = 8$ and $\alpha\beta = k$.
   Why: Read both values straight off the coefficients.
2. The condition says $8 = k$.
   Why: Sometimes the 'hard' question is one line long — if your bridge is strong.

ANSWER: k = 8.

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Olympiad Challenges 7 and 8

CHALLENGE 7

Form a quadratic polynomial whose zeroes are the SQUARES of the zeroes of $x^2 - 5x + 6$.

1. For the old polynomial: $\alpha + \beta = 5$, $\alpha\beta = 6$.
2. The new zeroes are α^2 and β^2. Their sum: $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 25 - 12 = 13$.
3. Their product: $\alpha^2\beta^2 = (\alpha\beta)^2 = 36$.
4. Form it: $x^2 - 13x + 36$.

   Why: Check with the actual zeroes 2, 3: squares are 4, 9; $(x - 4)(x - 9) = x^2 - 13x + 36$. Match.

ANSWER: $x^2 - 13x + 36$.

CHALLENGE 8

One zero of $x^3 - 3x^2 - 10x + 24$ is 2. Find all its zeroes.

1. Since 2 is a zero, $(x - 2)$ is a factor. Divide $p(x)$ by $(x - 2)$: quotient $x^2 - x - 12$, remainder 0.

   Why: The division algorithm turns one known zero into a quadratic problem.
2. Factorise: $x^2 - x - 12 = (x - 4)(x + 3)$, giving zeroes 4 and -3.
3. All zeroes: 2, 4, -3. Quick check of the sum: $2 + 4 - 3 = 3 = -b/a = 3$. Consistent.

ANSWER: The zeroes are 2, 4 and -3.

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Olympiad Challenges 9 and 10

CHALLENGE 9

If α, β are the zeroes of $2x^2 - 7x + 3$, find α/β + β/α.

1. Combine: $\alpha/\beta + \beta/\alpha = (\alpha^2 + \beta^2) / (\alpha\beta)$.
2. Bridge: $\alpha + \beta = 7/2$, $\alpha\beta = 3/2$. So $\alpha^2 + \beta^2 = (7/2)^2 - 2(3/2) = 49/4 - 3 = 37/4$.
3. Divide: $(37/4) / (3/2) = (37/4) \times (2/3) = 37/6$.

ANSWER: α/β + β/α = 37/6.

CHALLENGE 10

Find all values of k for which $x^2 + kx + k$ has equal zeroes.

1. Equal zeroes means α = β. Then $(\alpha - \beta)^2 = 0$, i.e. $(\alpha + \beta)^2 = 4\alpha\beta$.
   Why: Equal-zeroes questions always pass through this gate.
2. Bridge: $\alpha + \beta = -k$, $\alpha\beta = k$. Substitute: $(-k)^2 = 4k$, so $k^2 - 4k = 0$, i.e. $k(k - 4) = 0$.
3. So k = 0 or k = 4. Check k = 4: $x^2 + 4x + 4 = (x + 2)^2$ — equal zeroes −2, −2. Check k = 0: the polynomial becomes x^2, whose zeroes are 0 and 0 — also equal, and still a genuine quadratic (coefficient of x^2 is 1 ≠ 0).
   Why: Do not throw away k = 0 carelessly — examine what it does to the polynomial before rejecting it.

ANSWER: k = 0 or k = 4.

TOPPER TIP: *Ten challenges, and not one of them needed the actual zeroes found first. That is the Olympiad lesson of this chapter: sum and product are handles strong enough to lift the whole polynomial.*

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Practice Plus — Find the Error (1 of 3)

Each item shows a student's written solution (in handwriting) containing **exactly one error**. Your job: spot it, name it, correct it. This is how examiners think — learn to mark like one.

E1. Find the sum of the zeroes of $2x^2 - 5x + 3$.

Sum of zeroes = b/a = −5/2.

Find the error — then write the corrected solution. (Corrections: Answer Key.)

E2. Find the zeroes of $x^2 - 2x - 8$.

$x^2 - 2x - 8 = (x - 4)(x + 2)$. So $x - 4 = 0$ or $x + 2 = 0$, giving $x = 4$ or $x = 2$.

Find the error — then write the corrected solution. (Corrections: Answer Key.)

E3. Find a quadratic polynomial with sum of zeroes 1/4 and product of zeroes −1.

$p(x) = x^2 + (1/4)x - 1$.

Find the error — then write the corrected solution. (Corrections: Answer Key.)

E4. Find the product of the zeroes of $3x^2 + 6x - 9$.

Product of zeroes = −c/a = −(−9)/3 = 3.

Find the error — then write the corrected solution. (Corrections: Answer Key.)

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Practice Plus — Find the Error (2 of 3)

E5. Find the zeroes of $t^2 - 15$.

$t^2 = 15$, so $t = \sqrt{15}$. The zero is $\sqrt{15}$.

Find the error — then write the corrected solution. (Corrections: Answer Key.)

E6. A graph meets the y-axis at (0, 3) and meets the x-axis at x = 1 and x = 4. How many zeroes does p(x) have?

The graph meets the axes at 3 points in total, so p(x) has 3 zeroes.

Find the error — then write the corrected solution. (Corrections: Answer Key.)

E7. Verify the sum of zeroes for $6x^2 - 3 - 7x$, whose zeroes are 3/2 and −1/3.

Here $b = 7$, so $-b/a = -7/6$. But $3/2 - 1/3 = 7/6$. The relationship fails!

Find the error — then write the corrected solution. (Corrections: Answer Key.)

E8. Divide $x^2 + 5x + 6$ by $x + 2$.

Quotient = $x + 3$, remainder = 6.

Find the error — then write the corrected solution. (Corrections: Answer Key.)

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Practice Plus — Find the Error (3 of 3)

E9. Is -2 a zero of $p(x) = x^2 - 3x + 2$?

$p(-2) = (-2)^2 - 3(-2) + 2 = 4 - 6 + 2 = 0$. Yes, -2 is a zero.

Find the error — then write the corrected solution. (Corrections: Answer Key.)

E10. State the sum of the zeroes of $x^3 - 6x^2 + 11x - 6$.

Sum of zeroes $= b/a = -6$.

Find the error — then write the corrected solution. (Corrections: Answer Key.)

TOPPER TIP: *If you found all ten quickly, your error-detector is exam-grade. These ten errors are the ten most common real errors in this chapter — examiners meet them thousands of times.*

Board Exam Hall — Previous Year Questions (1 of 6)

Every question below is a **real CBSE board examination question** on Polynomials. Where the examination paper and year are verified, we print them; where they are not, the question is labelled simply *CBSE Board Exam Question*. We never invent a year. Solve each before reading the steps — this is the real thing.

CBSE 2024 • STANDARD • MCQ

PYQ 1

If α and β (α > β) are the zeroes of the polynomial $-x^2 + 8x + 9$, then (α − β) is equal to: (A) −10 (B) 10 (C) ±10 (D) 8

THINK FIRST: The condition α > β removes the ± confusion — but only if you find both zeroes with their signs.

1. Factorise: $-x^2 + 8x + 9 = -(x^2 - 8x - 9) = -(x - 9)(x + 1)$.
   Why: Take −1 out first; then split −8x with −9 and +1.
2. Zeroes: x = 9 and x = −1. Since α > β: α = 9, β = −1.
3. α − β = 9 − (−1) = 10.
   Why: Subtracting a negative adds — the last trap of the question.

ANSWER: Option (B): α − β = 10.

CBSE-STYLE MARKING SCHEME: 1 mark — correct option. (Zeroes 1/2 + difference 1/2 in a worked version.)

Previous Year Questions (2 of 6)

CBSE 2024 • BASIC

PYQ 2

If α, β are the zeroes of the quadratic polynomial $2x^2 + 7x + 5$, find the value of $\alpha^2 + \beta^2 + \alpha\beta$.

THINK FIRST: Do NOT find α, β. Build $\alpha^2 + \beta^2$ from the bridge, then add αβ.

1. Bridge: $\alpha + \beta = -7/2$ and $\alpha\beta = 5/2$.
2. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 49/4 - 5 = 29/4$.
3. Add αβ: $29/4 + 5/2 = 29/4 + 10/4 = 39/4$.

ANSWER: $\alpha^2 + \beta^2 + \alpha\beta = 39/4$.

CBSE-STYLE MARKING SCHEME: 1 mark for sum and product from coefficients • 1 mark for the identity • 1 mark for the final value.

CBSE 2024 • COMPARTMENT

PYQ 3

If α and β are the zeroes of the polynomial $p(x) = x^2 - (k + 5)x + (5k + 1)$ such that $\alpha + \beta = \alpha\beta/3$, find the value of k.

THINK FIRST: Translate the condition into an equation in k using the bridge — one equation, one unknown.

1. Bridge: $\alpha + \beta = k + 5$ and $\alpha\beta = 5k + 1$.
   Why: Read −b/a and c/a carefully: $b = -(k + 5)$, so $-b/a = k + 5$.
2. The condition: $k + 5 = (5k + 1)/3$.
3. Multiply by 3: $3k + 15 = 5k + 1$. So $15 - 1 = 5k - 3k$, giving $14 = 2k$, hence $k = 7$.

ANSWER: $k = 7$.

CBSE-STYLE MARKING SCHEME: 1 mark for sum and product • 1 mark for the equation • 1 mark for $k = 7$.

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Previous Year Questions (3 of 6)

CBSE 2023 • STANDARD

PYQ 4

If one zero of the polynomial $p(x) = 6x^2 + 37x - (k - 2)$ is reciprocal of the other, find the value of k.

THINK FIRST: 'Reciprocal of the other' is a product condition in disguise: if the zeroes are α and 1/α, their product is 1.

1. Let the zeroes be α and 1/α. Their product is α × (1/α) = 1.
2. But product of zeroes = c/a = −(k − 2)/6. So −(k − 2)/6 = 1.
   Why: Watch the constant term: it is −(k − 2), brackets and all.
3. Solve: −(k − 2) = 6, so k − 2 = −6, hence k = −4.

ANSWER: k = −4.

CBSE-STYLE MARKING SCHEME: 1 mark for product = 1 • 1 mark for the equation • 1 mark for k = −4.

CBSE 2023 • BASIC

PYQ 5

Find the value of k such that the polynomial $p(x) = 3x^2 + 2kx + x - k - 5$ has the sum of its zeroes equal to half of their product.

THINK FIRST: First tidy the polynomial: group the x-terms, group the constants.

1. Rewrite: $p(x) = 3x^2 + (2k + 1)x - (k + 5)$. So a = 3, b = 2k + 1, c = −(k + 5).
   Why: Grouping like terms is step zero here.
2. Sum of zeroes = −(2k + 1)/3. Product of zeroes = −(k + 5)/3.
3. Condition: −(2k + 1)/3 = (1/2)(−(k + 5)/3). Multiply by −6: 2(2k + 1) = k + 5, so 4k + 2 = k + 5, giving 3k = 3 and k = 1.

ANSWER: k = 1.

CBSE-STYLE MARKING SCHEME: 1 mark for tidying and reading a, b, c • 1 mark for the equation • 1 mark for k = 1.

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Previous Year Questions (4 of 6)

CBSE 2023 • BASIC • MCQ

PYQ 6

If one zero of the quadratic polynomial $kx^2 + 3x + k$ is 2, then the value of k is: (A) −6/5 (B) 6/5 (C) 5/6 (D) −5/6

THINK FIRST: '2 is a zero' means p(2) = 0. Substitute — one line.

1. Substitute x = 2: k(4) + 3(2) + k = 0, so 5k + 6 = 0.
2. Hence k = −6/5.

ANSWER: Option (A): k = −6/5.

CBSE-STYLE MARKING SCHEME: 1 mark — correct option.

CBSE 2023 • COMPARTMENT

PYQ 7

Find the zeroes of the polynomial $4x^2 + 17x − 15$ and verify the relationship between the zeroes and the coefficients.

THINK FIRST: The Section 07 rhythm, in a real paper. a × c = −60.

1. Split pair: product −60, sum 17 — they are +20 and −3.
2. $4x^2 + 20x − 3x − 15 = 4x(x + 5) − 3(x + 5) = (4x − 3)(x + 5)$.
3. Zeroes: x = 3/4 and x = −5.
4. Verify sum: 3/4 − 5 = −17/4, and −b/a = −17/4. Verify product: (3/4)(−5) = −15/4, and c/a = −15/4. Both match.

ANSWER: Zeroes: 3/4 and −5; sum = −17/4 = −b/a, product = −15/4 = c/a. Verified.

CBSE-STYLE MARKING SCHEME: 1 mark for zeroes • 1 mark for sum verification • 1 mark for product verification.

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Previous Year Questions (5 of 6)

CBSE 2023 • COMPARTMENT

PYQ 8

Find the zeroes of the polynomial $x^2 - 11x + 18$ and verify the relationship between the zeroes and the coefficients.

THINK FIRST: Split pair: product 18, sum −11.

1. $x^2 - 11x + 18 = (x - 2)(x - 9)$, so the zeroes are 2 and 9.
2. Verify sum: $2 + 9 = 11 = -(-11)/1$. Verify product: $2 \times 9 = 18 = 18/1$. Both match.

ANSWER: Zeroes: 2 and 9; sum = 11 = −b/a, product = 18 = c/a. Verified.

CBSE-STYLE MARKING SCHEME: 1 mark for zeroes • 1 mark for sum verification • 1 mark for product verification.

CBSE BOARD EXAM QUESTION

PYQ 9

If α and β are the zeroes of the polynomial $x^2 - 3x + k$ such that $\alpha - \beta = 1$, find the value of k.

THINK FIRST: Use the identity $(\alpha + \beta)^2 - (\alpha - \beta)^2 = 4\alpha\beta$ — it connects all three quantities at once.

1. Bridge: $\alpha + \beta = 3$ and $\alpha\beta = k$.
2. Substitute into the identity: $3^2 - 1^2 = 4k$, so $8 = 4k$.
3. Hence $k = 2$. (Check: $x^2 - 3x + 2$ has zeroes 2 and 1, differing by 1.)

ANSWER: k = 2.

CBSE-STYLE MARKING SCHEME: 1 mark for sum and product • 1 mark for the identity • 1 mark for k = 2.

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Previous Year Questions (6 of 6)

CBSE BOARD EXAM QUESTION

PYQ 10

If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - p(x + 1) - c$, show that $(\alpha + 1)(\beta + 1) = 1 - c$.

THINK FIRST: First expand f(x) into standard form — the coefficients are hiding inside the brackets. (Here p and c are constants, not the polynomial.)

1. Expand: $f(x) = x^2 - px - p - c$. So the coefficient of x is $-p$ and the constant term is $-(p + c)$.
2. Bridge: $\alpha + \beta = p$ and $\alpha\beta = -(p + c)$.
3. Expand the target: $(\alpha + 1)(\beta + 1) = \alpha\beta + \alpha + \beta + 1$.
4. Substitute: $-(p + c) + p + 1 = -p - c + p + 1 = 1 - c$. Shown. ∎
   Why: The p's cancel — the examiner designed it so.

ANSWER: $(\alpha + 1)(\beta + 1) = 1 - c$. ∎

CBSE-STYLE MARKING SCHEME: 1 mark for standard form • 1 mark for sum and product • 1 mark for the expansion and conclusion.

16 Chapter Mind Map

THE WHOLE CHAPTER ON ONE PAGE — REVISE FROM HERE

POLYNOMIALS

Heart of the chapter: **the zeroes–coefficients bridge** — the zeroes of a polynomial and its coefficients determine each other.

Zeroes — the Picture

Zeroes = x-coordinates where y = p(x) meets the x-axis. Linear: 1. Quadratic: 0, 1 (touch) or 2. Cubic: up to 3. Degree n → at most n zeroes.

Finding Zeroes

Split the middle term (pair: sum b, product ac) → factorise → zero-product rule. b = 0? Use the difference of squares — zeroes come in ± pairs.

The Quadratic Bridge

$\alpha + \beta = -b/a$; $\alpha\beta = c/a$. Derived by expanding $a(x - \alpha)(x - \beta)$ and comparing coefficients.

The Cubic Bridge

$\alpha + \beta + \gamma = -b/a$; $\alpha\beta + \beta\gamma + \gamma\alpha = c/a$; $\alpha\beta\gamma = -d/a$. Signs: −, +, −.

Forming a Polynomial

$k[x^2 - Sx + P]$. Minus the sum, plus the product. Choose k to clear fractions.

Division Algorithm

$p(x) = g(x)q(x) + r(x)$; stop when deg r < deg g. Remainder 0 ⇒ g(x) is a factor.

Bridge Gymnastics (PYQ & Olympiad)

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$; $1/\alpha + 1/\beta = -b/c$; $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$; reciprocal zeroes ⇒ $\alpha\beta = 1$; equal zeroes ⇒ $(\alpha + \beta)^2 = 4\alpha\beta$.

Story Signals

Lands / touches / break-even → solve p(x) = 0. Area as one expression → factorise for the sides. Centre / highest point → midpoint of zeroes.

Key Results to Quote

$x^2 - 2x - 8$: zeroes 4, −2 • $4s^2 - 4s + 1$: equal zeroes 1/2, 1/2 • $t^2 - 15$: $\pm\sqrt{15}$ • Sum 1/4, product −1 → $4x^2 - x - 4$ • $(x^3 - 3x^2 + 5x - 3) \div (x^2 - 2)$: quotient $x - 3$, remainder $7x - 9$

TOPPER TIP: *Revise actively: cover a box, speak its contents aloud from memory, then uncover and check. A mind map you can REPRODUCE from memory is worth ten readings.*

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Pin-Point Flash Cards (1 of 2) — Cut Them Out

How to use these cards: cut along the dashed borders (ask an adult to help with scissors if needed). Keep the deck in your pocket or pencil box. On every Smart Memory day (Day 1, 3, 7, 14, 21, 30), shuffle the deck, read a card's title, and say the pin-point aloud BEFORE flipping your eyes to the fact. Cards you miss twice go into your Error Tracker. Sixteen cards — the sixteen most exam-important facts of this chapter.

Card 1 · Zero of a Polynomial

a is a zero of p(x) ⇔ p(a) = 0 ⇔ the graph meets the x-axis at x = a ⇔ (x − a) is a factor of p(x).

Card 2 · The Degree Rule

A polynomial of degree n has AT MOST n zeroes. Quadratic: at most 2. Cubic: at most 3. More zeroes than the degree = a wrong answer, always.

Card 3 · Quadratic — Sum of Zeroes

$\alpha + \beta = -b/a = -$(coefficient of x) / (coefficient of x^2). The MINUS belongs to the sum.

Card 4 · Quadratic — Product of Zeroes

$\alpha\beta = c/a$ = (constant term) / (coefficient of x^2). NO minus here — do not invent one.

Card 5 · Forming a Quadratic

$p(x) = k[x^2 -$ (sum of zeroes)x + (product of zeroes)], for any $k \neq 0$. Minus the sum, plus the product.

Card 6 · Cubic — Sum of Zeroes

For $ax^3 + bx^2 + cx + d$: $\alpha + \beta + \gamma = -b/a$.

Card 7 · Cubic — Pairwise Sum

$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$. Three pairs, each pair once, all plus.

Card 8 · Cubic — Product of Zeroes

$\alpha\beta\gamma = -d/a$. The product of THREE zeroes carries a MINUS — the most-lost mark in this chapter.

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Pin-Point Flash Cards (2 of 2)

Card 9 · Graph Shapes

Linear → straight line. Quadratic → parabola (U or ∩). Cubic → S-curve. Zeroes = meetings with the X-axis only.

Card 10 · No Touch, No Zero

If the graph never meets the x-axis — like $y = x^2 + 1$, whose smallest value is 1 — the polynomial has NO real zeroes.

Card 11 · Equal Zeroes

Zeroes are equal ⇔ the graph TOUCHES the x-axis at one point ⇔ $(\alpha + \beta)^2 = 4\alpha\beta$. Example: $4s^2 - 4s + 1$, zeroes 1/2, 1/2.

Card 12 · Splitting the Middle Term

For $ax^2 + bx + c$: split b into two parts whose SUM is b and whose PRODUCT is a × c. Check both — then group.

Card 13 · Difference of Squares

$x^2 - m = (x - \sqrt{m})(x + \sqrt{m})$. When b = 0, zeroes come as a ± pair: $t^2 - 15 \rightarrow \pm\sqrt{15}$. Never report only the plus.

Card 14 · Division Algorithm

$p(x) = g(x) \times q(x) + r(x)$, where r(x) = 0 or deg r(x) < deg g(x). Remainder 0 ⇒ g(x) is a factor of p(x).

Card 15 · Reciprocal Zeroes

If one zero is the reciprocal of the other, their product is 1: c/a = 1. (CBSE 2023 asked exactly this.)

Card 16 · Standard Form First

ALWAYS rewrite in the form $ax^2 + bx + c$ before reading a, b, c. In $6x^2 - 3 - 7x$, b is −7 — not 7.

TOPPER TIP: *Mastered all 16? Test a friend. Teaching a card to someone else is the deepest form of revision — if you can explain Card 8's minus sign, you truly own it.*

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17 Active Recall

8D • RECALL — CLOSE THE BOOK, THEN ANSWER

How this works: close every other page. Answer all ten questions on rough paper, from memory alone. Only then open the Answer Key. Anything you could not recall is exactly what your Day-3 revision must target — write it in the Error Tracker (Section 18).

Ten rapid questions

R1. What is the degree of $7x^3 - 4x + 9$?

R2. Find the zeroes of $x^2 - 5x + 6$.

R3. Find the sum of the zeroes of $2x^2 - 3x + 7$.

R4. Find the product of the zeroes of $5x^2 + 9x - 2$.

R5. What is the maximum number of zeroes of a cubic polynomial?

R6. For $p(x) = x^2 - 4x + 3$, find $p(0)$.

R7. Form a quadratic polynomial with sum of zeroes 3 and product of zeroes -4.

R8. Find the zeroes of $x^2 - 16$.

R9. For the polynomial $2x^2 + bx + c$, the sum of the zeroes is 5. Find b.

R10. Find k if $x = 1$ is a zero of $x^2 + kx - 5$.

Five recall prompts — say each answer aloud, in a full sentence

P1. Define a zero of a polynomial.

P2. State both zeroes–coefficients relations for a quadratic $ax^2 + bx + c$.

P3. State the geometrical meaning of the zeroes of a polynomial.

P4. State the division algorithm for polynomials, including the condition on the remainder.

P5. Explain how to form a quadratic polynomial when the sum S and product P of its zeroes are given.

> **TOPPER TIP:** *Scoring: 13–15 correct — chapter mastered, move on. 9–12 — revise the missed topics, retake in 3 days. Below 9 — re-study Sections 05–07, then retake. Record your score in the Mastery Tracker.*

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18 Self-Assessment & Mastery Tracker

KALVIYOGI SMART MEMORY SYSTEM — LEARN IT, THEN KEEP IT

Memory fades on a predictable curve — unless you revisit at the right moments. This schedule is your revisit plan for THIS chapter. Tick each day off when done. The Day 7, Day 21 and Day 30 mini-tests follow this section.

Day	Your activity for Chapter 2	Done
Day 0	Pre-learning assessment: attempt the Quick Check (Section 02) and Rapid Questions R1–R10 (Section 17) BEFORE studying. Note your score.	☐
Day 1	Learn the chapter fully (Sections 01–10) and complete Level 1–3 practice plus the 25 MCQs.	☐
Day 3	Without-book recall: reproduce the Mind Map (Section 16) on a blank page; shuffle the Flash Cards; finish Level 4.	☐
Day 7	Independent problem-solving: Level 5 + Olympiad Challenges, timed, book closed. Then take **Mini-Test 1** (next pages).	☐
Day 14	Mixed practice: re-attempt every question in your Error Tracker; redo the PYQ set (Section: Board Exam Hall) fresh.	☐
Day 21	Long-term recall test: take **Mini-Test 2**; write the quadratic and cubic bridge derivations from memory, then verify line by line.	☐
Day 30	Chapter mastery assessment: take **Mini-Test 3** and one full Model Test under exam conditions; update your Concept Mastery Score below.	☐

Formula Recall Tracker — tick when you can say it with the book closed

Formula / Statement	Day 1	Day 3	Day 7	Day 21
Zeroes = x-axis meetings of the graph	☐	☐	☐	☐
Quadratic: $\alpha + \beta = -b/a$; $\alpha\beta = c/a$	☐	☐	☐	☐
Cubic: $-b/a$; c/a; $-d/a$ (signs −, +, −)	☐	☐	☐	☐
Forming: $k[x^2 - Sx + P]$	☐	☐	☐	☐
Division algorithm + degree rule for r(x)	☐	☐	☐	☐
Degree rule: degree n → at most n zeroes	☐	☐	☐	☐

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Mastery Tracker — Errors and Concept Scores

Error Tracker — every mistake becomes a lesson

Date	Question	What went wrong	The correct idea

Concept Mastery Score — rate yourself honestly, out of 5

Concept	Score / 5	Status
Reading zeroes from graphs (Ex 2.1 skill)	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Splitting the middle term & factorising	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Verifying the bridge (Ex 2.2 Q1 skill)	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Forming polynomials from sum & product	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Cubic relations (all three, signs correct)	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Division algorithm (long division)	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Bridge gymnastics ($\alpha^2 + \beta^2$, $1/\alpha + 1/\beta$, $\alpha - \beta$)	___ / 5	☐ Weak ☐ Okay ☐ Mastered

Learning Time Tracker

Day / Date	Minutes studied	What I studied

Mastery rule: a concept counts as MASTERED only when you score 4 or 5 AND can reproduce it on Day 21 without the book. Anything less — revisit, don't just re-read.

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MINI-TEST 1 • DAY 7 — CORE SKILLS • 10 QUESTIONS • BOOK CLOSED

Answer all ten on rough paper. Marks and answers are in the Answer Key — check only after finishing. Record your score in the Mastery Tracker.

T1. Find the zeroes of $x^2 - 9x + 20$.

T2. Find the sum of the zeroes of $7x^2 - 3x + 1$.

T3. For $p(x) = x^3 - x + 2$, find $p(2)$.

T4. Form a quadratic polynomial with sum of zeroes 2 and product of zeroes -15.

T5. Find the product of the zeroes of $6x^2 - x - 2$.

T6. Find the zeroes of $2x^2 + 3x - 2$.

T7. Is -1 a zero of $x^2 + 3x + 2$? Justify in one line.

T8. A parabola touches the x-axis at exactly one point. How many distinct zeroes does its polynomial have?

T9. Find k if $x = 1$ is a zero of $x^2 + kx - 4$.

T10. Find the zeroes of $x^2 - 2x - 3$.

> ***TOPPER TIP:*** *Target: 8/10 or more. Below that, your Day 14 work is decided for you: Sections 05–07 again, slowly.*

MINI-TEST 2 • DAY 21 — MIXED RECALL • 10 QUESTIONS • BOOK CLOSED

Answer all ten on rough paper. Marks and answers are in the Answer Key — check only after finishing. Record your score in the Mastery Tracker.

T1. For $4x^2 + 8x$, write the sum and the product of the zeroes.

T2. Find the zeroes of $5x^2 - 20$.

T3. Form a quadratic polynomial whose zeroes are -5 and 1.

T4. If α, β are the zeroes of $x^2 - 7x + 12$, find $1/\alpha + 1/\beta$.

T5. Divide $x^2 + 6x + 8$ by $x + 4$. Give the quotient and remainder.

T6. If α, β are the zeroes of $x^2 - x - 6$, find $\alpha^2 + \beta^2$.

T7. Find the sum of the zeroes of the cubic $x^3 - 2x^2 + 3x - 4$.

T8. The product of the zeroes of $x^2 - kx + 9$ is 9 and their sum is 6. Find k.

T9. Find the zeroes of $3x^2 - 12x$.

T10. A polynomial's graph does not meet the x-axis at all. How many real zeroes does it have?

> ***TOPPER TIP:*** *Target: 8/10. Any cubic or division slip goes straight into the Error Tracker — those are Day-30 deciders.*

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MINI-TEST 3 • DAY 30 — MASTERY CHECK • 10 QUESTIONS • BOOK CLOSED

Answer all ten on rough paper. Marks and answers are in the Answer Key — check only after finishing. Record your score in the Mastery Tracker.

T1. If α, β are the zeroes of $x^2 - 2x - 3$, find $\alpha^3 + \beta^3$.

T2. Find k so that $x^2 - 10x + k$ has equal zeroes.

T3. The zeroes of $x^2 - 8x + k$ differ by 2. Find k.

T4. If α, β are the zeroes of $x^2 - 4x + 3$, form a quadratic with zeroes 2α and 2β.

T5. For the cubic $x^3 + 3x^2 - 5x + 2$, write the value of αβ + βγ + γα.

T6. For which k do the zeroes of $4x^2 - 3x + k$ become reciprocals of each other?

T7. If α, β are the zeroes of $x^2 - 2x - 8$, find α − β (taking α > β).

T8. Divide $x^3 + 1$ by x + 1. Give the quotient and remainder.

T9. Form a quadratic polynomial whose zeroes satisfy α + β = αβ = 4.

T10. What is the maximum number of zeroes of a polynomial of degree 5?

> ***TOPPER TIP:*** *Target: 9/10 — this is board standard. Score it honestly, update the Mastery Tracker, and close the chapter.*

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Chapter Model Test A — Basics

CHAPTER MODEL TEST A • BASICS • BUILD THE FOUNDATIONS • MAX. MARKS: 20 • TIME: 45 MINUTES

General instructions: All questions are compulsory. Section A: 5 MCQs × 1 mark. Section B: 3 questions × 2 marks. Section C: 3 questions × 3 marks. Show all steps — step marks apply, exactly as in the board exam. Answers: Answer Key.

Section A — Multiple Choice (1 mark each)

A1. The degree of $4x^2 - 3x + 7$ is

(A) 1 (B) 2 (C) 3 (D) 7

A2. The zero of the polynomial $2x - 4$ is

(A) 4 (B) -2 (C) 2 (D) 0

A3. The sum of the zeroes of $x^2 - 5x + 6$ is

(A) 6 (B) -5 (C) -6 (D) 5

A4. The product of the zeroes of $3x^2 + 2x - 1$ is

(A) 1/3 (B) $-1/3$ (C) $-2/3$ (D) 3

A5. The number of zeroes of a linear polynomial is

(A) 2 (B) 0 (C) 3 (D) 1

Section B — Short Answers (2 marks each)

A6. For $p(x) = x^2 - 4x + 3$, find $p(1)$ and $p(-1)$.

A7. Find the zeroes of $x^2 - 8x + 15$.

A8. Write the sum and the product of the zeroes of $5x^2 - 9x + 4$.

Section C — Long Answers (3 marks each)

A9. Find the zeroes of $2x^2 - 7x + 3$ and verify the relationship between the zeroes and the coefficients.

A10. Find a quadratic polynomial whose sum of zeroes is 7 and product of zeroes is 12.

A11. The graph of $y = p(x)$ crosses the x-axis at $x = -1$, $x = 0$ and $x = 3$, and nowhere else. How many zeroes does $p(x)$ have, and what are they?

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Chapter Model Test B — Standard

CHAPTER MODEL TEST B • STANDARD • BOARD PATTERN • MAX. MARKS: 20 • TIME: 45 MINUTES

General instructions: All questions are compulsory. Section A: 5 MCQs × 1 mark. Section B: 3 questions × 2 marks. Section C: 3 questions × 3 marks. Show all steps — step marks apply, exactly as in the board exam. Answers: Answer Key.

Section A — Multiple Choice (1 mark each)

B1. The zeroes of $x^2 + 5x + 6$ are

(A) 2 and 3 (B) −2 and −3 (C) −2 and 3 (D) 2 and −3

B2. A quadratic polynomial with sum of zeroes −3 and product of zeroes 2 is

(A) $x^2 + 3x - 2$ (B) $x^2 - 3x + 2$ (C) $x^2 + 3x + 2$ (D) $x^2 - 3x - 2$

B3. The product of the zeroes of $-4x^2 + x + 3$ is

(A) 3/4 (B) 1/4 (C) −1/4 (D) −3/4

B4. If $x = 3$ is a zero of $x^2 - kx + 6$, then k equals

(A) 3 (B) 5 (C) −5 (D) 2

B5. The zeroes of $9x^2 - 9$ are

(A) 1 and −1 (B) 1 (C) −1 (D) 3 and −3

Section B — Short Answers (2 marks each)

B6. Find the zeroes of $4x^2 - 4x - 3$.

B7. Form a quadratic polynomial whose zeroes are 1/2 and −1/2.

B8. If α, β are the zeroes of $x^2 - 4x + 3$, find $\alpha + \beta - \alpha\beta$.

Section C — Long Answers (3 marks each)

B9. Find the zeroes of $3x^2 - x - 4$ and verify the relationship between the zeroes and the coefficients.

B10. Divide $x^2 + 7x + 10$ by $x + 2$. Give the quotient and the remainder, and verify your answer.

B11. The sum of the zeroes of $x^2 - kx + 8$ is 6. Find k, and hence find the zeroes.

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Chapter Model Test C — Advanced

CHAPTER MODEL TEST C • ADVANCED • FOR THE 95+ CLUB • MAX. MARKS: 20 • TIME: 45 MINUTES

General instructions: All questions are compulsory. Section A: 5 MCQs × 1 mark. Section B: 3 questions × 2 marks. Section C: 3 questions × 3 marks. Show all steps — step marks apply, exactly as in the board exam. Answers: Answer Key.

Section A — Multiple Choice (1 mark each)

C1. If α, β are the zeroes of $x^2 - 3x + 2$, then $\alpha^2 + \beta^2$ equals

(A) 9 (B) 13 (C) 3 (D) 5

C2. The value of k for which $x^2 + 6x + k$ has equal zeroes is

(A) 6 (B) 36 (C) 9 (D) −9

C3. If α, β are the zeroes of $x^2 - 5x + 6$, then $1/\alpha + 1/\beta$ equals

(A) 6/5 (B) 5/6 (C) 1/6 (D) −5/6

C4. The zeroes of $x^3 - 4x$ are

(A) 0 only (B) 0, 2 and −2 (C) 2 and −2 (D) 0 and 4

C5. The product of the zeroes of $2x^2 + 5x + 2$ is

(A) 5/2 (B) 1 (C) 2/5 (D) −1

Section B — Short Answers (2 marks each)

C6. If α, β are the zeroes of $x^2 - 6x + 5$ with $\alpha > \beta$, find $\alpha - \beta$.

C7. If α, β are the zeroes of $x^2 - 3x + 2$, form a quadratic polynomial whose zeroes are $\alpha + 2$ and $\beta + 2$.

C8. One zero of $x^2 + 3x + k$ is the reciprocal of the other. Find k.

Section C — Long Answers (3 marks each)

C9. If α, β are the zeroes of $x^2 - 4x + 1$, find $\alpha^3 + \beta^3$.

C10. Divide $x^3 - 3x^2 + 5x - 3$ by $x^2 - 2$. Give the quotient and the remainder, and verify.

C11. If α, β are the zeroes of $x^2 - px + q$, prove that $\alpha^2 + \beta^2 = p^2 - 2q$.

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Answer Key — Graded Practice, Levels 1 and 2

Level 1 — Foundation

L1.1 $p(1) = 2 - 3 + 5 = 4$. **L1.2** Yes: $p(2) = 4 - 10 + 6 = 0$. **L1.3** $x^2 - 9 = (x - 3)(x + 3)$, zeroes 3 and −3. **L1.4** Sum = $-(-3)/5 = 3/5$; product = 2/5. **L1.5** $(x - 3)(x - 4)$, zeroes 3 and 4. **L1.6** $x^2 - 5x + 6$ (or any non-zero multiple).

Level 2 — Skill Builder

L2.1 $2x^2 - 7x + 3 = (2x - 1)(x - 3)$; zeroes 3 and 1/2. Sum = $7/2 = -(-7)/2$; product = 3/2. Verified.

L2.2 $4x^2 + 4x - 3 = (2x - 1)(2x + 3)$; zeroes 1/2 and −3/2. Sum = $-1 = -4/4$; product = −3/4. Verified.

L2.3 $x^2 - (-3)x + 2 = x^2 + 3x + 2$. **L2.4** $x^2 - 0x + (-4) = x^2 - 4$.

L2.5 $(x + 2)(x + 5)$; zeroes −2 and −5. Sum = −7; product = 10. Verified. **L2.6** $(3x - 1)(x + 2)$; zeroes 1/3 and −2. Sum = −5/3; product = −2/3. Verified.

TOPPER TIP: *Marked yourself wrong at Level 2? Compare your working with the twin solution in Section 07 step by step — find the FIRST step where you differ. That step is the lesson.*

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Answer Key — Levels 3, 4, 5 and Active Recall

Level 3 — Application

L3.1 $-5t(t-6)=0 \rightarrow t=6$ seconds (ignoring $t=0$). **L3.2** $(x+5)(x+6)$ — split pair 5, 6. **L3.3** $-x^2+6x-5=0 \rightarrow x^2-6x+5=0 \rightarrow (x-1)(x-5)$; meets the ground at $x=1$ and $x=5$; width = 4 m. **L3.4** $x^2-11x+30=(x-5)(x-6)$; the numbers are 5 and 6.

Level 4 — Advanced Reasoning

L4.1 $\alpha+\beta=5$, $\alpha\beta=6$, so $\alpha^2+\beta^2=25-12=13$. **L4.2** Let the zeroes be m and 7m: sum $8m=8/3 \rightarrow m=1/3$; product $7m^2=7/9=(2k+1)/3 \rightarrow 2k+1=7/3 \rightarrow k=2/3$. **L4.3** $\alpha+\beta=-b/a$ and $\alpha\beta=c/a$; the condition gives $-b/a=c/a$, so $-b=c$, i.e. $b+c=0$. ∎

Level 5 — Mathematical Genius

L5.1 Equal zeroes: $(\alpha+\beta)^2=4\alpha\beta$ gives $k^2=36$, so $k=6$ or $k=-6$ (polynomials $(x+3)^2$ and $(x-3)^2$ up to scale). **L5.2** New sum $=(\alpha+\beta)+2=8$; new product $=\alpha\beta+(\alpha+\beta)+1=5+6+1=12$; polynomial $x^2-8x+12$.

Active Recall (Section 17)

R1 3. **R2** 2 and 3. **R3** 3/2. **R4** −2/5. **R5** 3. **R6** 3. **R7** x^2-3x-4. **R8** 4 and −4. **R9** $-b/2=5 \rightarrow b=-10$. **R10** $1+k-5=0 \rightarrow k=4$.

P1 A zero of p(x) is a real number a with p(a) = 0. **P2** $\alpha+\beta=-b/a$ and $\alpha\beta=c/a$. **P3** The zeroes are the x-coordinates of the points where the graph of y = p(x) meets the x-axis. **P4** $p(x)=g(x)q(x)+r(x)$ with $r(x)=0$ or $\deg r(x)<\deg g(x)$. **P5** $p(x)=k[x^2-Sx+P]$ for any non-zero k.

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Answer Key — Concept MCQs with Explanations (1 of 2)

M1 (C) Degree = highest power of x = 3. **M2 (B)** (x − 4)(x + 1) → zeroes 4, −1. **M3 (B)** p(2) = 4 − 14 + 10 = 0. **M4 (A)** −b/a = 5/3. **M5 (C)** c/a = −5/2. **M6 (D)** Sum = 1, product = −12 → x^2 − x − 12. **M7 (B)** Degree rule: at most 3. **M8 (A)** One touching point = one distinct zero (counted twice). **M9 (C)** Zeroes summing to 0 are opposites: −3.

M10 (A) x^2 + 1 ≥ 1 > 0 always; the graph never meets the axis. **M11 (D)** p(1) = 0 → 1 + k − 6 = 0 → k = 5. **M12 (B)** −b/a = −5/−1 = 5. **M13 (C)** c/a = −9/4.

Answer Key — Concept MCQs with Explanations (2 of 2)

M14 (A) Split −9, +2: (2x − 3)(3x + 1) → 3/2, −1/3. **M15 (B)** A line meets the x-axis exactly once. **M16 (D)** (α + β) + αβ = 6 + 8 = 14. **M17 (C)** Degree rule. **M18 (B)** p(3) = 9 − 15 + 6 = 0.

M19 (A) x(x − 2) = 0 → 0, 2. **M20 (C)** q = product of zeroes = 2 × 3 = 6. **M21 (B)** Two crossings = two zeroes. **M22 (A)** A product is zero only if a factor is zero. **M23 (D)** k[x^2 + (1/2)x − 3] with k = 2. **M24 (B)** 3x = 6 → x = 2. **M25 (C)** (α + β)/(αβ) = 1/−6 = −1/6.

Assertion–Reason — answers

AR1 (a) x^2 − 4 = (x − 2)(x + 2) crosses at ±2, and R is the very definition used. **AR2 (c)** Sum = −(−6)/3 = 2 is true; R states the product formula, so R is false. **AR3 (a)** p(2) = 0, and R is the definition that settles it. **AR4 (d)** A is false (at most 2); R is the true degree rule. **AR5 (b)** Both true, but R speaks of the SUM — it does not explain the product in A.

AR6 (a) R is the forming formula applied directly. **AR7 (a)** $(2x - 1)^2$ = 0 gives x = 1/2 twice — R explains A. **AR8 (d)** A is false; R is true. **AR9 (c)** The sum is indeed 2, but R is false: the sum is −b/a, not b/a. **AR10 (b)** Both true — but a matching sum alone cannot prove the zeroes (many triples sum to 6), so R is not the correct explanation.

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Answer Key — Find-the-Error Corrections

E1. The formula carries a minus: sum = −b/a = −(−5)/2 = **5/2**. **E2.** x + 2 = 0 gives x = **−2**, not 2. Zeroes: 4 and −2. **E3.** The sum is SUBTRACTED: $x^2 - (1/4)x - 1$ (i.e. $4x^2 - x - 4$). **E4.** Product = c/a with no extra minus: −9/3 = **−3**. **E5.** t = **±√15** — two zeroes, not one.

E6. Only X-axis meetings count: the zeroes are **2** (at x = 1 and x = 4); the y-axis meeting is p(0), not a zero. **E7.** Standard form first: $6x^2 - 7x - 3$, so b = −7 and −b/a = 7/6 — the relationship HOLDS. **E8.** $(x + 2)(x + 3) = x^2 + 5x + 6$ exactly, so the remainder is **0**. **E9.** −3 × (−2) = +6, so p(−2) = 4 + 6 + 2 = **12 ≠ 0** — −2 is NOT a zero. **E10.** Sum of zeroes of a cubic = **−b/a = 6**, not b/a.

Smart Memory Mini-Tests — answers

Mini-Test 1 (Day 7): 1. 4, 5. 2. 3/7. 3. 8. 4. $x^2 - 2x - 15$. 5. −1/3. 6. 1/2, −2. 7. Yes — p(−1) = 1 − 3 + 2 = 0. 8. One distinct zero. 9. k = 3. 10. 3, −1.

Mini-Test 2 (Day 21): 1. Sum −2, product 0. 2. 2, −2. 3. $x^2 + 4x - 5$. 4. 7/12. 5. Quotient x + 2, remainder 0. 6. 13. 7. 2. 8. k = 6. 9. 0, 4. 10. Zero real zeroes.

Mini-Test 3 (Day 30): 1. 26. 2. k = 25. 3. k = 15. 4. $x^2 - 8x + 12$. 5. −5. 6. k = 4. 7. 6. 8. Quotient $x^2 - x + 1$, remainder 0. 9. $x^2 - 4x + 4$. 10. Five.

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Answer Key — Model Tests A, B and C

Model Test A — Basics

Section A: A1 (B) 2. A2 (C) 2. A3 (D) 5. A4 (B) −1/3. A5 (D) 1.

Section B: A6 $p(1) = 0$; $p(-1) = 8$. A7 $(x - 3)(x - 5)$: zeroes 3, 5. A8 Sum 9/5; product 4/5.

Section C: A9 Zeroes 3 and 1/2; sum $7/2 = -b/a$, product $3/2 = c/a$ — verified. A10 $x^2 - 7x + 12$. A11 Three zeroes: −1, 0 and 3 (every x-axis crossing is a zero).

Model Test B — Standard

Section A: B1 (B) −2 and −3. B2 (C) $x^2 + 3x + 2$. B3 (D) −3/4. B4 (B) 5. B5 (A) 1 and −1.

Section B: B6 $(2x - 3)(2x + 1)$: zeroes 3/2, −1/2. B7 $x^2 - 1/4$, i.e. $4x^2 - 1$. B8 $(\alpha + \beta) - \alpha\beta = 4 - 3 = 1$.

Section C: B9 Zeroes 4/3 and −1; sum 1/3, product −4/3 — verified. B10 Quotient $x + 5$, remainder 0; check $(x + 2)(x + 5) = x^2 + 7x + 10$. B11 $-b/a = k = 6$; then $x^2 - 6x + 8 = (x - 2)(x - 4)$: zeroes 2, 4.

Model Test C — Advanced

Section A: C1 (D) 5. C2 (C) 9. C3 (B) 5/6. C4 (B) 0, 2 and −2. C5 (B) 1.

Section B: C6 $(\alpha - \beta)^2 = 36 - 20 = 16$, so $\alpha - \beta = 4$. C7 New sum $= 3 + 4 = 7$; new product $= 2 + 2(3) + 4 = 12$; $x^2 - 7x + 12$. C8 Product $= 1 \rightarrow k/1 = 1 \rightarrow k = 1$.

Section C: C9 $\alpha^3 + \beta^3 = 4^3 - 3(1)(4) = 64 - 12 = 52$. C10 Quotient $x - 3$, remainder $7x - 9$ (verify by multiplying back, as in Division Example 2). C11 $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = p^2 - 2q$, since $\alpha + \beta = p$ and $\alpha\beta = q$ for $x^2 - px + q$. ∎

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Revision Card — Chapter 2 on One Page

The one picture

Zeroes of p(x) = x-coordinates where the graph of y = p(x) meets the x-axis. Degree n → at most n zeroes. Parabola up if a > 0, down if a < 0; vertex on the line x = −b/(2a), midway between the zeroes.

The bridge

Quadratic $ax^2 + bx + c$: α + β = −b/a, αβ = c/a. **Cubic** $ax^3 + bx^2 + cx + d$: sum −b/a, pairs c/a, product −d/a. Both come from expanding a(x − α)(x − β)… and comparing coefficients.

The two recipes

Finding zeroes: standard form → split pair (sum b, product ac) → group → zero-product rule. **Forming:** $k[x^2 - Sx + P]$, k chosen to clear fractions. **Division:** p = gq + r, stop when deg r < deg g; always multiply back to check.

Story signals

Lands / touches / break-even / height zero → solve p(x) = 0. **Area as one expression** → factorise for sides. **Centre / highest point** → midpoint of the zeroes. **Reciprocal zeroes** → product = 1. **Equal zeroes** → $(\alpha + \beta)^2 = 4\alpha\beta$.

Exam-eve checklist

☐ Standard form written before reading a, b, c ☐ Minus sign spoken in −b/a ☐ Both verifications written in Q1-type questions ☐ ± pair reported when b = 0 ☐ Cubic product has its minus ☐ Answer translated back into the story's words

Chapter 2 complete. You did not memorise Polynomials — you saw them, derived them, and mastered them.

Next: Chapter 3 — Pair of Linear Equations in Two Variables, in the same 8D format. • KalviYogi • Mathematics – Beyond the Formula • 369 Tesla × KalviYogi • © Edunivra Pvt Ltd

CBSE • CLASS 10 • MATHEMATICS

MATHEMATICS BEYOND THE FORMULA

CHAPTER 3 • PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Don't Memorize Mathematics. Understand, Visualise and Discover It.

The KalviYogi 8D Way: Discover • Visualise • Understand • Derive • Solve • Apply • Challenge • Recall

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01 Chapter Introduction

8D • DISCOVER — WHY SHOULD I LEARN THIS?

A single equation like x + y = 10 has **endless answers**: (1, 9), (2, 8), (3, 7), (4.5, 5.5)... Life, however, rarely gives us only one clue. The shopkeeper gives you **two clues** — the total number of items *and* the total cost. The taxi meter follows **two rules** — a fixed charge *and* a per-kilometre charge. When **two clues** act on **two unknowns** at the same time, the answer becomes exact and unique. That is the world of this chapter.

Here is the beautiful part: every linear equation in two variables is secretly a **straight line**. Two equations = two lines. And two lines on a page can do only three things — **cross once, run parallel forever, or lie exactly on top of each other**. By the end of this chapter you will be able to *see* which of the three will happen **before** solving, just by comparing three small ratios.

What you will master in this chapter

- Turning a real-life story into a **pair of linear equations** — the modelling step every word problem begins with.
- The **graphical method**: tables of values, plotted lines, and reading the solution off the meeting point.
- **Consistent vs inconsistent** pairs, and dependent pairs with infinitely many solutions.
- The **ratio test** — comparing a_1/a_2, b_1/b_2, c_1/c_2 — and **why it works** (derived from slopes in Section 06, never just memorised).
- The **substitution method** and the **elimination method**, step by step, and how to choose between them.
- Word problems of every board type: ages, fractions, two-digit numbers, taxi fares, bank notes, library charges.

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Your Chapter Map — 18 Stops, One Journey

This chapter is a journey with 18 stops, plus a full practice stadium at the end. Tick each stop as you finish it.

Discover & Visualise	Understand & Derive	Solve, Apply, Recall
☐ 01 Chapter Introduction ☐ 02 Prerequisite Knowledge ☐ 03 Real-Life Story ☐ 04 Visual Concepts (CPA)	☐ 05 Mathematical Foundations ☐ 06 Derivations: Why the Ratio Test Works ☐ 08 Multiple Solution Methods ☐ 10 Common Mistakes	☐ 07 Detailed Solved Examples (NCERT) ☐ 09 Hands-on Activities ☐ 11 Graded Practice (5 Levels) ☐ 12 Mathematical Reasoning
☐ 13 Real-Life Applications ☐ 14 Competency: 5 Case Studies + Assertion–Reason	☐ 15 Olympiad Challenges ☐ 16 Chapter Mind Map	☐ 17 Active Recall ☐ 18 Mastery Tracker

How to use this chapter (read this first!)

1. **Never read a solution passively.** Cover the steps, try yourself, then uncover one step at a time.
2. Every solution shows **Step 1, Step 2, Step 3...** with a small *Why* note in purple handwriting — the reason behind the step. Examiners award marks for these steps, not just for the final answer.
3. For every graphical solution, the **table of values** is shown before the graph — in the board exam, that table itself carries marks.
4. After Section 07, test yourself in Sections 11–15 and the Model Tests **before** opening the Answer Key.
5. Finish with the Mind Map (16), closed-book Recall (17) and your Mastery Tracker (18) — and return on Days 3, 7, 14, 21 and 30 as the Smart Memory schedule directs.

DIGITAL COMPANION: Simulations, video lessons and online tests for this chapter are part of the KalviYogi Digital Ecosystem. QR code and link activate with the KalviYogi app/website at print.

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02 Prerequisite Knowledge

THE TOOLS YOU ALREADY OWN — ONE VARIABLE, POINTS AND LINES

This chapter stands on three earlier skills. Refresh them here — five minutes now will save you fifty minutes later.

1. Solving a linear equation in ONE variable

You already know how to undo operations one by one. Example: $3x - 5 = 10 \rightarrow 3x = 15 \rightarrow x = 5$. The golden rules: whatever you do to one side, do to the other; a term that jumps across the = sign changes its sign. In this chapter you will use this skill *inside* bigger problems — every method finally lands on one equation in one variable.

2. Points on the Cartesian plane

Every pair (x, y) is one point: x tells you how far to walk along the horizontal axis, y how far up the vertical axis. The four regions are the quadrants: (+, +) first, (−, +) second, (−, −) third, (+, −) fourth. The meeting point of the axes is the origin O(0, 0).

3. One equation in two variables = one straight line

Take $x + y = 5$. Its solutions are pairs: (0, 5), (1, 4), (2, 3), (5, 0), (7, −2)... infinitely many. Plot them and they fall exactly on one straight line. **Every solution is a point on the line; every point on the line is a solution.** This single fact powers the whole graphical method.

4. Building a table of values

To draw the line of $ax + by = c$: choose an easy x, compute y; repeat for a second x; a third point is your safety check. Favourite easy choices: $x = 0$ (gives the y-intercept) and $y = 0$ (gives the x-intercept). Two points fix the line — the third point proves you made no arithmetic slip.

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Quick Check — Are You Ready? (5 Questions)

Try all five without looking back. Answers are given below — cover them with your hand first!

Q1. Solve: $2x + 3 = 11$.

Q2. Solve: $5y - 3 = 2y + 9$.

Q3. In which quadrant does the point $(-3, -2)$ lie? And the point $(4, -1)$?

Q4. If $y = 2x - 1$, find y when $x = 3$, and find x when $y = 9$.

Q5. Does the point (2, 5) satisfy $x + y = 7$? Does it satisfy $x - y = 3$? What does your answer mean on the graph?

Answers — check only after trying

Q1. $2x = 8$, so **$x = 4$**.

Q2. $5y - 2y = 9 + 3 \rightarrow 3y = 12$, so **$y = 4$**.

Q3. $(-3, -2)$: both negative — **third quadrant**. $(4, -1)$: x positive, y negative — **fourth quadrant**.

Q4. $x = 3 \rightarrow y = 2(3) - 1 = \mathbf{5}$. $y = 9 \rightarrow 9 = 2x - 1 \rightarrow 2x = 10 \rightarrow \mathbf{x = 5}$.

Q5. $2 + 5 = 7$ — **yes**, (2, 5) lies on the line $x + y = 7$. $2 - 5 = -3 \neq 3$ — **no**, (2, 5) does NOT lie on the line $x - y = 3$. A point satisfies an equation exactly when it lies on that equation's line.

TOPPER TIP: Got 4 or 5 right? You are fully ready. Got fewer? Re-read the four cards on the previous page once, then come back — do not skip this step.

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03 Real-Life Mathematical Story

DISCOVER • VISUALISE — THE FRUIT MARKET PUZZLE

Before any formula, live the problem. Read this story slowly — the two equations will walk out of the story by themselves.

*Saturday morning. The fruit market near **Anitha's** home is loud with bargaining. Anitha has exactly ₹340 in her purse and a list from her mother: buy some mangoes and some oranges for the family.*

*At the first stall she buys **3 kg of mangoes and 2 kg of oranges** — and the purse is empty. Total: ₹340. She notes the purchase in her phone and walks home, thinking: what is the price of one kilogram of each fruit? One bill, two unknown prices. The bill gives her one equation — but one equation with two unknowns has endless answers. Mango at ₹100 and orange at ₹20 fits. Mango at ₹60 and orange at ₹80 also fits! She needs a **second clue**.*

*Next morning, her brother **Ravi** goes to the same stall and buys **2 kg of mangoes and 3 kg of oranges** for **₹310**. Two bills. Two clues. Anitha smiles — now the prices have nowhere to hide.*

Let the price of 1 kg of mangoes be ₹x and of 1 kg of oranges be ₹y. Anitha's bill says: **3x + 2y = 340**. Ravi's bill says: **2x + 3y = 310**. Two equations, two unknowns — a **pair of linear equations in two variables**. Let us catch the prices, one step at a time.

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Solving the Story — Step by Step

STORY PROBLEM

Anitha: 3 kg mangoes + 2 kg oranges cost ₹340. Ravi: 2 kg mangoes + 3 kg oranges cost ₹310. Find the price per kg of each fruit.

THINK FIRST: Two unknown prices, two bills. If I can make the two equations talk to each other — add them, subtract them — one unknown should disappear.

1. Write the two clues as equations. Let mangoes cost ₹x per kg and oranges ₹y per kg. Anitha's bill: 3x + 2y = 340 ...(1). Ravi's bill: 2x + 3y = 310 ...(2).
Why: A word problem is solved twice — first in words, then in symbols. Choosing clear letters is Step Zero of every solution.

2. Add the two equations, left side with left side and right side with right side: (3x + 2x) + (2y + 3y) = 340 + 310, so **5x + 5y = 650**. Divide by 5: **x + y = 130** ...(3).
Why: Adding is legal because equals added to equals stay equal. Notice the trick: together, the two bills bought 5 kg of each fruit — the sum equation has a real meaning!

3. Subtract equation (2) from equation (1): (3x − 2x) + (2y − 3y) = 340 − 310, so **x − y = 30** ...(4).
Why: Anitha bought 1 kg more mangoes and 1 kg less oranges than Ravi, and paid ₹30 more — so 1 kg of mangoes costs ₹30 more than 1 kg of oranges.

4. Add (3) and (4): 2x = 160, so **x = 80**.
Why: The y cancels perfectly — this 'add and subtract' dance is the seed of the elimination method you will master in Section 07.

5. Put x = 80 in (3): 80 + y = 130, so **y = 50**.
Why: Always finish the pair — a solution means BOTH values.

6. Verify with Ravi's bill (the equation you did not use last): 2(80) + 3(50) = 160 + 150 = 310 and Anitha's: 3(80) + 2(50) = 240 + 100 = 340.
Why: Checking in BOTH original bills takes ten seconds and makes the answer bullet-proof.

ANSWER: Mangoes cost ₹80 per kg and oranges cost ₹50 per kg.

TOPPER TIP: *Remember the shape of this story. Ages, fares, notes, digits — nearly every word problem in this chapter is the Fruit Market Puzzle wearing a different costume.*

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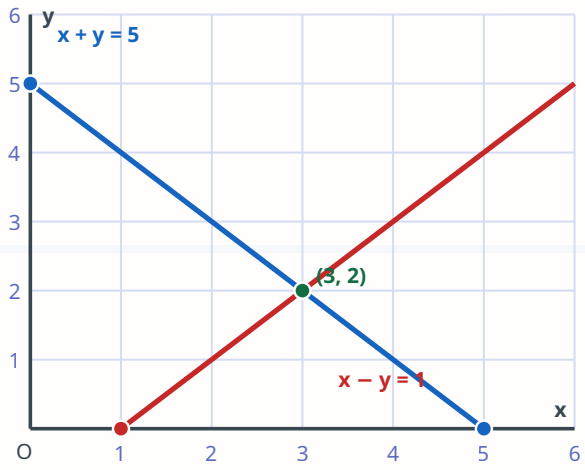
04 Visual Concept Explanation

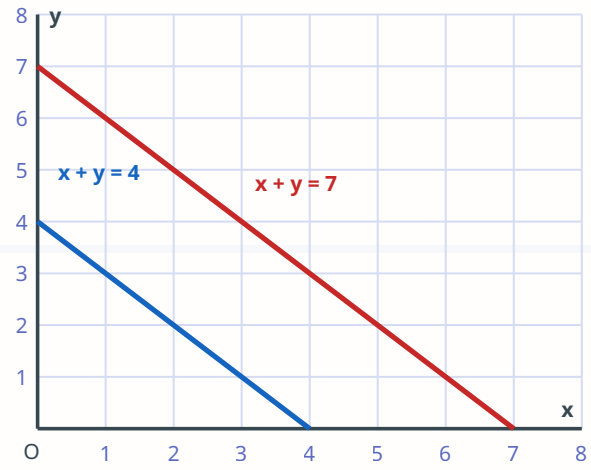
SINGAPORE CPA — CONCRETE → PICTORIAL → ABSTRACT

Concrete. Hold the story in your hands: two shopping bills, each mixing the same two fruits. Every bill is a balance — the left pan (fruit) exactly balances the right pan (money).

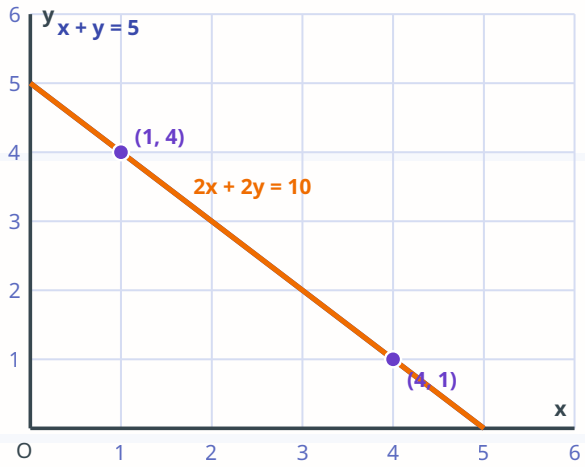
Pictorial. Now draw each equation as a line. A solution of the *pair* must satisfy *both* equations, so it must lie on *both* lines. Two lines on a plane can meet in only three ways — and each way has a different story to tell:



Picture 1 — The lines CROSS once.
One common point (3, 2): exactly **one solution**. The pair is **consistent**.



Picture 2 — The lines are PARALLEL.
They never meet: **no solution**. The pair is **inconsistent** — the two clues contradict each other.



Picture 3 — The lines COINCIDE.
One line drawn twice: **infinitely many solutions**. The pair is consistent and **dependent** — the second clue repeats the first.

The three pictures at a glance

Cross → 1 solution → consistent.
Parallel → 0 solutions → inconsistent.
Coincide → infinitely many → consistent, dependent.

There is no fourth picture. Two straight lines in a plane have no other way to be — Reasoning 3 (Section 12) proves even 'exactly two solutions' is impossible.

Abstract. Pictures are honest but slow. In Section 05 you will learn to read the same three stories directly from the coefficients — and in Section 06 you will *derive* why that reading can never fail.

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Visual Concepts — Reading a Meeting Point

Look again at Picture 1. The blue line x + y = 5 carries all pairs that add to 5. The red line x − y = 1 carries all pairs whose difference is 1. The green point (3, 2) is the **only** point sitting on both lines — 3 + 2 = 5 and 3 − 2 = 1. That is why "solving a pair" and "finding the meeting point" are the same sentence in two languages.

The three pictures — your permanent memory hook

Cross → one solution → the two clues agree on exactly one answer.
Parallel → no solution → the two clues can never be true together.
Coincide → infinitely many solutions → the two clues are really one clue said twice.

Why a graph alone is not enough

A graph is drawn by hand, on squared paper, with a pencil of some thickness. If the true meeting point were (22/5, 8/5) = (4.4, 1.6) — a real CBSE 2023 board question you will meet in the PYQ section — reading it exactly off a drawing is nearly impossible. So we use the graph to **understand**, and algebra (substitution and elimination) to be **exact**. This chapter gives you both.

TOPPER TIP: *In the exam, when a question says 'solve graphically', the drawing IS the answer — draw it carefully, with a table of values and a sharp pencil. When it just says 'solve', reach for algebra.*

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05 Mathematical Foundations

UNDERSTAND — THE PRECISE IDEAS, WORD BY WORD

A **linear equation in two variables** is any equation that can be written as **ax + by + c = 0**, where a, b, c are real numbers and a, b are not both zero. Its graph is always a straight line. A **pair** of such equations, considered together, is:

$$a_1x + b_1y + c_1 = 0 \quad \text{and} \quad a_2x + b_2y + c_2 = 0$$

The four foundation definitions

Solution of the pair: a pair (x, y) that satisfies BOTH equations at the same time — geometrically, a point lying on both lines.
Consistent pair: a pair that has at least one solution (the lines cross, or coincide).
Inconsistent pair: a pair that has no solution at all (the lines are parallel and distinct).
Dependent pair: a consistent pair with infinitely many solutions — the two equations carry the same information (coincident lines).

The Ratio Test — the whole chapter in one table

Compare	Lines	Solutions	Type
$a_1/a_2 \neq b_1/b_2$	Intersecting	Exactly one	Consistent
$a_1/a_2 = b_1/b_2 \neq c_1/c_2$	Parallel	None	Inconsistent
$a_1/a_2 = b_1/b_2 = c_1/c_2$	Coincident	Infinitely many	Consistent, dependent

Warning: the c-values must be read from the form ax + by + c = 0, signs included. 2x + 3y = 7 becomes 2x + 3y − 7 = 0, so c = −7, not 7!

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Foundations — The Three Methods at a Glance

You will solve pairs by three methods. Each is a different road to the same meeting point.

Method 1 — Graphical

For EACH equation: build a table of at least three (x, y) pairs → plot the points → draw the line. The coordinates of the meeting point are the solution. If the lines are parallel, report "no solution"; if they coincide, report "infinitely many solutions".

Method 2 — Substitution

1. From one equation, write one variable in terms of the other (y = ... or x = ...). 2. Substitute this expression into the OTHER equation — now it has one variable only. 3. Solve it. 4. Put the value back into the expression from Step 1 to get the second variable. 5. Verify in both original equations.

Method 3 — Elimination

1. Multiply one or both equations by suitable numbers so that the coefficients of one variable become equal (or opposite). 2. Add or subtract the equations — that variable is eliminated. 3. Solve the resulting one-variable equation. 4. Substitute back into either original equation for the second variable. 5. Verify in both original equations.

TOPPER TIP: *Method choice in one line: a coefficient of 1 or −1 somewhere? Substitute. Coefficients already matching or easy multiples? Eliminate. The question demands a picture? Go graphical.*

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06 Formula Derivations

8D • DERIVE — WHY THE RATIO TEST CAN NEVER FAIL

Most guides order you to memorise the ratio table. We will **earn** it — twice. First from the geometry of slopes, then from pure algebra. After these two pages, you will never be able to forget the table, because you will be able to rebuild it from scratch in an empty room.

Derivation A — The Slope Story

1 Take one line in the form $ax + by + c = 0$, with $b \neq 0$. Shift terms: $by = -ax - c$. Divide by b: **$y = -(a/b)\,x - (c/b)$**.

Why: Every non-vertical line can be written as y = (slope) x + (intercept). Comparing the two forms reads off both numbers at once.

2 So for our line: **slope $m = -a/b$** and **y-intercept $k = -c/b$**. For the pair, $m_1 = -a_1/b_1$, $k_1 = -c_1/b_1$, and similarly m_2, k_2 with subscript 2.

Why: The slope measures steepness and direction; the intercept measures where the line crosses the y-axis. Two numbers completely describe a line's position.

3 Two lines **intersect** exactly when they point in different directions — that is, when $m_1 \neq m_2$. Now $m_1 \neq m_2$ means $a_1/b_1 \neq a_2/b_2$; cross-multiplying gives $a_1b_2 \neq a_2b_1$, which is the same as **$a_1/a_2 \neq b_1/b_2$**.

Why: Cross-multiplication is just multiplying both sides by b_1b_2 — legal, and it turns the slope condition into the ratio condition. This is the first row of the table, derived.

4 Two lines are **parallel** exactly when $m_1 = m_2$ but $k_1 \neq k_2$ (same direction, different heights — they can never meet). $m_1 = m_2$ gives $a_1/a_2 = b_1/b_2$; and $k_1 \neq k_2$ means $c_1/b_1 \neq c_2/b_2$, i.e. $c_1/c_2 \neq b_1/b_2$. Together: **$a_1/a_2 = b_1/b_2 \neq c_1/c_2$**.

Why: Second row of the table, derived. Parallel is 'same slope, shifted up or down'.

5 Two lines **coincide** exactly when $m_1 = m_2$ AND $k_1 = k_2$ — same direction, same height, same line. That gives **$a_1/a_2 = b_1/b_2 = c_1/c_2$**.

Why: Third row, derived. (Vertical lines, where b = 0, obey the same ratio rules — the algebra of Derivation B covers every case without exception.)

DERIVED: Intersecting ⇔ $a_1/a_2 \neq b_1/b_2$ | Parallel ⇔ $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ | Coincident ⇔ all three ratios equal.

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Derivation B — The Algebra Story (Where Elimination is Born)

Now forget pictures. Start with the pair in the form $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$, and try to force y to disappear.

1. Multiply the first equation by b_2: $a_1b_2\,x + b_1b_2\,y = c_1b_2$ ...(i)
   Why: Multiplying every term of an equation by the same non-zero number never changes its solutions — the line stays the same line. We choose b_2 to make the y-coefficients match.

2. Multiply the second equation by b_1: $a_2b_1\,x + b_1b_2\,y = c_2b_1$ ...(ii)
   Why: Now both equations carry the identical y-term $b_1b_2\,y$ — the trap is set.

3. Subtract (ii) from (i). The y-terms cancel exactly, leaving ONE equation in x alone: $\mathbf{(a_1b_2 - a_2b_1)\,x = c_1b_2 - c_2b_1}$.
   Why: This is the elimination method itself, written in general letters! Every elimination you will ever do is this step with numbers instead of letters.

4. Case 1: if $a_1b_2 - a_2b_1 \neq 0$, we can divide and get exactly ONE value of x (and then exactly one y). Note $a_1b_2 \neq a_2b_1$ is precisely $a_1/a_2 \neq b_1/b_2$.
   Why: Unique solution ⇔ the first row of the ratio table — derived a second time, with no geometry at all.

5. Case 2: if $a_1b_2 - a_2b_1 = 0$ but $c_1b_2 - c_2b_1 \neq 0$, the equation becomes $0 \cdot x =$ (a non-zero number) — impossible for every x. No solution; the pair is inconsistent, and the ratios say $a_1/a_2 = b_1/b_2 \neq c_1/c_2$.
   Why: 0 = 5 is a false statement no x can rescue — that is exactly what 'parallel' means in algebra.

6. Case 3: if BOTH quantities are 0, the equation becomes $0 \cdot x = 0$ — true for every x. Infinitely many solutions; the ratios are all equal.
   Why: 0 = 0 gives no information — the second equation was the first one in disguise all along.

DERIVED: The ratio table is not a rule to memorise — it is the three possible fates of one subtraction.

TOPPER TIP: *Forgot the table in the exam hall? Redo Derivation B with letters on your rough sheet — thirty seconds, and the whole table is back.*

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07 Detailed Solved Examples

8D • SOLVE — NCERT CHAPTER EXAMPLES & EXERCISES 3.1–3.3, EVERY STEP SHOWN

This is the heart of the chapter: the complete NCERT set for the rationalised textbook — the chapter's own representation examples, then **Exercise 3.1** (graphical method and the ratio test), **Exercise 3.2** (substitution) and **Exercise 3.3** (elimination). Every solution shows each step in order, with the reason for the step. For graphical questions you always get the **table of values first, then the graph** — exactly as the examiner wants to see it.

How NCERT builds this chapter

Chapter Examples: three real situations (ages, sports equipment, fruit prices) are first written as pairs of equations and drawn — algebra and geometry side by side.
Exercise 3.1: solve graphically; judge pairs by comparing ratios.
Exercise 3.2: the substitution method, including six story problems.
Exercise 3.3: the elimination method, including five story problems.

PIN POINT

One pair, three fates. Before solving anything, glance at the ratios: a_1/a_2 vs b_1/b_2. Different → one answer exists. Equal → check c_1/c_2 to separate 'never' from 'always'.

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NCERT Chapter Example 1 — Aftab and His Daughter (Ages)

EXAMPLE 1

Aftab tells his daughter: "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." Represent this situation algebraically and graphically.

THINK FIRST: Two unknown present ages. Each sentence about the past/future becomes one equation. Then each equation becomes one line.

1. Let Aftab's present age be x years and his daughter's present age be y years.
Why: Always declare your letters with units — the examiner reads this line first.

2. Seven years ago: Aftab was (x − 7) and his daughter was (y − 7). The first sentence says x − 7 = 7(y − 7). Expand: x − 7 = 7y − 49, so **x − 7y = −42** ...(1).
Why: 'Seven times as old as you were THEN' — both ages must travel back 7 years. Forgetting to subtract from the daughter's age is the classic slip here.

3. Three years from now: Aftab will be (x + 3) and his daughter (y + 3). The second sentence says x + 3 = 3(y + 3). Expand: x + 3 = 3y + 9, so **x − 3y = 6** ...(2).
Why: Same care in the future direction: both ages gain 3 years.

4. Build a table for (1): x = 7y − 42. Taking y = 6, 7, 12 gives x = 0, 7, 42 — points (0, 6), (7, 7), (42, 12).
Why: Choose y-values that make x a whole number — plotting stays clean.

5. Build a table for (2): x = 3y + 6. Taking y = 0, 4, 12 gives x = 6, 18, 42 — points (6, 0), (18, 4), (42, 12).
Why: Notice (42, 12) appearing in BOTH tables — the meeting point is announcing itself.

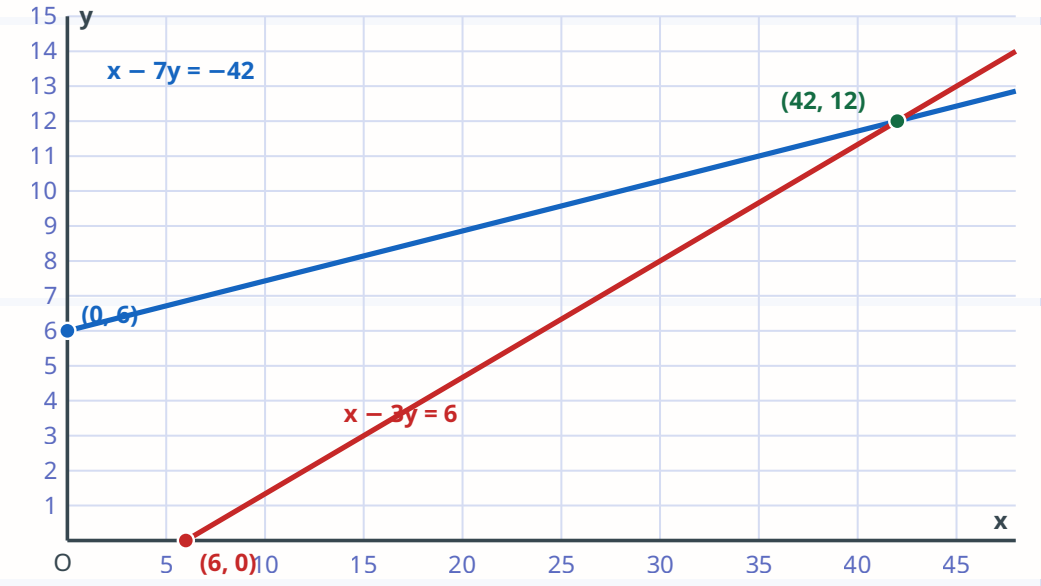
6. Plot both lines (graph below). They intersect at exactly one point: **(42, 12)**.
Why: One crossing point = one pair of ages that fits both sentences.

7. Check: 7 years ago they were 35 and 5, and 35 = 7 × 5. Three years from now they will be 45 and 15, and 45 = 3 × 15.
Why: Verifying against the STORY (not just the equations) catches modelling errors, not only arithmetic errors.

ANSWER: Aftab is 42 years old and his daughter is 12 years old. Algebraically: x − 7y = −42 and x − 3y = 6; graphically the two lines meet at (42, 12).

CBSE-STYLE MARKING SCHEME: 1 mark for the two correct equations • 1 mark for the two tables of values • 1 mark for the graph and the meeting point (3 marks).

NCERT Chapter Example 1 — The Graph



The two age-lines meet only at (42, 12) — one father, one daughter, one answer.

Read the graph like an examiner

The blue line carries every (present age of Aftab, present age of daughter) pair satisfying the 'seven years ago' sentence; the red line carries the 'three years hence' sentence. Their crossing point (42, 12) is the ONLY pair of ages satisfying both sentences at once — that is what 'solving a pair' means, drawn.

TOPPER TIP: *On graph paper, always mark the meeting point with a dot and label its coordinates — an unlabelled crossing earns no mark for the answer.*

NCERT Chapter Example 2 — The Cricket Coach (Representation)

EXAMPLE 2

The coach of a cricket team buys 3 bats and 6 balls for ₹3900. Later, she buys another bat and 3 more balls of the same kind for ₹1300. Represent this situation algebraically and geometrically.

THINK FIRST: Two unknown prices again — but watch the ending: this pair teaches that a graph can also reveal a 'strange' answer.

1. Let one bat cost ₹x and one ball cost ₹y. First purchase: **3x + 6y = 3900** ...(1). Second purchase: **x + 3y = 1300** ...(2).
   Why: Each purchase is one balance: items on the left, money on the right.
2. Table for (1): y = (3900 − 3x)/6. Taking x = 0, 300, 500 gives y = 650, 500, 400 — points (0, 650), (300, 500), (500, 400).
3. Table for (2): y = (1300 − x)/3. Taking x = 100, 400, 1300 gives y = 400, 300, 0 — points (100, 400), (400, 300), (1300, 0).
4. Plot both lines (graph below). They meet at **(1300, 0)** — on the x-axis itself.
   Why: Mathematically the pair is consistent with the single solution x = 1300, y = 0.
5. Read the meaning honestly: the 'solution' says a ball costs ₹0, which no shop allows. NCERT uses this example to show that a pair of equations always has a graphical story — but a real-life answer must also make real-life sense.
   Why: A model is a mirror of the story, not the story itself. When the mirror shows nonsense, the data itself was artificial — a lesson every modeller must learn.

ANSWER: Algebraically: 3x + 6y = 3900 and x + 3y = 1300. Geometrically the lines meet at (1300, 0).

CBSE-STYLE MARKING SCHEME: 1 mark for the two equations • 1 mark for tables • 1 mark for the graph (3 marks).

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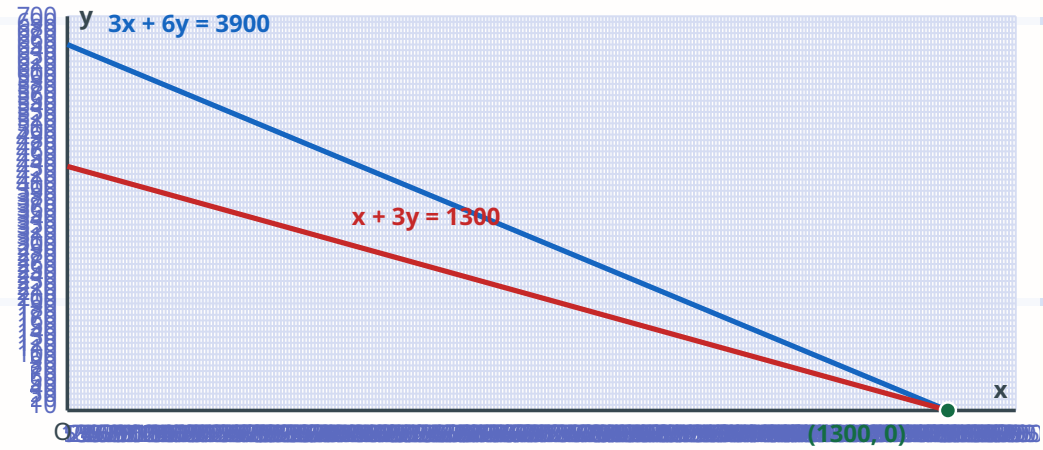
NCERT Chapter Example 2 — The Graph



The two purchase-lines meet at (1300, 0) — on the x-axis itself.

When the graph 'lies' politely

The mathematics is flawless: the lines DO meet at (1300, 0). The story is what protests — a ball cannot cost ₹0. NCERT's lesson: always ask whether a mathematical answer is also a sensible answer. In board word problems the data is honest, but your own modelling errors can produce the same kind of nonsense — the sense-check is your alarm bell.

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NCERT Chapter Example 3 — Apples and Grapes (Parallel Lines)

EXAMPLE 3

The cost of 2 kg of apples and 1 kg of grapes on a day was found to be ₹160. After a month, the cost of 4 kg of apples and 2 kg of grapes is ₹300. Represent the situation algebraically and geometrically.

THINK FIRST: The second bill looks like double the first — 4 kg and 2 kg — but the money is NOT double (₹300 ≠ ₹320). Prices changed during the month! The graph should expose the contradiction.

1. Let apples cost ₹x per kg and grapes ₹y per kg. Day one: **2x + y = 160** ...(1). After a month: **4x + 2y = 300** ...(2).

2. Divide (2) throughout by 2: **2x + y = 150** ...(2′). Now compare (1) and (2′): identical left sides, different right sides.
Why: Simplifying an equation before comparing is legal and often reveals the secret immediately.

3. Table for (1): y = 160 − 2x. Taking x = 0, 40, 80 gives y = 160, 80, 0. Table for (2′): y = 150 − 2x. Taking x = 0, 50, 75 gives y = 150, 50, 0.

4. Plot both lines (graph below): they are **parallel** — they never meet.
Why: Same left side means same slope (both fall by 2 for every 1 step right); different constants keep them apart forever.

5. Ratio check: a_1/a_2 = 4/2... using (1) and (2) directly: 2/4 = 1/2, 1/2, and 160/300 = 8/15. Since $a_1/a_2 = b_1/b_2 \neq$ c-ratio, the pair is **inconsistent** — no single pair of prices can satisfy both days, exactly because prices changed.
Why: The ratio test and the picture agree — they always do. That agreement is what Section 06 proved.

ANSWER: Algebraically: 2x + y = 160 and 4x + 2y = 300. Geometrically the lines are parallel — the pair has no solution (inconsistent).

CBSE-STYLE MARKING SCHEME: 1 mark for the equations • 1 mark for tables • 1 mark for the parallel graph and conclusion (3 marks).

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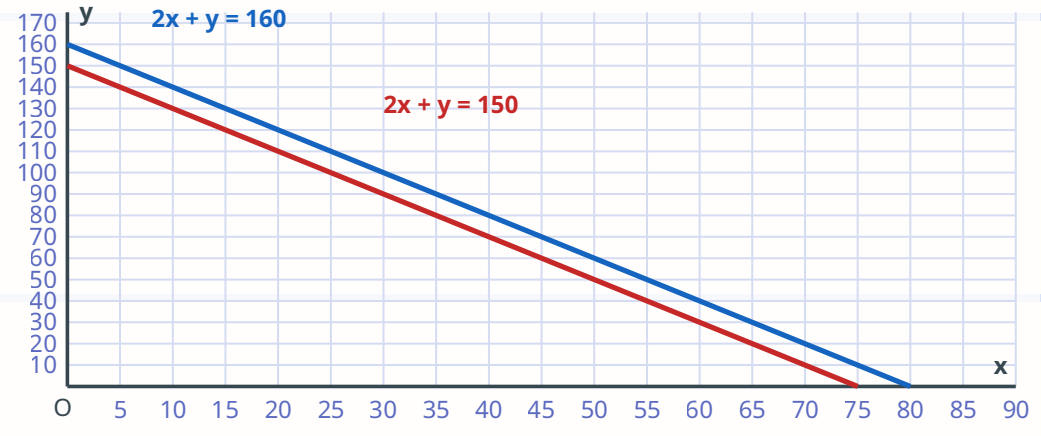
NCERT Chapter Example 3 — The Graph



Two parallel price-lines: the two days can never agree on one pair of prices.

Parallel lines are information, not failure

A pair with no solution is the mathematics RAISING ITS HAND: 'your two clues cannot describe one fixed situation.' Here it detected that prices changed during the month. Whenever your equations turn out parallel in a word problem, re-read the story — something in it changed, or something in your translation did.

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Exercise 3.1 Q1(i) — The Mathematics Quiz (Graphical)

EXERCISE 3.1 • Q1(i)

Form the pair of linear equations and find the solution graphically: 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

THINK FIRST: Total students give one equation; '4 more girls than boys' gives the second. Then draw both lines.

1. Let the number of boys be x and the number of girls be y. Total: **x + y = 10** ...(1). Girls are 4 more than boys: **y = x + 4**, i.e. **−x + y = 4** ...(2).
   Why: Translate each sentence separately — one sentence, one equation. Never try to swallow a whole problem in one gulp.
2. Table for (1): y = 10 − x. x = 0, 3, 10 gives y = 10, 7, 0 — points (0, 10), (3, 7), (10, 0).
3. Table for (2): y = x + 4. x = 0, 3, 6 gives y = 4, 7, 10 — points (0, 4), (3, 7), (6, 10).
   Why: The point (3, 7) sits in both tables — expect it on both lines.
4. Plot the two lines (graph below). They intersect at **(3, 7)**.
5. Read off: x = 3 boys, y = 7 girls. Check: 3 + 7 = 10 and 7 = 3 + 4.
   Why: The graphical answer must be stated in the words of the question — 'boys' and 'girls', not just x and y.

ANSWER: 3 boys and 7 girls took part in the quiz.

CBSE-STYLE MARKING SCHEME: 1 mark for forming the equations • 1 mark for tables • 1 mark for graph and answer (3 marks).

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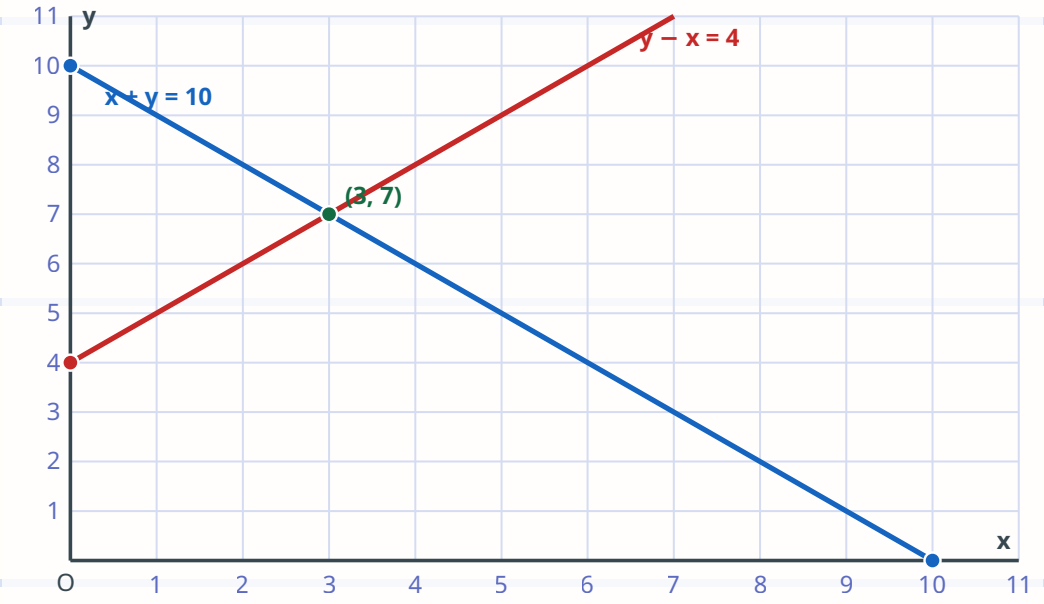
Exercise 3.1 Q1(i) — The Graph



x + y = 10 (blue) and y − x = 4 (red) cross at (3, 7): 3 boys, 7 girls.

PIN POINT

Table first, graph second, answer third. In graphical questions the examiner's marks are split exactly in that order — a beautiful graph with no table still loses its first mark.

Exercise 3.1 Q1(ii) — Pencils and Pens (Graphical)

EXERCISE 3.1 • Q1(ii)

Form the pair of linear equations and find the solution graphically: 5 pencils and 7 pens together cost ₹50, whereas 7 pencils and 5 pens together cost ₹46. Find the cost of one pencil and one pen.

THINK FIRST: Prices are the unknowns. Each shopping combination is one equation. The numbers 5 and 7 swap places between the two bills — a pattern worth noticing.

1. Let one pencil cost ₹x and one pen cost ₹y. First bill: **5x + 7y = 50** ...(1). Second bill: **7x + 5y = 46** ...(2).
2. Table for (1): y = (50 − 5x)/7. x = 10, 3, −4 gives y = 0, 5, 10 — points (10, 0), (3, 5), (−4, 10). *Why: Choose x-values that make the division exact. A negative x is useless as a price but perfectly legal as a point on the line — the line does not stop where the story stops.*
3. Table for (2): y = (46 − 7x)/5. x = 3, 8, −2 gives y = 5, −2, 12 — points (3, 5), (8, −2), (−2, 12).
4. Plot both lines (graph below). They intersect at **(3, 5)**.
5. So one pencil costs ₹3 and one pen costs ₹5. Check: 5(3) + 7(5) = 15 + 35 = 50; 7(3) + 5(5) = 21 + 25 = 46.

ANSWER: One pencil costs ₹3 and one pen costs ₹5.

CBSE-STYLE MARKING SCHEME: 1 mark for the equations • 1 mark for tables • 1 mark for graph and answer (3 marks).



Exercise 3.1 Q2 — Intersecting, Parallel or Coincident?

EXERCISE 3.1 • Q2

On comparing the ratios a_1/a_2, b_1/b_2 and c_1/c_2, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident: (i) $5x - 4y + 8 = 0$, $7x + 6y - 9 = 0$ (ii) $9x + 3y + 12 = 0$, $18x + 6y + 24 = 0$ (iii) $6x - 3y + 10 = 0$, $2x - y + 9 = 0$.

THINK FIRST: No solving, no drawing — just read the three ratios for each pair and let the table from Section 05 speak.

1. (i) Read the coefficients: $a_1 = 5$, $b_1 = -4$, $c_1 = 8$; $a_2 = 7$, $b_2 = 6$, $c_2 = -9$. Ratios: $a_1/a_2 = 5/7$ and $b_1/b_2 = -4/6 = -2/3$.
   Why: Write both equations in the form $ax + by + c = 0$ FIRST, and copy the signs with the numbers — b_1 is -4, not 4.

2. Since $5/7 \neq -2/3$, i.e. $a_1/a_2 \neq b_1/b_2$, the lines **intersect at exactly one point**. (The c-ratio is not even needed.)
   Why: The moment the first two ratios differ, the verdict is 'intersecting' — stop there and save time.

3. (ii) $a_1 = 9$, $b_1 = 3$, $c_1 = 12$; $a_2 = 18$, $b_2 = 6$, $c_2 = 24$. Ratios: $9/18 = 1/2$, $3/6 = 1/2$, $12/24 = 1/2$.
   Why: Every coefficient of the second equation is exactly double the first — expect coincidence.

4. All three ratios are equal, so the lines are **coincident**.

5. (iii) $a_1 = 6$, $b_1 = -3$, $c_1 = 10$; $a_2 = 2$, $b_2 = -1$, $c_2 = 9$. Ratios: $6/2 = 3$, $-3/-1 = 3$, $10/9$. So $a_1/a_2 = b_1/b_2 = 3 \neq 10/9 = c_1/c_2$.
   Why: $-3/-1 = +3$ — two negatives make a positive. Sign slips in this step are the number-one cause of wrong verdicts in Q2-type questions.

6. First two ratios equal but the c-ratio different → the lines are **parallel**.

ANSWER: (i) Intersecting at a point (ii) Coincident (iii) Parallel.

CBSE-STYLE MARKING SCHEME: 1 mark per part: ½ for the correct ratios, ½ for the correct conclusion (3 marks).

PIN POINT

The ratio test is a 20-second question. $a_1/a_2 \neq b_1/b_2$ → intersecting. First two equal → compare with c_1/c_2: equal → coincident, not equal → parallel.

Exercise 3.1 Q3 — Consistent or Inconsistent?

EXERCISE 3.1 • Q3

On comparing the ratios, find out whether the following pairs of linear equations are consistent or inconsistent: (i) 3x + 2y = 5; 2x − 3y = 7 (ii) 2x − 3y = 8; 4x − 6y = 9 (iii) (3/2)x + (5/3)y = 7; 9x − 10y = 14 (iv) 5x − 3y = 11; −10x + 6y = −22 (v) (4/3)x + 2y = 8; 2x + 3y = 12.

THINK FIRST: 'Consistent' only asks: does at least one solution exist? Intersecting AND coincident pairs are both consistent; only the parallel pair is inconsistent. Convert each equation to ax + by + c = 0 form mentally as you read the ratios.

1. (i) $a_1/a_2 = 3/2$; $b_1/b_2 = 2/-3 = -2/3$. Not equal → intersecting → **consistent** (unique solution).

2. (ii) $a_1/a_2 = 2/4 = 1/2$; $b_1/b_2 = -3/-6 = 1/2$; $c_1/c_2 = -8/-9 = 8/9$. First two equal, third different → parallel → **inconsistent**.
Why: The second equation is 'almost' double the first — 2×(2x − 3y) = 4x − 6y would need 16 on the right, not 9. Almost-double is the parallel trap.

3. (iii) $a_1/a_2 = (3/2)/9 = 1/6$; $b_1/b_2 = (5/3)/-10 = -1/6$. Not equal → intersecting → **consistent**.
Why: With fractions, divide carefully: (3/2) ÷ 9 = 3/18 = 1/6.

4. (iv) $a_1/a_2 = 5/-10 = -1/2$; $b_1/b_2 = -3/6 = -1/2$; $c_1/c_2 = -11/22 = -1/2$. All equal → coincident → **consistent** (infinitely many solutions).
Why: The second equation is exactly −2 times the first — same line, new costume.

5. (v) $a_1/a_2 = (4/3)/2 = 2/3$; $b_1/b_2 = 2/3$; $c_1/c_2 = -8/-12 = 2/3$. All equal → coincident → **consistent** (infinitely many solutions).
Why: Multiply the first equation by 3/2 and you get the second exactly — spotting the multiple is the fastest ratio test of all.

ANSWER: (i) Consistent (ii) Inconsistent (iii) Consistent (iv) Consistent (v) Consistent.

CBSE-STYLE MARKING SCHEME: ½ mark per part for ratios and ½ for the verdict (5 parts).

Exercise 3.1 Q4 — Consistent? If Yes, Solve Graphically

EXERCISE 3.1 • Q4

Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically: (i) x + y = 5, 2x + 2y = 10 (ii) x − y = 8, 3x − 3y = 16 (iii) 2x + y − 6 = 0, 4x − 2y − 4 = 0 (iv) 2x − 2y − 2 = 0, 4x − 4y − 5 = 0.

THINK FIRST: Run the ratio test on all four first. Only the consistent ones earn a graph.

1. (i) Ratios: 1/2, 1/2, −5/−10 = 1/2 — all equal → coincident → **consistent**, infinitely many solutions.

2. (i) Graph: both equations draw the SAME line x + y = 5 (left graph below). Every point of the line is a solution: (0, 5), (1, 4), (2, 3), (3, 2), (4, 1), (5, 0), ...
Why: For a coincident pair, 'the solution' is the whole line — listing a few of its points is the correct way to answer.

3. (ii) Ratios: 1/3, −1/−3 = 1/3, −8/−16 = 1/2. First two equal, third different → parallel → **inconsistent**. No graph needed.
Why: 3x(x − y) would give 24, not 16 — the second equation contradicts the first.

4. (iii) Ratios: 2/4 = 1/2, 1/−2 = −1/2. Not equal → intersecting → **consistent** with a unique solution.

5. (iii) Tables: for 2x + y = 6: x = 0, 2, 3 gives y = 6, 2, 0. For 4x − 2y = 4, i.e. 2x − y = 2: x = 0, 2, 3 gives y = −2, 2, 4. The lines (right graph below) meet at **(2, 2)**.
Why: Check (2, 2): 2(2) + 2 = 6 and 4(2) − 2(2) = 4.

6. (iv) Ratios: 2/4 = 1/2, −2/−4 = 1/2, −2/−5 = 2/5. First two equal, third different → parallel → **inconsistent**.

ANSWER: (i) Consistent — infinitely many solutions, e.g. (1, 4), (2, 3), (3, 2) (ii) Inconsistent (iii) Consistent — x = 2, y = 2 (iv) Inconsistent.

CBSE-STYLE MARKING SCHEME: 1 mark per part (ratio verdict; plus graph where consistent).

Exercise 3.1 Q5 — The Rectangular Garden (Graphical)

EXERCISE 3.1 • Q5

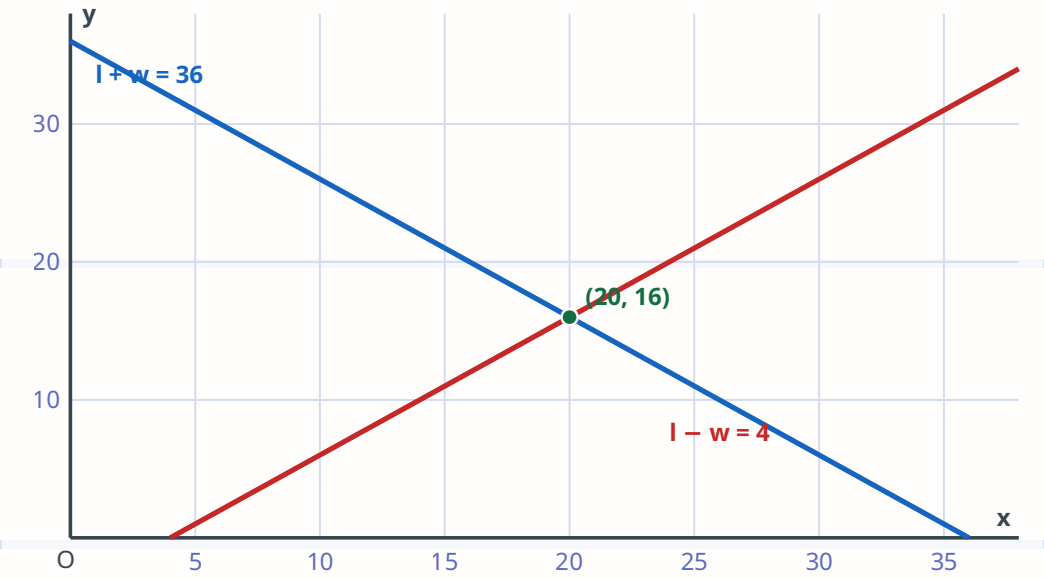
Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden (graphically).

THINK FIRST: 'Half the perimeter' of a rectangle is simply length + width. The story hands you both equations in one breath.

1. Let the length be l m and the width be w m. Length is 4 more than width: **l = w + 4**, i.e. l − w = 4 ...(1). Half perimeter: **l + w = 36** ...(2).
   Why: For a rectangle, perimeter = 2(l + w), so half the perimeter is l + w — a fact worth remembering for every garden/field problem.
2. Table for (2): w = 36 − l. l = 0, 20, 36 gives w = 36, 16, 0 — points (0, 36), (20, 16), (36, 0).
   Why: Here x-axis carries l and y-axis carries w.
3. Table for (1): w = l − 4. l = 4, 20, 36 gives w = 0, 16, 32 — points (4, 0), (20, 16), (36, 32).
4. Plot both lines (graph below): they meet at **(20, 16)**.
5. So length = 20 m and width = 16 m. Check: 20 = 16 + 4 and half perimeter = 20 + 16 = 36 m.

ANSWER: The garden is 20 m long and 16 m wide.

CBSE-STYLE MARKING SCHEME: 1 mark for the equations • 1 mark for tables • 1 mark for graph and dimensions (3 marks).



Exercise 3.1 Q6 — Inventing a Partner Equation

EXERCISE 3.1 • Q6

Given the linear equation 2x + 3y − 8 = 0, write another linear equation in two variables such that the geometrical representation of the pair so formed is: (i) intersecting lines (ii) parallel lines (iii) coincident lines.

THINK FIRST: This time YOU choose the second equation — the ratio test tells you exactly what freedom you have in each case.

1. The given line has $a_1 = 2$, $b_1 = 3$, $c_1 = -8$. Let your invented equation be $a_2x + b_2y + c_2 = 0$.

2. (i) Intersecting needs $a_1/a_2 \neq b_1/b_2$. Choose $a_2 = 1$, $b_2 = 1$: then $2/1 \neq 3/1$. Answer (one of many): **x + y − 4 = 0**.
Why: Any equation whose x:y coefficient pattern is NOT 2:3 will cross the given line.

3. (ii) Parallel needs $a_1/a_2 = b_1/b_2 \neq c_1/c_2$. Keep the 2:3 pattern but change the constant: **2x + 3y − 5 = 0**. Check: $2/2 = 3/3 = 1 \neq -8/-5$.
Why: Same left side, different constant — the simplest parallel partner of all.

4. (iii) Coincident needs all three ratios equal. Multiply the given equation by 2: **4x + 6y − 16 = 0**. Check: $2/4 = 3/6 = -8/-16 = 1/2$.
Why: Any non-zero multiple of the given equation draws the same line.

ANSWER: (i) x + y − 4 = 0 (many answers possible) (ii) 2x + 3y − 5 = 0 (iii) 4x + 6y − 16 = 0.

CBSE-STYLE MARKING SCHEME: 1 mark per part, with the ratio condition stated (3 marks).

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Exercise 3.1 Q7 — The Triangle on the x-axis

EXERCISE 3.1 • Q7

Draw the graphs of the equations x − y + 1 = 0 and 3x + 2y − 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

THINK FIRST: A triangle needs three vertices: the two points where the lines cut the x-axis, and the point where the lines cut each other.

1. Table for x − y + 1 = 0, i.e. y = x + 1: x = −1, 0, 2 gives y = 0, 1, 3 — points (−1, 0), (0, 1), (2, 3).

2. Table for 3x + 2y − 12 = 0, i.e. y = (12 − 3x)/2: x = 0, 2, 4 gives y = 6, 3, 0 — points (0, 6), (2, 3), (4, 0).
Why: The shared point (2, 3) appears in both tables again — it must be the top vertex.

3. Vertex 1 — first line meets the x-axis where y = 0: from its table, **(−1, 0)**. Vertex 2 — second line meets the x-axis where y = 0: **(4, 0)**. Vertex 3 — the lines meet each other at **(2, 3)**.
Why: On the x-axis, y is always 0 — that single fact finds two of the three vertices.

4. Draw both lines, mark the three vertices and shade the triangle they form with the x-axis (graph below).
Why: Optional check: base = 5 units (from −1 to 4), height = 3, so area = ½ × 5 × 3 = 7.5 sq. units.

ANSWER: Vertices of the triangle: (−1, 0), (4, 0) and (2, 3).

CBSE-STYLE MARKING SCHEME: 1 mark for the two tables • 1 mark for the graph • 1 mark for the vertices and shading (3 marks).

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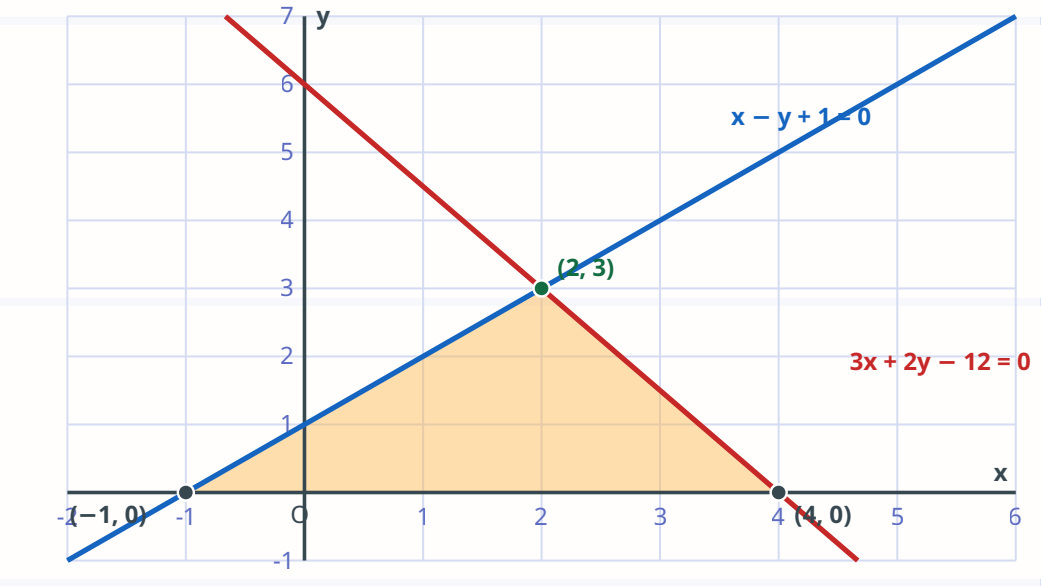
Exercise 3.1 Q7 — The Graph and the Shaded Triangle



Vertices: (−1, 0), (4, 0) and (2, 3). The shaded triangle has base 5 units and height 3 units — area 7.5 sq. units.

TOPPER TIP: *Shade with light, even strokes and outline the triangle firmly — in the board exam the shading itself carries the final mark of this question.*

Exercise 3.2 Q1 (i)–(iii) — Substitution Method

EXERCISE 3.2 • Q1(i)

Solve by the substitution method: x + y = 14; x − y = 4.

THINK FIRST: The coefficient of y in the first equation is 1 — y is begging to be isolated.

1. From (1): **y = 14 − x** ...(3).
   Why: Step 1 of substitution: express ONE variable through the other, choosing the equation where this is cheapest.
2. Substitute (3) into (2): x − (14 − x) = 4, so x − 14 + x = 4, giving 2x = 18.
   Why: The brackets matter: minus a bracket flips every sign inside — −(14 − x) = −14 + x.
3. Solve: **x = 9**.
4. Put x = 9 in (3): y = 14 − 9 = **5**.
   Why: Use the SIMPLE expression (3) for the back-substitution, not a complicated original.
5. Verify: 9 + 5 = 14 and 9 − 5 = 4.

ANSWER: x = 9, y = 5.

CBSE-STYLE MARKING SCHEME: ½ mark for expressing y • 1 mark for the one-variable equation and x • ½ mark for y (2 marks).

PIN POINT

Substitution cue: reach for substitution when some variable stands with coefficient 1 or −1 — it can be isolated without fractions.

EXERCISE 3.2 • Q1(ii)

Solve by the substitution method: s − t = 3; s/3 + t/2 = 6.

THINK FIRST: Isolate s from the first equation (coefficient 1), then clear the fractions in the second by multiplying through by 6.

1. From (1): **s = t + 3** ...(3).
2. Substitute (3) into (2): (t + 3)/3 + t/2 = 6. Multiply every term by 6 (the LCM of 3 and 2): 2(t + 3) + 3t = 36.
   Why: Multiplying an equation by the LCM of the denominators deletes all fractions in one move — do it BEFORE substituting if you prefer; the order is your choice.
3. Expand and solve: 2t + 6 + 3t = 36 → 5t = 30 → **t = 6**.
4.

Exercise 3.2 Q1 (iii)–(iv) — When Substitution Reveals a Secret

EXERCISE 3.2 • Q1(iii)

Solve by the substitution method: 3x − y = 3; 9x − 3y = 9.

THINK FIRST: Isolate y from the first equation. Then watch carefully — something unusual is about to happen, and it is the most instructive moment of this exercise.

1. From (1): **y = 3x − 3** ...(3).
2. Substitute (3) into (2): 9x − 3(3x − 3) = 9. Expand: 9x − 9x + 9 = 9.
3. The x-terms cancel completely and we are left with **9 = 9** — a statement that is ALWAYS true, whatever x may be.
   Why: When substitution ends in an identity like 9 = 9, the second equation gave no new information: indeed 9x − 3y = 9 is exactly 3 × (3x − y = 3). The two 'equations' are one equation wearing two coats.
4. So the pair has **infinitely many solutions**: every pair (x, 3x − 3) works — for example (0, −3), (1, 0), (2, 3), (3, 6)...
   Why: Geometrically the two lines coincide — ratio check: 3/9 = −1/−3 = −3/−9 = 1/3, all equal, exactly as Section 05 promised.

ANSWER: Infinitely many solutions: all pairs (x, y) with y = 3x − 3.

CBSE-STYLE MARKING SCHEME: 1 mark for the substitution • 1 mark for recognising the identity and concluding infinitely many solutions (2 marks).

EXERCISE 3.2 • Q1(iv)

Solve by the substitution method: 0.2x + 0.3y = 1.3; 0.4x + 0.5y = 2.3.

THINK FIRST: Decimals are fractions in disguise. Multiply both equations by 10 first and the problem becomes whole-number friendly.

1. Multiply (1) and (2) by 10: **2x + 3y = 13** ...(1′) and **4x + 5y = 23** ...(2′).
   Why: Scaling an equation by 10 changes nothing about its solutions — but it changes everything about your comfort.
2. From (1′): 3y = 13 − 2x, so **y = (13 − 2x)/3** ...(3).
3. Substitute (3) into (2′): 4x + 5(13 − 2x)/3 = 23. Multiply by 3: 12x + 65 − 10x = 69, so 2x = 4 and **x = 2**.
4.

Exercise 3.2 Q1 (v)–(vi) — Surds and Fractions

EXERCISE 3.2 • Q1(v)

Solve by the substitution method: √2 x + √3 y = 0; √3 x − √8 y = 0.

THINK FIRST: Do not fear the surds — √2, √3, √8 are just numbers. Treat them as coefficients and walk the same road. (Recall √8 = 2√2.)

1. From (1): √3 y = −√2 x, so **y = −√2 x/√3** ...(3).

2. Substitute (3) into (2): √3 x − √8 · (−√2 x/√3) = 0. The minus signs cancel: √3 x + (√16/√3) x = 0, since √8 × √2 = √16 = 4. So √3 x + 4x/√3 = 0.
Why: √8 · √2 = √(8 × 2) = √16 = 4 — surds multiply inside one root. This is the only 'new' skill in the question.

3. Multiply by √3: 3x + 4x = 0, so 7x = 0 and **x = 0**.

4. Put x = 0 in (3): **y = 0**. Verify: √2(0) + √3(0) = 0; √3(0) − √8(0) = 0.
Why: Both lines pass through the origin with different slopes — they can meet only at (0, 0). The algebra agrees with the picture.

ANSWER: x = 0, y = 0.

CBSE-STYLE MARKING SCHEME: 1 mark for the substitution with correct surd handling • 1 mark for x = 0, y = 0 (2 marks).

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Exercise 3.2 Q1 (vi) — Fractions, Cleared First

EXERCISE 3.2 • Q1(vi)

Solve by the substitution method: 3x/2 − 5y/3 = −2; x/3 + y/2 = 13/6.

THINK FIRST: Two fraction-heavy equations. Multiply each by 6 — the LCM of 2, 3 and 6 — before doing anything else.

1. (1) × 6: **9x − 10y = −12** …(1′). (2) × 6: **2x + 3y = 13** …(2′).
   Why: Check the multiplication term by term: (3x/2)×6 = 9x; (−5y/3)×6 = −10y; −2×6 = −12. One missed term is the classic error here.
2. From (2′): **x = (13 − 3y)/2** …(3).
3. Substitute (3) into (1′): 9(13 − 3y)/2 − 10y = −12. Multiply by 2: 117 − 27y − 20y = −24, so −47y = −141 and **y = 3**.
4. Put y = 3 in (3): x = (13 − 9)/2 = **2**. Verify: 3(2)/2 − 5(3)/3 = 3 − 5 = −2; 2/3 + 3/2 = 4/6 + 9/6 = 13/6.

ANSWER: x = 2, y = 3.

CBSE-STYLE MARKING SCHEME: ½ mark for clearing fractions • 1 mark for substitution and y • ½ mark for x (2 marks).

Exercise 3.2 Q2 — Solve, Then Find m

EXERCISE 3.2 • Q2

Solve 2x + 3y = 11 and 2x − 4y = −24, and hence find the value of m for which y = mx + 3.

THINK FIRST: The word 'hence' is an instruction: first solve the pair, then feed the solution into the line y = mx + 3, which must pass through that very point.

1. From (1): **x = (11 − 3y)/2** ...(3).
Why: Substitution is the named method of this exercise — use it even if elimination looks tempting.

2. Substitute (3) into (2): (11 − 3y) − 4y = −24, because 2x = 11 − 3y. So 11 − 7y = −24, giving 7y = 35 and **y = 5**.
Why: Substituting 2x as a block (instead of x) halves the fraction work — watch for such gifts whenever a coefficient repeats.

3. Put y = 5 in (3): x = (11 − 15)/2 = **−2**. Verify: 2(−2) + 3(5) = −4 + 15 = 11; 2(−2) − 4(5) = −4 − 20 = −24.

4. Now use the solution in y = mx + 3: the point (−2, 5) lies on this line, so 5 = m(−2) + 3, giving −2m = 2 and **m = −1**.
Why: 'y = mx + 3 passes through the solution point' — a point lies on a line exactly when its coordinates satisfy the line's equation (Section 02, Quick Check Q5!).

ANSWER: x = −2, y = 5, and m = −1.

CBSE-STYLE MARKING SCHEME: 1 mark for x and y by substitution • 1 mark for using the point in y = mx + 3 • 1 mark for m = −1 (3 marks).

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Exercise 3.2 Q3 (i)–(ii) — Numbers and Angles

EXERCISE 3.2 • Q3(i)

Form the pair of linear equations and solve by substitution: The difference between two numbers is 26 and one number is three times the other. Find them.

1. Let the larger number be x and the smaller be y. Difference: **x − y = 26** ...(1). One is three times the other: **x = 3y** ...(2).
   Why: Equation (2) is ALREADY in substituted form — this is the friendliest word problem in the chapter.
2. Substitute (2) into (1): 3y − y = 26, so 2y = 26 and **y = 13**.
3. From (2): x = 3(13) = **39**. Check: 39 − 13 = 26 and 39 = 3 × 13.

ANSWER: The numbers are 39 and 13.

CBSE-STYLE MARKING SCHEME: 1 mark for the pair of equations • 1 mark for substitution • 1 mark for the answer with check (3 marks).

EXERCISE 3.2 • Q3(ii)

Form the pair of linear equations and solve by substitution: The larger of two supplementary angles exceeds the smaller by 18°. Find them.

1. Let the larger angle be x° and the smaller be y°. Supplementary angles sum to 180°: **x + y = 180** ...(1). Larger exceeds smaller by 18°: **x = y + 18** ...(2).
   Why: 'Supplementary' is a vocabulary word carrying an equation: x + y = 180. (Complementary would carry 90.)
2. Substitute (2) into (1): (y + 18) + y = 180, so 2y = 162 and **y = 81**.
3. From (2): x = 81 + 18 = **99**. Check: 99 + 81 = 180 and 99 − 81 = 18.

ANSWER: The angles are 99° and 81°.

CBSE-STYLE MARKING SCHEME: Same split as Q3(i) (3 marks).

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Exercise 3.2 Q3 (iii)–(iv) — Bats, Balls and Taxi Fares

EXERCISE 3.2 • Q3(iii)

Form the pair of linear equations and solve by substitution: The coach of a cricket team buys 7 bats and 6 balls for ₹3800. She buys 3 bats and 5 balls for ₹1750 later. Find the cost of each bat and each ball.

1. Let a bat cost ₹x and a ball cost ₹y. Purchases: **7x + 6y = 3800** ...(1) and **3x + 5y = 1750** ...(2).
   Why: Compare with Chapter Example 2 — same costume, but this time the numbers are honest and the answer is sensible.
2. From (2): 3x = 1750 − 5y, so **x = (1750 − 5y)/3** ...(3).
3. Substitute (3) into (1): 7(1750 − 5y)/3 + 6y = 3800. Multiply by 3: 12250 − 35y + 18y = 11400, so −17y = −850 and **y = 50**.
   Why: Multiply through by 3 BEFORE expanding — fractions first, arithmetic second.
4. Put y = 50 in (3): x = (1750 − 250)/3 = 1500/3 = **500**. Check: 7(500) + 6(50) = 3500 + 300 = 3800; 3(500) + 5(50) = 1500 + 250 = 1750.

ANSWER: One bat costs ₹500 and one ball costs ₹50.

CBSE-STYLE MARKING SCHEME: 1 mark for equations • 1 mark for substitution working • 1 mark for both prices with check (3 marks).

EXERCISE 3.2 • Q3(iv)

Form the pair of linear equations and solve by substitution: In a city, taxi charges consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹105 and for a journey of 15 km, the charge paid is ₹155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

1. Let the fixed charge be ₹x and the charge per km be ₹y. For 10 km: **x + 10y = 105** ...(1). For 15 km: **x + 15y = 155** ...(2).
   Why: Total fare = fixed part + (rate × distance). Every fare problem in every exam is this one line.
2. From (1): **x = 105 − 10y** ...(3).
3. Substitute (3) into (2): (105 − 10y) + 15y = 155, so 5y = 50 and **y = 10**.
   Why: Notice how the fixed charge vanishes on subtraction — the extra 5 km cost exactly the extra ₹50. The algebra is just that observation, formalised.
4. Put y = 10 in (3): x = 105 − 100 = **5**. So fixed charge = ₹5 and rate = ₹10 per km.

Exercise 3.2 Q3 (v)–(vi) — A Fraction and Two Ages

EXERCISE 3.2 • Q3(v)

Form the pair of linear equations and solve by substitution: A fraction becomes 9/11 if 2 is added to both the numerator and the denominator. If 3 is added to both the numerator and the denominator, it becomes 5/6. Find the fraction.

1. Let the fraction be x/y. First condition: (x + 2)/(y + 2) = 9/11. Cross-multiply: 11(x + 2) = 9(y + 2), so **11x − 9y = −4** ...(1).
   Why: Cross-multiplication turns a fraction condition into a linear equation — always expand both sides fully before collecting terms.
2. Second condition: (x + 3)/(y + 3) = 5/6. Cross-multiply: 6(x + 3) = 5(y + 3), so **6x − 5y = −3** ...(2).
3. From (2): 6x = 5y − 3, so **x = (5y − 3)/6** ...(3). Substitute (3) into (1): 11(5y − 3)/6 − 9y = −4. Multiply by 6: 55y − 33 − 54y = −24, so **y = 9**.
4. Put y = 9 in (3): x = (45 − 3)/6 = 42/6 = **7**. Check: (7 + 2)/(9 + 2) = 9/11; (7 + 3)/(9 + 3) = 10/12 = 5/6.
   Why: Verify fraction answers in the ORIGINAL fraction language — it is the fastest check in this whole family of problems.

ANSWER: The fraction is 7/9.

CBSE-STYLE MARKING SCHEME: 1 mark for the two equations • 1 mark for substitution • 1 mark for the fraction with check (3 marks).

EXERCISE 3.2 • Q3(vi)

Form the pair of linear equations and solve by substitution: Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

1. Let Jacob's present age be x years and his son's be y years. Five years hence: Jacob x + 5, son y + 5, and x + 5 = 3(y + 5), so **x − 3y = 10** ...(1).
   Why: Both people age together — add 5 to BOTH ages, subtract 5 from BOTH ages. (Remember Aftab from the chapter examples!)
2. Five years ago: x − 5 = 7(y − 5), so **x − 7y = −30** ...(2).
3. From (1): **x = 3y + 10** ...(3). Substitute (3) into (2): (3y + 10) − 7y = −30, so −4y = −40 and **y = 10**.
4. From (3): x = 3(10) + 10 = **40**. Check: in 5 years, 45 and 15, and 45 = 3 × 15; 5 years ago, 35 and 5, and 35 = 7 × 5.

Exercise 3.3 Q1 (i)–(ii) — Elimination Method

EXERCISE 3.3 • Q1(i)

Solve by the elimination method (and substitution method): x + y = 5; 2x − 3y = 4.

THINK FIRST: Elimination plan: make the y-coefficients match. Multiply (1) by 3 so both equations carry 3y — with opposite signs, ready to be added away.

1. (1) × 3: **3x + 3y = 15** …(1′). Keep (2): 2x − 3y = 4.
   Why: Multiply EVERY term, including the right side. A half-multiplied equation is the most common elimination error in the country.
2. Add (1′) and (2): 5x = 19, so **x = 19/5**.
   Why: +3y and −3y destroy each other — that was the whole point of Step 1.
3. Put x = 19/5 in (1): 19/5 + y = 5, so y = 5 − 19/5 = **6/5**.
4. Substitution cross-check (NCERT asks for both methods): from (1), y = 5 − x. In (2): 2x − 3(5 − x) = 4 → 5x − 15 = 4 → x = 19/5, then y = 6/5. Same answer, second road.
   Why: Two methods agreeing is the strongest verification there is.
5. Final verify: 19/5 + 6/5 = 25/5 = 5; 2(19/5) − 3(6/5) = (38 − 18)/5 = 4.

ANSWER: x = 19/5, y = 6/5.

CBSE-STYLE MARKING SCHEME: 1 mark for making coefficients equal • 1 mark for x • 1 mark for y (3 marks). Fractional answers are welcome — do not 'round' them away.

PIN POINT

Elimination cue: choose the variable whose coefficients are closest to a common multiple. Match the coefficients, then ADD if signs are opposite, SUBTRACT if signs are the same.

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Exercise 3.3 Q1 (ii) — Elimination, Second Pair

EXERCISE 3.3 • Q1(ii)

Solve by the elimination method (and substitution method): 3x + 4y = 10; 2x − 2y = 2.

1. (2) × 2: **4x − 4y = 4** ...(2′). Now the y-terms are +4y and −4y.
2. Add (1) and (2′): 7x = 14, so **x = 2**.
3. Put x = 2 in (1): 6 + 4y = 10, so 4y = 4 and **y = 1**.
4. Substitution cross-check: from (2), x = 1 + y. In (1): 3(1 + y) + 4y = 10 → 7y = 7 → y = 1, x = 2.
   Final verify: 3(2) + 4(1) = 10; 2(2) − 2(1) = 2.

ANSWER: x = 2, y = 1.

CBSE-STYLE MARKING SCHEME: Same split as Q1(i) (3 marks).

Exercise 3.3 Q1 (iii)–(iv) — Harder Coefficients

EXERCISE 3.3 • Q1(iii)

Solve by the elimination method (and substitution method): 3x − 5y − 4 = 0; 9x = 2y + 7.

THINK FIRST: First drag both equations into the open: 3x − 5y = 4 and 9x − 2y = 7. The x-coefficients 3 and 9 are natural multiples — eliminate x.

1. Rewrite: **3x − 5y = 4** ...(1) and **9x − 2y = 7** ...(2).
   Why: 9x = 2y + 7 means 9x − 2y = 7 — move the y-term across with its sign flipped.
2. (1) × 3: **9x − 15y = 12** ...(1′). Subtract (2) from (1′): (9x − 15y) − (9x − 2y) = 12 − 7, so −13y = 5 and **y = −5/13**.
   Why: Subtracting a negative term adds it: −15y − (−2y) = −13y. Say this line aloud as you write it — sign errors live here.
3. Put y = −5/13 in (1): 3x − 5(−5/13) = 4, so 3x = 4 − 25/13 = 27/13 and **x = 9/13**.
4. Verify in (2): 9(9/13) = 81/13; 2(−5/13) + 7 = −10/13 + 91/13 = 81/13. Substitution method (from (2): x = (2y + 7)/9, substituted in (1)) leads to the same pair.
   Why: An answer with 13 in the denominator looks suspicious but is perfectly correct — NCERT deliberately includes one such question so you learn to trust correct working over pretty numbers.

ANSWER: x = 9/13, y = −5/13.

CBSE-STYLE MARKING SCHEME: 1 mark for standard form • 1 mark for elimination • 1 mark for both values (3 marks).

EXERCISE 3.3 • Q1(iv)

Solve by the elimination method (and substitution method): x/2 + 2y/3 = −1; x − y/3 = 3.

THINK FIRST: Clear the fractions first: multiply (1) by 6 and (2) by 3. Then the x-coefficients match for free.

1. (1) × 6: **3x + 4y = −6** ...(1′). (2) × 3: **3x − y = 9** ...(2′).
2. Subtract (2′) from (1′): 4y − (−y) = −6 − 9, so 5y = −15 and **y = −3**.
3. Put y = −3 in (2): x − (−3)/3 = 3, so x + 1 = 3 and **x = 2**.
   Why: Work in the ORIGINAL equation (2) here — it is shorter than its multiplied twin.
4.

Exercise 3.3 Q2 (i)–(ii) — A Fraction and Two Ages, Again

EXERCISE 3.3 • Q2(i)

Form the pair of linear equations and solve by elimination: If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes 1/2 if we only add 1 to the denominator. What is the fraction?

1. Let the fraction be x/y. First condition: (x + 1)/(y − 1) = 1, so x + 1 = y − 1, giving **x − y = −2** ...(1).
   Why: 'Reduces to 1' means numerator EQUALS denominator — no cross-multiplication even needed.
2. Second condition: x/(y + 1) = 1/2, so **2x − y = 1** ...(2).
3. Subtract (1) from (2): (2x − y) − (x − y) = 1 − (−2), so **x = 3**.
   Why: The y-terms were already identical — elimination needed no multiplication at all. Always look before you multiply!
4. Put x = 3 in (1): 3 − y = −2, so **y = 5**. Check: (3 + 1)/(5 − 1) = 4/4 = 1; 3/(5 + 1) = 3/6 = 1/2.

ANSWER: The fraction is 3/5.

CBSE-STYLE MARKING SCHEME: 1 mark for the equations • 1 mark for elimination • 1 mark for the fraction with check (3 marks).

EXERCISE 3.3 • Q2(ii)

Form the pair of linear equations and solve by elimination: Nuri is 3 times as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

1. Let Nuri's present age be x years and Sonu's be y years. Now: **x = 3y**, i.e. x − 3y = 0 ...(1).
2. Ten years later: Nuri x + 10, Sonu y + 10, and x + 10 = 2(y + 10), so **x − 2y = 10** ...(2).
   Why: x + 10 = 2y + 20 → x − 2y = 10. Expand the bracket fully before collecting.
3. Subtract (1) from (2): (x − 2y) − (x − 3y) = 10 − 0, so **y = 10**.
4. From (1): x = 3(10) = **30**. Check: 30 = 3 × 10; in ten years, 40 and 20, and 40 = 2 × 20.

ANSWER: Nuri is 30 years old and Sonu is 10 years old.

CBSE-STYLE MARKING SCHEME: Same split as Q2(i) (3 marks).

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Exercise 3.3 Q2 (iii) — The Two-Digit Number

EXERCISE 3.3 • Q2(iii)

Form the pair of linear equations and solve by elimination: The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

1. Let the tens digit be x and the units digit be y. The number itself is **10x + y**, and the reversed number is **10y + x**.
   Why: A two-digit number is NOT xy and NOT x + y: the tens digit carries a weight of 10. This single line is the whole secret of digit problems.
2. Sum of digits: **x + y = 9** ...(1).
3. Nine times the number is twice the reversed number: 9(10x + y) = 2(10y + x). Expand: 90x + 9y = 20y + 2x. Collect: 88x − 11y = 0. Divide by 11: **8x − y = 0** ...(2).
   Why: Divide a whole equation by a common factor (11 here) whenever you can — smaller numbers, fewer slips.
4. Add (1) and (2): 9x = 9, so **x = 1**. From (1): y = 9 − 1 = **8**.
   Why: The y-coefficients are +1 and −1 — a ready-made elimination pair.
5. The number is 10(1) + 8 = **18**. Check: digits sum to 9; 9 × 18 = 162 and 2 × 81 = 162.
   Why: State the NUMBER (18), not merely the digits — the question asked for the number.

ANSWER: The number is 18.

CBSE-STYLE MARKING SCHEME: 1 mark for expressing the number and its reverse • 1 mark for the equations • 1 mark for elimination and the number (3 marks).

Exercise 3.3 Q2 (iv)–(v) — Bank Notes and Library Charges

EXERCISE 3.3 • Q2(iv)

Form the pair of linear equations and solve by elimination: Meena went to a bank to withdraw ₹2000. She asked the cashier to give her ₹50 and ₹100 notes only. Meena got 25 notes in all. Find how many notes of ₹50 and ₹100 she received.

1. Let the number of ₹50 notes be x and of ₹100 notes be y. Count of notes: **x + y = 25** ...(1). Value of notes: **50x + 100y = 2000** ...(2).
   Why: Two different totals — a count total and a value total — give the two equations. Confusing them is the classic error in note problems.
2. Divide (2) by 50: **x + 2y = 40** ...(2′). Subtract (1) from (2′): y = 40 − 25 = **15**.
3. From (1): x = 25 − 15 = **10**. Check: 10 + 15 = 25 notes; 10(50) + 15(100) = 500 + 1500 = ₹2000.

ANSWER: Meena received ten ₹50 notes and fifteen ₹100 notes.

CBSE-STYLE MARKING SCHEME: 1 mark for the two equations • 1 mark for elimination • 1 mark for both counts with check (3 marks).

Exercise 3.3 Q2 (v) — The Library's Fixed Charge

EXERCISE 3.3 • Q2(v)

Form the pair of linear equations and solve by elimination: A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹27 for a book kept for seven days, while Susy paid ₹21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

1 Let the fixed charge (first three days) be ₹x and the extra charge per day be ₹y. Saritha kept the book 7 days = 3 + 4 extra days: **x + 4y = 27** ...(1). Susy kept it 5 days = 3 + 2 extra days: **x + 2y = 21** ...(2).

Why: Only the days AFTER the first three are charged at the daily rate; 7 days means 4 extra days, not 7. This is the trap the question is built around.

2 Subtract (2) from (1): 2y = 6, so **y = 3**.

Why: The fixed charge cancels on subtraction — the difference in bills is purely the difference in extra days.

3 Put y = 3 in (2): x = 21 − 6 = **15**. Check: Saritha: 15 + 4(3) = 27; Susy: 15 + 2(3) = 21.

ANSWER: The fixed charge is ₹15 and the charge for each extra day is ₹3.

CBSE-STYLE MARKING SCHEME: 1 mark for the equations (with correct extra days) • 1 mark for elimination • 1 mark for both charges (3 marks).

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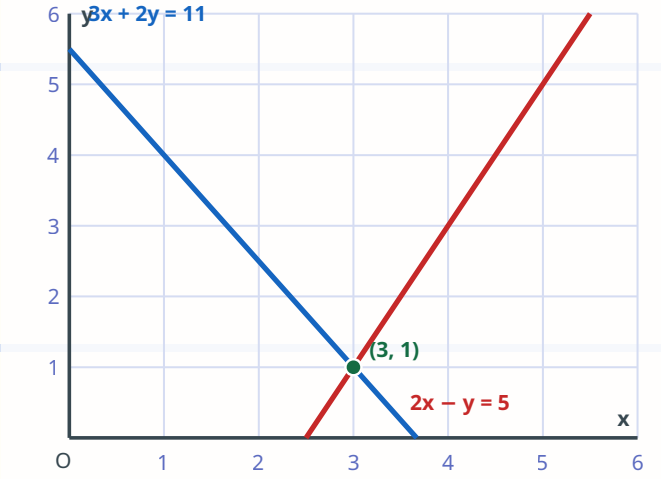
08 Multiple Solution Methods

8D • CHALLENGE — ONE PAIR, THREE ROADS

Take one pair and walk all three roads to its answer: **3x + 2y = 11 ...(1)** and **2x − y = 5 ...(2)**.

Road 1 — Graphical

For (1): y = (11 − 3x)/2. x = 1, 3 gives y = 4, 1 — points (1, 4), (3, 1).
For (2): y = 2x − 5. x = 2, 3 gives y = −1, 1 — points (2, −1), (3, 1).
The lines meet at **(3, 1)**.



Road 2 — Substitution

From (2): y = 2x − 5 ...(3).
In (1): 3x + 2(2x − 5) = 11 → 3x + 4x − 10 = 11 → 7x = 21 → **x = 3**.
In (3): y = 2(3) − 5 = **1**.

Road 3 — Elimination

(2) × 2: 4x − 2y = 10 ...(2′).
Add (1) and (2′): 7x = 21 → **x = 3**.
In (1): 9 + 2y = 11 → 2y = 2 → **y = 1**.

Three roads, one meeting point: **x = 3, y = 1**. Verify: 3(3) + 2(1) = 11; 2(3) − 1 = 5.

PIN POINT

Same answer, three roads — that is your safety net. In the exam, solve by the fastest road; if time remains, cross-check by a second road instead of re-reading the first.

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Multiple Methods — Which Road, When?

Situation	Best road	Why
The question says "graphically"	Graphical — no choice!	The drawing, table and reading-off ARE the marks.
A variable has coefficient 1 or −1	Substitution	Isolating it creates no fractions (e.g. y = 2x − 5 above).
Coefficients are equal, opposite, or easy multiples	Elimination	One multiplication (or none) clears a variable.
Big swapped coefficients like 62x + 43y, 43x + 62y	Add & subtract trick	Adding gives x + y, subtracting gives x − y (a real CBSE 2024 pattern — see PYQs).
You only need consistency, not the solution	Ratio test — no solving at all	Three divisions answer the question in seconds.
Decimals or fractions everywhere	First scale the equations clean, then eliminate	Whole numbers make every later step safer.

The add–subtract trick, once more with feeling

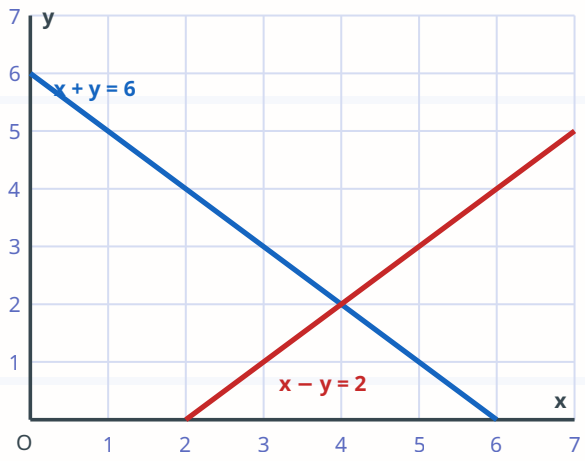
Whenever the coefficients of x and y are **swapped** between the two equations — ax + by = p and bx + ay = q — do not multiply anything. **Add** the equations to get (a + b)(x + y) = p + q, and **subtract** them to get (a − b)(x − y) = p − q. Two tiny equations replace one messy elimination. You used it in the Fruit Market Story; CBSE used it in 2024. It will serve you for years.

TOPPER TIP: *In a 3-mark question, the method named in the question is compulsory — 'by substitution' means substitution, even if elimination is quicker. Marks follow instructions.*

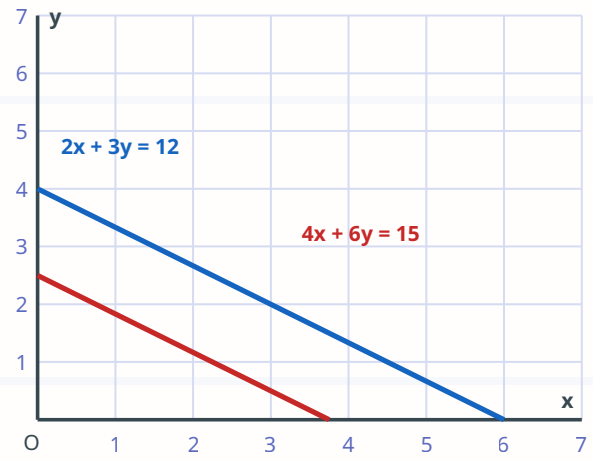
09 Interactive Simulations & Hands-on Activities

TRY IT YOURSELF — LEARN BY DOING

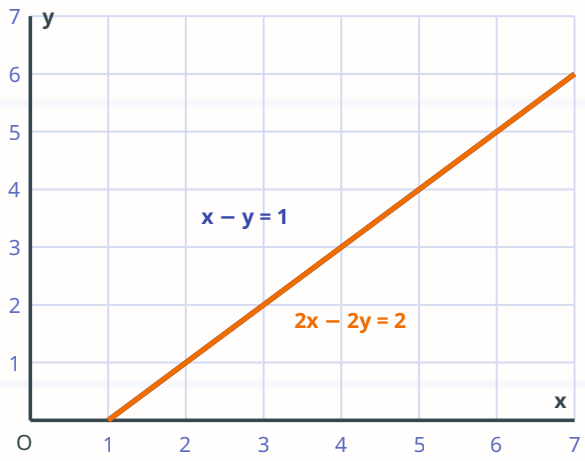
Activity 1 — The Graph Lab. On graph paper, draw all three pairs below yourself FIRST (tables of values, sharp pencil). Only then compare with the printed graphs. For each pair, write: intersecting / parallel / coincident, and the number of solutions.



Pair A: x + y = 6; x − y = 2



Pair B: 2x + 3y = 12; 4x + 6y = 15



Pair C: x − y = 1; 2x − 2y = 2

Your verdict table — fill it in

Pair	Picture	Solutions
A		
B		
C		

Write 'intersecting', 'parallel' or 'coincident' in the Picture column — then check yourself on the Activity Answers page.

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Activities 2 and 3 — Detectives and Surveys

Activity 2 — The Ratio Detective. Without drawing anything, classify these pairs by ratios alone, then check one of them by drawing: (a) $3x + 2y = 8$, $6x + 4y = 16$ (b) $3x + 2y = 8$, $6x + 4y = 20$ (c) $3x + 2y = 8$, $2x + 3y = 8$.

Activity 3 — The Price Survey. Visit (or imagine) two shops. Note the price of the SAME two items at both shops — for example, a notebook and a pen. Pretend you only know two mixed bills (like Anitha's). Form the pair of equations, solve, and recover both unit prices. Then check against the real price tags. You have just done market research with Class 10 algebra.

DIGITAL COMPANION: An interactive line-explorer (drag the coefficients, watch the lines cross, separate or merge) accompanies this chapter in the KalviYogi Digital Ecosystem. QR code and link activate with the KalviYogi app/website at print.

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Activity Answers — Check After Trying

Activity 1

Pair A: Intersecting at (4, 2) — one solution: x = 4, y = 2. Ratios: 1/1 ≠ 1/−1 confirms it.

Pair B: Parallel — no solution. Ratios: 2/4 = 3/6 = 1/2 ≠ 12/15 = 4/5.

Pair C: Coincident — infinitely many solutions; the second equation is exactly twice the first. Ratios: 1/2 = −1/−2 = −1/−2.

Activity 2

(a) 3/6 = 2/4 = −8/−16 = 1/2 — coincident, infinitely many solutions. **(b)** 3/6 = 2/4 = 1/2 ≠ −8/−20 = 2/5 — parallel, no solution. **(c)** 3/2 ≠ 2/3 — intersecting, one solution (in fact x = y = 8/5 — try solving it!).

TOPPER TIP: *Activities feel like play, but they are secretly the Day-3 and Day-7 recall tasks of your Smart Memory plan. Doing them properly now makes revision almost free later.*

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10 Common Mistakes

LEARN FROM OTHER STUDENTS' ERRORS — BEFORE THEY BECOME YOURS

COMMON MISTAKE: Reading c with the wrong sign in the ratio test. For $2x + 3y = 7$, students write $c_1 = 7$. **Right way:** first rewrite as $2x + 3y - 7 = 0$, so $c_1 = -7$. The ratio test only works on the $ax + by + c = 0$ form.

COMMON MISTAKE: Multiplying only SOME terms during elimination. From $x + y = 5$, students write $(1)\times 3$ as $3x + y = 15$. **Right way:** EVERY term is multiplied: $3x + 3y = 15$. Touch every term with the multiplier — no exceptions.

COMMON MISTAKE: Sign collapse when subtracting equations. $(9x - 15y) - (9x - 2y)$ written as $-17y$. **Right way:** subtracting a negative adds: $-15y - (-2y) = -13y$. Write the bracket, flip every sign inside, then combine.

COMMON MISTAKE: In age problems, moving only ONE person through time: 'five years ago Jacob was $x - 5$ and his son was y'. **Right way:** time moves everyone: son was $y - 5$. If one age travels, all ages travel.

COMMON MISTAKE: Writing a two-digit number as xy or as $x + y$. For digits 4 and 7, the number is not 47 by luck of writing — it is $10(4) + 7$. **Right way:** number = 10 × (tens digit) + (units digit); reversed = 10 × (units) + (tens).

COMMON MISTAKE: Library/hostel problems: charging the daily rate for ALL days. Saritha's 7 days billed as $x + 7y$. **Right way:** the fixed charge already covers the first three days; only $7 - 3 = 4$ days earn the daily rate: $x + 4y$.

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Common Mistakes 7–8 — and the Verification Habit

COMMON MISTAKE: Stopping after finding x. A 'solution' with only x = 3 written earns half the marks and zero satisfaction. **Right way:** a pair is solved only when BOTH values stand together, and the final line names them: x = 3, y = 1.

COMMON MISTAKE: Verifying in a MODIFIED equation instead of the original. If you multiplied wrongly at Step 1, checking inside the multiplied equation blesses your own error. **Right way:** substitute into the two ORIGINAL equations printed in the question — every time, both of them.

The 10-second habit that fixes half of all mistakes

Before you turn the page on any solution, spend ten seconds: put your (x, y) into the first original equation — does it balance? Into the second — does it balance? Toppers are not students who never err; they are students whose errors never survive ten seconds.

PIN POINT

0 = 0 means infinitely many solutions. 0 = 5 means no solution. When everything cancels during elimination or substitution, do not panic — the 'empty' result IS the answer; read it.

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11 Graded Practice Problems

FIVE LEVELS — CLIMB ONE LEVEL AT A TIME • ANSWERS IN THE ANSWER KEY

All questions in this section are original KalviYogi practice questions. Solve them in your notebook with full steps — the Answer Key at the end of the chapter gives the final answers and, for the harder levels, the key first step.

LEVEL 1 • FOUNDATION

L1.1 Solve: $x + y = 8$ and $x - y = 2$.

L1.2 Solve: $2x + y = 11$ and $x + y = 7$.

L1.3 Without solving, classify the pair $2x + 3y = 7$ and $4x + 6y = 10$: intersecting, parallel or coincident? How many solutions?

L1.4 Solve: $3x - y = 5$ and $x + y = 3$.

L1.5 The point (2, 1) is a solution of $3x + ky = 7$. Find k.

L1.6 Write any one linear equation whose graph is parallel to the graph of $x + 2y = 4$. Justify your choice with ratios.

LEVEL 2 • SKILL BUILDER

L2.1 Solve by substitution: $3x + 2y = 13$ and $x + y = 5$.

L2.2 Solve by elimination: $5x - 3y = 11$ and $2x + 3y = 17$.

L2.3 Solve: $x/3 + y/4 = 3$ and $x - y = -5$.

L2.4 The sum of two numbers is 74 and their difference is 26. Find the numbers.

L2.5 Solve: $4x + 3y = 20$ and $x - y = -2$.

L2.6 For what values of k does $kx + 2y = 5$ and $3x + y = 1$ have a unique solution?

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Graded Practice — Levels 3, 4 and 5

LEVEL 3 • APPLICATION

L3.1 A cab charges a fixed booking fee plus a per-km rate. A ride of 8 km costs ₹148 and a ride of 12 km costs ₹212. Find the fixed fee and the per-km rate, and the cost of a 20 km ride.

L3.2 A father is three times as old as his son. In 12 years, he will be twice as old as his son. Find their present ages.

L3.3 Three notebooks and two pens cost ₹95. Two notebooks and three pens cost ₹80. Find the cost of each.

L3.4 The length of a rectangle exceeds its width by 5 m and its perimeter is 54 m. Find its length and width.

LEVEL 4 • ADVANCED REASONING

L4.1 Solve without long multiplication: $99x + 101y = 499$ and $101x + 99y = 501$.

L4.2 Find k so that $2x + 3y = 7$ and $(k + 1)x + (2k - 1)y = 4k + 1$ have infinitely many solutions.

L4.3 Solve: $2/x + 3/y = 13$ and $5/x - 4/y = -2$. (Hint: set $u = 1/x$ and $v = 1/y$ first.)

LEVEL 5 • MATHEMATICAL GENIUS

L5.1 Given $x + y = 5$ and $x - y = 1$, first find x and y; then, WITHOUT finding them again, evaluate $x^2 - y^2$, xy and $x^3 - y^3$. Which identity made each evaluation one line long?

L5.2 A two-digit number is 4 times the sum of its digits. The number obtained by reversing the digits is 27 more than the original number. Find the number.

> **TOPPER TIP:** *Stuck at a level? Drop one level down, solve two questions there, and climb back. Staircases are climbed one step at a time — that is true for this chapter and for the staircase in the logo of mathematics itself.*

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Concept MCQs 1–13 — One Mark Each, Board Style

Choose the single correct option. Work them out — guessing teaches nothing. Answers with one-line explanations are in the Answer Key.

MCQ 1. The pair $x + y = 5$ and $x - y = 1$ has the solution
(A) (2, 3) (B) (3, 2) (C) (4, 1) (D) (5, 0)

MCQ 2. The lines $2x + 3y = 7$ and $4x + 6y = 15$ are
(A) intersecting (B) parallel (C) coincident (D) perpendicular

MCQ 3. A pair of linear equations has a unique solution exactly when
(A) $a_1/a_2 = b_1/b_2$ (B) $a_1/a_2 \neq b_1/b_2$ (C) $a_1/a_2 = b_1/b_2 = c_1/c_2$ (D) $a_1 + a_2 = b_1 + b_2$

MCQ 4. The pair $x + 2y = 6$ and $2x + 4y = 12$ has
(A) no solution (B) a unique solution (C) infinitely many solutions (D) exactly two solutions

MCQ 5. If (2, 1) is a solution of $3x + ky = 8$, then k equals
(A) 1 (B) 2 (C) 3 (D) 5

MCQ 6. The lines $x = 3$ and $y = -2$
(A) are parallel (B) intersect at (3, −2) (C) are coincident (D) intersect at (−2, 3)

MCQ 7. The pair $kx + 2y = 5$ and $3x + y = 1$ has a unique solution for
(A) $k = 6$ (B) $k = 3$ (C) every k except 6 (D) $k = 2$ only

MCQ 8. Two numbers have sum 35 and difference 13. The numbers are
(A) 24 and 11 (B) 25 and 10 (C) 23 and 12 (D) 26 and 9

MCQ 9. If $2x + y = 7$ and $x - y = 2$, then x equals
(A) 2 (B) 3 (C) 4 (D) 5

MCQ 10. Which pair has NO solution?
(A) $x + y = 4$; $x - y = 2$ (B) $2x + y = 6$; $4x + 2y = 12$ (C) $x + 2y = 5$; $2x + 4y = 8$ (D) $3x - y = 2$; $x + y = 6$

MCQ 11. A consistent pair whose lines coincide is called
(A) inconsistent (B) dependent (C) impossible (D) perpendicular

MCQ 12. If $x + y = 0$ and $x - y = 0$, then (x, y) is
(A) (1, 1) (B) (0, 0) (C) (1, −1) (D) (−1, 1)

MCQ 13. To eliminate y from $3x + 2y = 7$ and $5x - 3y = 1$, multiply the first equation by ___ and the second by ___
(A) 3 and 2 (B) 2 and 3 (C) 5 and 3 (D) 3 and 5

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Concept MCQs 14–25

MCQ 14. If $2x + 3y = 12$ and $x = 3$, then y equals
(A) 1 (B) 2 (C) 3 (D) 4

MCQ 15. Which of the following is NOT a linear equation in two variables?
(A) $2x + 3y = 5$ (B) $x + y^2 = 4$ (C) $3x - y = 0$ (D) $x/2 + y/3 = 1$

MCQ 16. The graph of $ax + by + c = 0$ (a, b not both zero) is always
(A) a circle (B) a parabola (C) a straight line (D) a curve through the origin

MCQ 17. The lines $5x - 4y + 8 = 0$ and $7x + 6y - 9 = 0$ are
(A) parallel (B) coincident (C) intersecting (D) the same as the axes

MCQ 18. The value of k for which $x + 2y = 5$ and $2x + ky = 10$ represent coincident lines is
(A) 2 (B) 3 (C) 4 (D) 5

MCQ 19. Two supplementary angles are equal. Each angle measures
(A) 45° (B) 60° (C) 90° (D) 120°

MCQ 20. The pair $3x + 2y = 5$ and $2x - 3y = 7$ is
(A) inconsistent (B) consistent (C) dependent (D) parallel

MCQ 21. A single linear equation in two variables, by itself, has
(A) no solution (B) exactly one solution (C) exactly two solutions (D) infinitely many solutions

MCQ 22. For substitution in $x + 3y = 6$, the cleanest first step is
(A) $x = 6 - 3y$ (B) $x = 6 + 3y$ (C) $y = 6 + x$ (D) $3y = x - 6$

MCQ 23. A two-digit number whose tens digit is x and units digit is y equals
(A) xy (B) $x + y$ (C) $10x + y$ (D) $10y + x$

MCQ 24. A taxi charges a fixed ₹20 plus ₹12 per km. The fare for 9 km is
(A) ₹108 (B) ₹128 (C) ₹148 (D) ₹96

MCQ 25. If a pair of linear equations is consistent, its lines are
(A) always parallel (B) always intersecting (C) intersecting or coincident (D) always coincident

TOPPER TIP: *Finished all 25? Score yourself honestly in the Mastery Tracker (Section 18). Below 20 means: revisit Sections 05 and 07 before the Model Tests.*

12 Mathematical Reasoning

JUSTIFY AND PROVE — FULL SOLUTIONS

REASONING 1

For which value(s) of k does the pair kx + 3y = k − 3 and 12x + ky = k have NO solution? Justify every step.

THINK FIRST: The first two ratios give candidates; only the third ratio can separate 'no solution' from 'infinitely many'. Never announce k before testing both candidates in all three ratios.

1. Write the condition for no solution: $a_1/a_2 = b_1/b_2 \neq c_1/c_2$. Here $a_1 = k$, $b_1 = 3$, $c_1 = -(k - 3) = 3 - k$; $a_2 = 12$, $b_2 = k$, $c_2 = -k$.
   Why: Move everything to the left FIRST: kx + 3y − (k − 3) = 0. The c-term is −(k−3), the single most-lost mark in this famous question.
2. From k/12 = 3/k: cross-multiply to get $k^2 = 36$, so **k = 6 or k = −6**.
   Why: Two candidates survive the first two ratios. The third ratio must now judge them — this is where the question separates toppers from the rest.
3. Test k = 6 in the third ratio: $c_1/c_2 = (3 - 6)/(-6) = 1/2$, and k/12 = 1/2 too. All three ratios equal → the lines are COINCIDENT — infinitely many solutions, NOT 'no solution'. Reject k = 6.
4. Test k = −6: $c_1/c_2 = (3 + 6)/6 = 3/2$, while k/12 = −1/2. First two ratios equal (−1/2), third different → parallel lines → no solution. Accept k = −6.

ANSWER: k = −6 only. (For k = 6 the pair has infinitely many solutions — the same pair of equations is a famous board favourite in BOTH costumes.)

PIN POINT

When k^2 appears, BOTH roots must face the third ratio. k = 6 and k = −6 look like twins, but one gives coincident lines and the other parallel lines.

REASONING 2

Solve the pair 148x + 231y = 527 and 231x + 148y = 610 — without multiplying any equation by a two-digit number.

1. Add the equations: (148 + 231)x + (231 + 148)y = 527 + 610, i.e. 379x + 379y = 1137. Divide by 379: **x + y = 3** ...(3).
   Why: The coefficients are swapped (148 ↔ 231) — the add-subtract trick of Section 08 was built for exactly this shape.
2.

Reasoning 3 — Why 'Exactly Two Solutions' is Impossible

REASONING 3

A student claims: "My pair of linear equations has exactly two solutions." Prove that the student must be wrong.

THINK FIRST: This is why the ratio table has exactly three rows — geometry forbids a fourth.

1. Suppose, for contradiction, that the pair has two DIFFERENT solutions, say (x_1, y_1) and (x_2, y_2).
   Why: Proof by contradiction: assume the claim, follow it honestly, and let it defeat itself. (You met this weapon in Chapter 1, against √2.)

2. A solution of the pair lies on BOTH lines. So both points lie on the first line, and both points lie on the second line.
   Why: A point satisfies an equation exactly when it lies on that equation's line — Section 02.

3. But through two distinct points there passes exactly ONE straight line. So the 'first line' and the 'second line' are the same line — the lines coincide.
   Why: Two points fix a line completely; neither line has anywhere else to go.

4. Coincident lines share EVERY point — so the pair has infinitely many solutions, contradicting 'exactly two'.
   Why: Therefore a pair can have 0, 1 or infinitely many solutions — never exactly two, never exactly seventeen. The trichotomy of Section 04 is complete.

ANSWER: A pair of linear equations can never have exactly two solutions. The only possibilities are: none, one, or infinitely many.

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13 Real-Life Applications

8D • APPLY — THE CHAPTER AT WORK IN THE WORLD

APPLICATION 1

Railway fares: 2 adult tickets and 3 child tickets on a train cost ₹740 in total, while 3 adult tickets and 2 child tickets cost ₹860. Find the fare of one adult ticket and one child ticket.

THINK FIRST: Two mixed bills, two unknown fares — the Fruit Market Puzzle at the railway station.

1. Let an adult ticket cost ₹x and a child ticket ₹y. Then **2x + 3y = 740** ...(1) and **3x + 2y = 860** ...(2).
2. Add: 5x + 5y = 1600, so x + y = 320 ...(3). Subtract (1) from (2): x − y = 120 ...(4).
   Why: Swapped coefficients again (2,3 and 3,2) — the add-subtract trick is becoming a reflex.
3. Add (3) and (4): 2x = 440, so **x = 220**. From (3): y = 320 − 220 = **100**.
4. Check: 2(220) + 3(100) = 440 + 300 = 740; 3(220) + 2(100) = 660 + 200 = 860.

ANSWER: An adult ticket costs ₹220 and a child ticket costs ₹100.

APPLICATION 2

A boat goes 40 km downstream in 2 hours and returns 24 km upstream in 2 hours. Find the speed of the boat in still water and the speed of the stream.

1. Let the boat's speed in still water be x km/h and the stream's speed be y km/h.
   Downstream speed = x + y; upstream speed = x − y.
   Why: The stream is a helper going down and an opponent coming up — that is why it adds once and subtracts once.
2. Downstream: 2(x + y) = 40, so **x + y = 20** ...(1). Upstream: 2(x − y) = 24, so **x − y = 12** ...(2).
   Why: Distance = speed × time, applied twice.
3. Add (1) and (2): 2x = 32, so **x = 16**. From (1): **y = 4**.
4. Check: downstream 20 km/h × 2 h = 40 km; upstream 12 km/h × 2 h = 24 km.

ANSWER: The boat's speed in still water is 16 km/h and the stream's speed is 4 km/h.

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Real-Life Applications 3 and 4

APPLICATION 3

A hostel charges a fixed monthly fee plus a daily mess charge. A student who stays 20 days pays ₹1800 in all, while a student who stays 30 days pays ₹2500. Find the fixed fee and the daily mess charge.

1. Let the fixed fee be ₹x and the daily charge be ₹y. Then **x + 20y = 1800** ...(1) and **x + 30y = 2500** ...(2).
   Why: Same skeleton as the taxi problem — a fixed part plus a rate part. Recognising skeletons is the real skill of this chapter.
2. Subtract (1) from (2): 10y = 700, so **y = 70**.
   Why: Ten extra days cost ₹700 — the fixed fee never even appears in this step.
3. Put y = 70 in (1): x = 1800 − 1400 = **400**. Check: 400 + 30(70) = 400 + 2100 = 2500.

ANSWER: The fixed fee is ₹400 and the daily mess charge is ₹70.

APPLICATION 4

A rectangular plot is fenced with 70 m of wire (its perimeter). Its length is 5 m more than twice its width. Find the length and width of the plot.

1. Let the length be l m and the width be w m. Perimeter: 2(l + w) = 70, so **l + w = 35** ...(1). Length condition: **l = 2w + 5** ...(2).
   Why: '5 more than twice the width' = 2w + 5 — translate word by word, in order.
2. Substitute (2) into (1): (2w + 5) + w = 35, so 3w = 30 and **w = 10**.
3. From (2): l = 2(10) + 5 = **25**. Check: perimeter = 2(25 + 10) = 70 m and 25 = 2(10) + 5.

ANSWER: The plot is 25 m long and 10 m wide.

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14 Competency-Based Questions

FIVE CASE STUDIES • ASSERTION–REASON — BOARD EXAM STYLE, FULLY SOLVED

CBSE case-study questions give you a real situation and ask four linked questions. Read the passage once, build the model, and the four answers usually fall one after another. All five cases below are original KalviYogi cases in board format.

Case Study 1 — The City Metro Card

A city metro charges a fixed boarding charge plus a per-kilometre fare. Ravi's card shows a deduction of ₹46 for a ride of 8 km. Sana's card shows ₹70 for a ride of 14 km. Let the fixed charge be ₹x and the per-km fare be ₹y.

CASE 1 (i)

Form the pair of linear equations for the two rides.

1 Ravi: fixed + 8 km of fare: **x + 8y = 46** ...(1). Sana: **x + 14y = 70** ...(2).
Why: Total fare = fixed charge + rate × distance — the taxi skeleton in a metro uniform.

ANSWER: x + 8y = 46 and x + 14y = 70.

CASE 1 (ii)

Find the per-kilometre fare.

1 Subtract (1) from (2): 6y = 24, so **y = 4**.
Why: Six extra kilometres cost ₹24 — the fixed charge cancels, as always.

ANSWER: The per-kilometre fare is ₹4.

CASE 1 (iii)

Find the fixed boarding charge.

1 Put y = 4 in (1): x = 46 − 8(4) = 46 − 32 = **14**.

ANSWER: The fixed boarding charge is ₹14.

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Case Study 1 — The City Metro Card (continued)

CASE 1 (iv)

How much will a ride of 20 km cost?

1

Fare = x + 20y = 14 + 20(4) = 14 + 80 = **94**.

Why: Once the model is built, new questions are one-line substitutions.

ANSWER: A 20 km ride costs ₹94.

Extend the case — try before you check

1. What does a 30 km ride cost? 2. Two friends ride 8 km and 14 km on one card — what is their combined fare, and why is it NOT simply 46 + 70 counted twice over? 3. The metro introduces a day pass at ₹120 for unlimited rides. For what total distance does the pay-per-ride model cost the same as the pass?

Answers: 1. 14 + 30(4) = ₹134. 2. ₹46 + ₹70 = ₹116 — it IS the sum of the two separate fares, because each ride pays its own fixed charge; the trap is to merge the distances first (22 km on one ride would cost 14 + 88 = ₹102, which is cheaper — two fixed charges versus one). 3. 14 + 4d = 120 → d = 26.5 km.

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Case Study 2 — The Sports Academy

Case Study 2 — Admission and Monthly Fees

A sports academy charges a one-time admission fee plus a monthly training fee. Karthik paid ₹2900 in total at the end of 2 months (admission + 2 months). Meera paid ₹5300 in total at the end of 5 months. Let the admission fee be ₹x and the monthly fee be ₹y.

CASE 2 (i)

Form the pair of linear equations.

1 Karthik: **x + 2y = 2900** ...(1). Meera: **x + 5y = 5300** ...(2).

ANSWER: x + 2y = 2900 and x + 5y = 5300.

CASE 2 (ii)

Find the monthly training fee.

1 Subtract (1) from (2): 3y = 2400, so **y = 800**.

ANSWER: The monthly fee is ₹800.

CASE 2 (iii)

Find the one-time admission fee.

1 Put y = 800 in (1): x = 2900 − 1600 = **1300**.

ANSWER: The admission fee is ₹1300.

CASE 2 (iv)

What will a student pay in total for 8 months of training?

1 Total = x + 8y = 1300 + 8(800) = 1300 + 6400 = **7700**.

ANSWER: ₹7700 in total for 8 months.

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Case Study 3 — Furniture for the New Classroom

Case Study 3 — Chairs and Tables

A school is furnishing a new classroom. The bill for 3 chairs and 2 tables is ₹2450. Another bill for 2 chairs and 3 tables is ₹2550. Let one chair cost ₹x and one table cost ₹y.

CASE 3 (i)

Form the pair of linear equations.

1. First bill: **3x + 2y = 2450** ...(1). Second bill: **2x + 3y = 2550** ...(2).

ANSWER: 3x + 2y = 2450 and 2x + 3y = 2550.

CASE 3 (ii)

Find the cost of one chair.

1. Add (1) and (2): 5x + 5y = 5000, so x + y = 1000 ...(3). Subtract (1) from (2): −x + y = 100, so y − x = 100 ...(4).
   Why: Swapped coefficients (3,2)/(2,3) — add and subtract, as the Story taught.
2. Add (3) and (4): 2y = 1100, so y = 550. From (3): x = 1000 − 550 = **450**.

ANSWER: One chair costs ₹450.

CASE 3 (iii)

Find the cost of one table.

1. From the working above, **y = 550**. Check: 3(450) + 2(550) = 1350 + 1100 = 2450; 2(450) + 3(550) = 900 + 1650 = 2550.

ANSWER: One table costs ₹550.

CASE 3 (iv)

The school later buys 4 chairs and 4 tables. Find the bill.

1. 4x + 4y = 4(x + y) = 4(1000) = **4000** — equation (3) answers this directly, without separate prices!
   Why: Sometimes the question asks for a COMBINATION you already own. Look before you compute.

ANSWER: The bill is ₹4000.

Case Study 4 — The Parking Lot

Case Study 4 — Parking Charges

A mall parking charges a car ₹x and a bike ₹y for the day. On Monday, 3 cars and 5 bikes paid ₹200 in total. On Tuesday, 5 cars and 3 bikes paid ₹280 in total.

CASE 4 (i)

Form the pair of linear equations.

1. Monday: **3x + 5y = 200** ...(1). Tuesday: **5x + 3y = 280** ...(2).

ANSWER: 3x + 5y = 200 and 5x + 3y = 280.

CASE 4 (ii)

Find the charge for one car.

1. Add: 8x + 8y = 480, so x + y = 60 ...(3). Subtract (1) from (2): 2x − 2y = 80, so x − y = 40 ...(4).
2. Add (3) and (4): 2x = 100, so **x = 50**.

ANSWER: One car is charged ₹50.

CASE 4 (iii)

Find the charge for one bike.

1. From (3): y = 60 − 50 = **10**. Check: 3(50) + 5(10) = 200; 5(50) + 3(10) = 280.

ANSWER: One bike is charged ₹10.

CASE 4 (iv)

A family arrives in 2 cars with 4 bikes. What do they pay?

1. 2x + 4y = 2(50) + 4(10) = 100 + 40 = **140**.

ANSWER: They pay ₹140.

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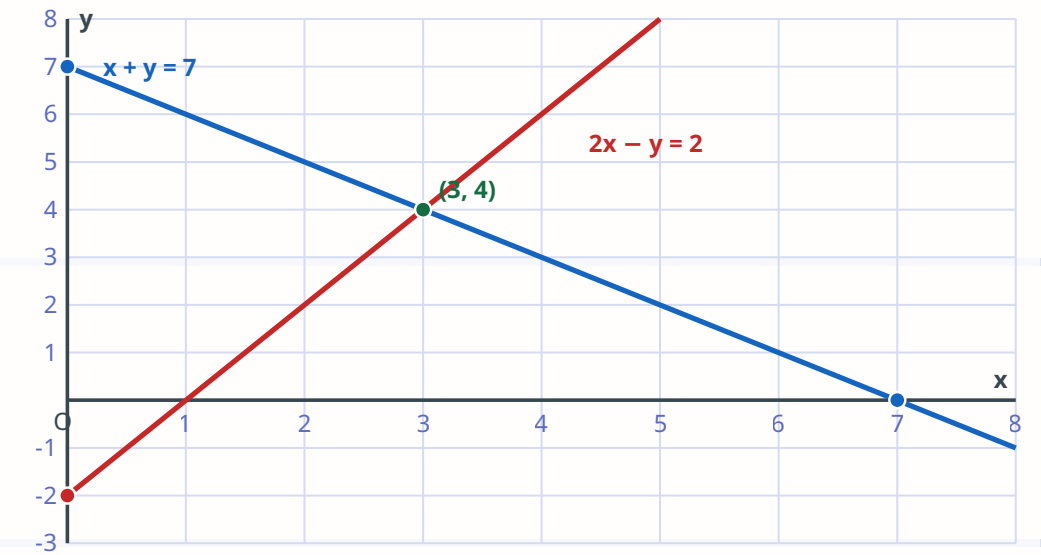
Case Study 5 — Reading a Graph (Visual Case)

Case Study 5 — Two Lines in a Weather Chart

A Class 10 student plotted two straight lines on graph paper while studying this chapter: line L_1 passes through (0, 7) and (7, 0); line L_2 passes through (0, −2) and (1, 0). The lines intersect at (3, 4), as shown.



CASE 5 (i)

Write the equation of line L_1.

1

L_1 joins (0, 7) and (7, 0): at every point of it, the coordinates add to 7 — check (0, 7): 0 + 7 = 7; (7, 0): 7 + 0 = 7. So L_1 is **x + y = 7**.

Why: When intercepts are this friendly, the equation often reads itself off the axis crossings.

ANSWER: L_1: x + y = 7.

CASE 5 (ii)

Write the equation of line L_2.

1

Test the candidate 2x − y = 2 at both known points: (0, −2): 0 + 2 = 2; (1, 0): 2 − 0 = 2. So L_2 is **2x − y = 2**.

Why: Verifying a guessed equation at TWO points proves it — one point could be luck, two points fix a line.

ANSWER: L_2: 2x − y = 2.

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Case Study 5 — Reading a Graph (continued)

CASE 5 (iii)

State the solution of the pair, and verify it algebraically.

1 From the graph, the lines meet at **(3, 4)**. Verify: $3 + 4 = 7$ and $2(3) - 4 = 2$.

ANSWER: The solution is x = 3, y = 4.

CASE 5 (iv)

Is the pair consistent? Give one more point that lies on L_1 only.

1 The lines intersect, so the pair is **consistent** with a unique solution. Any point with coordinates summing to 7 but not satisfying $2x - y = 2$ lies on L_1 only — for example **(2, 5)**: $2 + 5 = 7$, but $2(2) - 5 = -1 \neq 2$.

ANSWER: Consistent. Example point on L_1 only: (2, 5).

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Assertion–Reason Questions 1–10

For each question choose: **(a)** Both A and R are true and R is the correct explanation of A. **(b)** Both A and R are true but R is NOT the correct explanation of A. **(c)** A is true but R is false. **(d)** A is false but R is true. Answers with reasons are in the Answer Key.

AR 1. Assertion (A): The pair $2x + 3y = 5$ and $4x + 6y = 10$ has infinitely many solutions. **Reason (R):** For this pair, $a_1/a_2 = b_1/b_2 = c_1/c_2$.

AR 2. Assertion (A): The pair $x + y = 4$ and $x - y = 2$ has a unique solution. **Reason (R):** The lines representing the pair are parallel.

AR 3. Assertion (A): The pair $3x + 2y = 8$ and $6x + 4y = 20$ has no solution. **Reason (R):** Here $a_1/a_2 = b_1/b_2 \neq c_1/c_2$.

AR 4. Assertion (A): If a pair of linear equations is consistent, then its lines always intersect. **Reason (R):** Coincident lines also form a consistent pair.

AR 5. Assertion (A): (3, 2) is a solution of $x + y = 5$ and $2x - y = 4$. **Reason (R):** A point that satisfies both equations lies at the intersection of their lines.

AR 6. Assertion (A): $kx + 2y = 5$ and $3x + y = 1$ have a unique solution for every value of k. **Reason (R):** The pair has a unique solution exactly when $k \neq 6$.

AR 7. Assertion (A): The graph of every linear equation in two variables is a straight line. **Reason (R):** All solutions of $ax + by + c = 0$, when plotted, lie on one straight line.

AR 8. Assertion (A): The pair $5x - 3y = 11$ and $-10x + 6y = -22$ represents coincident lines. **Reason (R):** The second equation is exactly -2 times the first.

AR 9. Assertion (A): The substitution method cannot be applied to $3x - y = 3$ and $9x - 3y = 9$. **Reason (R):** This pair has infinitely many solutions.

AR 10. Assertion (A): The pair $x = 5$ and $y = 3$ has exactly one solution. **Reason (R):** A vertical line and a horizontal line intersect at exactly one point.

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15 Olympiad & Foundation Challenges

8D • CHALLENGE — TEN PROBLEMS FOR STUDENTS WHO THINK DEEP

OLYMPIAD 1

Solve: 6x + 5y = 37 and 5x + 6y = 40.

1. Add: 11x + 11y = 77, so **x + y = 7** ...(3). Subtract the second from the first: **x − y = −3** ...(4).
Why: Swapped coefficients (6,5)/(5,6) — by now your hand should reach for add-subtract before your brain finishes reading.

2. Add (3) and (4): 2x = 4, so **x = 2**; from (3), **y = 5**. Check: 12 + 25 = 37; 10 + 30 = 40.

ANSWER: x = 2, y = 5.

OLYMPIAD 2

The sum of the digits of a two-digit number is 11. The number obtained by reversing the digits is 27 less than the original number. Find the number.

1. Let the tens digit be x and the units digit be y. Digits: **x + y = 11** ...(1). Original number 10x + y; reversed 10y + x.

2. Reversed = original − 27: 10y + x = 10x + y − 27, so 9y − 9x = −27, i.e. **x − y = 3** ...(2).
Why: Reversing changes a number by 9 × (difference of digits) — a jewel worth remembering: (10x + y) − (10y + x) = 9(x − y).

3. Add (1) and (2): 2x = 14, so x = 7; then y = 4. The number is **74**. Check: 7 + 4 = 11; reversed 47, and 74 − 47 = 27.

ANSWER: The number is 74.

OLYMPIAD 3

A chemist has a 20% acid solution and a 50% acid solution. How many litres of each must she mix to obtain 10 litres of a 35% acid solution?

1. Let x litres of the 20% solution and y litres of the 50% solution be mixed. Total volume: **x + y = 10** ...(1).

2. Count the pure acid: 0.20x + 0.50y must equal 35% of 10 = 3.5 litres. So **0.2x + 0.5y = 3.5**, i.e. 2x + 5y = 35 ...(2).
Why: In mixture problems, write one equation for the total amount and one for the precious ingredient — here, the acid itself.

3. From (1): x = 10 − y. In (2): 2(10 − y) + 5y = 35, so 3y = 15 and **y = 5**; then **x = 5**. Check acid: 0.2(5) + 0.5(5) = 1 + 2.5 = 3.5.

Olympiad Challenges 4–6

OLYMPIAD 4

Solve: 49x + 51y = 499 and 51x + 49y = 501.

1. Add: 100x + 100y = 1000, so **x + y = 10** ...(3). Subtract the first from the second: 2x − 2y = 2, so **x − y = 1** ...(4).

2. Add (3) and (4): 2x = 11, so **x = 11/2**; from (3), **y = 9/2**. Check: 49(11/2) + 51(9/2) = (539 + 459)/2 = 998/2 = 499.
Why: Fractional answers in an olympiad problem are not a warning sign — they are often the design.

ANSWER: x = 11/2, y = 9/2.

OLYMPIAD 5

The sum of the digits of a two-digit number is 7. If 27 is added to the number, the digits are reversed. Find the number.

1. Let the digits be x (tens) and y (units): **x + y = 7** ...(1). The number is 10x + y; adding 27 gives the reversed number 10y + x.

2. 10x + y + 27 = 10y + x, so 9x − 9y = −27, i.e. **y − x = 3** ...(2).
Why: Adding a multiple of 9 reverses or preserves digit order in a controlled way — the 9(x − y) jewel from Olympiad 2 again.

3. Add (1) and (2): 2y = 10, so y = 5; then x = 2. The number is **25**. Check: 2 + 5 = 7; 25 + 27 = 52, the reverse.

ANSWER: The number is 25.

OLYMPIAD 6

Solve for x and y: 3/x + 2/y = 12 and 1/x + 1/y = 5 (x, y ≠ 0).

1. Set u = 1/x and v = 1/y. The pair becomes **3u + 2v = 12** ...(1) and **u + v = 5** ...(2) — an ordinary linear pair in disguise.
Why: Substitution of VARIABLES (not just expressions) is a higher form of the substitution method: rename the awkward objects until the problem looks familiar.

2. From (2): u = 5 − v. In (1): 3(5 − v) + 2v = 12, so 15 − v = 12 and **v = 3**; then **u = 2**.

3. Translate back: x = 1/u = **1/2** and y = 1/v = **1/3**. Check: 3/(1/2) + 2/(1/3) = 6 + 6 = 12; 1/(1/2) + 1/(1/3) = 2 + 3 = 5.
Why: Never forget the return journey: the question asked for x and y, not for u and v.

Olympiad Challenges 7–8

OLYMPIAD 7

Ten years ago, a father was four times as old as his son. Ten years from now, he will be twice as old as his son. Find their present ages.

1. Let the father's present age be x and the son's be y. Ten years ago: x − 10 = 4(y − 10), so **x − 4y = −30** ...(1).
2. Ten years hence: x + 10 = 2(y + 10), so **x − 2y = 10** ...(2).
3. Subtract (1) from (2): 2y = 40, so **y = 20**. From (2): x = 10 + 2(20) = **50**.
   Why: The x-coefficients were already equal — elimination asked for a single subtraction.
4. Check: ten years ago, 40 and 10 — 40 = 4 × 10. Ten years hence, 60 and 30 — 60 = 2 × 30.

ANSWER: The father is 50 years old and the son is 20 years old.

Olympiad Challenge 8 — Discussing a Parameter

OLYMPIAD 8

Discuss the nature of the pair 2x + 3y = 5 and 4x + ky = 10 for different values of k: for which k is the solution unique, and for which k are there infinitely many? Can the pair ever have no solution?

1. Compare ratios: $a_1/a_2 = 2/4 = 1/2$ and $c_1/c_2 = -5/-10 = 1/2$ — these two never change with k. Only $b_1/b_2 = 3/k$ moves.
   Why: A 'discuss' question is answered by cases. Fix what is fixed; watch what moves.

2. Case k = 6: $b_1/b_2 = 3/6 = 1/2$. All three ratios equal → coincident lines → **infinitely many solutions**.
   Why: Indeed at k = 6 the second equation is exactly twice the first.

3. Case k ≠ 6: $b_1/b_2 \neq 1/2 = a_1/a_2$ → intersecting lines → **unique solution**.

4. No solution would need $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ — but a_1/a_2 and c_1/c_2 are BOTH permanently 1/2, so the '≠' can never happen. **No value of k gives 'no solution'.**
   Why: Sometimes the deepest answer in mathematics is 'that case is impossible' — with a reason.

ANSWER: Unique solution for every k ≠ 6; infinitely many solutions for k = 6; no solution for no value of k.

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Olympiad Challenges 9–10

OLYMPIAD 9

Two cars start at the same time from towns A and B, 300 km apart, and travel towards each other. They meet after 3 hours. If the faster car travels 20 km/h faster than the slower one, find the speed of each car.

1. Let the speeds be x km/h (faster) and y km/h (slower). Moving towards each other, their speeds ADD: in 3 hours together they cover 3(x + y) = 300, so **x + y = 100** ...(1).
Why: For objects moving towards each other, the gap closes at the SUM of the speeds — the physical picture writes the equation.
2. Speed difference: **x − y = 20** ...(2). Add (1) and (2): 2x = 120, so **x = 60**; then **y = 40**.
3. Check: in 3 hours the cars cover 180 km and 120 km — total 300 km , and 60 − 40 = 20.

ANSWER: The faster car travels at 60 km/h and the slower at 40 km/h.

OLYMPIAD 10

If the length of a rectangle is increased by 3 m and its width decreased by 2 m, its area decreases by 8 m². If the length is decreased by 2 m and the width increased by 3 m, its area increases by 12 m². Find the length and width.

1. Let the length be l m and the width be w m; the area is lw. First change: (l + 3)(w − 2) = lw − 8. Expand the left side: lw − 2l + 3w − 6 = lw − 8. The lw cancels! We get **−2l + 3w = −2** ... (1).
Why: An area problem that becomes LINEAR — the product terms cancel because both sides contain the same lw. Expansion is the key that unlocks it.
2. Second change: (l − 2)(w + 3) = lw + 12. Expand: lw + 3l − 2w − 6 = lw + 12, so **3l − 2w = 18** ...(2).
3. Eliminate l: (1) × 3 gives −6l + 9w = −6; (2) × 2 gives 6l − 4w = 36. Add: 5w = 30, so **w = 6**.
4. Put w = 6 in (1): −2l + 18 = −2, so **l = 10**. Check areas: original 10 × 6 = 60. First change: 13 × 4 = 52 = 60 − 8. Second change: 8 × 9 = 72 = 60 + 12.

ANSWER: The rectangle is 10 m long and 6 m wide.

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Find the Error — 10 Broken Solutions (Part 1)

Each solution below contains exactly one error. Your job: find it, name it, and write the corrected step. Answers are discussed right under each item — cover them while you hunt!

FIND THE ERROR 1. Solve $x + y = 7$ and $x - y = 3$. Shown: adding gives $2x = 10$, so $x = 5$. Then from $x + y = 7$, $y = 5 - 7 = -2$. Answer $(5, -2)$.

SHOWN SOLUTION: $y = 5 - 7 = -2$ — the rearrangement was done in the wrong direction.

The error: From $x + y = 7$ we get $y = 7 - x = 7 - 5 = \mathbf{2}$, not $x - 7$. Correct answer: **(5, 2)**. When you rearrange, the term that moves changes side AND the variable stays on its own side: $y = 7 - x$.

FIND THE ERROR 2. Classify $2x - 3y = 8$ and $4x - 6y = 9$. Shown: $a_1/a_2 = 1/2$, $b_1/b_2 = -3/6 = -1/2$. Since the ratios differ, the lines intersect.

SHOWN SOLUTION: b_1/b_2 was computed as $-3/6$ — but b_2 is -6, not 6.

The error: $b_1/b_2 = (-3)/(-6) = \mathbf{1/2}$. So $a_1/a_2 = b_1/b_2 = 1/2 \neq 8/9 = c_1/c_2$ (with signs, $-8/-9$) — the lines are **parallel**; the pair is inconsistent. Copy coefficients WITH their signs.

FIND THE ERROR 3. Solve $x + y = 5$ and $2x - 3y = 4$ by elimination. Shown: multiply the first equation by 3: $3x + y = 15$.

SHOWN SOLUTION: Only two of the three terms were multiplied — the y-term was left behind.

The error: Multiplying an equation multiplies EVERY term: $3x + \mathbf{3y} = 15$. Adding $2x - 3y = 4$ then gives $5x = 19$, $x = 19/5$, $y = 6/5$ (Exercise 3.3 Q1(i)).

FIND THE ERROR 4. Ages: 'Five years ago, a father was 7 times as old as his son.' Shown: the equation is $x - 5 = 7y$.

SHOWN SOLUTION: Only the father travelled back in time; the son stayed in the present.

The error: Five years ago the son was $y - 5$, so the equation is $x - 5 = 7(\mathbf{y - 5})$. In age problems, time moves every person in the story.

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Find the Error — Part 2

FIND THE ERROR 5. Digits: 'The sum of the digits of a two-digit number is 9.' Shown: the number is x + y, so x + y = 9 is also the number's value equation; with the second condition the student writes 9(9) = 2(9) and gets stuck.

SHOWN SOLUTION: x + y is the SUM of the digits, not the number. The number's value was never written down.

The error: If the digits are x (tens) and y (units), the number is **10x + y** and its reverse is 10y + x. The second condition becomes 9(10x + y) = 2(10y + x), leading to the number 18 (Exercise 3.3 Q2(iii)).

FIND THE ERROR 6. Solve x + y = 14 and x − y = 4 by substitution. Shown: y = 14 − x; then x − (14 − x) = x − 14 − x = −14 = 4, which is false, so the pair has no solution.

SHOWN SOLUTION: The bracket was opened wrongly: −(14 − x) became −14 − x.

The error: Minus in front of a bracket flips EVERY sign inside: x − (14 − x) = x − 14 **+ x** = 2x − 14 = 4, so x = 9 and y = 5. A 'false' result from substitution usually means a sign error, not an inconsistent pair.

FIND THE ERROR 7. Library: Saritha kept a book for 7 days and paid ₹27. Shown: x + 7y = 27, where x is the fixed charge for the first three days and y the daily charge.

SHOWN SOLUTION: The daily rate was charged for all 7 days, double-charging the first three days that the fixed charge already covers.

The error: Only 7 − 3 = 4 days are extra: **x + 4y = 27**. (Answer: fixed ₹15, daily ₹3 — Exercise 3.3 Q2(v).)

FIND THE ERROR 8. Graph of x + y = 5. Shown table: x = 0 gives y = 0; x = 5 gives y = 5; the student plots (0, 0) and (5, 5) and draws the line through them.

SHOWN SOLUTION: Both table entries are wrong — neither point even satisfies x + y = 5 (0 + 0 = 0; 5 + 5 = 10).

The error: y = 5 − x: x = 0 → y = **5**; x = 5 → y = **0**. Points (0, 5) and (5, 0). Test every table entry IN the equation before plotting — a 5-second habit.

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Find the Error — Part 3

FIND THE ERROR 9. Elimination: subtract 9x − 2y = 7 from 9x − 15y = 12. Shown: (9x − 15y) − (9x − 2y) = −17y = 5.

SHOWN SOLUTION: −15y − (−2y) was written as −17y — the double negative was added instead of flipped.

The error: −15y + 2y = **−13y**, so −13y = 5 and y = −5/13 (Exercise 3.3 Q1(iii)). Subtracting a negative term is the single most expensive sign error in this chapter.

FIND THE ERROR 10. Solve 3x − y = 3 and 9x − 3y = 9. Shown: substitution gives 9 = 9; the student concludes 'the equations cancel out, so there is NO solution.'

SHOWN SOLUTION: The conclusion is backwards. 9 = 9 is an identity — true for every x — not a contradiction like 9 = 5.

The error: An identity after substitution means the second equation carried no new information (it is 3 × the first): the pair has **infinitely many solutions**. A FALSE statement (0 = 5) would mean no solution; a TRUE statement (0 = 0) means infinitely many.

TOPPER TIP: If you found 8 or more errors unaided, your sign-and-sense checking is exam-ready. Note the ones that fooled you in your Error Tracker (Section 18).

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Previous Year Questions — Real CBSE Board Questions

These questions are reproduced from actual CBSE Class 10 board papers, with the year and paper identified wherever verified. Solve them exactly as you would in the hall — then check the worked solutions.

CBSE 2023 • MAIN EXAM, BASIC (430/4/3, Q1) • 1 MARK MCQ

The value of k for which the equations 3x − y + 8 = 0 and 6x − ky + 16 = 0 represent coincident lines is: (A) 1/2 (B) −1/2 (C) 2 (D) −2.

THINK FIRST: Coincident means all three ratios equal. The a-ratio and c-ratio are fixed numbers — they will tell us what the b-ratio must be.

1. $a_1/a_2 = 3/6 = 1/2$ and $c_1/c_2 = 8/16 = 1/2$.
   Why: Read c from the +8 and +16 already sitting in standard form — no sign work needed here.
2. Coincidence needs $b_1/b_2 = 1/2$ as well: $-1/-k = 1/2$, so $1/k = 1/2$ and **k = 2**.

ANSWER: Option (C): k = 2.

CBSE-STYLE MARKING SCHEME: 1 mark for the correct option.

CBSE 2023 • COMPARTMENT, STANDARD (30/C/1, Q4) • 1 MARK MCQ

The condition for which the pair of equations ax + 2y = 7 and 3x + by = 16 represent parallel lines is: (A) ab = 7/16 (B) ab = 6 (C) ab = 3 (D) ab = 2.

THINK FIRST: Parallel begins with $a_1/a_2 = b_1/b_2$. Both unknowns live in that one equation.

1. Parallel lines: a/3 = 2/b. Cross-multiply: **ab = 6**.
   Why: Cross-multiplication: a · b = 3 · 2. (The c-ratio 7/16 ≠ a/3 automatically since ab = 6 forces a/3 = 2/b and the constants are not in that proportion — the board's condition is the standard one.)

ANSWER: Option (B): ab = 6.

CBSE-STYLE MARKING SCHEME: 1 mark for the correct option.

Previous Year Questions — MCQ 3: The 'Not Equal To' Pattern

CBSE 2024 • MAIN EXAM, BASIC (430/2/3, Q14) • 1 MARK MCQ

The value of p for which the pair of equations $-2x + 3y - 9 = 0$ and $4x + py + 7 = 0$ has a unique solution is: (A) $p \neq 6$ (B) $p = 6$ (C) $p = -6$ (D) $p \neq -6$.

THINK FIRST: Unique solution needs $a_1/a_2 \neq b_1/b_2$ — an inequality, so the answer will be a 'not equal to' statement.

1 $-2/4 \neq 3/p$, i.e. $-1/2 \neq 3/p$. Cross-multiplying the equality case: $p = 3 \div (-1/2) = -6$ would BREAK uniqueness, so uniqueness holds exactly when **$p \neq -6$**.

Why: Solve the forbidden case first (where the ratios WOULD be equal), then exclude it — the standard way to handle every '≠' condition.

ANSWER: Option (D): $p \neq -6$.

CBSE-STYLE MARKING SCHEME: 1 mark for the correct option.

Previous Year Questions — Short Answer

CBSE 2023 • MAIN EXAM, BASIC (430/1/1, Q24)

Solve for x and y: x + y = 6 and 2x − 3y = 4.

THINK FIRST: Substitution from the first equation — and do not be surprised by fractions; this board answer is fractional by design.

1. From (1): **y = 6 − x** ...(3). Substitute in (2): 2x − 3(6 − x) = 4, so 2x − 18 + 3x = 4, giving 5x = 22 and **x = 22/5**.
2. From (3): y = 6 − 22/5 = **8/5**. Verify: 22/5 + 8/5 = 30/5 = 6; 2(22/5) − 3(8/5) = (44 − 24)/5 = 4.
   Why: This is the question Section 04 warned about: no graph could ever give you 22/5 exactly. Algebra earns its keep.

ANSWER: x = 22/5, y = 8/5.

CBSE-STYLE MARKING SCHEME: 1 mark for the one-variable equation • 1 mark for both values (2 marks).

CBSE 2024 • MAIN EXAM, STANDARD (30/1/1, Q21)

Solve the following system of linear equations: 7x − 2y = 5 and 8x + 7y = 15, and verify your answer.

THINK FIRST: The word 'verify' is printed in the question — the verification is part of the answer, not an optional extra.

1. (1) × 7: 49x − 14y = 35. (2) × 2: 16x + 14y = 30. Add: 65x = 65, so **x = 1**.
   Why: Eliminate y: coefficients 2 and 7 become 14 and 14 with opposite signs.
2. Put x = 1 in (1): 7 − 2y = 5, so **y = 1**.
3. Verify in (1): 7(1) − 2(1) = 5. Verify in (2): 8(1) + 7(1) = 15.
   Why: Both originals checked, as ordered.

ANSWER: x = 1, y = 1 — verified in both equations.

CBSE-STYLE MARKING SCHEME: 1 mark for elimination • ½ mark for x • ½ mark for y and verification (2 marks).

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Previous Year Questions — The Swapped-Coefficient Specials

CBSE 2024 • COMPARTMENT, STANDARD (30/S/1, Q30)

Find the values of x and y from the following pair of linear equations: 62x + 43y = 167 and 43x + 62y = 148.

THINK FIRST: 62 and 43 are swapped between the equations — CBSE is testing whether you see the add–subtract door instead of the multiplication wall.

1. Add: 105x + 105y = 315. Divide by 105: **x + y = 3** ...(3).
2. Subtract the second from the first: 19x − 19y = 19. Divide by 19: **x − y = 1** ...(4).
3. Add (3) and (4): 2x = 4, so **x = 2**; from (3), **y = 1**. Verify: 62(2) + 43(1) = 124 + 43 = 167; 43(2) + 62(1) = 86 + 62 = 148.
   Why: Ninety seconds with structure, versus ten minutes with 62 × 43. The board rewards the eye, not the arm.

ANSWER: x = 2, y = 1.

CBSE-STYLE MARKING SCHEME: 1 mark for forming x + y and x − y • 1 mark for both values (2–3 marks).

CBSE 2023 • COMPARTMENT, BASIC (430/C/2, Q33)

36 pens and 24 pencils together cost ₹780, while 24 pens and 36 pencils together cost ₹720. Find the cost of one pen and of one pencil.

1. Let a pen cost ₹x and a pencil ₹y: **36x + 24y = 780** ...(1) and **24x + 36y = 720** ...(2). Divide both by 12: **3x + 2y = 65** ...(1′) and **2x + 3y = 60** ...(2′).
   Why: Divide FIRST — both equations share the factor 12. Small numbers are a gift; accept gifts.
2. Add (1′) and (2′): 5x + 5y = 125, so x + y = 25 ...(3). Subtract (2′) from (1′): x − y = 5 ...(4).
3. Add (3) and (4): 2x = 30, so **x = 15**; then **y = 10**. Check: 36(15) + 24(10) = 540 + 240 = 780; 24(15) + 36(10) = 360 + 360 = 720.

ANSWER: One pen costs ₹15 and one pencil costs ₹10.

CBSE-STYLE MARKING SCHEME: 1 mark for the equations • 1 mark for simplifying/solving • 1 mark for both prices (3 marks).

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Previous Year Questions — The Famous k

CBSE BOARD EXAM QUESTION (asked in the board examination; a perennial favourite)

For what value of k will the following pair of linear equations have infinitely many solutions? kx + 3y − (k − 3) = 0 and 12x + ky − k = 0.

THINK FIRST: This is the mirror image of Reasoning 1 (Section 12), where the same pair was asked for NO solution. Together they are the most repeated parameter question in CBSE history.

1. Infinitely many solutions need $a_1/a_2 = b_1/b_2 = c_1/c_2$: here that is $k/12 = 3/k = (k - 3)/k$.
   Why: The equations are already in standard form — $c_1 = -(k - 3)$ and $c_2 = -k$, and the two minus signs cancel inside the ratio.
2. From $k/12 = 3/k$: $k^2 = 36$, so $k = 6$ or $k = -6$.
3. Test both in $3/k = (k - 3)/k$, i.e. $3 = k - 3$ (for $k \neq 0$): this forces **k = 6**. ($k = -6$ fails: $3/-6 = -1/2$ but $(-6 - 3)/-6 = 3/2$.)
   Why: Both candidates must face the third ratio — the same lesson as Reasoning 1, from the other side.
4. Conclusion: for **k = 6**, all three ratios equal 1/2 and the lines coincide.

ANSWER: k = 6.

CBSE-STYLE MARKING SCHEME: 1 mark for the condition • 1 mark for $k^2 = 36$ • 1 mark for rejecting −6 via the third ratio (3 marks).

PIN POINT

Board pattern recognition: swapped coefficients (62, 43) → add & subtract. Parameter k in two places → ratios + test BOTH roots. The words 'and verify' → verification carries marks. CBSE repeats PATTERNS, not just questions.

16

Chapter Mind Map

THE WHOLE CHAPTER ON ONE PAGE — REVISE FROM HERE

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

$a_1x + b_1y + c_1 = 0$ • $a_2x + b_2y + c_2 = 0$ • a solution = a point on BOTH lines

THE THREE PICTURES

Intersecting → 1 solution → consistent
Parallel → 0 solutions → inconsistent
Coincident → infinitely many → consistent, dependent
Never exactly two — proved in Reasoning 3.

↔

THE RATIO TEST

$a_1/a_2 \neq b_1/b_2$ → unique
$a_1/a_2 = b_1/b_2 \neq c_1/c_2$ → none
all three equal → infinitely many
Derived from slopes AND from elimination (Sec. 06).
Read c from ax + by + c = 0!

↔

GRAPHICAL METHOD

Table of values (3 points per line) → plot → draw → read the meeting point.
Parallel → 'no solution'.
Coincident → list points of the line.

SUBSTITUTION

Isolate a variable (coefficient 1 is cheapest) → substitute into the OTHER equation → solve → back-substitute → verify.
Ends in 0 = 0? Infinitely many. Ends in 0 = 5? None.

↔

ELIMINATION

Scale to equal coefficients → add (opposite signs) or subtract (same signs) → solve → back-substitute → verify.
Swapped coefficients? Add & subtract directly.

↔

WORD PROBLEMS

Declare x, y → one sentence = one equation.
Ages: move everyone through time. Digits: 10x + y.
Fares/fees: fixed + rate × quantity.
Notes: one equation counts notes, one counts value.

The one-line summary

Two lines can only cross, run parallel, or coincide — the ratios tell you which, substitution and elimination find the point, and verification in BOTH original equations makes the answer unbreakable.

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Pin-Point Flash Cards — Cards 1–8

How to use these cards

Cut along the dashed borders and keep the 16 cards in your pocket or pencil box. Revise them on your Smart Memory days (Day 3, Day 7, Day 14, Day 21, Day 30): read the title, SAY the fact aloud from memory, then flip to check. Any card you cannot say instantly goes back into the 'today' pile. Ninety seconds a day — and the whole chapter lives in your head.

UNIQUE SOLUTION

$a_1/a_2 \neq b_1/b_2$ → the lines intersect at exactly ONE point.

NO SOLUTION

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$ → the lines are PARALLEL and never meet.

INFINITELY MANY SOLUTIONS

$a_1/a_2 = b_1/b_2 = c_1/c_2$ → the lines COINCIDE — one line drawn twice.

CONSISTENT

A pair with AT LEAST ONE solution: intersecting or coincident lines.

INCONSISTENT

A pair with NO solution: parallel, distinct lines.

DEPENDENT PAIR

A consistent pair with infinitely many solutions — the second equation repeats the first.

WHAT IS A SOLUTION?

A pair (x, y) satisfying BOTH equations — the point lying on BOTH lines.

EVERY EQUATION IS A LINE

The graph of ax + by + c = 0 is always a straight line; every solution is a point on it.

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Pin-Point Flash Cards — Cards 9–16

THE c-SIGN TRAP

Read c only from ax + by + c = 0. For 2x + 3y = 7, c = −7, never 7.

SUBSTITUTION CUE

Use it when a variable has coefficient 1 or −1 — isolate it without fractions.

ELIMINATION CUE

Make one variable's coefficients equal: ADD if signs are opposite, SUBTRACT if they are the same.

SWAPPED COEFFICIENTS?

ax + by = p, bx + ay = q → ADD for x + y, SUBTRACT for x − y. No big multiplications!

THE 0 = 0 / 0 = 5 RULE

Everything cancels? 0 = 0 → infinitely many solutions. 0 = 5 → no solution.

VERIFY — ALWAYS

Substitute your (x, y) into BOTH ORIGINAL equations. Ten seconds; unbreakable answers.

GRAPHICAL SOLUTION

Solution = coordinates of the intersection point. Your table of values carries marks — show 3 points per line.

WORD-PROBLEM FIRST STEP

Declare x and y in words, with units. One sentence of the story = one equation.

TOPPER TIP: *Make your own 17th card: the mistake YOU made most recently, written as a rule. Personal cards are remembered longest — that is why the Error Tracker exists.*

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17 Active Recall

8D • RECALL — CLOSE THE BOOK, THEN ANSWER

No peeking. Answer all fifteen in your notebook from memory alone — the struggle to remember IS the learning. Answers are in the Answer Key.

Rapid Questions (10)

R1. State the ratio conditions for (a) a unique solution, (b) no solution, (c) infinitely many solutions.

R2. Solve mentally: $x + y = 13$ and $x - y = 7$.

R3. Is (4, 3) a solution of $2x + y = 11$ and $x - y = 1$? How do you check?

R4. Classify the pair $5x + 2y = 9$ and $10x + 4y = 15$ without solving.

R5. Solve by substitution: $y = 3x$ and $x + y = 16$.

R6. Solve by elimination: $5x + 2y = 19$ and $5x - 2y = 11$.

R7. Find k if $x + 3y = 7$ and $2x + ky = 14$ are coincident.

R8. Two numbers differ by 5 and add up to 31. Find them.

R9. A taxi charges a fixed ₹40 plus ₹15 per km. Write the fare F for a ride of d km as an equation.

R10. The lines of a pair are parallel and distinct. How many solutions does the pair have? What is such a pair called?

Recall Prompts (5) — say each aloud, in full sentences

P1. Define a consistent pair and an inconsistent pair.

P2. Define a dependent pair. Give one example.

P3. When is substitution the smartest method? When is elimination?

P4. State the add–subtract trick: for which shape of pair does it work, and what does adding give?

P5. Write the general form of a pair of linear equations in two variables, and say what a 'solution' of the pair means geometrically.

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18 Self-Assessment & Mastery Tracker

KALVIYOGI SMART MEMORY SYSTEM — LEARN IT, THEN KEEP IT

Learning once is not learning forever. Your brain keeps what it revisits on a schedule. Follow this plan — each visit is short, and each has a named job.

Day	Your activity	Done
Day 0	Pre-learning assessment: attempt the Quick Check (Section 02) and Rapid Questions R1–R5 cold. Note your score honestly.	☐
Day 1	Learn the chapter (Sections 01–08) and complete Level 1–2 practice.	☐
Day 3	Without-book recall: Flash Cards 1–16 aloud + Active Recall (Section 17).	☐
Day 7	Independent problem-solving: **Day 7 Mini-Test** (next pages) + Level 3 practice.	☐
Day 14	Mixed practice: Model Test A + Find-the-Error re-check + Flash Cards.	☐
Day 21	Long-term recall test: **Day 21 Mini-Test** + Model Test B.	☐
Day 30	Chapter mastery assessment: **Day 30 Mini-Test** + Model Test C + PYQ set, timed.	☐

Error Tracker — every error is a lesson with your name on it

Question / source	What went wrong	The rule I now follow

Method Recall Tracker — can you still do it without the book?

☐ Ratio test (all three cases) ☐ Graphical method with table ☐ Substitution ☐ Elimination ☐ Add–subtract trick ☐ Ages ☐ Digits ☐ Fares & fees ☐ Notes & value ☐ Verification habit

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Mastery Tracker — Concept Scores and Learning Time

Concept Mastery Score — rate yourself 0 (lost), 1 (shaky), 2 (solid) after each test day

Concept	Day 7	Day 21	Day 30
Forming equations from stories			
Graphical method & tables of values			
Ratio test & consistency			
Substitution method			
Elimination method			
Word problems: ages, digits, fares, notes			
Parameter (k) questions			

Mastery target: every row at 2 by Day 30. Any row still at 0 or 1 → revisit that section, redo its NCERT questions, and re-test after two days.

Learning Time Tracker

Session	Minutes	Session	Minutes
Learning (Day 1)		Day 14 mixed practice	
Day 3 recall		Day 21 test	
Day 7 test		Day 30 mastery	

TOPPER TIP: *Your Day 7, Day 21 and Day 30 Mini-Tests are printed on the next three pages. Sit them like real exams: timed (20 minutes each), no book, no phone. Answers are in the Answer Key.*

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Smart Memory Mini-Test — Day 7

Day 7 Mini-Test — 10 questions, 20 minutes, closed book. Full steps in your notebook; answers in the Answer Key.

1. Solve: $x + y = 9$ and $x - y = 5$.

2. Classify the pair $3x + 2y = 6$ and $6x + 4y = 12$ (intersecting / parallel / coincident) and state the number of solutions.

3. Solve by substitution: $y = 2x$ and $x + y = 12$.

4. Find k so that $2x + ky = 6$ and $4x + 6y = 14$ represent parallel lines.

5. Solve by elimination: $3x + 2y = 12$ and $x + 2y = 8$.

6. Two numbers add up to 50 and differ by 14. Find them.

7. Does the point (4, 1) satisfy BOTH $2x - y = 7$ and $x + y = 6$? Justify.

8. Write the x-intercept and y-intercept points of the line $2x + 3y = 6$.

9. Two supplementary angles differ by 40°. Find them.

10. Solve: $x + y = 5$ and $3x - y = 3$.

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Smart Memory Mini-Test — Day 21

Day 21 Mini-Test — 10 questions, 20 minutes, closed book. Full steps in your notebook; answers in the Answer Key.

1. Solve: $5x - 2y = 8$ and $3x + 2y = 8$.

2. Classify the pair $4x - 2y = 6$ and $6x - 3y = 9$.

3. A fraction becomes 5/9 if 2 is added to both the numerator and the denominator. If 1 is subtracted from the numerator alone, it becomes 2/7. Find the fraction.

4. Find k so that $kx + 2y = 4$ and $2x + y = 3$ have no solution.

5. Solve: $x + y = 11$ and $2x - y = 4$.

6. A mother is four times as old as her daughter. In 6 years, she will be three times as old. Find their present ages.

7. Solve: $0.5x + 0.5y = 2.5$ and $0.3x - 0.1y = 0.7$. (Clear the decimals first.)

8. Write down one inconsistent pair of linear equations and prove it is inconsistent using ratios.

9. The sum of the digits of a two-digit number is 13. The number is 27 more than the number obtained by reversing its digits. Find the number.

10. Solve: $4x + 3y = 24$ and $2x - 3y = -6$.

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Smart Memory Mini-Test — Day 30 (Mastery)

Day 30 Mini-Test — 10 questions, 20 minutes, closed book. Full steps in your notebook; answers in the Answer Key.

1. Solve: 5x + 3y = 21 and 2x + 3y = 12.

2. Classify the pair x − 2y = 3 and 3x − 6y = 12.

3. Solve: 8x + 5y = 44 and 3x + 5y = 29.

4. A taxi's fixed charge plus 6 km costs ₹92; the same fixed charge plus 10 km costs ₹132. Find the fixed charge and the per-km rate.

5. Find k so that 3x + ky = 9 and 6x + 6y = 18 represent coincident lines.

6. Solve: x/2 + y/3 = 8 and x − y = −9.

7. The perimeter of a rectangle is 44 m and its length exceeds its width by 4 m. Find both dimensions.

8. Solve using the add–subtract trick: 53x + 47y = 247 and 47x + 53y = 253.

9. Is (0, 5) the solution of x + y = 5 and 2x + y = 5? Is it the ONLY solution? Justify both answers.

10. The sum of two numbers is 96. The larger number is 8 more than three times the smaller. Find both numbers.

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Chapter Model Test A — Basics (20 Marks, 40 Minutes)

Section A — MCQs (1 mark each)

A1. The solution of $x + y = 4$ and $x - y = 2$ is (A) (3, 1) (B) (1, 3) (C) (2, 2) (D) (4, 0)

A2. The lines $2x + y = 5$ and $4x + 2y = 10$ are (A) parallel (B) coincident (C) intersecting (D) perpendicular

A3. A pair has a unique solution exactly when (A) $a_1/a_2 = b_1/b_2$ (B) $a_1/a_2 \neq b_1/b_2$ (C) all ratios are equal (D) $c_1 = c_2$

A4. The point (1, 2) is a solution of (A) $x + y = 4$ (B) $x - y = 2$ (C) $x + y = 3$ (D) $2x + y = 6$

Section B — Short answers (2 marks each)

B1. Solve: $3x + y = 11$ and $x - y = 1$.

B2. Classify the pair $5x - 2y = 8$ and $10x - 4y = 20$ using ratios.

B3. Solve by substitution: $x = 2y$ and $x + y = 9$.

Section C — (3 marks each)

C1. Solve graphically: $x + y = 5$ and $x - y = 1$. (Show tables of values.)

C2. The sum of two numbers is 64 and their difference is 18. Find the numbers.

Section D — (4 marks)

D1. Four chairs and three tables cost ₹3100. Two chairs and one table cost ₹1300. Find the cost of one chair and one table, solving by elimination.

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Chapter Model Test B — Standard (20 Marks, 40 Minutes)

Section A — MCQs (1 mark each)

A1. The pair x + ky = 4 and 2x + 6y = 8 is coincident when k equals (A) 2 (B) 3 (C) 4 (D) 6

A2. Which pair has no solution?
(A) x + y = 3; 2x + 2y = 6 (B) 3x + y = 7; 6x + 2y = 9 (C) x − y = 1; x + y = 2 (D) 2x − y = 4; x + y = 5

A3. The solution of 2x − y = 3 and x + y = 6 is (A) (2, 4) (B) (3, 3) (C) (4, 2) (D) (1, 5)

A4. A fare is ₹30 fixed plus ₹15 per km. The fare for 7 km is
(A) ₹105 (B) ₹120 (C) ₹135 (D) ₹150

Section B — Short answers (2 marks each)

B1. Solve by elimination: 4x + 3y = 19 and 2x − 3y = 5.

B2. Solve 3x + 2y = 13 and x + y = 5, and hence find m if y = mx + 1 passes through the solution point.

B3. Determine, with ratios, whether 6x − 3y = 12 and 2x − y = 4 are consistent. If yes, state the number of solutions.

Section C — (3 marks each)

C1. A fraction becomes 8/11 if 3 is added to both the numerator and the denominator. If 2 is subtracted from both, it becomes 1/2. Find the fraction.

C2. A is 27 years older than B. Ten years from now, A will be twice as old as B. Find their present ages.

Section D — Case (4 × 1 mark)

D1. A city bus charges a fixed boarding fee plus a per-km fare. A 5 km ride costs ₹65 and a 9 km ride costs ₹105. (i) Form the pair of equations, taking the fixed fee as ₹x and the per-km fare as ₹y. (ii) Find the per-km fare. (iii) Find the fixed fee. (iv) Find the fare for 15 km.

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Chapter Model Test C — Advanced (20 Marks, 40 Minutes)

Section A — MCQs (1 mark each)

A1. For the pair 6x + 5y = 37 and 5x + 6y = 40, the value of x − y is (A) 3 (B) −3 (C) 7 (D) −7

A2. Which pair has the solution (2, 3)?
(A) 3x + 2y = 11; x − y = 1 (B) 3x + 2y = 12; x − y = −1 (C) x + y = 6; 2x − y = 0 (D) 2x + 3y = 12; x + y = 4

A3. The pair 2x + 3y = 7 and 4x + 6y = k has infinitely many solutions when k equals
(A) 7 (B) 10 (C) 14 (D) 21

A4. If x + y = 10 and x − y = 4, then xy equals (A) 40 (B) 24 (C) 21 (D) 14

Section B — Short answers (2 marks each)

B1. Solve: 4/x + 3/y = 18 and 1/x − 1/y = 1 (x, y ≠ 0).

B2. Find k so that kx + 2y = 7 and 6x + 3y = 11 have no solution.

B3. Solve using the add–subtract trick: 37x + 43y = 197 and 43x + 37y = 203.

Section C — (3 marks each)

C1. The sum of a two-digit number and the number obtained by reversing its digits is 110. The digits differ by 2, and the tens digit is the larger one. Find the number.

C2. Show that the three lines x + y = 5, 2x − y = 4 and 3x + y = 11 are concurrent (pass through a single common point), and find that point.

Section D — Case (4 × 1 mark)

D1. A sweet maker mixes a 30% sugar syrup with a 60% sugar syrup to prepare 12 litres of 45% syrup. (i) Form the pair of equations for the litres x (of 30%) and y (of 60%). (ii) Find x. (iii) Find y. (iv) Verify the sugar content of your mixture.

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Answer Key — Graded Practice (Section II)

Level 1 — Foundation

L1.1 x = 5, y = 3. **L1.2** x = 4, y = 3. **L1.3** Parallel (1/2 = 1/2 ≠ 7/10 in c-ratio, using standard form −7/−10): no solution. **L1.4** x = 2, y = 1. **L1.5** 3(2) + k(1) = 7 → k = 1. **L1.6** Any equation with x:y pattern 1:2 but a different constant, e.g. 2x + 4y = 9 (ratios 1/2 = 2/4 ≠ −4/−9).

Level 2 — Skill Builder

L2.1 x = 3, y = 2. **L2.2** Add the equations: 7x = 28 → x = 4, y = 3. **L2.3** x = 3, y = 8 (clear fractions: 4x + 3y = 36 with x − y = −5). **L2.4** 50 and 24. **L2.5** x = 2, y = 4. **L2.6** Unique solution iff k/3 ≠ 2/1, i.e. all k ≠ 6.

Level 3 — Application

L3.1 Fixed fee ₹20; per-km rate ₹16; 20 km ride = 20 + 20(16) = ₹340. **L3.2** Father 36 years, son 12 years. **L3.3** Notebook ₹25, pen ₹10. **L3.4** Length 16 m, width 11 m.

Level 4 — Advanced Reasoning

L4.1 Add: 200(x + y) = 1000 → x + y = 5. Subtract: −2x + 2y = −2 → y − x = −1. Hence x = 3, y = 2. **L4.2** Coincidence needs 2/(k + 1) = 3/(2k − 1) = 7/(4k + 1). From the first and third: 2(4k + 1) = 7(k + 1) → k = 5 (check: 2/6 = 3/9 = 7/21 = 1/3). **L4.3** With u = 1/x, v = 1/y: 2u + 3v = 13, 5u − 4v = −2 → u = 2, v = 3 → x = 1/2, y = 1/3.

Level 5 — Mathematical Genius

L5.1 x = 3, y = 2. $x^2 - y^2 = (x - y)(x + y) = 1 \times 5 =$ **5**. $xy = ((x + y)^2 - (x - y)^2)/4 = (25 - 1)/4 =$ **6**. $x^3 - y^3 = (x - y)(x^2 + xy + y^2) = 1 \times (9 + 6 + 4) =$ **19** (with $x^2 + y^2 = (x + y)^2 - 2xy = 25 - 12 = 13$, so $x^2 + xy + y^2 = 13 + 6 = 19$). **L5.2** Digits x, y: 10x + y = 4(x + y) → y = 2x; reversed exceeds original by 27 → 9(y − x) = 27 → y − x = 3. Hence x = 3, y = 6 — the number is **36**.

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Answer Key — Concept MCQs (with one-line explanations)

MCQ 1. (B) — 3 + 2 = 5 and 3 − 2 = 1.

MCQ 2. (B) — Ratios 1/2, 1/2, but constants 7 vs 15 are not in ratio 1/2 — parallel.

MCQ 3. (B) — Unique solution ⇔ $a_1/a_2 \neq b_1/b_2$.

MCQ 4. (C) — The second equation is exactly twice the first — coincident.

MCQ 5. (B) — 3(2) + k(1) = 8 → k = 2.

MCQ 6. (B) — A vertical and a horizontal line meet at (3, −2).

MCQ 7. (C) — Unique iff k/3 ≠ 2, i.e. k ≠ 6 — every other k works.

MCQ 8. (A) — x = (35 + 13)/2 = 24, y = 11.

MCQ 9. (B) — Adding: 3x = 9 → x = 3 (then y = 4).

MCQ 10. (C) — Ratios 1/2, 1/2, 5/8 — parallel, no solution.

MCQ 11. (B) — Coincident + consistent = dependent.

MCQ 12. (B) — Adding gives 2x = 0; then y = 0.

MCQ 13. (A) — First × 3 gives 6y; second × 2 gives −6y — they cancel on adding.

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Answer Key — MCQs 14–25 and Assertion–Reason

MCQ 14. (B) — $6 + 3y = 12 \rightarrow y = 2$.

MCQ 15. (B) — y^2 makes $x + y^2 = 4$ non-linear.

MCQ 16. (C) — Every linear equation in two variables graphs as a straight line.

MCQ 17. (C) — $5/7 \neq -4/6$ — intersecting (Exercise 3.1 Q2(i)).

MCQ 18. (C) — $1/2 = 2/k \rightarrow k = 4$, and the c-ratio $5/10 = 1/2$ agrees.

MCQ 19. (C) — $x + x = 180 \rightarrow x = 90°$.

MCQ 20. (B) — $3/2 \neq 2/-3$ — intersecting, hence consistent.

MCQ 21. (D) — One equation, two unknowns: a whole line of solutions.

MCQ 22. (A) — x stands with coefficient 1 — isolate it directly.

MCQ 23. (C) — Tens digit × 10 + units digit.

MCQ 24. (B) — $20 + 12 \times 9 = 20 + 108 = 128$.

MCQ 25. (C) — Consistent = intersecting OR coincident.

Assertion–Reason (Section 14)

AR1 (a) — ratios are 1/2, 1/2, 1/2; R states exactly why A holds.

AR2 (c) — A is true ($1/1 \neq 1/-1$, solution (3, 1)); R is false — the lines intersect, they are not parallel.

AR3 (a) — $3/6 = 2/4 = 1/2 \neq 8/20 = 2/5$; R is the correct explanation.

AR4 (d) — A is false (coincident pairs are consistent without intersecting); R is true.

AR5 (a) — $3 + 2 = 5$ and $6 - 2 = 4$; R explains the geometry of it.

AR6 (d) — A is false: $k = 6$ destroys uniqueness. R is true.

AR7 (a) — R is precisely the reason A holds.

AR8 (a) — $-2 \times (5x - 3y = 11)$ gives $-10x + 6y = -22$ exactly; R explains A.

AR9 (d) — A is false: substitution works and reveals the identity $9 = 9$. R is true.

AR10 (a) — $x = 5$ (vertical) and $y = 3$ (horizontal) meet only at (5, 3); R explains A.

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Answer Key — Active Recall and Mini-Tests

Active Recall (Section 17)

R1 Unique: $a_1/a_2 \neq b_1/b_2$; none: $a_1/a_2 = b_1/b_2 \neq c_1/c_2$; infinitely many: all three equal. **R2** (10, 3). **R3** Yes — 2(4) + 3 = 11 and 4 − 3 = 1; check by substituting into both equations. **R4** Parallel: 5/10 = 2/4 = 1/2 ≠ 9/15 = 3/5 (c-ratio with signs: −9/−15). **R5** x = 4, y = 12. **R6** x = 3, y = 2. **R7** k = 6. **R8** 18 and 13. **R9** F = 40 + 15d. **R10** Zero solutions; the pair is inconsistent.

P1 Consistent: at least one solution exists. Inconsistent: none exists. **P2** Dependent: consistent with infinitely many solutions, e.g. x + y = 5 with 2x + 2y = 10. **P3** Substitution when a variable has coefficient ±1; elimination when coefficients are equal/opposite/easy multiples. **P4** For swapped coefficients ax + by = p, bx + ay = q: adding gives (a + b)(x + y) = p + q. **P5** $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$; a solution is a point lying on both lines.

Day 7 Mini-Test

1 (7, 2). **2** Coincident — infinitely many solutions. **3** x = 4, y = 8. **4** k = 3 (a-ratio 2/4 = 1/2; parallel needs k/6 = 1/2, and the c-ratio −6/−14 = 3/7 ≠ 1/2 confirms parallel, not coincident). **5** x = 2, y = 3. **6** 32 and 18. **7** No: 2(4) − 1 = 7 but 4 + 1 = 5 ≠ 6 — it satisfies only the first equation. **8** x-intercept (3, 0); y-intercept (0, 2). **9** 110° and 70°. **10** x = 2, y = 3.

Day 21 Mini-Test

1 x = 2, y = 1. **2** Coincident (all ratios 2/3). **3** 3/7. **4** k = 4 (k/2 = 2/1; c-ratio 4/3 ≠ 2 confirms 'no solution', not coincidence). **5** x = 5, y = 6. **6** Mother 48, daughter 12. **7** x = 3, y = 2. **8** Example: x + y = 4 and 2x + 2y = 10 — ratios 1/2 = 1/2 ≠ 4/10. **9** 85. **10** x = 3, y = 4.

Day 30 Mini-Test

1 x = 3, y = 2. **2** Parallel (1/3 = 1/3 ≠ 3/12). **3** x = 3, y = 4. **4** Fixed ₹32; ₹10 per km. **5** k = 3. **6** x = 6, y = 15. **7** Length 13 m, width 9 m. **8** x = 2, y = 3. **9** Yes, it satisfies both (0 + 5 = 5; 0 + 5 = 5); and it is the ONLY solution because the ratios 1/2 ≠ 1/1 show the lines intersect. **10** 74 and 22.

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Answer Key — Chapter Model Tests

Model Test A — Basics

A1 (A). **A2** (B). **A3** (B). **A4** (C). **B1** x = 3, y = 2. **B2** Ratios 1/2, 1/2, 2/5 — parallel (inconsistent). **B3** y = 3, x = 6. **C1** Lines meet at (3, 2): x = 3, y = 2. **C2** 41 and 23. **D1** Chair ₹400, table ₹500.

Model Test B — Standard

A1 (B) k = 3. **A2** (B) — ratios 1/2, 1/2, 7/9. **A3** (B). **A4** (C) ₹135. **B1** x = 4, y = 1. **B2** x = 3, y = 2; then 2 = 3m + 1 → m = 1/3. **B3** All ratios equal 3 — consistent, infinitely many solutions (coincident). **C1** 5/8. **C2** A is 44, B is 17. **D1** (i) x + 5y = 65, x + 9y = 105. (ii) ₹10 per km. (iii) Fixed ₹15. (iv) 15 + 15(10) = ₹165.

Model Test C — Advanced

A1 (B) −3. **A2** (B). **A3** (C) k = 14. **A4** (C) 21. **B1** u = 1/x = 3, v = 1/y = 2 → x = 1/3, y = 1/2. **B2** k = 4 (k/6 = 2/3; c-ratio 7/11 ≠ 2/3). **B3** x = 3, y = 2 (add: 80(x + y) = 400; subtract: −6x + 6y = −6). **C1** 64 (digits sum to 10 and differ by 2: 6 and 4). **C2** The first two lines meet at (3, 2), and 3(3) + 2 = 11, so the third line passes through the same point — concurrent at (3, 2). **D1** (i) x + y = 12 and 0.3x + 0.6y = 5.4. (ii) x = 6 litres. (iii) y = 6 litres. (iv) Sugar = 0.3(6) + 0.6(6) = 1.8 + 3.6 = 5.4 litres = 45% of 12.

TOPPER TIP: Scored below 14/20 on a Model Test? The test has told you exactly which section to revisit — that is not failure, that is a map.

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Revision Card — Chapter 3 on One Page

The pair and its three fates

$a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$.

Intersecting ($a_1/a_2 \neq b_1/b_2$): one solution, consistent.

Parallel ($a_1/a_2 = b_1/b_2 \neq c_1/c_2$): no solution, inconsistent.

Coincident (all ratios equal): infinitely many solutions, consistent and dependent.

The three methods

Graphical: table (3 points) → plot → read the meeting point.
Substitution: isolate (coefficient ±1 first) → substitute → solve → back-substitute.
Elimination: equalise coefficients → add/subtract → solve → back-substitute.
Add–subtract: swapped coefficients → add for x + y, subtract for x − y.
Always: verify in BOTH original equations.

Word-problem dictionary

Ages: past/future shifts apply to EVERY person. **Digits:** number = 10x + y; reverse = 10y + x; reversal changes the number by 9(x − y). **Fares & fees:** total = fixed + rate × quantity; watch for 'first three days free inside the fixed charge'. **Notes & coins:** one equation counts items, the other counts value. **Fractions:** cross-multiply each condition into a linear equation.

Traps that cost marks every year

c read without its sign • multiplying only some terms • −(a − b) opened wrongly • answering with only x • verifying in a modified equation • announcing both k-roots without testing them in the third ratio.

End of Chapter 3 — Pair of Linear Equations in Two Variables.

Next: Chapter 4 — Quadratic Equations. | 369 Tesla • KalviYogi • Edunivra Pvt Ltd

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CBSE • CLASS 10 • MATHEMATICS

MATHEMATICS BEYOND THE FORMULA

CHAPTER 4 • QUADRATIC EQUATIONS

Don't Memorize Mathematics. Understand, Visualise and Discover It.

The KalviYogi 8D Way: Discover • Visualise • Understand • Derive • Solve • Apply • Challenge • Recall

Every solution with COMPLETE steps — one by one, nothing skipped

Factorisation • Completing the Square • The Quadratic Formula — DERIVED by you

The Discriminant • Nature of Roots • Story Problems: Ages, Trains, Taps, Boats, Areas

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Written like a topper's own notebook — so you can learn

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01 Chapter Introduction

8D • DISCOVER — WHY SHOULD I LEARN THIS?

A ball thrown into the air does not travel in a straight line — it rises, pauses, and falls along a curve. A gardener fencing a rectangular plot, a train driver planning a journey, two taps filling a tank together — all of them are secretly solving the same kind of equation: a **quadratic equation**, an equation where the unknown appears with power 2.

In Class 9 you solved *linear* equations — one unknown, one answer. This chapter is the next level: an x^2 appears, and suddenly an equation can have **two answers, one answer, or no real answer at all**. Your job in this chapter is not just to find those answers — it is to understand **why** each method works, so deeply that you could rebuild the famous quadratic formula yourself if you ever forgot it. And in Section 06, you will do exactly that.

What you will master in this chapter

- Recognising a quadratic equation — and catching the impostors that only *look* quadratic.
- Turning real-life situations (areas, ages, trains, taps, boats) into quadratic equations.
- Solving by **factorisation** — splitting the middle term, step by step.
- Solving by **completing the square** — every rearrangement shown, nothing hidden.
- **Deriving** the quadratic formula from $ax^2 + bx + c = 0$ — then using it like a master key.
- The **discriminant** $D = b^2 - 4ac$ — predicting the nature of roots before solving.
- Finding k when roots are equal, and judging whether a given situation is even *possible*.

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Your Chapter Map — 18 Stops, One Journey

Tick each stop as you finish it. The journey is the same 8D path as every KalviYogi chapter.

Discover & Visualise	Understand & Derive	Solve, Apply, Recall
☐ 01 Chapter Introduction ☐ 02 Prerequisite Knowledge ☐ 03 Real-Life Story ☐ 04 Visual Concepts (CPA)	☐ 05 Mathematical Foundations ☐ 06 Formula Derivations ☐ 08 Multiple Solution Methods ☐ 10 Common Mistakes	☐ 07 Detailed Solved Examples ☐ 09 Hands-on Activities ☐ 11 Graded Practice (5 Levels) ☐ 12 Mathematical Reasoning
☐ 13 Real-Life Applications ☐ 14 Competency-Based Questions	☐ 15 Olympiad Challenges ☐ 16 Chapter Mind Map	☐ 17 Active Recall ☐ 18 Mastery Tracker

How to use this chapter (read this first!)

1. **Never read a solution passively.** Cover the steps, try yourself, then uncover one step at a time.
2. Every solution shows **Step 1, Step 2, Step 3...** with a small *Why* note in purple — the reason behind the step. Examiners award marks for these steps, not just the final answer.
3. In word problems, watch the moment where a root is **rejected** (a negative length, an impossible age). That rejection, with its reason, is worth a full mark.
4. After Section 07, test yourself in Sections 11–15 **before** opening the Answer Key.
5. Finish with the Mind Map (16), closed-book Recall (17) and your Mastery Tracker (18) — and return on Days 3, 7, 14, 21 and 30 as the tracker directs.

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02 Prerequisite Knowledge

CLASS 9 RECAP — FACTORISATION & SQUARE ROOTS

Quadratic equations stand on three Class 9 pillars: expanding brackets, factorising by splitting the middle term, and square roots. Rebuild them here.

1. The identities you must own

$(a + b)^2 = a^2 + 2ab + b^2$ • $(a - b)^2 = a^2 - 2ab + b^2$ • $a^2 - b^2 = (a + b)(a - b)$

Example: $(x + 5)^2 = x^2 + 10x + 25$. Example: $x^2 - 49 = (x + 7)(x - 7)$.

These two directions — expanding and factorising — are the engine of the whole chapter.

2. Splitting the middle term (the Class 9 superpower)

To factorise $x^2 + 7x + 12$: find two numbers whose **product is 12** (the constant) and whose **sum is 7** (the middle coefficient). The pair is 3 and 4.

$x^2 + 7x + 12 = x^2 + 3x + 4x + 12 = x(x + 3) + 4(x + 3) =$ **$(x + 3)(x + 4)$**.

For $ax^2 + bx + c$, the pair must have product **a × c** and sum **b**. Section 07 uses this hundreds of times — slowly, every time.

3. Square roots — and the ± rule

$\sqrt{144} = 12$, $\sqrt{225} = 15$, $\sqrt{625} = 25$, $\sqrt{1225} = 35$, $\sqrt{2025} = 45$, $\sqrt{4225} = 65$, $\sqrt{7225} = 85$.

But if $x^2 = 49$, then $x =$ **± 7** — BOTH 7 and -7 square to 49. Forgetting the ± is the single most common mark-loser in this chapter.

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Quick Check — Are You Ready? (5 Questions)

Try all five without looking back. Cover the answers with your hand first!

Q1. Factorise $x^2 + 9x + 20$ by splitting the middle term.

Q2. Expand $(x - 6)^2$.

Q3. Solve $x^2 = 81$.

Q4. Factorise $2x^2 + 11x + 12$ (hint: the pair has product 24 and sum 11).

Q5. Find $\sqrt{361}$ and $\sqrt{1024}$.

Answers — check only after trying

Q1. Pair with product 20, sum 9: 4 and 5. $x^2 + 4x + 5x + 20 = x(x + 4) + 5(x + 4) =$ **$(x + 4)(x + 5)$**.

Q2. $(x - 6)^2 = x^2 - 12x + 36$.

Q3. $x =$ **± 9** (both square to 81).

Q4. Pair 8 and 3: $2x^2 + 8x + 3x + 12 = 2x(x + 4) + 3(x + 4) =$ **$(x + 4)(2x + 3)$**.

Q5. $\sqrt{361} = 19$; $\sqrt{1024} = 32$.

TOPPER TIP: Got 4 or 5 right? You are fully ready. Got fewer? Practise splitting the middle term on five Class 9 examples first — Section 07 will feel twice as easy.

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03 Real-Life Mathematical Story

DISCOVER • VISUALISE — THE GARDEN THAT WOULD NOT FIT

Before any formula, live the problem. Read this story slowly — the equation will walk out of it by itself.

*Behind her grandmother's house in Madurai, **Meena** has been given a small rectangular patch of land to grow a vegetable garden. She has measured it carefully: the patch must have an area of exactly **24 square metres** — no more, no less — because that is all the land there is.*

*But the patch comes with a condition. The land sits beside a long wall, and the shape is fixed by the plot: **the length of the garden must be 5 metres more than its width**. Meena cannot choose the length and width freely — choose the width, and the length is decided for her.*

Meena starts guessing. Width 2 m? Then the length is 7 m, and the area is $2 \times 7 = 14$ m^2 — too small. Width 4 m? Length 9 m, area 36 m^2 — too big. Width 3.5 m? Length 8.5 m, area 29.75 m^2 — still not 24.

*She sits down with her notebook. "Every guess is a different width. Let me stop guessing numbers and give the width a name." She writes: **let the width be x metres**. Then the length, which is always 5 more, becomes **$x + 5$**. And the area — length × width — must equal 24.*

*One line of writing has replaced all her guessing: **$x(x + 5) = 24$**. Now the question is no longer "which number should I try?" but "how do I solve this?" — and that is exactly what this chapter teaches.*

Notice what Meena did: she turned **words into algebra**. That single move — name the unknown, translate the condition, write the equation — is the heart of every word problem in Sections 07, 13 and 14.

Solving the Story — Step by Step

STORY PROBLEM

A rectangular garden has area 24 m^2. Its length is 5 m more than its width. Find the length and the width.

THINK FIRST: Area + a relation between length and width always produces a quadratic equation. Watch for it in Section 07.

1. Let the width of the garden be **x** metres. Then the length = **x + 5** metres.
   Why: One unknown is enough, because the condition ties the length to the width.

2. Area of a rectangle = length × width, so $x(x + 5) = 24$.
   Why: This is the translation step: the story's condition becomes an equation.

3. Expand the bracket: $x^2 + 5x = 24$.
   Why: Multiply x by both terms inside the bracket.

4. Bring every term to one side: $x^2 + 5x - 24 = 0$.
   Why: A quadratic equation is always arranged with 0 on one side before solving.

5. Factorise by splitting the middle term. We need two numbers with product −24 and sum +5: they are **+8 and −3**. So $x^2 + 8x - 3x - 24 = 0$.
   Why: Product must be a × c = 1 × (−24) = −24; sum must be b = +5.

6. Group and factorise: $x(x + 8) - 3(x + 8) = (x + 8)(x - 3) = 0$.
   Why: Take x out of the first pair and −3 out of the second pair; the bracket (x + 8) is common.

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Solving the Story — The Last Steps and the Lesson

7 So either $x + 8 = 0$ or $x - 3 = 0$, giving $x = -8$ or $x = 3$.

Why: If a product of two factors is 0, at least one factor must be 0.

8 **Reject $x = -8$**: a width cannot be negative. So the width is $x =$ **3 m**, and the length = 3 + 5 = **8 m**.

Why: Both roots satisfy the equation, but only one satisfies the story. Always check roots against real life.

9 Final check: $3 \times 8 = 24\ m^2$. Correct!

Why: Ten seconds of checking protects the full answer mark.

ANSWER: Width = 3 m and Length = 8 m (Area = 24 m^2).

The story's three gifts — remember them

1. Name the unknown: "Let the width be x" turns a puzzle into algebra.
2. Translate the condition: "5 m more than" becomes x + 5; "area is 24" becomes x(x + 5) = 24.
3. Judge the roots: an equation may give two roots, but the story accepts only those that make sense. A negative width is mathematics, not a garden.

PIN POINT **ROOT REJECTION RULE: lengths, widths, speeds, ages and counts can never be negative. Reject such roots — and always write the reason. The reason carries a mark.**

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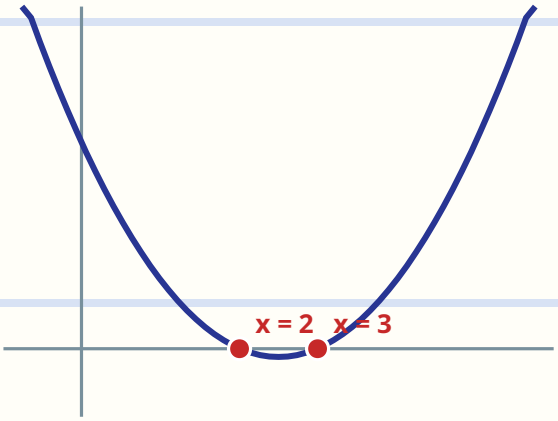
04 Visual Concept Explanation

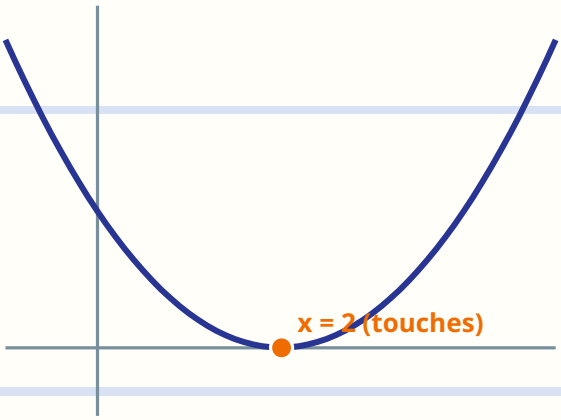
VISUALISE — SEE THE ROOTS BEFORE YOU SOLVE

Every quadratic equation $ax^2 + bx + c = 0$ has a picture: the curve $y = ax^2 + bx + c$, called a **parabola**. The **roots** of the equation are exactly the points where this curve **crosses the x-axis**. Look at the three pictures — they are the whole "nature of roots" story, told without a single formula.



y = x² − 5x + 6 — crosses twic

Picture 1 — TWO crossings. The parabola of $y = x^2 - 5x + 6$ cuts the x-axis at x = 2 and x = 3. So the equation $x^2 - 5x + 6 = 0$ has **two distinct real roots**. (Check: $(x - 2)(x - 3) = x^2 - 5x + 6$.)



y = x² − 4x + 4 — touches onc

Picture 2 — ONE touch. The parabola of $y = x^2 - 4x + 4$ just kisses the x-axis at x = 2 and turns back. The equation has **two equal real roots** (x = 2 and x = 2), because $x^2 - 4x + 4 = (x - 2)^2$.

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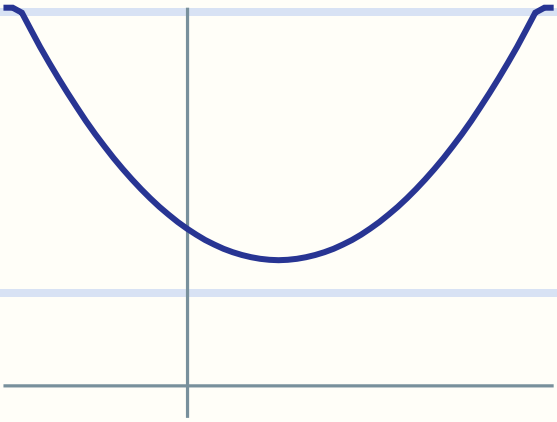
The Third Picture — and the Area Model

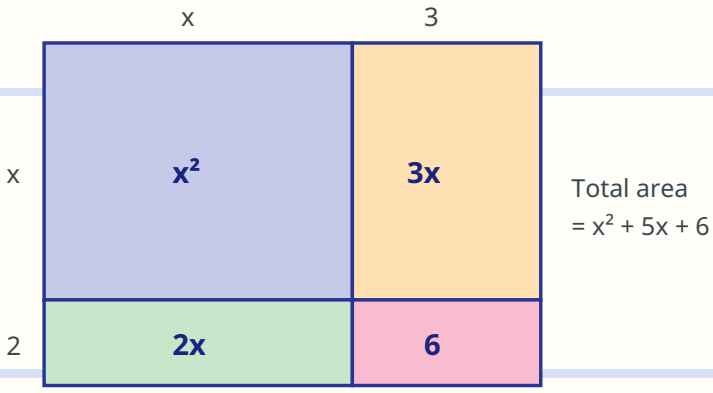


$y = x^2 - 2x + 5$ — never reach

Picture 3 — NO crossing. The parabola of $y = x^2 - 2x + 5$ floats above the axis (its lowest point is y = 4). It never meets the x-axis, so $x^2 - 2x + 5 = 0$ has **no real roots**. In Section 06 you will meet the number D that predicts which of the three pictures you get — before you draw or solve anything.

Factorisation as an Area Picture (Concrete → Pictorial)

Why does $(x + 3)(x + 2) = x^2 + 5x + 6$? Because it is the area of one rectangle cut into four pieces:



The big square is $x \times x = x^2$. The two strips are 3x and 2x — together they make the middle term **5x**. The small corner is $3 \times 2 = 6$. So splitting the middle term is not a trick: **the two parts of the middle term are the two strips of the rectangle**. When you split 5x into 3x + 2x, you are just naming the strips.

> **TOPPER TIP:** *When a factorisation feels hard, draw the rectangle in the margin. The picture tells you which two numbers to hunt for: their product is the corner (c), their sum is the middle coefficient (b).*

05 Mathematical Foundations

UNDERSTAND — THE EXACT MEANING OF EVERY WORD

1. Standard form

A **quadratic equation** in the variable x is an equation that can be written as $\mathbf{ax^2 + bx + c = 0}$, where a, b, c are real numbers and $\mathbf{a \neq 0}$. This is the **standard form**. If a were 0, the x^2 term would vanish and the equation would become linear — that is why $a \neq 0$ is part of the definition.

2. What is a root?

A real number α is called a **root** (or solution) of $ax^2 + bx + c = 0$ if it **satisfies** the equation: $a\alpha^2 + b\alpha + c = 0$. Equivalently, α is a zero of the polynomial $ax^2 + bx + c$. Example: $x = 2$ is a root of $x^2 - 5x + 6 = 0$ because $4 - 10 + 6 = 0$. But $x = 4$ is not, because $16 - 20 + 6 = 2 \neq 0$.

3. Why at most two roots?

If α and β are roots, the polynomial splits as $a(x - \alpha)(x - \beta)$. A product can be zero only when a factor is zero — and there are only two factors. So a quadratic equation has **at most two roots**: two distinct, two equal ($\alpha = \beta$), or none in real numbers.

4. The discriminant

The number $\mathbf{D = b^2 - 4ac}$ is called the **discriminant** of $ax^2 + bx + c = 0$. Section 06 will show you where it is born: it is the number sitting under the square-root sign in the quadratic formula, and its sign alone decides the nature of the roots.

5. The three solving methods of this chapter

Method 1 — Factorisation: split the middle term, factorise, set each factor to zero. Fastest when it works.

Method 2 — Completing the square: force the left side into a perfect square $(x + p)^2$, then take square roots. Always works; also the mother of the formula.

Method 3 — Quadratic formula: $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$. The master key — works on every quadratic, and you will derive it yourself in the next section.

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06 Formula Derivations

DERIVE — BUILD THE FORMULA YOURSELF

Most students memorise the quadratic formula. You will **build** it. The tool is **completing the square** — applied once to the general equation $ax^2 + bx + c = 0$. Follow every step; each one is a move you already know.

DERIVATION 1

Derive the quadratic formula: solve $ax^2 + bx + c = 0$ ($a \neq 0$) by completing the square.

THINK FIRST: One method (completing the square) + one general equation = a formula that solves every quadratic ever written. That is the power of algebra.

1. Start: $ax^2 + bx + c = 0$.
   Why: The general quadratic equation in standard form.

2. Divide every term by a: $x^2 + (b/a)x + c/a = 0$.
   Why: Making the coefficient of x^2 equal to 1 — completing the square is easy only in this form. This is legal because $a \neq 0$.

3. Move the constant term to the right: $x^2 + (b/a)x = -c/a$.
   Why: We want all x-terms on the left, ready to be packed into a square.

4. Find the number that completes the square: half of the coefficient of x is $(1/2)(b/a) = b/2a$. Square it: $(b/2a)^2 = b^2/4a^2$.
   Why: Recall $(x + p)^2 = x^2 + 2px + p^2$: the added constant must be the square of half the x-coefficient.

5. Add $b^2/4a^2$ to BOTH sides: $x^2 + (b/a)x + b^2/4a^2 = -c/a + b^2/4a^2$.
   Why: Adding the same number to both sides keeps the equation balanced — this is the step students most often forget on the right side.

The square is packed. Turn the page — steps 6 to 9 finish the derivation.

Derivation 1 — continued: The Formula Appears (Steps 6 to 9)

6 The left side is now a perfect square: $(x + b/2a)^2 = -c/a + b^2/4a^2$.

Why: Check by expanding: $(x + b/2a)^2 = x^2 + (b/a)x + b^2/4a^2$. Exactly our left side.

7 Combine the right side over the common denominator $4a^2$: $(x + b/2a)^2 = (b^2 - 4ac) / 4a^2$.

Why: Multiply $-c/a$ by $4a/4a$ to get $-4ac/4a^2$, then add. Watch the expression $b^2 - 4ac$ appear — it will soon run the whole chapter.

8 Take square roots on both sides: $x + b/2a = \pm \sqrt{(b^2 - 4ac)} / 2a$.

Why: Two facts used together: $\sqrt{(4a^2)} = 2a$, and a square root always carries ± — both signs square to the same number. (This step needs $b^2 - 4ac \geq 0$; see Derivation 2.)

9 Isolate x: $x = -b/2a \pm \sqrt{(b^2 - 4ac)} / 2a = \mathbf{[-b \pm \sqrt{(b^2 - 4ac)}] / 2a}$.

Why: Subtract $b/2a$ from both sides and join over the common denominator $2a$. The derivation is complete.

ANSWER: THE QUADRATIC FORMULA: for $ax^2 + bx + c = 0$ ($a \neq 0$), $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$.

PIN POINT **The formula was BORN by completing the square on $ax^2 + bx + c = 0$. If you ever forget the formula in the exam hall, rebuild it — Derivation 1 shows every step.**

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Derivation 2 — Where the Discriminant Gets Its Power

Look again at the last line of Derivation 1: $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$. Everything in the formula is plain arithmetic — except one thing: the square root. Whether real roots exist depends entirely on the number under it. That number deserves its name:

$$D = b^2 - 4ac \quad \text{(the Discriminant)}$$

DERIVATION 2

Derive the nature-of-roots rules from the formula $x = [-b \pm \sqrt{D}] / 2a$.

THINK FIRST: Do not memorise the table — re-derive it from the $\pm \sqrt{D}$ in the formula whenever you are unsure. It takes ten seconds.

1. **Case 1: D > 0.** Then $\sqrt{D}$ is a positive real number, so $-b + \sqrt{D}$ and $-b - \sqrt{D}$ are two **different** real numbers. Dividing by 2a gives two different roots: $x = (-b + \sqrt{D})/2a$ and $x = (-b - \sqrt{D})/2a$.

   Why: This is Picture 1 of Section 04: the parabola crosses the axis twice.

2. **Case 2: D = 0.** Then $\sqrt{D} = 0$, and the $\pm$ makes no difference: both branches give the same value $x = -b/2a$. The two roots have **merged into one repeated value**.

   Why: Picture 2 of Section 04: the parabola touches the axis at exactly one point.

3. **Case 3: D < 0.** Then $\sqrt{D}$ would be the square root of a negative number, which is **not a real number**. So neither branch gives a real value of x.

   Why: Picture 3 of Section 04: the parabola never reaches the axis. The equation simply has no real roots.

4. Conclusion: the sign of D, computed from a, b, c alone, tells you the nature of the roots **before** you solve anything.

   Why: Three cases, one number. That is why D is called the discriminant — it discriminates (separates) the three cases.

ANSWER: D > 0: two distinct real roots • D = 0: two equal real roots • D < 0: no real roots.

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Derivation 2 — The Summary Table

Discriminant	Nature of roots	Graph (Section 04)	Example
$D > 0$	Two distinct real roots	Crosses twice	$x^2 - 5x + 6 = 0$, $D = 1$
$D = 0$	Two equal real roots	Touches once	$x^2 - 4x + 4 = 0$, $D = 0$
$D < 0$	No real roots	Never reaches	$x^2 - 2x + 5 = 0$, $D = -16$

Watch D being computed — the three table examples, step by step

$x^2 - 5x + 6$: $a = 1$, $b = -5$, $c = 6 \rightarrow D = (-5)^2 - 4(1)(6) = 25 - 24 =$ **$1 > 0$** → two distinct real roots (2 and 3).

$x^2 - 4x + 4$: $a = 1$, $b = -4$, $c = 4 \rightarrow D = (-4)^2 - 4(1)(4) = 16 - 16 =$ **0** → two equal real roots (2 and 2).

$x^2 - 2x + 5$: $a = 1$, $b = -2$, $c = 5 \rightarrow D = (-2)^2 - 4(1)(5) = 4 - 20 =$ **$-16 < 0$** → no real roots.

Three computations, three fates. This is the complete nature-of-roots method — Section 07 now puts it to work on every NCERT question.

TOPPER TIP: *Exam shortcut with a conscience: for 'nature of roots' questions, the board expects D computed step by step — write a, b, c first, then $D = b^2 - 4ac$ with the values substituted, then the conclusion. All three lines carry marks.*

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07 Detailed Solved Examples

SOLVE — EVERY NCERT QUESTION, EVERY STEP SHOWN

This is the heart of the chapter: **every question of the rationalised NCERT Chapter 4**, solved with complete one-by-one steps. A quadratic equation must reduce to the form $ax^2 + bx + c = 0$ with $a \neq 0$. Exercise 4.1 first tests whether you can recognise one — including the impostors.

EXERCISE 4.1 — Q1(i)

Check whether the following is a quadratic equation: $(x + 1)^2 = 2(x - 3)$

THINK FIRST: Expand both sides and see whether an x^2 term survives in standard form.

1. Expand the left side using $(a + b)^2 = a^2 + 2ab + b^2$: $(x + 1)^2 = x^2 + 2x + 1$.
   Why: The most common error here is writing $(x + 1)^2 = x^2 + 1$. The middle term $2x$ must be there.
2. Expand the right side: $2(x - 3) = 2x - 6$. So the equation is $x^2 + 2x + 1 = 2x - 6$.
3. Bring all terms to the left: $x^2 + 2x + 1 - 2x + 6 = 0$, which simplifies to $x^2 + 7 = 0$.
   Why: The $2x$ and $-2x$ cancel — but the x^2 term does not.
4. Compare with $ax^2 + bx + c = 0$: $a = 1$, $b = 0$, $c = 7$, and $a \neq 0$.
   Why: $b = 0$ is perfectly allowed. Only a must not be zero.

ANSWER: Yes — $x^2 + 7 = 0$ is of the form $ax^2 + bx + c = 0$, so it IS a quadratic equation.

CBSE-STYLE MARKING SCHEME: 1 mark: correct expansion (1/2) + standard form and conclusion (1/2).

EXERCISE 4.1 — Q1(ii)

Check whether the following is a quadratic equation: $x^2 - 2x = (-2)(3 - x)$

THINK FIRST: Expand the right side carefully — the minus sign is the trap.

1. Expand the right side: $(-2)(3 - x) = -6 + 2x$.
   Why: $(-2) \times 3 = -6$ and $(-2) \times (-x) = +2x$. Both signs matter.
2. The equation becomes $x^2 - 2x = -6 + 2x$.
3. Bring all terms to the left: $x^2 - 2x - 2x + 6 = 0$, i.e. $x^2 - 4x + 6 = 0$.

Exercise 4.1 Q1 — Check (iii): the Impostor Test

EXERCISE 4.1 — Q1(iii)

Check whether the following is a quadratic equation: $(x - 2)(x + 1) = (x - 1)(x + 3)$

THINK FIRST: Both sides are products. Expand both fully before judging — this one is an impostor.

1. Expand the left side: $(x - 2)(x + 1) = x^2 + x - 2x - 2 = x^2 - x - 2$.
   Why: Multiply each term of the first bracket by each term of the second.
2. Expand the right side: $(x - 1)(x + 3) = x^2 + 3x - x - 3 = x^2 + 2x - 3$.
3. Set them equal and bring everything to the left: $(x^2 - x - 2) - (x^2 + 2x - 3) = 0$.
4. Simplify: $x^2 - x^2 - x - 2x - 2 + 3 = -3x + 1 = 0$.
   Why: The x^2 terms cancel completely.
5. The result $-3x + 1 = 0$ has degree 1 — it is a **linear** equation, not of the form $ax^2 + bx + c = 0$ with $a \neq 0$.
   Why: An equation is judged by its final simplified form, not by how it first looks.

ANSWER: No — it is NOT a quadratic equation (it reduces to a linear equation).

CBSE-STYLE MARKING SCHEME: 1 mark: both expansions (1/2) + cancellation and conclusion (1/2).

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Exercise 4.1 Q1 — Checks (iv) to (vi)

EXERCISE 4.1 — Q1(iv)

Check whether the following is a quadratic equation: $(x - 3)(2x + 1) = x(x + 5)$

THINK FIRST: Expand both sides; watch whether x^2 survives.

1. Expand the left side: $(x - 3)(2x + 1) = 2x^2 + x - 6x - 3 = 2x^2 - 5x - 3$.
2. Expand the right side: $x(x + 5) = x^2 + 5x$.
3. Bring all terms to the left: $2x^2 - 5x - 3 - x^2 - 5x = 0$, i.e. $x^2 - 10x - 3 = 0$.
   Why: $2x^2 - x^2 = x^2$ survives.
4. This is $ax^2 + bx + c = 0$ with $a = 1$, $b = -10$, $c = -3$.

ANSWER: Yes — it IS a quadratic equation.

CBSE-STYLE MARKING SCHEME: 1 mark: expansions (1/2) + standard form and conclusion (1/2).

EXERCISE 4.1 — Q1(v)

Check whether the following is a quadratic equation: $(2x - 1)(x - 3) = (x + 5)(x - 1)$

THINK FIRST: Same routine: expand, collect, judge the final form.

1. Expand the left side: $(2x - 1)(x - 3) = 2x^2 - 6x - x + 3 = 2x^2 - 7x + 3$.
2. Expand the right side: $(x + 5)(x - 1) = x^2 - x + 5x - 5 = x^2 + 4x - 5$.
3. Bring all terms to the left: $2x^2 - 7x + 3 - x^2 - 4x + 5 = 0$, i.e. $x^2 - 11x + 8 = 0$.
4. Standard form with $a = 1 \neq 0$.

ANSWER: Yes — it IS a quadratic equation.

CBSE-STYLE MARKING SCHEME: 1 mark: expansions (1/2) + standard form and conclusion (1/2).

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Exercise 4.1 Q1 — Checks (vi) and (vii)

EXERCISE 4.1 — Q1(vi)

Check whether the following is a quadratic equation: $x^2 + 3x + 1 = (x - 2)^2$

THINK FIRST: Expand the square on the right first.

1. Expand the right side: $(x - 2)^2 = x^2 - 4x + 4$.
   Why: Use $(a - b)^2 = a^2 - 2ab + b^2$.
2. The equation becomes $x^2 + 3x + 1 = x^2 - 4x + 4$.
3. Bring all terms to the left: $x^2 - x^2 + 3x + 4x + 1 - 4 = 0$, i.e. $7x - 3 = 0$.
   Why: Again the x^2 terms cancel.
4. $7x - 3 = 0$ is linear (degree 1).

ANSWER: No — it is NOT a quadratic equation.

CBSE-STYLE MARKING SCHEME: 1 mark: expansion (1/2) + cancellation and conclusion (1/2).

EXERCISE 4.1 — Q1(vii)

Check whether the following is a quadratic equation: $(x + 2)^3 = 2x(x^2 - 1)$

THINK FIRST: Cubes appear — expand patiently and compare the highest power.

1. Expand the left side using $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$: $(x + 2)^3 = x^3 + 6x^2 + 12x + 8$.
2. Expand the right side: $2x(x^2 - 1) = 2x^3 - 2x$.
3. Bring all terms to the left: $x^3 - 2x^3 + 6x^2 + 12x + 2x + 8 = 0$, i.e. $-x^3 + 6x^2 + 14x + 8 = 0$.
4. The highest power of x is 3 — this is a **cubic** equation, not a quadratic one.
   Why: A quadratic equation must have degree exactly 2 after simplification.

ANSWER: No — it is NOT a quadratic equation (it is cubic).

CBSE-STYLE MARKING SCHEME: 1 mark: expansions (1/2) + degree observation and conclusion (1/2).

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Exercise 4.1 Q1 — Check (viii) and the Verdict Gallery

EXERCISE 4.1 — Q1(viii)

Check whether the following is a quadratic equation: $x^3 - 4x^2 - x + 1 = (x - 2)^3$

THINK FIRST: It starts with x^3 on both sides — do not judge too fast. Expand and watch the cubes.

1. Expand the right side: $(x - 2)^3 = x^3 - 6x^2 + 12x - 8$.
   Why: Use $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.
2. Bring all terms to the left: $(x^3 - 4x^2 - x + 1) - (x^3 - 6x^2 + 12x - 8) = 0$.
3. Simplify term by term: $x^3 - x^3 = 0$; $-4x^2 + 6x^2 = 2x^2$; $-x - 12x = -13x$; $1 + 8 = 9$.
   Result: $2x^2 - 13x + 9 = 0$.
   Why: The cubes cancel, and a genuine x^2 term remains.
4. This is $ax^2 + bx + c = 0$ with $a = 2 \neq 0$.

ANSWER: Yes — it IS a quadratic equation (the x^3 terms cancel).

CBSE-STYLE MARKING SCHEME: 1 mark: expansion (1/2) + simplified standard form and conclusion (1/2).

The Verdict Gallery — all eight checks at a glance

Check	Simplifies to	Quadratic?
(i)	$x^2 + 7 = 0$	YES
(ii)	$x^2 - 4x + 6 = 0$	YES
(iii)	$-3x + 1 = 0$	NO — linear
(iv)	$x^2 - 10x - 3 = 0$	YES
(v)	$x^2 - 11x + 8 = 0$	YES
(vi)	$7x - 3 = 0$	NO — linear
(vii)	$-x^3 + 6x^2 + 14x + 8 = 0$	NO — cubic
(viii)	$2x^2 - 13x + 9 = 0$	YES

Exercise 4.1 Q2 — From Situations to Equations (Solved Fully)

NCERT asks you to *represent* each situation as a quadratic equation. We go one step further and **solve each one fully** — because the board's later questions expect exactly that.

EXERCISE 4.1 — Q2(i)

The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. Find the length and the breadth of the plot.

THINK FIRST: Length is tied to breadth: if breadth = x, length = 2x + 1. Area gives the equation.

1. Let the breadth be x m. Then the length = (2x + 1) m.
   Why: 'One more than twice the breadth' translates word by word: 2x + 1.
2. Area = length × breadth: $x(2x + 1) = 528$.
3. Expand: $2x^2 + x = 528$, so $2x^2 + x - 528 = 0$.
   Why: This is the required representation.
4. Factorise: split the middle term using product $2 \times (-528) = -1056$ and sum +1. The pair is +33 and −32 ($33 \times -32 = -1056$; $33 - 32 = 1$). So $2x^2 + 33x - 32x - 528 = 0$.
5. Group: $x(2x + 33) - 16(2x + 33) = (2x + 33)(x - 16) = 0$.
   Why: From the second pair, −16 × 33 = −528. Check the common bracket (2x + 33) matches.
6. So $x = 16$ or $x = -33/2$. **Reject x = −33/2** — breadth cannot be negative.
7. Breadth = 16 m; length = 2(16) + 1 = 33 m. Check: $16 \times 33 = 528$ m^2. Correct!

ANSWER: Breadth = 16 m and Length = 33 m. (Representation: $2x^2 + x - 528 = 0$.)

CBSE-STYLE MARKING SCHEME: Representation 1 mark. Full solution: factorisation steps (1) + rejecting the negative root with reason (1/2) + both dimensions (1/2).

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Exercise 4.1 Q2 — The Consecutive Integers (ii)

EXERCISE 4.1 — Q2(ii)

The product of two consecutive positive integers is 306. Find the integers.

THINK FIRST: Consecutive integers differ by 1: x and x + 1.

1. Let the smaller integer be x. Then the next consecutive integer is x + 1.
2. Product: $x(x + 1) = 306$.
   Why: This is the required representation.
3. Expand and rearrange: $x^2 + x - 306 = 0$.
4. Factorise: pair with product −306 and sum +1 is +18 and −17. So $x^2 + 18x - 17x - 306 = (x + 18)(x - 17) = 0$.
   Why: 18 × −17 = −306 and 18 − 17 = 1.
5. So x = 17 or x = −18. **Reject −18** — the integers are stated to be positive.
6. The integers are 17 and 17 + 1 = 18. Check: 17 × 18 = 306. Correct!

ANSWER: The two consecutive positive integers are 17 and 18. (Representation: $x^2 + x - 306 = 0$.)

CBSE-STYLE MARKING SCHEME: Representation 1 mark. Full solution: factorisation (1) + positive-root selection (1/2) + answer pair (1/2).

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Exercise 4.1 Q2 — Ages and the Train

EXERCISE 4.1 — Q2(iii)

Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. Find Rohan's present age.

THINK FIRST: Ages move together: after 3 years, BOTH ages increase by 3.

1. Let Rohan's present age be x years. Then his mother's present age = (x + 26) years.
2. Three years from now: Rohan will be (x + 3) and his mother will be (x + 26 + 3) = (x + 29).
   Why: Add 3 to each age — a classic slip is to add 3 only once.
3. Product after 3 years: (x + 3)(x + 29) = 360.
   Why: This is the required representation.
4. Expand: $x^2 + 29x + 3x + 87 = 360$, so $x^2 + 32x - 273 = 0$.
5. Factorise: pair with product −273 and sum +32 is +39 and −7. So (x + 39)(x − 7) = 0.
   Why: 39 × −7 = −273; 39 − 7 = 32.
6. x = 7 or x = −39. **Reject −39** — age cannot be negative.
7. Check: Rohan is 7, mother is 33. After 3 years: 10 and 36; 10 × 36 = 360. Correct!

ANSWER: Rohan's present age is 7 years. (Representation: $x^2 + 32x - 273 = 0$.)

CBSE-STYLE MARKING SCHEME: Representation 1 mark. Full solution: correct future ages (1/2) + expansion (1/2) + factorisation and rejection (1).

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Exercise 4.1 Q2 — The Train (iv)

EXERCISE 4.1 — Q2(iv)

A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. Find the speed of the train.

THINK FIRST: Time = distance / speed. The SLOWER journey takes LONGER — set up the time difference carefully.

1. Let the usual speed be x km/h. Usual time = 480/x hours.
   Why: Time = distance ÷ speed.
2. Reduced speed = (x − 8) km/h, so the slower time = 480/(x − 8) hours.
3. The slower journey takes 3 hours MORE: 480/(x − 8) − 480/x = 3.
   Why: Bigger time minus smaller time = 3. This is the required representation.
4. Combine the left side over the common denominator x(x − 8): 480[x − (x − 8)] / [x(x − 8)] = 3, i.e. 480 × 8 / [x(x − 8)] = 3.
   Why: The numerator simplifies beautifully: x − x + 8 = 8.
5. So x(x − 8) = 480 × 8 / 3 = 1280, giving $x^2 - 8x - 1280 = 0$.
6. Factorise: pair with product −1280 and sum −8 is −40 and +32. So (x − 40)(x + 32) = 0.
   Why: −40 × 32 = −1280; −40 + 32 = −8.
7. x = 40 or x = −32. **Reject −32** — speed cannot be negative.
8. Check: at 40 km/h, time = 480/40 = 12 h. At 32 km/h, time = 480/32 = 15 h. Difference = 3 h. Correct!

ANSWER: The speed of the train is 40 km/h. (Representation: 480/(x − 8) − 480/x = 3.)

CBSE-STYLE MARKING SCHEME: Representation 1 mark. Full solution: time equation (1) + simplification to quadratic (1) + roots with rejection (1).

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Exercise 4.2 — Roots by Factorisation

Exercise 4.2 is the factorisation workshop. The method never changes: **standard form → split the middle term (product ac, sum b) → group → set each factor = 0**. Watch it run, question after question.

EXERCISE 4.2 — Q1(i)

Find the roots of the equation by factorisation: $x^2 - 3x - 10 = 0$

THINK FIRST: Pair with product −10 and sum −3.

1. Find the pair: product = 1 × (−10) = −10, sum = −3. The numbers are −5 and +2.
   Why: −5 × 2 = −10 and −5 + 2 = −3.
2. Split: $x^2 - 5x + 2x - 10 = 0$.
3. Group: $x(x - 5) + 2(x - 5) = (x - 5)(x + 2) = 0$.
4. Set each factor to zero: $x - 5 = 0$ gives $x = 5$; $x + 2 = 0$ gives $x = -2$.

ANSWER: The roots are x = 5 and x = −2.

CBSE-STYLE MARKING SCHEME: 2 marks: correct factorisation (1) + both roots (1).

EXERCISE 4.2 — Q1(ii)

Find the roots of the equation by factorisation: $2x^2 + x - 6 = 0$

THINK FIRST: Here a = 2, so the pair needs product a × c = −12, sum = +1.

1. Find the pair: product = 2 × (−6) = −12, sum = +1. The numbers are +4 and −3.
   Why: 4 × −3 = −12; 4 − 3 = 1.
2. Split: $2x^2 + 4x - 3x - 6 = 0$.
3. Group: $2x(x + 2) - 3(x + 2) = (x + 2)(2x - 3) = 0$.
4. Set each factor to zero: $x + 2 = 0$ gives $x = -2$; $2x - 3 = 0$ gives $x = 3/2$.

ANSWER: The roots are x = 3/2 and x = −2.

CBSE-STYLE MARKING SCHEME: 2 marks: correct splitting ac (1) + both roots (1).

Exercise 4.2 Q1 — The Surd Question (iii) and the First Perfect Square (iv)

EXERCISE 4.2 — Q1(iii)

Find the roots of the equation by factorisation: $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

THINK FIRST: Surds obey the same rule: pair with product $\sqrt{2} \times 5\sqrt{2} = 10$ and sum 7.

1. Compute the target product: $a \times c = \sqrt{2} \times 5\sqrt{2} = 5 \times (\sqrt{2})^2 = 10$. Sum needed: 7. The pair is 5 and 2.
   Why: √2 × √2 = 2 — the surd becomes a whole number.
2. Split: $\sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$.
3. Group: $x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = (\sqrt{2}x + 5)(x + \sqrt{2}) = 0$.
   Why: Check the second group: √2 × √2 x = 2x and √2 × 5 = 5√2. Correct.
4. Set each factor to zero: $x + \sqrt{2} = 0$ gives $x = -\sqrt{2}$; $\sqrt{2}x + 5 = 0$ gives $x = -5/\sqrt{2} = -5\sqrt{2}/2$.
   Why: Rationalise −5/√2 by multiplying top and bottom by √2.

ANSWER: The roots are $x = -\sqrt{2}$ and $x = -5\sqrt{2}/2$.

CBSE-STYLE MARKING SCHEME: 2 marks: handling the surd product (1) + both roots, rationalised (1).

Exercise 4.2 Q1 — The First Perfect Square (iv)

EXERCISE 4.2 — Q1(iv)

Find the roots of the equation by factorisation: $2x^2 - x + 1/8 = 0$

THINK FIRST: Fractions clear if you multiply through by 8 first.

1. Multiply every term by 8: $16x^2 - 8x + 1 = 0$.
   Why: Multiplying an equation by a non-zero number does not change its roots.

2. Recognise the perfect square: $16x^2 - 8x + 1 = (4x)^2 - 2(4x)(1) + 1^2 = (4x - 1)^2$.
   Why: It matches $(a - b)^2 = a^2 - 2ab + b^2$ with $a = 4x$, $b = 1$.

3. So $(4x - 1)^2 = 0$, which means $4x - 1 = 0$ (twice).
   Why: A squared factor set to zero gives the same root twice.

4. Therefore $x = 1/4$ and $x = 1/4$ — two equal roots.

ANSWER: The roots are $x = 1/4$ and $x = 1/4$ (equal roots).

CBSE-STYLE MARKING SCHEME: 2 marks: clearing fractions / spotting the square (1) + the repeated root stated twice (1).

Exercise 4.2 Q1 — The Second Perfect Square (v)

EXERCISE 4.2 — Q1(v)

Find the roots of the equation by factorisation: $100x^2 - 20x + 1 = 0$

THINK FIRST: Another perfect square in disguise: $100x^2 = (10x)^2$ and $1 = 1^2$.

1. Check the middle term against the identity: $2 \times (10x) \times (1) = 20x$. It matches the given $-20x$.
   Why: If the middle term equals 2ab, the trinomial is a perfect square.
2. So $100x^2 - 20x + 1 = (10x - 1)^2 = 0$.
3. Hence $10x - 1 = 0$ (twice), giving $x = 1/10$ and $x = 1/10$.

ANSWER: The roots are x = 1/10 and x = 1/10 (equal roots).

CBSE-STYLE MARKING SCHEME: 2 marks: perfect-square recognition (1) + repeated root (1).

PIN POINT **EQUAL ROOTS LEAVE A FINGERPRINT: when the trinomial is a perfect square like $(4x - 1)^2$, the roots are equal — and, looking ahead, its discriminant D is exactly 0. Two languages, one fact.**

TOPPER TIP: In Q1(iv) and (v), splitting the middle term also works, but spotting the perfect square is faster. Train your eye: check whether the first and last terms are squares and whether the middle is twice their product.

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Exercise 4.2 — Word Problems Q2

EXERCISE 4.2 — Q2(i)

John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. Find out how many marbles they had to start with.

THINK FIRST: Let John's starting count be x; Jivanti's is whatever remains of 45.

1. Let John have started with x marbles. Then Jivanti started with (45 − x) marbles.
   Why: Together they total 45 — one unknown describes both.
2. After losing 5 each: John has (x − 5) and Jivanti has (45 − x − 5) = (40 − x).
3. Product now is 124: (x − 5)(40 − x) = 124.
4. Expand: $40x - x^2 - 200 + 5x = 124$, i.e. $-x^2 + 45x - 200 = 124$.
5. Rearrange and multiply by −1: $x^2 - 45x + 324 = 0$.
6. Use the formula (or factorise): $D = 45^2 - 4(324) = 2025 - 1296 = 729 = 27^2$. So x = (45 ± 27)/2 = 36 or 9.
   Why: Recognising $729 = 27^2$ makes the formula route clean.
7. If John started with 36, Jivanti started with 45 − 36 = 9. If John started with 9, Jivanti started with 36.
   Why: Both roots are acceptable here — the problem does not say who had more. Check: after losing 5 each: 31 and 4; 31 × 4 = 124. Correct!

ANSWER: John and Jivanti started with 36 and 9 marbles (in some order).

CBSE-STYLE MARKING SCHEME: 3 marks: set-up with one unknown (1) + equation (1) + both cases with check (1).

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Exercise 4.2 — Word Problems Q2 (continued)

EXERCISE 4.2 — Q2(ii)

A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs 750. Find the number of toys produced on that day.

THINK FIRST: Total cost = (number of toys) × (cost per toy). Both parts depend on the same x.

1. Let the number of toys produced be x. Then the cost of each toy = Rs (55 − x).
   Why: '55 minus the number of toys' translates directly.
2. Total cost: x(55 − x) = 750.
3. Expand and rearrange: $55x - x^2 = 750$, so $x^2 - 55x + 750 = 0$.
4. Factorise: pair with product 750 and sum −55 is −25 and −30. So (x − 25)(x − 30) = 0.
   Why: −25 × −30 = 750; −25 − 30 = −55.
5. x = 25 or x = 30. **Both are valid**: if 25 toys are made, each costs Rs 30 and 25 × 30 = 750; if 30 toys are made, each costs Rs 25 and 30 × 25 = 750.
   Why: Unlike lengths, both roots describe real, possible days. Do not reject a root just because there are two!

ANSWER: The industry produced 25 toys (at Rs 30 each) or 30 toys (at Rs 25 each) — both fit the given total cost of Rs 750.

CBSE-STYLE MARKING SCHEME: 3 marks: cost expression (1) + equation and factorisation (1) + both valid answers with reasoning (1).

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Exercise 4.2 — Q3 and Q4

EXERCISE 4.2 — Q3

Find two numbers whose sum is 27 and product is 182.

THINK FIRST: If one number is x, the other is forced: 27 − x.

1. Let one number be x. Then the other number = 27 − x.
   Why: Their sum must be 27.
2. Product: x(27 − x) = 182.
3. Expand and rearrange: $27x - x^2 = 182$, so $x^2 - 27x + 182 = 0$.
4. Factorise: pair with product 182 and sum −27 is −13 and −14. So (x − 13)(x − 14) = 0.
   Why: 13 × 14 = 182.
5. x = 13 or x = 14. If x = 13, the other number is 27 − 13 = 14; if x = 14, the other is 13. Same pair.
6. Check: 13 + 14 = 27 and 13 × 14 = 182. Correct!

ANSWER: The two numbers are 13 and 14.

CBSE-STYLE MARKING SCHEME: 2 marks: equation (1) + factorisation and the pair (1).

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Exercise 4.2 — Q4: Sum of Squares 365

EXERCISE 4.2 — Q4

Find two consecutive positive integers, sum of whose squares is 365.

THINK FIRST: Consecutive: x and x + 1. Square EACH before adding.

1. Let the integers be x and x + 1.
2. Sum of squares: $x^2 + (x + 1)^2 = 365$.
   Why: Note: it is $x^2 + (x + 1)^2$, NOT $(x + x + 1)^2$ — the squares are separate.
3. Expand $(x + 1)^2 = x^2 + 2x + 1$: total $2x^2 + 2x + 1 = 365$, so $2x^2 + 2x - 364 = 0$.
4. Divide by 2: $x^2 + x - 182 = 0$.
   Why: Simplifying first keeps the numbers small.
5. Factorise: pair with product −182 and sum +1 is +14 and −13. So $(x + 14)(x - 13) = 0$.
6. $x = 13$ or $x = -14$. **Reject −14** — positive integers are required.
7. The integers are 13 and 14. Check: 169 + 196 = 365. Correct!

ANSWER: The two consecutive positive integers are 13 and 14.

CBSE-STYLE MARKING SCHEME: 2 marks: correct sum-of-squares equation (1) + solution with rejection (1).

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Exercise 4.2 — Q5 and Q6

EXERCISE 4.2 — Q5

The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

THINK FIRST: Pythagoras connects the three sides: base^2 + altitude^2 = hypotenuse^2.

1. Let the base be x cm. Then the altitude = (x − 7) cm.
2. By Pythagoras' theorem: $x^2 + (x - 7)^2 = 13^2 = 169$.
3. Expand $(x - 7)^2 = x^2 - 14x + 49$: total $2x^2 - 14x + 49 = 169$.
4. Rearrange and divide by 2: $2x^2 - 14x - 120 = 0$, so $x^2 - 7x - 60 = 0$.
5. Factorise: pair with product −60 and sum −7 is −12 and +5. So $(x - 12)(x + 5) = 0$.
6. $x = 12$ or $x = -5$. **Reject −5** — a side cannot be negative.
7. Base = 12 cm; altitude = 12 − 7 = 5 cm. Check: $5^2 + 12^2 = 25 + 144 = 169 = 13^2$. Correct — the famous 5-12-13 triangle!

ANSWER: Base = 12 cm and altitude = 5 cm.

CBSE-STYLE MARKING SCHEME: 3 marks: Pythagoras set-up (1) + simplification (1) + sides with rejection and check (1).

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Exercise 4.2 — Q6: The Pottery Articles

EXERCISE 4.2 — Q6

A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.

THINK FIRST: Cost per article = 2x + 3 this time. Total = x(2x + 3).

1. Let the number of articles be x. Then cost per article = Rs (2x + 3).
   Why: '3 more than twice the number' = 2x + 3.
2. Total cost: x(2x + 3) = 90.
3. Expand: $2x^2 + 3x - 90 = 0$.
4. Solve by formula: $D = 3^2 - 4(2)(-90) = 9 + 720 = 729 = 27^2$. So $x = (-3 \pm 27) / 4$.
   Why: Note the sign: −4ac with c = −90 becomes +720.
5. $x = 24/4 = 6$ or $x = -30/4 = -7.5$. **Reject −7.5** — a count of articles cannot be negative (or fractional).
6. Number of articles = 6; cost per article = 2(6) + 3 = Rs 15. Check: 6 × 15 = 90. Correct!

ANSWER: 6 articles were produced, at a cost of Rs 15 per article.

CBSE-STYLE MARKING SCHEME: 3 marks: cost expression (1) + equation and discriminant (1) + both answers with rejection (1).

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Method 2 in Action — Completing the Square

The rationalised NCERT builds its formula questions on factorisation and the quadratic formula — but completing the square is the method that **created** the formula (Section 06), and top students own it. Here it is on three equations, with **every rearrangement shown**. The recipe never changes: make x^2 coefficient 1 → move the constant → add (half of the x-coefficient)^2 to both sides → write as a square → take ± square roots.

COMPLETING THE SQUARE — EXAMPLE 1

Solve by completing the square: $x^2 + 6x + 5 = 0$

THINK FIRST: The x-coefficient is 6; half of it is 3; $3^2 = 9$ is the completing number.

1. Move the constant to the right: $x^2 + 6x = -5$.
2. Add 9 to BOTH sides: $x^2 + 6x + 9 = -5 + 9 = 4$.
   Why: $9 = (6/2)^2$. Adding to only one side is the classic error — see Find-the-Error E7.
3. Write the left side as a square: $(x + 3)^2 = 4$.
   Why: Check: $(x + 3)^2 = x^2 + 6x + 9$. Correct.
4. Take square roots with ±: $x + 3 = \pm 2$.
   Why: Both +2 and −2 square to 4.
5. So $x = -3 + 2 = -1$ or $x = -3 - 2 = -5$.

ANSWER: The roots are $x = -1$ and $x = -5$.

CBSE-STYLE MARKING SCHEME: 2 marks: completing step with the number added to both sides (1) + both roots via ± (1).

Completing the Square — Example 2: Fractions

COMPLETING THE SQUARE — EXAMPLE 2

Solve by completing the square: $2x^2 - 7x + 3 = 0$

THINK FIRST: First divide by 2 — completing the square demands coefficient 1 on x^2.

1. Divide every term by 2: $x^2 - (7/2)x + 3/2 = 0$.
2. Move the constant: $x^2 - (7/2)x = -3/2$.
3. Half of 7/2 is 7/4; its square is 49/16. Add 49/16 to both sides: $x^2 - (7/2)x + 49/16 = -3/2 + 49/16 = (-24 + 49)/16 = 25/16$.
   Why: Convert −3/2 to −24/16 before adding.
4. Write as a square: $(x - 7/4)^2 = 25/16$.
5. Take square roots: $x - 7/4 = \pm 5/4$.
   Why: $\sqrt{(25/16)} = 5/4$.
6. $x = 7/4 + 5/4 = 12/4 = 3$, or $x = 7/4 - 5/4 = 2/4 = 1/2$.

ANSWER: The roots are x = 3 and x = 1/2. (Compare with the formula solution in Exercise 4.3 Q1(i) — same answer, as it must be.)

CBSE-STYLE MARKING SCHEME: 3 marks: division by 2 (1) + completing with fractions (1) + both roots (1).

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Completing the Square — When the Square Refuses

COMPLETING THE SQUARE — EXAMPLE 3

Solve by completing the square: $2x^2 + x + 4 = 0$

THINK FIRST: Follow the same recipe and watch what happens on the right side.

1. Divide by 2: $x^2 + (1/2)x + 2 = 0$.
2. Move the constant: $x^2 + (1/2)x = -2$.
3. Half of 1/2 is 1/4; its square is 1/16. Add to both sides: $x^2 + (1/2)x + 1/16 = -2 + 1/16 = -31/16$.
4. Write as a square: $(x + 1/4)^2 = -31/16$.
5. STOP. A square of a real number can never be negative — but the right side is $-31/16 < 0$. No real value of x can satisfy this.

   Why: This is exactly the D < 0 case wearing a different costume: here D = 1 − 32 = −31.

ANSWER: The equation has NO real roots.

CBSE-STYLE MARKING SCHEME: 2 marks: correct completion (1) + the impossibility argument (1).

PIN POINT **COMPLETING THE SQUARE KEY STEP: for $x^2 + px$, add $(p/2)^2$ — the square of HALF the x-coefficient — to BOTH sides. That one move creates the perfect square $(x + p/2)^2$.**

TOPPER TIP: *Notice Examples 2 and 3 are the same equations as Exercise 4.3 Q1(i) and Q1(iv). Different methods, same roots — use one method to solve and the other to check in the exam if time permits.*

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Exercise 4.3 — The Quadratic Formula at Work

Formula discipline: **write a, b, c first — then D — then substitute**. Every solution below follows that order, because the marking scheme rewards it.

EXERCISE 4.3 — Q1(i)

Find the roots using the quadratic formula: $2x^2 - 7x + 3 = 0$

THINK FIRST: a = 2, b = −7, c = 3. Mind the double negative in −b.

1. Identify: a = 2, b = −7, c = 3.
2. Discriminant: $D = b^2 - 4ac = (-7)^2 - 4(2)(3) = 49 - 24 = 25$.
   Why: $(-7)^2 = +49$. Squaring a negative gives a positive.
3. Since D = 25 > 0, two distinct real roots exist, and √D = 5.
4. Apply the formula: $x = [-(-7) \pm 5] / (2 \times 2) = (7 \pm 5)/4$.
   Why: −b = −(−7) = +7 — the step where most sign errors are born.
5. x = (7 + 5)/4 = 12/4 = 3, or x = (7 − 5)/4 = 2/4 = 1/2.

ANSWER: The roots are x = 3 and x = 1/2.

CBSE-STYLE MARKING SCHEME: 2 marks: a, b, c and D (1) + substitution and both roots (1).

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Exercise 4.3 Q1 — (ii) Surd Roots and (iii) Equal Roots

EXERCISE 4.3 — Q1(ii)

Find the roots using the quadratic formula: $2x^2 + x - 4 = 0$

THINK FIRST: a = 2, b = 1, c = −4. D will not be a perfect square — that is fine; leave the answer in surd form.

1. Identify: a = 2, b = 1, c = −4.
2. Discriminant: $D = 1^2 - 4(2)(-4) = 1 + 32 = 33$.
   Why: −4ac with c negative becomes +32.
3. D = 33 > 0, so two distinct real roots exist.
4. Apply the formula: $x = [-1 \pm \sqrt{33}] / (2 \times 2) = (-1 \pm \sqrt{33})/4$.
   Why: 33 = 3 × 11 has no square factor, so √33 cannot be simplified.

ANSWER: The roots are $x = (-1 + \sqrt{33})/4$ and $x = (-1 - \sqrt{33})/4$.

CBSE-STYLE MARKING SCHEME: 2 marks: D with correct sign handling (1) + roots in exact surd form (1).

EXERCISE 4.3 — Q1(iii)

Find the roots using the quadratic formula: $4x^2 + 4\sqrt{3}x + 3 = 0$

THINK FIRST: a = 4, b = 4√3, c = 3. Compute b^2 carefully: $(4\sqrt{3})^2 = 16 \times 3 = 48$.

1. Identify: a = 4, b = 4√3, c = 3.
2. Discriminant: $D = (4\sqrt{3})^2 - 4(4)(3) = 48 - 48 = 0$.
   Why: $(\sqrt{3})^2 = 3$, so $(4\sqrt{3})^2 = 16 \times 3 = 48$.
3. D = 0 means two equal real roots: $x = -b/2a$.
   Why: With √D = 0, the ± branch makes no difference.
4. $x = -4\sqrt{3} / (2 \times 4) = -4\sqrt{3}/8 = -\sqrt{3}/2$.

ANSWER: The roots are $x = -\sqrt{3}/2$ and $x = -\sqrt{3}/2$ (equal roots).

CBSE-STYLE MARKING SCHEME: 2 marks: b^2 computed correctly (1) + equal root stated (1).

Exercise 4.3 Q1(iv) and Q2 — Disguised Quadratics

EXERCISE 4.3 — Q1(iv)

Find the roots using the quadratic formula: $2x^2 + x + 4 = 0$

THINK FIRST: a = 2, b = 1, c = 4. Compute D before touching the formula.

1. Identify: a = 2, b = 1, c = 4.
2. Discriminant: $D = 1^2 - 4(2)(4) = 1 - 32 = -31$.
3. $D = -31 < 0$. The formula would demand $\sqrt{(-31)}$, which is not a real number.
   Why: This is Derivation 2, Case 3, in action.

ANSWER: The equation has NO real roots. (Completing the square agreed — see Example 3 in this section.)

CBSE-STYLE MARKING SCHEME: 2 marks: D computed (1) + correct conclusion (1).

EXERCISE 4.3 — Q2(i)

Find the roots of the equation: $x - 1/x = 3$, $x \neq 0$

THINK FIRST: Multiply through by x to clear the fraction — the quadratic is hiding underneath.

1. Multiply every term by x (allowed, since $x \neq 0$): $x^2 - 1 = 3x$.
   Why: x × (1/x) = 1 — the fraction vanishes.
2. Rearrange to standard form: $x^2 - 3x - 1 = 0$. Here a = 1, b = −3, c = −1.
3. Discriminant: $D = (-3)^2 - 4(1)(-1) = 9 + 4 = 13$.
4. Apply the formula: $x = [3 \pm \sqrt{13}] / 2$.
5. Both values are non-zero, so both are acceptable (neither breaks the $x \neq 0$ condition).
   Why: Always re-check roots against the original restriction.

ANSWER: The roots are $x = (3 + \sqrt{13})/2$ and $x = (3 - \sqrt{13})/2$.

CBSE-STYLE MARKING SCHEME: 2 marks: clearing the fraction (1) + formula application (1).

Exercise 4.3 Q2(ii) — The Collapsing Numerator

EXERCISE 4.3 — Q2(ii)

Find the roots of the equation: $1/(x + 4) - 1/(x - 7) = 11/30$, $x \neq -4, 7$

THINK FIRST: Combine the left fractions first — the numerator collapses to a constant.

1. Combine the left side over the common denominator $(x + 4)(x - 7)$: numerator = $(x - 7) - (x + 4) = -11$. So the equation becomes $-11 / [(x + 4)(x - 7)] = 11/30$.
   Why: The x terms cancel in the numerator — that is the designed beauty of this question.
2. Cancel 11 and cross-multiply: $-30 = (x + 4)(x - 7)$.
   Why: Divide both sides by 11 first: $-1/[(x+4)(x-7)] = 1/30$.
3. Expand the right: $(x + 4)(x - 7) = x^2 - 3x - 28$. So $x^2 - 3x - 28 = -30$.
4. Rearrange: $x^2 - 3x + 2 = 0$. Factorise: $(x - 1)(x - 2) = 0$.
   Why: Pair with product +2, sum −3: −1 and −2.
5. $x = 1$ or $x = 2$. Neither equals -4 or 7, so both respect the restrictions.

ANSWER: The roots are $x = 1$ and $x = 2$.

CBSE-STYLE MARKING SCHEME: 3 marks: combining fractions (1) + reduction to quadratic (1) + roots with restriction check (1).

TOPPER TIP: *Disguised quadratics: when fractions hide the quadratic, clear them first — multiply through by the denominators, then rearrange to standard form and check every root against the original restrictions.*

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Exercise 4.3 Q3 — Nature of Roots

EXERCISE 4.3 — Q3(i)

Find the nature of the roots of the quadratic equation. If real roots exist, find them: $2x^2 - 3x + 5 = 0$

THINK FIRST: Nature first (D), roots only if D ≥ 0.

1. Identify: a = 2, b = −3, c = 5.
2. $D = b^2 - 4ac = (-3)^2 - 4(2)(5) = 9 - 40 = -31$.
   Why: Careful: 9 − 40 is NEGATIVE — see Find-the-Error E5 for the classic slip.
3. Since D < 0, the equation has no real roots. Nothing further to find.
   Why: The question's second part simply does not arise.

ANSWER: Nature: NO real roots (D = −31 < 0).

CBSE-STYLE MARKING SCHEME: 2 marks: D with correct arithmetic (1) + conclusion (1).

EXERCISE 4.3 — Q3(ii)

Find the nature of the roots. If real roots exist, find them: $3x^2 - 4\sqrt{3}x + 4 = 0$

THINK FIRST: b = $-4\sqrt{3}$, so $b^2 = 16 \times 3 = 48$.

1. Identify: a = 3, b = $-4\sqrt{3}$, c = 4.
2. $D = (-4\sqrt{3})^2 - 4(3)(4) = 48 - 48 = 0$.
3. D = 0, so the roots are real and equal: $x = -b/2a = 4\sqrt{3} / 6 = 2\sqrt{3}/3$.
   Why: Simplify 4√3/6 by dividing numerator and denominator by 2. (Equivalently, 2/√3 rationalised.)

ANSWER: Nature: two EQUAL real roots, x = 2√3/3 (each).

CBSE-STYLE MARKING SCHEME: 2 marks: D = 0 shown (1) + the equal root (1).

Exercise 4.3 Q3 — (iii): Roots in Surd Form

EXERCISE 4.3 — Q3(iii)

Find the nature of the roots. If real roots exist, find them: $2x^2 - 6x + 3 = 0$

THINK FIRST: a = 2, b = −6, c = 3.

1. Identify: a = 2, b = −6, c = 3.
2. $D = (-6)^2 - 4(2)(3) = 36 - 24 = 12$.
3. D = 12 > 0, so the roots are real and distinct.
4. Apply the formula: $x = [6 \pm \sqrt{12}] / 4 = [6 \pm 2\sqrt{3}] / 4$.
   Why: Simplify √12 = √(4 × 3) = 2√3.
5. Divide numerator and denominator by 2: $x = (3 \pm \sqrt{3})/2$.
   Why: Never leave [6 ± 2√3]/4 unsimplified — it costs polish marks in strict marking.

ANSWER: Nature: two distinct real roots, $x = (3 + \sqrt{3})/2$ and $x = (3 - \sqrt{3})/2$.

CBSE-STYLE MARKING SCHEME: 2 marks: D and nature (1) + simplified roots (1).

Exercise 4.3 Q4 — Finding k for Equal Roots

EXERCISE 4.3 — Q4(i)

Find the value of k such that the following quadratic equation has two equal roots: $2x^2 + kx + 3 = 0$

THINK FIRST: Equal roots is code for D = 0.

1. Identify a = 2, b = k, c = 3. For equal roots, D = 0: $b^2 - 4ac = k^2 - 4(2)(3) = k^2 - 24 = 0$.
2. So $k^2 = 24$, giving $k = \pm\sqrt{24} = \pm 2\sqrt{6}$.
   Why: $\sqrt{24} = 2\sqrt{6}$. Keep the ± — see Find-the-Error E8.

ANSWER: $k = 2\sqrt{6}$ or $k = -2\sqrt{6}$.

CBSE-STYLE MARKING SCHEME: 2 marks: D = 0 condition (1) + both values of k with ± (1).

EXERCISE 4.3 — Q4(ii)

Find the value of k such that the following quadratic equation has two equal roots: $kx(x - 2) + 6 = 0$

THINK FIRST: First expand into standard form — k is hiding in two places.

1. Expand: $kx^2 - 2kx + 6 = 0$, so a = k, b = −2k, c = 6. For equal roots, D = 0: $(-2k)^2 - 4(k)(6) = 4k^2 - 24k = 0$.
   Why: Note: $k \neq 0$, else it is not quadratic at all.
2. Factorise: $4k(k - 6) = 0$, so k = 0 or k = 6.
3. **Reject k = 0**: it destroys the quadratic (the equation becomes 6 = 0). Hence k = 6.
   Check: $6(x - 1)^2 = 0$ — equal roots 1, 1. Correct!
   Why: Reject any parameter value that kills the x^2 term.

ANSWER: k = 6.

CBSE-STYLE MARKING SCHEME: 2 marks: standard form (1/2) + D = 0 and factorisation (1) + rejection of k = 0 (1/2).

PIN POINT **EQUAL ROOTS = D = 0 — first line $b^2 - 4ac = 0$, and keep the ±.**

Exercise 4.3 Q5 to Q7 — Is It Even Possible?

EXERCISE 4.3 — Q5

Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2? If so, find its length and breadth.

THINK FIRST: Set up the equation and let the discriminant judge possibility.

1. Let the breadth be x m. Then the length = 2x m.
2. Area: $x \times 2x = 800$, so $2x^2 = 800$, i.e. $x^2 = 400$.
3. Take square roots: $x = \pm 20$.
   Why: $x^2 = 400$ has two mathematical solutions.
4. **Reject x = −20** (breadth cannot be negative). So breadth = 20 m and length = 2×20 = 40 m.
5. Check: $20 \times 40 = 800$ m^2. The design works.

ANSWER: Yes, it is possible: breadth = 20 m and length = 40 m.

CBSE-STYLE MARKING SCHEME: 2 marks: equation (1) + positive root with rejection and both dimensions (1).

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Exercise 4.3 Q6 — The Friends' Ages: an Impossible Situation

EXERCISE 4.3 — Q6

Is the following situation possible? If so, determine their present ages: The sum of the ages of two friends is 20 years. Four years ago, the product of their ages was 48.

THINK FIRST: Build the equation, then let D deliver the verdict.

1. Let one friend's present age be x years. The other's is (20 − x) years.
2. Four years ago their ages were (x − 4) and (20 − x − 4) = (16 − x).
3. Product four years ago: (x − 4)(16 − x) = 48.
4. Expand: $16x - x^2 - 64 + 4x = 48$, i.e. $-x^2 + 20x - 64 = 48$.
5. Rearrange and multiply by −1: $x^2 - 20x + 112 = 0$.
6. Discriminant: $D = (-20)^2 - 4(1)(112) = 400 - 448 = -48$.
7. D < 0, so the equation has no real roots — no pair of real ages can satisfy both conditions.

   Why: The situation collapses not on a rejected root, but because the equation itself has no solution. Two different kinds of 'impossible' – know both.

ANSWER: No — the situation is NOT possible (D = −48 < 0).

CBSE-STYLE MARKING SCHEME: 3 marks: age set-up (1) + equation (1) + discriminant verdict (1).

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Exercise 4.3 Q7 — The Park: Possible or Not?

EXERCISE 4.3 — Q7

Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2? If so, find its length and breadth.

THINK FIRST: Perimeter gives l + b = 40; area gives lb = 400. One unknown does both jobs.

1. Let the length be x m. Perimeter 80 means length + breadth = 40, so breadth = (40 − x) m.
   Why: Perimeter = 2(l + b) = 80, so l + b = 40.
2. Area: $x(40 - x) = 400$.
3. Expand and rearrange: $40x - x^2 = 400$, so $x^2 - 40x + 400 = 0$.
4. Discriminant: $D = (-40)^2 - 4(1)(400) = 1600 - 1600 = 0$.
   Why: D = 0 — the design is possible, but only just: exactly one shape works.
5. $x = 40/2 = 20$. So length = 20 m and breadth = 40 − 20 = 20 m.
6. Check: perimeter = 2(20 + 20) = 80 m; area = 400 m^2. The 'rectangle' is in fact a square — a square IS a rectangle, so the design is valid.

ANSWER: Yes, it is possible: the park is 20 m by 20 m (a square).

CBSE-STYLE MARKING SCHEME: 3 marks: semi-perimeter set-up (1) + equation and D = 0 (1) + dimensions with check (1).

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08 Multiple Solution Methods

CHALLENGE — ONE EQUATION, THREE KEYS

Return to Meena's garden equation from Section 03: **$x^2 + 5x - 24 = 0$**. We solved it by factorisation there. Now solve it by all three methods and watch them agree — that agreement is your built-in exam checker.

Method 1 — Factorisation

1. Pair with product −24 and sum +5: +8 and −3.
2. $x^2 + 8x - 3x - 24 = x(x + 8) - 3(x + 8) = (x + 8)(x - 3) = 0$.
3. $x = -8$ or $x = 3$.

Method 2 — Completing the Square

1. Move the constant: $x^2 + 5x = 24$.
2. Half of 5 is 5/2; its square is 25/4. Add to both sides: $x^2 + 5x + 25/4 = 24 + 25/4 = 121/4$.
3. Write as a square: $(x + 5/2)^2 = 121/4$.
4. Take square roots: $x + 5/2 = \pm 11/2$.
   Why: Careful: √(121/4) = 11/2 — the root of the denominator 4 is 2, not 4.
5. $x = -5/2 + 11/2 = 6/2 = 3$, or $x = -5/2 - 11/2 = -16/2 = -8$.

Method 3 — Quadratic Formula

1. $a = 1, b = 5, c = -24$. $D = 25 - 4(1)(-24) = 25 + 96 = 121$.
2. $x = [-5 \pm \sqrt{121}] / 2 = (-5 \pm 11)/2$.
3. $x = 6/2 = 3$, or $x = -16/2 = -8$.

ANSWER (all three methods): x = 3 or x = −8 — and for the garden, width = 3 m (rejecting −8).

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Which Method, When? — The Honest Comparison

Method	Fastest when…	Struggles when…	Exam advice
Factorisation	D is a perfect square and factors are easy to spot	Roots are irrational or fractions with big denominators	Try first — 20 seconds of hunting can save 2 minutes
Completing the square	The question demands it, or you must prove something about roots	$a \neq 1$ with awkward fractions	Also the method behind the formula — and behind vertex/ graph questions
Quadratic formula	Always works — the universal key	Arithmetic slips with signs and big numbers	Your safety net: write a, b, c and D first, every time

The discriminant is the method-picker

Compute D in your head before choosing: **D a perfect square** (0, 1, 4, 9, 25, 49, 121, 729…) → factorisation will work. **D not a perfect square** → go straight to the formula and keep the surd form. **D < 0** → stop; there are no real roots to find. Thirty seconds of D saves five minutes of doomed factor-hunting.

TOPPER TIP: *In the exam, solve by your strongest method, then verify by substitution: put each root back into the original equation. A root that does not satisfy the equation is a wrong root, whatever the method said.*

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09 Hands-on Activities

APPLY • VISUALISE — THE GARDEN SIMULATION LAB

This is the print version of the KalviYogi simulation: you are the slider. Change the width x of Meena's garden, and watch the length (x + 5) and the area x(x + 5) respond. Complete the table by hand first — the answers are upside knowledge you should earn.

Width x (m)	Length x + 5 (m)	Area x(x + 5) (m^2)	Compared to target 24
1	6	6	Too small
2	7	14	Too small
3	8	**24**	**TARGET HIT**
4	9	36	Too big
5	10	50	Too big
6	11	66	Too big

What the simulation teaches: as x grows, the area does not grow evenly — it grows *faster and faster* (6, 14, 24, 36, 50, 66: the jumps are 8, 10, 12, 14, 16). That accelerating growth is the fingerprint of x^2. The target 24 is hit exactly once for positive x — at x = 3 — which is why the equation x(x + 5) = 24 has exactly one sensible garden answer even though it has two roots.

Activity 2 — The Discriminant Dial

Take the family $x^2 - 4x + c = 0$ and turn the dial c. For each c in the table, compute D = 16 − 4c and state the nature of roots. (Answers in the Answer Key.)

c	1	3	4	5	8
D = 16 − 4c	—	—	—	—	—
Nature	—	—	—	—	—

Watch the exact moment the roots merge (D = 0) and then vanish (D < 0) — you are watching Section 04's three pictures change into each other.

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10 Common Mistakes

CHALLENGE — LEARN FROM OTHER STUDENTS' SLIPS

COMMON MISTAKE: Dividing both sides by x. Wrong: $x^2 = 5x \rightarrow$ divide by x → x = 5. You have thrown away the root x = 0 — division by x is illegal when x might be 0, and here it IS one of the answers. **Right: $x^2 - 5x = 0 \rightarrow x(x - 5) = 0 \rightarrow$ x = 0 or 5.** (Full treatment: Find-the-Error E1.)

COMMON MISTAKE: Forgetting the ±. Wrong: $(x + 3)^2 = 4 \rightarrow x + 3 = 2 \rightarrow x = -1$ only. Right: x + 3 = ±2, so x = −1 or −5. Every square-root step in this chapter has two branches.

COMMON MISTAKE: The $(a + b)^2$ shortcut. Wrong: $(x + 1)^2 = x^2 + 1$. Right: $(x + 1)^2 = x^2 + 2x + 1$. The middle term 2ab is not optional decoration.

COMMON MISTAKE: Sign chaos in the formula. With b = −7, students write −b = −7. **−b means 'the negative of b': if b = −7 then −b = +7.** Write a, b, c on paper first, then substitute with brackets: x = [−(−7) ± ...]/...

COMMON MISTAKE: Accepting impossible roots. A width of −8 m, a speed of −32 km/h, an age of −39 years — each satisfies its equation and each is nonsense in the story. **Always finish word problems by judging the roots against real life, and write the rejection reason.**

COMMON MISTAKE: Setting D > 0 for equal roots. Equal roots need the two formula branches to coincide, which happens only when √D = 0. **Equal roots ↔ D = 0. Distinct roots ↔ D > 0. No real roots ↔ D < 0.**

COMMON MISTAKE: Splitting with the wrong target. For $2x^2 + x - 6$, students hunt for a pair with product −6. **The product target is a × c = 2 × (−6) = −12, not c alone.** When a = 1 the two coincide, which is why the slip survives unnoticed until a ≠ 1.

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Activity 3 — Graph Lab and Your Digital Companion

Activity 3 — Graph Lab: $y = x^2 - 4$

Complete the table, plot the points on squared paper, and read the roots off your graph where the curve crosses y = 0.

x	−3	−2	−1	0	1	2	3
$y = x^2 - 4$	5	__	−3	__	−3	__	5

Your curve should cross the axis at exactly the roots of $x^2 - 4 = 0$. Solve it by factorisation and compare.

DIGITAL COMPANION: Interactive simulations, video lessons and online tests for this chapter are linked by QR code in the printed edition. QR code and link activate with the KalviYogi app/website at print.

TOPPER TIP: *In the exam, 'read the roots off the graph' means: where the parabola crosses the x-axis. Two crossings, two roots; one touch, equal roots; no crossing, no real roots — the picture and the algebra always agree.*

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Find the Error — 10 Worked Solutions, 10 Hidden Slips

Each solution below was written by a student — and each contains **exactly one error**. Your job: find it, name it, and write the corrected finish. Answers follow each item (cover them first!).

E1. Solve $x^2 = 5x$. *Student:* "Divide both sides by x: x = 5."

Error: dividing by x throws away the root x = 0 (division by a quantity that may be zero is invalid). **Correction:** $x^2 - 5x = 0 \rightarrow x(x - 5) = 0 \rightarrow$ **x = 0 or x = 5**.

E2. Solve $(x - 3)^2 = 16$. *Student:* "x − 3 = 4, so x = 7."

Error: the ± is missing — both 4 and −4 square to 16. **Correction:** $x - 3 = \pm 4 \rightarrow$ **x = 7 or x = −1**.

E3. Solve $x^2 - 5x + 6 = 0$ by formula. *Student:* "D = 25 − 24 = 1; x = (−5 ± 1)/2, so x = −2 or −3."

Error: −b was taken as −5, but b = −5 so −b = +5. **Correction:** $x = (5 \pm 1)/2 \rightarrow$ **x = 3 or x = 2**. (Check by factorisation: (x − 2)(x − 3).)

E4. Factorise $x^2 - 5x + 6$. *Student:* "Split −5x as −6x + x: x(x − 6) + 1(x + 6) — stuck."

Error: the pair −6, +1 has product −6, but the target product is **+6** (and the brackets do not match). **Correction:** pair −2, −3 (product +6, sum −5): $x^2 - 2x - 3x + 6 = x(x - 2) - 3(x - 2)$ = **(x − 2)(x − 3)**.

E5. Nature of roots of $2x^2 - 3x + 5 = 0$. *Student:* "D = 9 − 40 = 31 > 0, so real and distinct roots."

Error: pure arithmetic: 9 − 40 = **−31**, not 31. **Correction:** D = −31 < 0 → **no real roots**. (This exact equation is NCERT Ex 4.3 Q3(i).)

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Find the Error — E6 to E10

E6. Garden: width x, length x + 5, area 24. *Student:* "x = 3 or −8, so the width is −8 m or 3 m."

Error: reporting a negative width as a possible answer. **Correction:** reject x = −8 with the reason (width cannot be negative) → **width 3 m, length 8 m**.

E7. Completing the square on $x^2 + 6x + 5 = 0$. *Student:* "$x^2 + 6x = -5$; add 9: $(x + 3)^2 = -5$; no solution."

Error: 9 was added to the left side only. **Correction:** add 9 to BOTH sides: $(x + 3)^2 = -5 + 9 = 4$ → $x + 3 = \pm 2$ → **x = −1 or −5**.

E8. Equal roots of $2x^2 + kx + 3 = 0$. *Student:* "$k^2 = 24$, so $k = 2\sqrt{6}$."

Error: half the answer — $k^2 = 24$ has two solutions. **Correction: k = ±2√6**. Both values genuinely give equal roots.

E9. Train (480 km, 8 km/h less, 3 h more). *Student:* "x = 40 or −32, so the speed is −32 km/h."

Error: choosing the impossible root (and ignoring the valid one). **Correction:** speed cannot be negative → **40 km/h**. Check: 12 h vs 15 h.

E10. Is $(x + 1)^2 = 2(x - 3)$ quadratic? *Student:* "$(x + 1)^2 = x^2 + 1$, so $x^2 + 1 = 2x - 6 \rightarrow x^2 - 2x + 7 = 0$."

Error: $(x + 1)^2 = x^2 + \mathbf{2x} + 1$ — the middle term was dropped (though the final verdict 'quadratic' accidentally stayed right). **Correction:** $x^2 + 2x + 1 = 2x - 6 \rightarrow \mathbf{x^2 + 7 = 0}$, still quadratic — but with b = 0, not −2.

PIN POINT **THE ERROR HUNTER'S RULE: after finishing any solution, re-read it once as an enemy. Check (1) signs, (2) ±, (3) both roots judged, (4) substitution check. Most lost marks are not unknown maths — they are unfound slips.**

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11 Graded Practice Problems

SOLVE — FIVE LEVELS, FOUNDATION TO GENIUS

All questions here are original KalviYogi practice. Climb the levels in order — each level assumes the one below. Answers are in the Answer Key at the end of the chapter. Attempt first, check later.

LEVEL 1 — FOUNDATION

L1.1 Write $2x^2 = 7x - 3$ in standard form and state a, b, c.

L1.2 Check whether $x = 2$ is a root of $x^2 - 3x + 2 = 0$.

L1.3 Solve by factorisation: $x^2 - 9 = 0$.

L1.4 Solve by factorisation: $x^2 + 5x + 6 = 0$.

L1.5 Find the discriminant of $2x^2 - 4x + 3 = 0$ and state the nature of its roots.

L1.6 Solve: $x^2 - 4x = 0$. (Careful — do not divide by x!)

LEVEL 2 — SKILL BUILDER

L2.1 Solve by factorisation: $3x^2 - 7x - 6 = 0$.

L2.2 Solve by factorisation: $6x^2 - x - 2 = 0$.

L2.3 Solve using the quadratic formula: $x^2 - 3x - 4 = 0$.

L2.4 Solve using the quadratic formula: $2x^2 - 5x + 2 = 0$.

L2.5 Find k if $x^2 + kx + 9 = 0$ has two equal roots.

L2.6 The sum of a number and its reciprocal is 10/3. Find the number. (Two answers!)

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Graded Practice — Levels 3, 4 and 5

LEVEL 3 — APPLICATION

L3.1 A rectangular garden has area 60 m^2. Its length is 7 m more than its width. Find the length and width.

L3.2 One tap can fill a tank in x hours; a second tap takes 3 hours longer. Together they fill the tank in 2 hours. Find how long each tap takes alone.

L3.3 A motorboat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream. Find the speed of the stream.

L3.4 Rani is 6 years older than her brother. The product of their ages is 72. Find their ages.

LEVEL 4 — ADVANCED REASONING

L4.1 Find the values of k for which $x^2 - 2kx + 7k - 12 = 0$ has equal roots.

L4.2 One root of $x^2 - 7x + k = 0$ is 2. Find k and the other root. (Hint: use sum of roots = −b/a.)

L4.3 Two numbers differ by 3 and their product is 504. Find the numbers.

LEVEL 5 — MATHEMATICAL GENIUS

L5.1 Solve the biquadratic equation $x^4 - 5x^2 + 4 = 0$. (Hint: substitute $y = x^2$, solve for y, then come back.)

L5.2 The equation $x^2 - 6x + k = 0$ has integer roots. Find ALL integer values of k from 0 to 9 for which this happens. (Hint: D must be a perfect square.)

> *TOPPER TIP: Stuck at a level? Drop one level down, solve two questions there, and climb again. Levels are stairs, not walls.*

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Concept MCQs — 25 Rapid Checks (Part 1)

One mark each, board style. Answer all 25 before opening the key — the key explains every answer in one line.

MCQ 1. The degree of a quadratic equation is

(a) 1 (b) 2 (c) 3 (d) any number

MCQ 2. Which of the following is a quadratic equation?

(a) $x^3 - x = 0$ (b) $1/x + 1 = 0$ (c) $x^2 - 2x + 1 = 0$ (d) $\sqrt{x} + 2 = 0$

MCQ 3. The roots of $x^2 - 7x + 10 = 0$ are

(a) 2 and 5 (b) -2 and -5 (c) 7 and 10 (d) 3 and 4

MCQ 4. The discriminant of $x^2 - 4x + 4 = 0$ is

(a) 16 (b) 8 (c) 0 (d) -16

MCQ 5. If $D < 0$ for a quadratic equation, its roots are

(a) real and distinct (b) real and equal (c) no real roots (d) always zero

MCQ 6. The sum of the roots of $2x^2 - 3x + 5 = 0$ is

(a) $-3/2$ (b) $3/2$ (c) $5/2$ (d) $-5/2$

MCQ 7. The product of the roots of $3x^2 - 6x + 2 = 0$ is

(a) 2 (b) -2 (c) $2/3$ (d) $3/2$

MCQ 8. The roots of $x^2 - 9x = 0$ are

(a) 0 and 9 (b) 9 only (c) 0 and -9 (d) 3 and -3

MCQ 9. If $x = 3$ is a root of $x^2 - 5x + k = 0$, then $k =$

(a) 4 (b) 6 (c) -6 (d) 15

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Concept MCQs — Part 2

MCQ 10. For equal roots of $4x^2 - 12x + k = 0$, k =

(a) 3 (b) 6 (c) 9 (d) 12

MCQ 11. The nature of roots of $3x^2 - 2x + 1 = 0$ is ($D = -8$)

(a) real and distinct (b) real and equal (c) no real roots (d) cannot say

MCQ 12. The equation whose roots are 3 and -2 is

(a) $x^2 + x - 6 = 0$ (b) $x^2 - x - 6 = 0$ (c) $x^2 - 5x - 6 = 0$ (d) $x^2 + 5x + 6 = 0$

MCQ 13. The roots of $x^2 - 16 = 0$ are

(a) 4 only (b) -4 only (c) ± 4 (d) 16

MCQ 14. One root of $x^2 - 5x + 6 = 0$ is 2. The other root is

(a) 6 (b) 5 (c) 3 (d) -3

MCQ 15. If $x = -1$ is a root of $x^2 + kx - 6 = 0$, then k =

(a) -5 (b) 5 (c) 7 (d) -7

MCQ 16. The discriminant of $2x^2 - 7x + 3 = 0$ is

(a) 25 (b) 49 (c) 73 (d) 1

MCQ 17. The roots of $100x^2 - 20x + 1 = 0$ are

(a) distinct (b) equal, $x = 1/10$ (c) equal, $x = 10$ (d) not real

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Concept MCQs — Part 3

MCQ 18. In standard form, $(x - 2)^2 = x + 3$ becomes $x^2 - 5x + 1 = 0$. Its constant term c is

(a) −5 (b) 3 (c) 1 (d) −1

MCQ 19. The equation $(x - 1)^2 = 0$ has

(a) no roots (b) two equal roots (c) two distinct roots (d) three roots

MCQ 20. If $\alpha + \beta = 4$ and $\alpha\beta = 3$, the quadratic equation with roots α, β is

(a) $x^2 + 4x + 3 = 0$ (b) $x^2 - 4x - 3 = 0$ (c) $x^2 - 4x + 3 = 0$ (d) $x^2 + 3x + 4 = 0$

MCQ 21. Which of the following is NOT a quadratic equation?

(a) $(x + 1)^2 = 2(x - 3)$ (b) $(x - 2)(x + 1) = (x - 1)(x + 3)$ (c) $x^2 + 7 = 0$ (d) $(x - 3)(2x + 1) = x(x + 5)$

MCQ 22. The roots of $x^2 - 3x = 0$ are

(a) 3 only (b) 0 and 3 (c) 0 and −3 (d) ±3

MCQ 23. For a quadratic equation to have real roots, D must satisfy

(a) $D < 0$ (b) $D = 0$ only (c) $D \geq 0$ (d) $D > 1$

MCQ 24. In $x^2 - 4x + k = 0$, the equation has no real roots when

(a) $k < 4$ (b) $k = 4$ (c) $k > 4$ (d) $k = 0$

MCQ 25. The quadratic formula for $ax^2 + bx + c = 0$ is

(a) $x = [b \pm \sqrt{(b^2 - 4ac)}]/2a$ (b) $x = [-b \pm \sqrt{(b^2 - 4ac)}]/2a$ (c) $x = [-b \pm \sqrt{(b^2 + 4ac)}]/2a$ (d) $x = [-b \pm (b^2 - 4ac)]/2a$

> **TOPPER TIP:** *Scoring guide: 22–25 — board-ready. 16–21 — revise the Discriminant and sum/product cards. Below 16 — redo Sections 05 and 06, then retake.*

12 Mathematical Reasoning

REASON — PROVE IT, DO NOT JUST COMPUTE IT

REASONING 1

Prove that if $ad \neq bc$, then the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ has no real roots.

THINK FIRST: This is the discriminant used as a proof machine: compute D, force it into a square, read its sign.

1. Compare with $Ax^2 + Bx + C = 0$: here $A = a^2 + b^2$, $B = -2(ac + bd)$, $C = c^2 + d^2$.
   Why: Name the coefficients before computing — with letters, one slip ruins everything.
2. Compute $D = B^2 - 4AC = 4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2)$.
   Why: B has a factor 2, so B^2 has a factor 4; factor the 4 out of the whole expression.
3. Expand the first square: $(ac + bd)^2 = a^2c^2 + 2abcd + b^2d^2$.
4. Expand the product: $(a^2 + b^2)(c^2 + d^2) = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2$.
5. Subtract: $D/4 = (a^2c^2 + 2abcd + b^2d^2) - (a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2) = 2abcd - a^2d^2 - b^2c^2 = -(a^2d^2 - 2abcd + b^2c^2) = -(ad - bc)^2$.
   Why: The a^2c^2 and b^2d^2 terms cancel; what remains is a perfect square with a minus sign in front.
6. So $D = -4(ad - bc)^2$. Since $ad \neq bc$, $(ad - bc)^2 > 0$, and therefore $D < 0$.
   Why: A square of a non-zero number is positive; with the minus sign, D is strictly negative.
7. By Derivation 2 ($D < 0$ case), the equation has no real roots. Proved.
   Why: Bonus insight: if $ad = bc$, then $D = 0$ and the roots are real and equal — the boundary case.

ANSWER: PROVED: $D = -4(ad - bc)^2 < 0$ whenever $ad \neq bc$, so there are no real roots.

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Reasoning 2 — One Equation, Two Proofs

REASONING 2

Show that $x^2 - 4x + 13 = 0$ has no real roots — in TWO different ways.

THINK FIRST: Way 2 is secretly the graph: $(x - 2)^2 + 9$ says the parabola's lowest point is 9 units above the axis.

1. **Way 1 — Discriminant:** a = 1, b = −4, c = 13, so D = 16 − 52 = −36 < 0. No real roots.
   Why: Thirty seconds. This is the board's expected method.

2. **Way 2 — Completing the square:** $x^2 - 4x + 13 = (x - 2)^2 + 9$, because $(x - 2)^2 = x^2 - 4x + 4$ and 4 + 9 = 13.
   Why: Rewrite the expression as a square plus a leftover constant.

3. Now $(x - 2)^2 \geq 0$ for every real x, so $(x - 2)^2 + 9 \geq 9 > 0$. The expression is always positive — it can never equal 0.
   Why: A sum of a square and a positive number is positive. No value of x can fix that.

4. Both ways agree: no real roots.
   Why: Two independent methods, one conclusion — this is what 'mathematical certainty' feels like.

ANSWER: SHOWN: D = −36 < 0, and equivalently $x^2 - 4x + 13 = (x - 2)^2 + 9 > 0$ for all real x.

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Reasoning 3 — A Pattern Worth Proving

REASONING 3

Prove that the product of two consecutive odd integers, increased by 1, is always a perfect square.

THINK FIRST: Numerical examples suggest; algebra proves. One letter n does the work of infinitely many examples.

1. Write the two consecutive odd integers as $(2n - 1)$ and $(2n + 1)$, where n is any integer.

   Why: Every odd number is one away from an even number 2n; consecutive odd numbers sit on either side of it.

2. Their product: $(2n - 1)(2n + 1) = (2n)^2 - 1^2 = 4n^2 - 1$.

   Why: Use the difference-of-squares identity $a^2 - b^2 = (a + b)(a - b)$ in reverse.

3. Increase by 1: $4n^2 - 1 + 1 = 4n^2 = (2n)^2$.

   Why: The −1 and +1 cancel exactly.

4. $(2n)^2$ is the square of the integer 2n — a perfect square. Proved.

   Why: Test it: $3 \times 5 + 1 = 16 = 4^2$; $7 \times 9 + 1 = 64 = 8^2$. The algebra explains the pattern you can see.

ANSWER: PROVED: $(2n - 1)(2n + 1) + 1 = (2n)^2$, a perfect square.

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13 Real-Life Applications

APPLY — QUADRATICS AT WORK

APPLICATION 1 — THE PARK

A rectangular park has perimeter 100 m and area 600 m^2. Find its length and breadth.

THINK FIRST: When both roots are positive in a length-breadth problem, they usually describe the same rectangle twice.

1. Let the length be x m. Since l + b = 100/2 = 50, the breadth = (50 − x) m.
   Why: Semi-perimeter first — the same move as NCERT Q7.
2. Area: $x(50 - x) = 600$, so $x^2 - 50x + 600 = 0$.
3. Factorise: pair with product 600, sum −50: −20 and −30. $(x - 20)(x - 30) = 0$.
4. x = 20 or x = 30. If length = 30, breadth = 20; if length = 20, breadth = 30. Same park, two namings.
   Why: Both roots are positive and both are correct — they swap the roles of length and breadth.
5. Check: 2(30 + 20) = 100 m; 30 × 20 = 600 m^2. Correct!

ANSWER: The park is 30 m by 20 m.

Application 2 — The Train (360 km)

APPLICATION 2 — THE TRAIN

A train covers 360 km at a uniform speed. If its speed had been 5 km/h more, the journey would have taken 1 hour less. Find the original speed.

THINK FIRST: Speed problems always come down to time = distance/speed and one honest subtraction.

1. Let the original speed be x km/h. Original time = 360/x h; faster time = 360/(x + 5) h.
2. The faster journey takes 1 hour LESS: 360/x − 360/(x + 5) = 1.
   Why: This time the ORIGINAL time is the bigger one — read the sentence direction carefully.
3. Combine over the common denominator x(x + 5): 360[(x + 5) − x] / [x(x + 5)] = 1, i.e. 360 × 5 = x(x + 5).
4. So $x^2 + 5x - 1800 = 0$. D = 25 + 7200 = 7225 = 85^2.
   Why: $7225 = 85^2$ — spotting it makes the formula painless.
5. x = (−5 ± 85)/2 = 40 or −45. **Reject −45** (speed).
6. Check: 360/40 = 9 h; 360/45 = 8 h. Difference exactly 1 h. Correct!

ANSWER: The original speed of the train is 40 km/h.

Applications 3 and 4 — Taps and the Field

APPLICATION 3 — THE TWO TAPS

Two taps together can fill a tank in 6 hours. The larger tap alone takes 5 hours less than the smaller tap alone. Find the time each tap takes separately.

THINK FIRST: In tap problems, reject any root that makes ANY time in the story negative — not just the one you named x.

1. Let the smaller tap take x hours alone. Then the larger takes (x − 5) hours.
   Why: '5 hours less' belongs to the larger, faster tap.
2. In one hour: smaller fills 1/x of the tank, larger fills 1/(x − 5), together they fill 1/6.
   Why: Rates add — that is the whole secret of tap problems.
3. Equation: $1/x + 1/(x - 5) = 1/6$.
4. Combine: $[(x - 5) + x] / [x(x - 5)] = 1/6$, i.e. $(2x - 5)/[x(x - 5)] = 1/6$.
5. Cross-multiply: $6(2x - 5) = x(x - 5)$, so $12x - 30 = x^2 - 5x$, giving $x^2 - 17x + 30 = 0$.
6. Factorise: $(x - 15)(x - 2) = 0$, so $x = 15$ or $x = 2$.
   Why: Pair with product 30, sum −17: −15, −2.
7. **Reject x = 2**: then the larger tap's time would be 2 − 5 = −3 hours — impossible. So x = 15.
   Why: Here the rejected root is positive but still impossible — judge roots against the WHOLE story, not just the sign.
8. Smaller tap: 15 h; larger tap: 15 − 5 = 10 h. Check: 1/15 + 1/10 = 2/30 + 3/30 = 5/30 = 1/6. Together: 6 h. Correct!

ANSWER: The smaller tap takes 15 hours and the larger tap takes 10 hours.

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Application 4 — The Field Diagonal

APPLICATION 4 — THE FIELD DIAGONAL

The diagonal of a rectangular field is 60 m more than its shorter side. The longer side is 30 m more than the shorter side. Find the sides of the field.

THINK FIRST: Pythagoras + a quadratic: the same partnership as Ex 4.2 Q5, in a field instead of a triangle.

1. Let the shorter side be x m. Then longer side = (x + 30) m and diagonal = (x + 60) m.
   Why: Three quantities, one unknown — the story ties them all to x.
2. Pythagoras: $x^2 + (x + 30)^2 = (x + 60)^2$.
   Why: The diagonal is the hypotenuse.
3. Expand: $x^2 + x^2 + 60x + 900 = x^2 + 120x + 3600$.
4. Collect: $2x^2 + 60x + 900 - x^2 - 120x - 3600 = 0$, i.e. $x^2 - 60x - 2700 = 0$.
5. Formula: $D = 3600 + 10800 = 14400 = 120^2$. $x = (60 \pm 120)/2 = 90$ or -30.
6. **Reject −30** (a side). Shorter side = 90 m; longer = 120 m; diagonal = 150 m.
7. Check: $90^2 + 120^2 = 8100 + 14400 = 22500 = 150^2$. Correct — a scaled 3-4-5 triangle!

ANSWER: The sides are 90 m and 120 m (diagonal 150 m).

14 Competency-Based Questions

APPLY — FIVE CASE STUDIES, TWENTY QUESTIONS

Board papers now test whether you can *use* mathematics inside a real situation. Each case below carries four questions. Solutions are shown in compact steps — the full one-by-one method is the one you practised in Section 07.

CASE STUDY 1 — The School Vegetable Garden

Green Valley School is making a rectangular vegetable garden. The length is 4 m more than the width, and the area is 96 m^2. Let the width be x m.

(i) Form the quadratic equation. Length = x + 4, so x(x + 4) = 96 → **$x^2 + 4x - 96 = 0$**.

(ii) Find the width. Factorise: (x + 12)(x − 8) = 0 → x = 8 or −12. Reject −12. **Width = 8 m**.

(iii) Find the length. Length = 8 + 4 = **12 m**. Check: 8 × 12 = 96. Correct.

(iv) Cost of fencing all four sides at Rs 50 per metre? Perimeter = 2(12 + 8) = 40 m. Cost = 40 × 50 = **Rs 2000**.

CASE STUDY 2 — The Thrown Ball

A ball is thrown straight up. Its height after t seconds is $h = 20t - 5t^2$ metres.

(i) Height after 1 second? h = 20(1) − 5(1) = **15 m**.

(ii) When is the height 20 m? $20t - 5t^2 = 20$ → divide by 5: $4t - t^2 = 4$ → $t^2 - 4t + 4 = 0$ → $(t - 2)^2 = 0$ → **t = 2 s** (the peak — equal roots!)

(iii) When does the ball return to the ground? h = 0: t(20 − 5t) = 0 → t = 0 (launch) or **t = 4 s**.

(iv) What is the greatest height? At t = 2: h = 40 − 20 = **20 m**. Notice (ii) and (iv) agree — 20 m is reached only at the peak, which is why the equation had equal roots.

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Case Studies 3, 4 and 5

CASE STUDY 3 — The Pottery Workshop

A workshop produces x pottery items in a day. The cost of production of each item is Rs (2x + 5), and the total cost of production for the day is Rs 250.

(i) Form the equation. $x(2x + 5) = 250 \rightarrow$ **$2x^2 + 5x - 250 = 0$**.

(ii) How many items are produced? $D = 25 + 2000 = 2025 = 45^2$. $x = (-5 \pm 45)/4 = 10$ or -12.5. Reject the negative/fractional count. **10 items**.

(iii) Cost per item? $2(10) + 5 =$ **Rs 25**. Check: $10 \times 25 = 250$. Correct.

(iv) If production doubles to 20 items, what is the new total cost under the same rule? Cost per item = $2(20) + 5 =$ Rs 45. Total = $20 \times 45 =$ **Rs 900**.

CASE STUDY 4 — The Even Numbers Puzzle

The product of two consecutive positive even integers is 168. Let the smaller be x.

(i) Form the equation. The next even integer is x + 2: **$x^2 + 2x - 168 = 0$**.

(ii) Solve it. $D = 4 + 672 = 676 = 26^2$. $x = (-2 \pm 26)/2 = 12$ or -14. Positive: **x = 12**.

(iii) The two integers? 12 and 14. Check: $12 \times 14 = 168$. Correct.

(iv) Sum of their squares? $144 + 196 =$ **340**.

CASE STUDY 5 — The River Trip

A boat travels 24 km downstream and then returns 24 km upstream, taking 5 hours in total. The speed of the stream is 2 km/h. Let the speed of the boat in still water be x km/h.

(i) Downstream and upstream speeds? Downstream: **(x + 2) km/h**; upstream: **(x − 2) km/h**. The stream helps one way and fights the other.

(ii) Form the equation. Time down + time up = 5: $24/(x + 2) + 24/(x - 2) = 5$.

(iii) Solve it. Combine: $24[(x - 2) + (x + 2)] / (x^2 - 4) = 5 \rightarrow 48x = 5(x^2 - 4) \rightarrow 5x^2 - 48x - 20 = 0$. $D = 2304 + 400 = 2704 = 52^2$. $x = (48 \pm 52)/10 = 10$ or -0.4. Reject the negative: **boat speed = 10 km/h**.

Assertion — Reason (10 Questions)

Choose: **(a)** both true, R explains A **(b)** both true, R does not explain A **(c)** A true, R false **(d)** A false, R true. Answers in the Answer Key.

AR1. Assertion: The equation $x^2 + 4x + 5 = 0$ has no real roots.
Reason: $D = 16 - 20 = -4 < 0$.

AR2. Assertion: The equation $2x^2 - 8x + 8 = 0$ has two distinct real roots.
Reason: $D = 64 - 64 = 0$.

AR3. Assertion: The roots of $x^2 - 5x + 6 = 0$ are 2 and 3.
Reason: For roots α, β: $\alpha + \beta = -b/a = 5$ and $\alpha\beta = c/a = 6$, and $2 + 3 = 5$, $2 \times 3 = 6$.

AR4. Assertion: The roots of $x^2 - 4 = 0$ are 2 and -2.
Reason: $x^2 - 4 = (x - 2)(x + 2)$.

AR5. Assertion: The equation $x^2 + x + 1 = 0$ has real roots.
Reason: $D = 1 - 4 = -3 < 0$.

AR6. Assertion: If $D > 0$, a quadratic equation has two distinct real roots.
Reason: $x = [-b \pm \sqrt{D}]/2a$ gives two different values when $\sqrt{D} \neq 0$.

AR7. Assertion: $(x - 2)(x + 1) = (x - 1)(x + 3)$ is a quadratic equation.
Reason: On simplification it becomes $-3x + 1 = 0$, which has degree 1.

AR8. Assertion: For equal roots of $x^2 - 6x + k = 0$, $k = 9$.
Reason: Equal roots require $D = 0$: $36 - 4k = 0$ gives $k = 9$.

AR9. Assertion: The sum of the roots of $3x^2 - 7x + 2 = 0$ is $7/3$.
Reason: Sum of roots $= -b/a = 7/3$.

AR10. Assertion: A quadratic equation can have three distinct real roots.
Reason: A quadratic polynomial has at most two zeros.

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15 Olympiad & Foundation Challenges

CHALLENGE — TEN PROBLEMS FOR MATHEMATICAL THINKERS

OLYMPIAD 1

Solve: $x^4 - 13x^2 + 36 = 0$.

THINK FIRST: Substitution is a general weapon: whenever an equation contains only x^2 and x^4, set $y = x^2$.

1. Substitute $y = x^2$: the equation becomes $y^2 - 13y + 36 = 0$.
   Why: A quartic in disguise — only even powers appear, so one substitution demotes it to a quadratic.
2. Factorise: $(y - 4)(y - 9) = 0$, so $y = 4$ or $y = 9$.
   Why: Pair with product 36, sum −13: −4, −9.
3. Come back to x: $x^2 = 4$ gives $x = \pm 2$; $x^2 = 9$ gives $x = \pm 3$.
   Why: Do not stop at y — the question asked for x. And remember ± both times.

ANSWER: The four roots are $x = 2, -2, 3, -3$.

OLYMPIAD 2

The sum of the squares of two consecutive natural numbers is 421. Find the numbers.

THINK FIRST: Compare NCERT Ex 4.2 Q4 (sum of squares 365) — same skeleton, bigger numbers.

1. Let the numbers be x and x + 1: $x^2 + (x + 1)^2 = 421$.
2. Expand: $2x^2 + 2x + 1 = 421$, so $2x^2 + 2x - 420 = 0$; divide by 2: $x^2 + x - 210 = 0$.
3. Factorise: $(x + 15)(x - 14) = 0$, so $x = 14$ or -15.
   Why: Pair with product −210, sum +1: +15, −14.
4. Reject −15 (natural numbers are positive). The numbers are 14 and 15. Check: 196 + 225 = 421. Correct!

ANSWER: The numbers are 14 and 15.

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Olympiad 3 and 4 — Roots in Disguise

OLYMPIAD 3

Find all values of k for which the roots of $x^2 - kx + (k - 1) = 0$ differ by exactly 1.

THINK FIRST: $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ — pin this identity next to sum and product in your memory.

1. Let the roots be α and β. Then $\alpha + \beta = k$ and $\alpha\beta = k - 1$ (sum and product of roots).
   Why: We use the roots' symmetric sums — no need to find the roots themselves.
2. Use the identity $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = k^2 - 4(k - 1) = k^2 - 4k + 4 = (k - 2)^2$.
   Why: The difference of roots, squared, is expressible through sum and product — a classic Olympiad move.
3. The roots differ by 1, so $(\alpha - \beta)^2 = 1$: $(k - 2)^2 = 1$, giving $k - 2 = \pm 1$, so $k = 3$ or $k = 1$.
4. Check $k = 3$: $x^2 - 3x + 2 = 0$ has roots 1, 2 — differ by 1. Check $k = 1$: $x^2 - x = 0$ has roots 0, 1 — differ by 1. Both work.

ANSWER: k = 1 or k = 3.

OLYMPIAD 4

If α and β are the roots of $x^2 - 3x + 1 = 0$, find (i) $\alpha^2 + \beta^2$ and (ii) $1/\alpha + 1/\beta$ — WITHOUT solving the equation.

THINK FIRST: The roots of this equation are irrational ($(3 \pm \sqrt{5})/2$) — yet their symmetric combinations are clean integers. That is why this technique exists.

1. From the equation: $\alpha + \beta = -b/a = 3$ and $\alpha\beta = c/a = 1$.
   Why: Sum and product come free from the coefficients.
2. (i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 3^2 - 2(1) = 9 - 2 = 7$.
   Why: Complete the square on the roots themselves: $(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$.
3. (ii) $1/\alpha + 1/\beta = (\alpha + \beta) / \alpha\beta = 3/1 = 3$.
   Why: Combine the fractions: the numerator is the sum, the denominator the product.

ANSWER: (i) $\alpha^2 + \beta^2 = 7$ (ii) $1/\alpha + 1/\beta = 3$.

Olympiad 5 and 6

OLYMPIAD 5

If α and β are the roots of $x^2 - 11x + 30 = 0$, find $\alpha^3 + \beta^3$ without finding α and β separately.

THINK FIRST: Symmetric expressions in α, β always reduce to α + β and αβ. Always.

1. α + β = 11 and αβ = 30.
2. Use the identity $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$.

   Why: Expand $(\alpha + \beta)^3 = \alpha^3 + \beta^3 + 3\alpha\beta(\alpha + \beta)$ and rearrange – verify this identity once and it is yours forever.
3. Substitute: $\alpha^3 + \beta^3 = 11^3 - 3(30)(11) = 1331 - 990 = 341$.
4. Sanity check by actually solving: roots are 5 and 6 (factorise!), and 125 + 216 = 341. Agreement!

   Why: When a shortcut and a direct method agree, confidence is total.

ANSWER: $\alpha^3 + \beta^3 = 341$.

Olympiad 6

OLYMPIAD 6

Two numbers differ by 2. The sum of their squares is 52 more than their product. Find the numbers.

THINK FIRST: Read the problem's exact wording: no positivity condition means negative solutions are welcome.

1. Let the numbers be x and $x - 2$ (they differ by 2).
2. Sum of squares = product + 52: $x^2 + (x - 2)^2 = x(x - 2) + 52$.
3. Expand: $2x^2 - 4x + 4 = x^2 - 2x + 52$.
4. Collect: $x^2 - 2x - 48 = 0$. Factorise: $(x - 8)(x + 6) = 0$.
   Why: Pair with product −48, sum −2: −8, +6.
5. $x = 8$ gives the pair (8, 6); $x = -6$ gives (−6, −8). Both pairs satisfy every condition — the problem did not demand positive numbers.
   Why: Check (8, 6): squares sum 100; product 48; 100 = 48 + 52. Check (−6, −8): 36 + 64 = 100; product 48. Both correct!

ANSWER: The numbers are 8 and 6, or −6 and −8.

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Olympiad 7 and 8

OLYMPIAD 7

If x = 2/3 and x = −3 are roots of the equation $ax^2 + 7x + b = 0$, find a and b.

THINK FIRST: Given the roots, sum and product rebuild the equation — the reverse direction of everything so far.

1. Sum of roots = 2/3 + (−3) = −7/3. But sum of roots also equals −(coefficient of x)/a = −7/a.
   Why: Two expressions for the same quantity — equate them.
2. −7/a = −7/3, so a = 3.
3. Product of roots = (2/3)(−3) = −2. Product also equals b/a = b/3. So b/3 = −2, giving b = −6.
4. Verify: $3x^2 + 7x − 6$ at x = 2/3: 3(4/9) + 14/3 − 6 = 4/3 + 14/3 − 6 = 6 − 6 = 0. At x = −3: 27 − 21 − 6 = 0. Both check out.

ANSWER: a = 3 and b = −6.

OLYMPIAD 8

Find k if the roots of $3x^2 − 10x + k = 0$ are reciprocals of each other.

THINK FIRST: 'Reciprocal roots' is code for product = 1, just as 'equal roots' is code for D = 0. Learn the code-words.

1. Roots are reciprocals: if one is α, the other is 1/α. Their product is α × (1/α) = 1.
   Why: Reciprocal pairs always multiply to 1 — that is the key observation.
2. But product of roots = c/a = k/3. So k/3 = 1, giving k = 3.
3. Verify: $3x^2 − 10x + 3 = 0$ factorises as (3x − 1)(x − 3) = 0, roots 3 and 1/3 — genuine reciprocals.

ANSWER: k = 3.

Olympiad 9 and 10

OLYMPIAD 9

The perimeter of a rectangular field is 46 m and its diagonal is 17 m. Find its length and breadth.

THINK FIRST: Sum-and-product thinking again — the same tool as Olympiad 4, 5 and 7, in a field this time.

1. $l + b = 46/2 = 23$, and by Pythagoras $l^2 + b^2 = 17^2 = 289$.

   Why: Two conditions, two unknowns — but one clever identity avoids solving simultaneously.

2. Square the first: $(l + b)^2 = 529$, i.e. $l^2 + 2lb + b^2 = 529$. Subtract $l^2 + b^2 = 289$: $2lb = 240$, so $lb = 120$.

   Why: The identity $(l + b)^2 = (l^2 + b^2) + 2lb$ bridges the two conditions.

3. Now l and b are roots of $t^2 - (\text{sum})t + (\text{product}) = t^2 - 23t + 120 = 0$.

   Why: Any two numbers are the roots of $t^2 - (\text{their sum})t + (\text{their product}) = 0$.

4. Factorise: $(t - 8)(t - 15) = 0$, so the sides are 8 m and 15 m. Check: $8 + 15 = 23$; $64 + 225 = 289 = 17^2$. Correct — an 8-15-17 triangle in disguise!

ANSWER: The field is 15 m by 8 m.

Olympiad 10

OLYMPIAD 10

For the equation $x^2 - 6x + k = 0$, find every integer k between 0 and 9 (inclusive) for which BOTH roots are integers.

THINK FIRST: This is Level 5.2 promoted to the Olympiad page — if you solved it there, you have already beaten an Olympiad problem.

1. For real roots, D = 36 − 4k must be ≥ 0, so k ≤ 9 — the whole search range survives.
2. For INTEGER roots, x = (6 ± √D)/2 must be an integer, so √D must be a whole number with the same parity as 6 — i.e. D must be a perfect square: D ∈ {36, 25, 16, 9, 4, 1, 0} with D = 36 − 4k, so D must be divisible by 4: D ∈ {36, 16, 4, 0}.
   Why: D = 36 − 4k is always a multiple of 4, which prunes the candidate squares at once.
3. D = 36 → k = 0: roots (6 ± 6)/2 = 6, 0. Integers. D = 16 → k = 5: roots (6 ± 4)/2 = 5, 1. Integers.
4. D = 4 → k = 8: roots (6 ± 2)/2 = 4, 2. Integers. D = 0 → k = 9: root 3 (twice). Integer.
5. No other k in 0–9 makes D a perfect square (k = 1..4, 6, 7 give D = 32, 28, 24, 20, 12, 8 — none squares).
   Why: Exhaustion is a valid proof when the search space is finite and fully checked.

ANSWER: k = 0, 5, 8 or 9.

Previous Year Questions — CBSE Board (Verified)

Real board questions, with the year and region as published in CBSE papers and marking schemes. Where a marking split is labelled **CBSE MARKING SCHEME**, it follows the official CBSE scheme for that paper; elsewhere the split is our CBSE-style guidance.

CBSE 2017 (DELHI) — 2 MARKS

Find the value of k for which the equation $x^2 + k(2x + k - 1) + 2 = 0$ has real and equal roots.

THINK FIRST: Expand first: k multiplies a whole bracket.

1. Expand: $x^2 + 2kx + k^2 - k + 2 = 0$. So $a = 1$, $b = 2k$, $c = k^2 - k + 2$.
   Why: $k(2x + k - 1) = 2kx + k^2 - k$.
2. For real and equal roots, $D = 0$: $(2k)^2 - 4(1)(k^2 - k + 2) = 0$.
3. Expand: $4k^2 - 4k^2 + 4k - 8 = 0$, so $4k - 8 = 0$.
4. $k = 2$.

ANSWER: $k = 2$.

CBSE MARKING SCHEME (2017): Official split: standard form (1/2) + D = 0 expanded (1) + k = 2 (1/2).

CBSE 2017 (DELHI) — 2 MARKS

Find the roots of the quadratic equation $\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$.

THINK FIRST: You have solved this — it is NCERT Exercise 4.2 Q1(iii), asked verbatim in the board.

1. Pair with product $\sqrt{2} \times 5\sqrt{2} = 10$ and sum 7: the pair is 5 and 2.
2. Split and group: $\sqrt{2}\,x^2 + 5x + 2x + 5\sqrt{2} = (\sqrt{2}\,x + 5)(x + \sqrt{2}) = 0$.
3. $x = -\sqrt{2}$ or $x = -5/\sqrt{2} = -5\sqrt{2}/2$.

ANSWER: The roots are $x = -\sqrt{2}$ and $x = -5\sqrt{2}/2$.

CBSE-STYLE MARKING SCHEME: CBSE-style: factorisation (1) + both roots (1).

Previous Year Questions — 2017 (continued)

CBSE 2017 — 3 MARKS

If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, show that $c^2 = a^2(1 + m^2)$.

THINK FIRST: Equal roots → D = 0. Here the coefficients are letters: A = $1 + m^2$, B = $2mc$, C = $c^2 - a^2$.

1. For equal roots, $B^2 - 4AC = 0$: $(2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$.

2. Expand: $4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$. Divide by 4: $m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2 = 0$.

   Why: Expand $(1 + m^2)(c^2 - a^2)$ into four terms, carefully.

3. The m^2c^2 terms cancel: $-c^2 + a^2 + m^2a^2 = 0$.

4. Rearrange: $c^2 = a^2 + m^2a^2 = a^2(1 + m^2)$. Shown.

   Why: Factor a^2 out of the right side to land exactly on the target form.

ANSWER: SHOWN: $c^2 = a^2(1 + m^2)$.

CBSE MARKING SCHEME (2017): Official split: D = 0 condition (1/2) + expansion (1) + simplification (1) + final form (1/2).

Previous Year Questions — 2017 Outside Delhi

CBSE 2017 (OUTSIDE DELHI)

Find the value of p for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other root.

THINK FIRST: Call the roots α and 6α — then use sum AND product.

1. Let the roots be α and 6α. Sum of roots: $7\alpha = 14/p$, so $\alpha = 2/p$.
   Why: Sum = −b/a = 14/p.
2. Product of roots: $\alpha \times 6\alpha = 6\alpha^2 = 8/p$.
   Why: Product = c/a = 8/p.
3. Substitute $\alpha = 2/p$: $6(2/p)^2 = 8/p$, i.e. $24/p^2 = 8/p$.
4. Multiply both sides by p^2: $24 = 8p$, so $p = 3$.
   Why: p = 0 also appears if you cross-multiply carelessly — but p = 0 would destroy the quadratic, so it is rejected.
5. Check with $p = 3$: $3x^2 - 14x + 8 = 0$. $D = 196 - 96 = 100$. $x = (14 \pm 10)/6 = 4$ or $2/3$. Indeed $4 = 6 \times (2/3)$. Correct!

ANSWER: p = 3.

CBSE-STYLE MARKING SCHEME: CBSE-style: root naming and sum (1) + product substitution (1) + p with check (1).

Previous Year Questions — 2012, 2011 and a Board Classic

CBSE 2012 (DELHI)

Find the value(s) of k so that the quadratic equation $x^2 - 4kx + k = 0$ has equal roots.

THINK FIRST: a = 1, b = −4k, c = k. Equal roots → D = 0.

1. $D = (-4k)^2 - 4(1)(k) = 16k^2 - 4k = 0$.
2. Factorise: $4k(4k - 1) = 0$, so k = 0 or k = 1/4.
3. Both values are genuine: k = 0 gives $x^2 = 0$ (root 0, twice); k = 1/4 gives $(x - 1/2)^2 = 0$. Equal roots in both cases.

   Why: Here k = 0 does NOT destroy the quadratic (k is not the x^2 coefficient) — never reject by habit.

ANSWER: k = 0 or k = 1/4.

CBSE-STYLE MARKING SCHEME: CBSE-style: D (1) + both values (1).

CBSE 2011 (OUTSIDE DELHI)

Find the roots of the equation $x^2 - 3x - m(m + 3) = 0$, where m is a constant.

THINK FIRST: Treat m as a number. The discriminant turns into a perfect square in m.

1. a = 1, b = −3, c = −m(m + 3). $D = 9 - 4(1)(-m(m + 3)) = 9 + 4m^2 + 12m$.
2. Recognise the square: $4m^2 + 12m + 9 = (2m + 3)^2$. So $\sqrt{D} = 2m + 3$.

   Why: Check: $(2m + 3)^2 = 4m^2 + 12m + 9$. Exact.
3. Apply the formula: $x = [3 \pm (2m + 3)]/2$.
4. $x = (3 + 2m + 3)/2 = m + 3$, or $x = (3 - 2m - 3)/2 = -m$.

ANSWER: The roots are x = m + 3 and x = −m.

CBSE-STYLE MARKING SCHEME: CBSE-style: D as a perfect square in m (1) + both roots (1).

Previous Year Questions — The Root-Substitution Classic

CBSE BOARD EXAM QUESTION

If one root of the quadratic equation $6x^2 - x - k = 0$ is 2/3, find the value of k.

THINK FIRST: A root satisfies the equation — substitute it.

1. Since x = 2/3 is a root, it must satisfy the equation: $6(2/3)^2 - (2/3) - k = 0$.
   Why: This is the definition of a root (Section 05) used as a tool.
2. Compute: 6 × (4/9) = 24/9 = 8/3. So 8/3 − 2/3 − k = 0, i.e. 6/3 − k = 0.
3. 2 − k = 0, so k = 2.

ANSWER: k = 2.

CBSE-STYLE MARKING SCHEME: CBSE-style: substitution (1) + simplification and answer (1).

What the Board Keeps Asking — the PYQ pattern

Seven real questions, four repeating shapes: **(1)** find k for equal roots (2017D, 2012D); **(2)** solve by factorisation, surds included (2017D); **(3)** prove a relation from D = 0 with letter coefficients (2017); **(4)** use sum & product of roots, or substitute a known root to find a parameter (2017OD, board classic). Master these four shapes and the board's quadratic questions hold no surprises.

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16 Chapter Mind Map

RECALL — THE WHOLE CHAPTER ON ONE PAGE

Read this map from the centre outward. If you can explain every branch aloud, in your own words, you own this chapter.

QUADRATIC EQUATIONS

$ax^2 + bx + c = 0$, $a \neq 0$ • at most two roots

Three Methods

1. Factorisation — split middle term (product ac, sum b)
2. Completing the square — add (half of x-coeff.)^2 to both sides
3. Formula — $x = [-b \pm \sqrt{D}]/2a$ (derived in Sec 06)

The Discriminant

$D = b^2 - 4ac$
$D > 0$ → two distinct real roots
$D = 0$ → two equal roots, $x = -b/2a$
$D < 0$ → no real roots

Root Facts

Sum of roots = $-b/a$
Product of roots = c/a
Equation from roots: $x^2 - (\text{sum})x + (\text{product}) = 0$
A root satisfies the equation — substitute to test or to find k

Word Problems

Translate: name x, tie the second quantity to x, write the condition
Areas • ages • consecutive numbers • trains (time = distance/speed) • taps (rates add) • boats (stream ±) • Pythagoras

Golden Rules

Never divide by x — factorise instead
Square root step always carries ±
Equal roots ↔ D = 0 (and keep the ± on k)
Judge every root against the story — reject with a reason

The Picture

$y = ax^2 + bx + c$ is a parabola
Roots = where it crosses the x-axis
Crosses twice / touches once / never — exactly the three D cases

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Pin-Point Flash Cards — Cut, Carry, Conquer (1 of 2)

How to use: cut along the dashed borders (ask an adult if needed), keep the 16 cards in your pocket or pencil box, and flip through them on your Smart Memory days — Day 3 recall, Day 7 test, Day 14 mixed practice, Day 21 and Day 30. Read the title, say the fact aloud from memory, then flip to check. If you miss one, that card goes to the front of the pile.

Standard Form

$ax^2 + bx + c = 0$ with $a \neq 0$. If $a = 0$, it is not quadratic at all.

The Quadratic Formula

$x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$ — derived by completing the square.

Discriminant

$D = b^2 - 4ac$. Compute D FIRST in every nature-of-roots question.

D > 0

Two distinct real roots. The parabola crosses the x-axis twice.

D = 0

Two equal real roots: $x = -b/2a$. The parabola just touches the axis.

D < 0

No real roots. The parabola never reaches the x-axis.

Sum of Roots

$\alpha + \beta = -b/a$. Mind the minus sign — it is part of the formula.

Product of Roots

$\alpha\beta = c/a$. Reciprocal roots ↔ product = 1.

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Pin-Point Flash Cards (2 of 2)

Splitting the Middle Term

Hunt a pair with product = a × c and sum = b. Not product c — product ac!

Completing the Square

For $x^2 + px$, add $(p/2)^2$ to BOTH sides. One move creates the square.

Never Divide by x

$x^2 = 5x \rightarrow$ dividing by x loses the root x = 0. Factorise: $x(x - 5) = 0$.

The ± Rule

$x^2 = 49 \rightarrow x = \pm 7$. Every square-root step has two branches.

Equal Roots Code

'Equal roots' always means D = 0. And when k^2 appears, k takes a ± too.

Root Rejection Rule

Lengths, speeds, ages, counts < 0 are impossible. Reject the root AND write the reason.

A Root Satisfies

If x = r is a root, substituting r makes the equation true. Use it to find k.

Build from Roots

Roots α, β → equation $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.

TOPPER TIP: *Exam-eve ritual: shuffle the 16 cards, lay them face down, and explain each one aloud in under 20 seconds. Sixteen cards, sixteen facts, zero surprises in the paper.*

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17 Active Recall

RECALL — BOOK CLOSED, BRAIN OPEN

Close the rest of the chapter. Answer from memory — struggling a little is exactly what builds the memory. Answers are in the Answer Key.

Ten Rapid Questions

R1. Write the standard form of a quadratic equation and state the condition on a.

R2. State the quadratic formula.

R3. What is the discriminant, and what do its three signs tell you?

R4. Solve $x^2 - 6x + 9 = 0$ by factorisation.

R5. For splitting the middle term of $ax^2 + bx + c$, what two conditions must the pair satisfy?

R6. Solve $x^2 = 25$. (Both answers!)

R7. Find D for $5x^2 - 3x + 1 = 0$ and state the nature of roots.

R8. Sum and product of the roots of $2x^2 + 7x - 4 = 0$?

R9. A rectangle's length is 3 more than its width and its area is 40. Write the equation (do not solve).

R10. Why must $x = -8$ be rejected in Meena's garden problem?

Five Formula & Definition Prompts

F1. Define a root of a quadratic equation.

F2. Write the completing-the-square completing number for $x^2 + px$, and for $x^2 - 10x$.

F3. Write the equation whose roots are 4 and -1.

F4. State the condition on D for (a) distinct roots, (b) equal roots, (c) no real roots.

F5. If one root of $x^2 - 8x + k = 0$ is 3, find k in one line. (Use the definition of a root.)

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18 Self-Assessment & Mastery Tracker

RECALL — THE SMART MEMORY SYSTEM

Learning once is not learning. The KalviYogi Smart Memory System revisits this chapter on a schedule — each visit is short, and each one locks the chapter deeper into long-term memory.

Day	Your Mission
Day 0	Pre-learning check: attempt the Quick Check (Section 02) and Rapid Questions R1–R5 cold. Note your score below.
Day 1	Learn the chapter fully (Sections 01–10) and complete Level 1–2 practice.
Day 3	Without-book recall: Flash Cards (all 16) + Active Recall (Section 17). Write formulas from memory, then verify.
Day 7	Independent problem-solving: **Day 7 Mini-Test** (next page) + Level 3 practice.
Day 14	Mixed practice: Level 4–5 + two case studies from Section 14, timed.
Day 21	Long-term recall: **Day 21 Mini-Test** + re-derive the quadratic formula from memory (Section 06).
Day 30	Chapter mastery: **Day 30 Mastery Test** + one Chapter Model Test (A, B or C) under exam conditions.

Error Tracker — every mistake, logged and killed

Date _____ Question _____ My error: _____

The correct idea: _____ Error type: ☐ sign ☐ ± ☐ method ☐ rejected-root ☐ arithmetic

☐ I have redone this question correctly on Day _____

Formula Recall Tracker — tick only if written from memory

☐ Quadratic formula ☐ $D = b^2 - 4ac$ ☐ Three D cases ☐ Sum = $-b/a$ ☐ Product = c/a ☐ Completing number $(p/2)^2$

Concept Mastery Score — rate yourself honestly (0 = lost, 1 = shaky, 2 = solid)

Recognising quadratics __/2 Factorisation __/2 Completing the square __/2 Formula __/2 Discriminant __/2 Finding k __/2 Word problems __/2 Reasoning & proofs __/2 **Total __/16**

Smart Memory Mini-Tests — Day 7 and Day 21

DAY 7 MINI-TEST — 10 Questions, 20 Minutes, Book Closed

1. Solve $x^2 - 8x + 15 = 0$.

2. Solve $2x^2 + 3x - 2 = 0$.

3. Find D of $x^2 - 6x + 2 = 0$ and state the nature of roots.

4. Solve $x^2 - 49 = 0$.

5. Solve by formula: $x^2 + 2x - 8 = 0$.

6. Find k for equal roots of $x^2 - 10x + k = 0$.

7. Check whether $x = 4$ is a root of $x^2 - 7x + 12 = 0$.

8. State the sum of the roots of $x^2 - 9x + 4 = 0$.

9. Two consecutive positive integers have product 72. Find them.

10. State the nature of roots of $3x^2 + 2x + 2 = 0$.

DAY 21 MINI-TEST — 10 Questions, 20 Minutes, Book Closed

1. Solve $3x^2 - 7x - 6 = 0$.

2. Solve $1/(x + 4) - 1/(x - 7) = 11/30$.

3. Find k for equal roots of $2x^2 + kx + 3 = 0$.

4. Solve $x^2 + 6x + 5 = 0$ by completing the square.

5. Two numbers have sum 27 and product 182. Find them.

6. Find D of $4x^2 - 4x + 1 = 0$ and state the nature of roots.

7. Solve $2x^2 + x - 6 = 0$.

8. Is $(x - 3)(2x + 1) = x(x + 5)$ a quadratic equation? Justify.

9. State the product of the roots of $7x^2 - 3x + 2 = 0$.

10. A train travels 480 km; at 8 km/h less it takes 3 hours more. Find its speed.

Answers for both tests are in the Answer Key. Score below 7/10? Revisit the matching section the same day — do not wait for the next scheduled day.

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Smart Memory Mini-Test — Day 30 Mastery

DAY 30 MASTERY TEST — 10 Questions, 20 Minutes, Book Closed

1. Solve $4x^2 + 4\sqrt{3}\,x + 3 = 0$.

2. Two friends' ages sum to 20; four years ago the product of their ages was 48. Is this possible? Justify.

3. A park has perimeter 80 m and area 400 m^2. Find its sides.

4. If α, β are roots of $x^2 - 3x + 1 = 0$, find $\alpha^2 + \beta^2$.

5. Solve $x^4 - 5x^2 + 4 = 0$.

6. Find k if the roots of $3x^2 - 10x + k = 0$ are reciprocals.

7. Two taps fill a tank together in 6 h; the larger takes 5 h less than the smaller. Find each time.

8. Prove that $(x - 2)(x + 1) = (x - 1)(x + 3)$ is NOT a quadratic equation.

9. Find k for equal roots of $kx(x - 2) + 6 = 0$.

10. A boat (still-water speed 18 km/h) takes 1 h more to go 24 km upstream than downstream. Find the stream's speed.

> **TOPPER TIP:** *The Day 30 test deliberately mixes NCERT, reasoning and Olympiad items. A score of 8+ means this chapter is truly yours — record it in the Mastery Tracker.*

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Chapter Model Test A — Basics

MODEL TEST A • FOUNDATION LEVEL Time: 45 minutes • Maximum Marks: 20 • Section A: 5 MCQ × 1 mark • Section B: 3 × 2 marks • Section C: 3 × 3 marks. Answers in the Answer Key.

Section A — MCQ (1 mark each)

A1. The roots of $x^2 - 5x + 6 = 0$ are

(a) 1 and 6 (b) 2 and 3 (c) −2 and −3 (d) 5 and 6

A2. The discriminant of $2x^2 - 4x + 2 = 0$ is

(a) 16 (b) 8 (c) 0 (d) −8

A3. The sum of the roots of $x^2 + 7x - 3 = 0$ is

(a) 7 (b) −7 (c) 3 (d) −3

A4. Which of the following is a quadratic equation?

(a) $x + 1 = 0$ (b) $x^3 - 2x = 0$ (c) $x^2 + 2x = 0$ (d) $1/x = 4$

A5. The product of the roots of $2x^2 - 8x + 3 = 0$ is

(a) 4 (b) 3/2 (c) −3/2 (d) 8

Section B — Short Answers (2 marks each)

B1. Solve: $x^2 - 11x + 30 = 0$.

B2. Solve: $x^2 - 36 = 0$.

B3. Find the discriminant of $x^2 - 2x - 8 = 0$ and state the nature of its roots.

Section C — Long Answers (3 marks each)

C1. Solve using the quadratic formula: $2x^2 - 5x + 2 = 0$.

C2. Find k if $x^2 + kx + 16 = 0$ has two equal roots.

C3. Check whether $(x + 2)^3 = 2x(x^2 - 1)$ is a quadratic equation. Justify.

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Chapter Model Test B — Standard

MODEL TEST B • BOARD LEVEL Time: 45 minutes • Maximum Marks: 20 • Section A: 5 MCQ × 1 mark • Section B: 3 × 2 marks • Section C: 3 × 3 marks. Answers in the Answer Key.

Section A — MCQ (1 mark each)

A1. The roots of $3x^2 - 5x + 2 = 0$ are

(a) 1 and 2/3 (b) 2 and 3 (c) −1 and −2/3 (d) 5 and 2

A2. If $x = 3$ is a root of $x^2 - 2x + k = 0$, then $k =$

(a) 3 (b) −3 (c) 9 (d) −9

A3. The nature of roots of $2x^2 - 3x + 5 = 0$ is

(a) real and distinct (b) real and equal (c) no real roots (d) cannot say

A4. The equation with roots −3 and 4 is

(a) $x^2 + x - 12 = 0$ (b) $x^2 - x - 12 = 0$ (c) $x^2 - 7x - 12 = 0$ (d) $x^2 + 7x + 12 = 0$

A5. The discriminant of $4x^2 + 4\sqrt{3}\,x + 3 = 0$ is

(a) 48 (b) 12 (c) 0 (d) −48

Section B — Short Answers (2 marks each)

B1. Solve: $\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$.

B2. Solve: $x - 1/x = 3$, $x \neq 0$.

B3. Find the nature of the roots of $2x^2 - 6x + 3 = 0$.

Section C — Long Answers (3 marks each)

C1. Find two numbers whose sum is 27 and product is 182.

C2. Find k for which $kx(x - 2) + 6 = 0$ has two equal roots.

C3. Is it possible to design a rectangular mango grove whose length is twice its breadth and whose area is 800 m^2? If so, find its length and breadth.

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Chapter Model Test C — Advanced

MODEL TEST C • CHALLENGE LEVEL Time: 45 minutes • Maximum Marks: 20 • Section A: 5 MCQ × 1 mark • Section B: 3 × 2 marks • Section C: 3 × 3 marks. Answers in the Answer Key.

Section A — MCQ (1 mark each)

A1. If α, β are roots of $x^2 - 3x + 1 = 0$, then $\alpha^2 + \beta^2 =$

(a) 9 (b) 7 (c) 11 (d) 3

A2. The number of real roots of $x^4 - 5x^2 + 4 = 0$ is

(a) 2 (b) 3 (c) 4 (d) 0

A3. The roots of $3x^2 - 10x + k = 0$ are reciprocals of each other when $k =$

(a) 10 (b) 1 (c) 3 (d) 30

A4. The discriminant of $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ is

(a) $4(ad - bc)^2$ (b) $-4(ad - bc)^2$ (c) 0 (d) $(ad - bc)^2$

A5. The roots of $x^2 - 8x + 16 = 0$ are

(a) 8 and 2 (b) equal, $x = 4$ (c) equal, $x = -4$ (d) not real

Section B — Short Answers (2 marks each)

B1. Solve: $x^4 - 13x^2 + 36 = 0$.

B2. Show that $x^2 + 4x + 13 = 0$ has no real roots, using completing the square.

B3. Find k for which $x^2 - 2kx + 7k - 12 = 0$ has equal roots.

Section C — Long Answers (3 marks each)

C1. Two taps together fill a tank in 6 hours. The larger tap takes 5 hours less than the smaller. Find the time taken by each tap separately.

C2. Show that $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ has no real roots when $ad \neq bc$.

C3. A motorboat of speed 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream. Find the speed of the stream.

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Answer Key — Practice Levels 1 to 5

Activities (Section 09)

Activity 2: c = 1: D = 12, distinct roots. c = 3: D = 4, distinct. c = 4: D = 0, equal roots. c = 5: D = −4, no real roots. c = 8: D = −16, no real roots.

Activity 3: missing values: x = −2 → y = 0; x = 0 → y = −4; x = 2 → y = 0. Roots of $x^2 - 4 = 0$ are ±2 — exactly the crossings.

Level 1 — Foundation

L1.1 $2x^2 - 7x + 3 = 0$; a = 2, b = −7, c = 3. **L1.2** 4 − 6 + 2 = 0 — yes, x = 2 is a root. **L1.3** x = ±3. **L1.4** x = −2, −3. **L1.5** D = 16 − 24 = −8 < 0 — no real roots. **L1.6** x(x − 4) = 0 → x = 0 or 4.

Level 2 — Skill Builder

L2.1 x = 3, −2/3. **L2.2** x = 2/3, −1/2. **L2.3** x = 4, −1. **L2.4** x = 2, 1/2. **L2.5** $k^2 - 36 = 0$ → k = ±6. **L2.6** $3x^2 - 10x + 3 = 0$ → the number is 3 or 1/3 (a reciprocal pair — both had to appear!).

Level 3 — Application

L3.1 $x^2 + 7x - 60 = 0$ → width 5 m, length 12 m. **L3.2** $x^2 - x - 6 = 0$ → taps take 3 h and 6 h. **L3.3** Stream speed 6 km/h (the other root, −54, is impossible). **L3.4** Brother 6 years, Rani 12 years.

Level 4 — Advanced Reasoning

L4.1 $D = 4k^2 - 4(7k - 12) = 0 \rightarrow k^2 - 7k + 12 = 0$ → k = 3 or 4. **L4.2** Sum of roots = 7; one root 2, so the other is 5; k = product = 2 × 5 = 10. **L4.3** $x^2 + 3x - 504 = 0$ → the numbers are 21 and 24.

Level 5 — Mathematical Genius

L5.1 $y = x^2$: $y^2 - 5y + 4 = 0 \rightarrow y = 1, 4 \rightarrow x = \pm 1, \pm 2$. **L5.2** k = 0, 5, 8, 9 (full proof in Olympiad 10).

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Answer Key — Concept MCQs Explained

MCQ 1: (b) Degree is the highest power of x: 2.

MCQ 2: (c) Only this one keeps degree 2 after simplification.

MCQ 3: (a) $(x - 2)(x - 5) = x^2 - 7x + 10$.

MCQ 4: (c) $D = 16 - 16 = 0$.

MCQ 5: (c) A negative number has no real square root.

MCQ 6: (b) Sum $= -b/a = 3/2$.

MCQ 7: (c) Product $= c/a = 2/3$.

MCQ 8: (a) $x(x - 9) = 0$ — never divide by x.

MCQ 9: (b) Substitute: $9 - 15 + k = 0$.

MCQ 10: (c) $D = 144 - 16k = 0$.

MCQ 11: (c) $D = 4 - 12 = -8 < 0$.

MCQ 12: (b) Sum 1, product −6: $x^2 - (\text{sum})x + (\text{product})$.

MCQ 13: (c) $x^2 = 16 \rightarrow x = \pm 4$.

MCQ 14: (c) Product of roots is 6, so the partner of 2 is 3.

MCQ 15: (a) Substitute: $1 - k - 6 = 0$.

MCQ 16: (a) $D = 49 - 24 = 25$.

MCQ 17: (b) $(10x - 1)^2 = 0 \rightarrow x = 1/10$ twice.

MCQ 18: (c) $x^2 - 4x + 4 - x - 3 = x^2 - 5x + 1$.

MCQ 19: (b) A squared factor gives one value, counted twice.

MCQ 20: (c) $x^2 - (\text{sum})x + (\text{product}) = 0$.

MCQ 21: (b) It collapses to $-3x + 1 = 0$ (Ex 4.1 Q1(iii)).

MCQ 22: (b) $x(x - 3) = 0$.

MCQ 23: (c) Real roots need $\sqrt{D}$ to be real: $D \geq 0$.

MCQ 24: (c) $D = 16 - 4k < 0$ exactly when $k > 4$.

MCQ 25: (b) The formula carries $-b$ and $-4ac$ under the root.

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Answer Key — Assertion-Reason, Recall and Mini-Tests

Assertion — Reason (Section 14)

AR1: (a) D < 0 is exactly why there are no real roots.

AR2: (d) A is false (roots are equal, x = 2, 2); R is true.

AR3: (a) The sum-product check is precisely how the roots are verified.

AR4: (a) Factorising gives both roots directly.

AR5: (d) A is false; R correctly computes D < 0.

AR6: (a) The ± √D produces two values exactly when D > 0.

AR7: (d) A is false (degree 1 after simplification); R is true.

AR8: (a) D = 0 is the equal-roots condition, and it yields k = 9.

AR9: (a) Sum = −b/a explains the value 7/3.

AR10: (d) A is false; a quadratic has at most two zeros, as R says.

Active Recall (Section 17)

R1 $ax^2 + bx + c = 0$, $a \neq 0$. **R2** $x = [-b \pm \sqrt{(b^2 - 4ac)}]/2a$. **R3** $D = b^2 - 4ac$; > 0 distinct, = 0 equal, < 0 none. **R4** $(x - 3)^2 = 0 \rightarrow x = 3, 3$. **R5** Product = ac, sum = b. **R6** x = ±5. **R7** D = 9 − 20 = −11 — no real roots. **R8** Sum = −7/2, product = −2. **R9** x(x + 3) = 40. **R10** A width cannot be negative.

F1 A value that makes the equation true when substituted. **F2** $(p/2)^2$; for $x^2 - 10x$ it is 25. **F3** $x^2 - 3x - 4 = 0$. **F4** (a) D > 0 (b) D = 0 (c) D < 0. **F5** 9 − 24 + k = 0 → k = 15.

Day 7 Mini-Test

1) 3, 5. 2) 1/2, −2. 3) D = 28 > 0 — distinct real roots. 4) ±7. 5) 2, −4. 6) k = 25. 7) Yes: 16 − 28 + 12 = 0. 8) 9. 9) 8 and 9. 10) D = 4 − 24 < 0 — no real roots.

Day 21 Mini-Test

1) 3, −2/3. 2) 1, 2. 3) k = ±2√6. 4) −1, −5. 5) 13, 14. 6) D = 0 — equal roots. 7) 3/2, −2. 8) Yes — it simplifies to $x^2 - 10x - 3 = 0$. 9) 2/7. 10) 40 km/h.

Day 30 Mastery Test

1) −√3/2 (equal roots). 2) Not possible — D = −48 < 0. 3) 20 m × 20 m. 4) 7. 5) ±1, ±2. 6) k = 3. 7) 15 h and 10 h. 8) It reduces to −3x + 1 = 0, degree 1. 9) k = 6. 10) 6 km/h.

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Answer Key — Chapter Model Tests

Model Test A — Basics (20 marks)

Section A: A1 (b) A2 (c) A3 (b) A4 (c) A5 (b).

Section B: B1: $(x - 5)(x - 6) = 0 \rightarrow 5, 6$. B2: $x = \pm 6$. B3: $D = 4 + 32 = 36 > 0$ — two distinct real roots.

Section C: C1: $D = 25 - 16 = 9$; $x = (5 \pm 3)/4 \rightarrow 2, 1/2$. C2: $k^2 - 64 = 0 \rightarrow k = \pm 8$. C3: Expands to $-x^3 + 6x^2 + 14x + 8 = 0$ — cubic, NOT quadratic.

Model Test B — Standard (20 marks)

Section A: A1 (a) A2 (b) A3 (c) A4 (b) A5 (c).

Section B: B1: $-\sqrt{2}, -5\sqrt{2}/2$. B2: $(3 \pm \sqrt{13})/2$. B3: $D = 36 - 24 = 12 > 0$ — real and distinct roots.

Section C: C1: 13 and 14. C2: $k = 6$ ($k = 0$ rejected — not quadratic). C3: Yes — breadth 20 m, length 40 m.

Model Test C — Advanced (20 marks)

Section A: A1 (b) A2 (c) A3 (c) A4 (b) A5 (b).

Section B: B1: $y = x^2 \rightarrow y = 4, 9 \rightarrow x = \pm 2, \pm 3$. B2: $x^2 + 4x + 13 = (x + 2)^2 + 9 > 0$ always — no real roots. B3: $k = 3$ or 4.

Section C: C1: Smaller tap 15 h, larger tap 10 h. C2: $D = -4(ad - bc)^2 < 0$ when $ad \neq bc$ — see Reasoning 1. C3: Stream speed 6 km/h.

TOPPER TIP: *Mark your tests honestly, log every error in the Error Tracker (Section 18), and re-attempt the missed questions after two days. A test you do not review is a test wasted.*

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Revision Card — Quadratic Equations on One Page

$$x = [\, -b \pm \sqrt{(b^2 - 4ac)}\,] / 2a \quad | \quad D = b^2 - 4ac$$

D > 0	D = 0	D < 0
Two distinct real roots	Two equal roots, x = −b/2a	No real roots

The 60-second method check

Factorise if D is a perfect square — pair: product ac, sum b.

Complete the square if asked, or to prove — add (half of x-coeff.)^2 to both sides.

Formula always — write a, b, c, then D, then substitute with brackets.

Sum of roots = −b/a • **Product** = c/a • **Build equation:** x^2 − (sum)x + (product) = 0.

Never again

Never divide by x. Never drop the ±. Never write $(a + b)^2 = a^2 + b^2$. Never report a negative length, speed, age or count — reject it with its reason. Never set D > 0 for equal roots.

Word-problem reflexes

Area → length × width. Ages → add the years to EVERY age. Train/boat → time = distance ÷ speed; stream adds downstream, subtracts upstream. Taps → rates (1/time) add. Triangle/field → Pythagoras. Consecutive → x, x + 1 (even/odd: x, x + 2).

End of Chapter 4 — Quadratic Equations.

Next: Chapter 5 — Arithmetic Progressions. Keep your Flash Cards; the Smart Memory days continue.

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CBSE • CLASS 10 • MATHEMATICS

MATHEMATICS BEYOND THE FORMULA

CHAPTER 5 • ARITHMETIC PROGRESSIONS

Don't Memorize Mathematics. Understand, Visualise and Discover It.

The KalviYogi 8D Way: Discover • Visualise • Understand • Derive • Solve • Apply • Challenge • Recall
Every solution with COMPLETE steps — one by one, nothing skipped
The n-th Term • Sum of n Terms • Both Formulas DERIVED, not dictated
Five-Level Practice • 25 Concept MCQs • Board PYQs • Model Tests • Smart Memory System

A chapter that teaches like a teacher beside you —
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01 Chapter Introduction

8D • DISCOVER — WHY SHOULD I LEARN THIS?

Look at these three lists of numbers:

2, 5, 8, 11, 14, ... | **100, 90, 80, 70, ...** | **3, 3, 3, 3, ...**

Did you spot it? In each list, you reach the next number by adding the **same fixed number** again and again: +3, −10, and 0. A list that grows (or shrinks) by **equal steps** is called an **Arithmetic Progression** — an **AP**.

Equal steps are everywhere in real life. A staircase climbs by the same height at every step. A worker's salary rises by the same increment every year. A savings plan adds the same extra amount every month. Theatre seats, taxi fares, loan instalments, production targets — wherever life moves in **equal jumps**, an AP is hiding inside it.

What you will master in this chapter

- How to **recognise** an AP and find its common difference d.
- The **n-th term formula** $a_n = a + (n - 1)d$ — and **where it comes from**.
- The **sum formula** $S_n = n/2\,[2a + (n - 1)d]$ — derived by a beautiful pairing trick.
- How to attack **any** AP problem: terms, sums, missing terms, real-life stories.

The 8D promise of this chapter

You will not be asked to memorise a single formula blindly. In Section 06 you will **derive** both formulas yourself, step by step. After that, every formula will feel like an old friend — because you built it.

Chapter map: Story (03) → Pictures (04) → Foundations (05) → Derivations (06) → All NCERT exercises solved (07) → Second methods (08) → Labs (09) → Mistakes and Find-the-Error (10) → Five-Level Practice + MCQs (11) → Reasoning + Assertion-Reason (12) → Applications (13) → Competency case studies (14) → Olympiad (15) → Mind Map (16) → Recall (17) → Mastery Tracker + Model Tests (18).

DIGITAL COMPANION: QR code and link activate with the KalviYogi app/website at print. Use it for the video lesson, the interactive AP simulator, online tests and learning analytics for this chapter.

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02 Prerequisite Knowledge

8D • UNDERSTAND — WHAT YOU ALREADY KNOW

You need only four small ideas from earlier classes. Let us refresh them quickly.

1. Sequences and patterns

A **sequence** is an ordered list of numbers written in a definite order: 1st, 2nd, 3rd, ... Each number is called a **term**. A **pattern** is the rule that produces the terms.

Sequence	Pattern (rule)	Next term
5, 10, 15, 20, ...	add 5 each time	25
40, 36, 32, 28, ...	subtract 4 each time	24
1, 4, 9, 16, ...	squares: 1^2, 2^2, 3^2, 4^2	25
2, 4, 8, 16, ...	multiply by 2 each time	32

Only the first two patterns — **adding or subtracting the same number** — give an AP. The square pattern and the doubling pattern are **not** APs, because their jumps keep changing.

2. Position numbers (n)

We count terms by position: the 1st term is written a_1, the 2nd is a_2, and the term in position n is a_n. Asking for a_{20} means: "what is the 20th number in the list?"

3. Negative numbers and fractions

Differences can be negative (a falling AP) or fractions like ½ or −3/2. Subtracting carefully is the only skill needed: e.g. 2 − 5 = −3, and −1 − (−4) = −1 + 4 = 3.

4. Simple equations

Many AP questions end in one small equation like 3 + 5(n − 1) = 78. You already know how to solve it: n − 1 = 15, so n = 16.

Quick check (answers at the bottom)

C1. Write the next two terms: 7, 12, 17, 22, ... **C2.** In 50, 47, 44, 41, ..., what is added each time? **C3.** What is the 6th term of 4, 9, 14, ...? **C4.** Is 1, 2, 4, 7, 11, ... an "equal steps" list? **C5.** Compute (−3) − (−8).

03 A Real-Life Mathematical Story

8D • DISCOVER — THE THEATRE OF EQUAL STEPS

In the village of Kattur, the school ground had one problem every Annual Day: people stood in the hot sun because there were no seats. So the village panchayat built a small open-air theatre — a set of wide stone steps facing a stage, where people could sit in rows.

*The mason, old Periyasamy, laid the first row with **18 seats**. "Each next row is a little wider," he said, "so I can fit **3 more seats** than the row before it." Row 2 got 21 seats. Row 3 got 24. Row 4 got 27.*

His granddaughter Divya, who is in Class 10, watched the rows grow and smiled. "Thatha," she said, "you are not building a theatre. You are building a mathematics chapter!"

*Periyasamy laughed. "Then answer me, Professor. I have stone for **12 rows**. How many seats will the last row have? And how many seats will my whole theatre hold?"*

*Divya took a stick and wrote in the dust: 18, 21, 24, 27, ... "Every row adds the same number — 3. This is an **Arithmetic Progression**. The last row and the total both have formulas... but first, let me understand why they work."*

By the end of this chapter, you will answer Periyasamy in seconds:

The theatre answers (you will derive them!)

Seats in the 12th row = 18 + (12 − 1) × 3 = 18 + 33 = **51 seats**.
Total seats in 12 rows = 12/2 × (18 + 51) = 6 × 69 = **414 seats**.

Notice the two questions hidden in this one story: **"the 12th row"** asks for one term (a_n), and **"the whole theatre"** asks for a sum (S_n). Almost every AP problem in your exam is one of these two questions wearing a costume!

TOPPER TIP: *Whenever a story adds or removes the SAME amount again and again — seats, salary, savings, bricks — it is an AP story. First find a (the starting value) and d (the fixed step). The rest is just the two formulas.*

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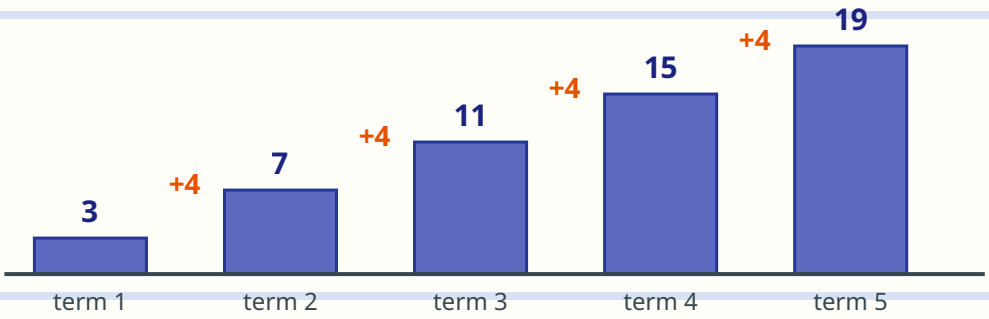
04 Visual Concept Explanation

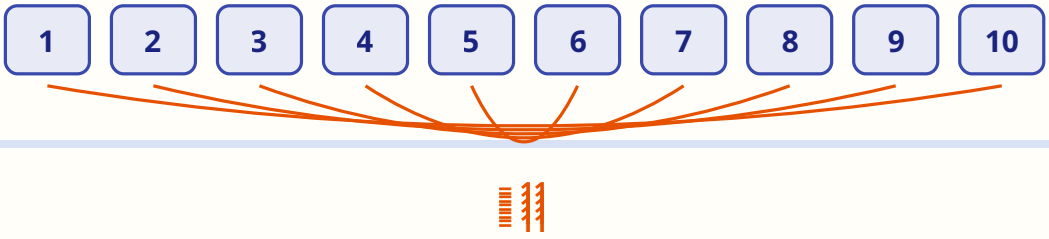
8D • VISUALISE — SINGAPORE CPA: CONCRETE → PICTORIAL → ABSTRACT

Concrete: Imagine actual bars of wood, each bar longer than the previous one by the same piece. The AP 3, 7, 11, 15, 19, ... looks like a staircase of bars:



Pictorial: See the pattern? Every bar top is exactly **+4** above the previous bar top. The bars climb like steps of a staircase with **equal risers**. That equal climb *is* the common difference d. If even one step climbed differently, the staircase would not be an AP.

The pairing picture (for sums): To add 1 + 2 + 3 + ... + 10, do not add left to right like a machine. Pair the ends:



1 pairs with 10, 2 with 9, 3 with 8, 4 with 7, 5 with 6 — and **every pair totals 11**! There are 5 pairs, so the sum is 5 × 11 = 55. This tiny picture is the seed of the great sum formula you will derive in Section 06.

Abstract: A list a, a + d, a + 2d, a + 3d, ... is just the bar staircase written in symbols: start at height a, climb d at every step.

> ***TOPPER TIP:*** *Draw the bar picture mentally for any AP question. If you can 'see' the staircase, you will never confuse a term (one bar) with a sum (all the bars together).*

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05 Mathematical Foundations

8D • UNDERSTAND — THE EXACT DEFINITIONS

Definition of an AP

An **Arithmetic Progression (AP)** is a list of numbers in which each term, except the first, is obtained by adding a fixed number to the preceding term. This fixed number is called the **common difference** and is denoted by **d**.

The language of AP

a = first term (also written a_1) • **d** = common difference = (any term) − (term just before it) • **n** = number of terms / position • $\mathbf{a_n}$ = n-th term (also called the **general term**) • **l** = last term of a finite AP • $\mathbf{S_n}$ = sum of the first n terms.

How to test whether a list is an AP

Compute the differences $a_2 - a_1$, $a_3 - a_2$, $a_4 - a_3$, … If **all** the differences are equal, the list is an AP and that equal value is d. If even one difference differs, it is **not** an AP. d can be positive (rising AP), negative (falling AP), zero (constant AP) or a fraction.

List	Differences	AP?	d
4, 10, 16, 22, …	6, 6, 6	Yes	6
21, 18, 15, 12, …	−3, −3, −3	Yes	−3
7, 7, 7, 7, …	0, 0, 0	Yes	0
1, 2, 4, 8, …	1, 2, 4	No	—
1, 4, 9, 16, …	3, 5, 7	No	—

The general form

If the first term is a and the common difference is d, the AP is:

a, a + d, a + 2d, a + 3d, a + 4d, …

Look at the third term: a + 2d. Look at the fifth: a + 4d. The multiplier of d is always **one less** than the position number. Hold that thought — it becomes the n-th term formula in the next section.

06 Formula Derivations

8D • DERIVE — WHERE THE FORMULAS COME FROM

Derivation I: The n-th term, $a_n = a + (n - 1)d$

Write the terms of an AP one by one, and count carefully *how many times d has been added*:

$a_1 = a$	→ d added **0** times
$a_2 = a + d$	→ d added **1** time
$a_3 = a + 2d$	→ d added **2** times
$a_4 = a + 3d$	→ d added **3** times
$a_5 = a + 4d$	→ d added **4** times
$a_n = a + (n - 1)d$	**→ d added $(n - 1)$ times**

Why (n − 1) and not n? Think of a fence: to walk from post 1 to post n, you cross only n − 1 gaps. The first term is the *starting post* — no jump has been taken yet. Each jump adds one d. Reaching term number n takes exactly n − 1 jumps.

THE N-TH TERM FORMULA: $a_n = a + (n - 1)d$

PIN POINT: Count the gaps, not the posts! Term number n needs only (n − 1) jumps of d.

FORMULA DERIVATIONS — CONTINUED

Derivation 2: The sum, $S_n = n/2\ [2a + (n - 1)d]$

There is a famous story that young Carl Friedrich Gauss, aged about seven, was asked by his teacher to add 1 + 2 + … + 100 — and answered 5050 almost instantly. His trick: **pair the ends**. We now use the same trick on a general AP. Let S be the sum of the first n terms.

Step 1 — write the sum forwards:
S = a + (a + d) + (a + 2d) + … + [a + (n − 2)d] + [a + (n − 1)d]

Step 2 — write the same sum backwards (same terms, reverse order):
S = [a + (n − 1)d] + [a + (n − 2)d] + … + (a + d) + a

Step 3 — add the two lines, column by column. First column: a + [a + (n − 1)d] = 2a + (n − 1)d. Second column: (a + d) + [a + (n − 2)d] = 2a + (n − 1)d. **Every** column gives the same total, 2a + (n − 1)d — and there are n columns. So:
2S = n [2a + (n − 1)d]

Step 4 — divide by 2:

THE SUM FORMULA: $S_n = n/2 \times [2a + (n - 1)d]$

Step 5 — a second dress for the same formula. Since the last term l = a_n = a + (n − 1)d, we get 2a + (n − 1)d = a + [a + (n − 1)d] = a + l. Substituting:

$S_n = n/2 \times (a + l)$ = (number of terms) × (average of first and last)

PIN POINT: S_n = n/2 (a + l) — use this form when the LAST term is known; use S_n = n/2 [2a + (n−1)d] when d is known instead.

TOPPER TIP: *Gauss did not add faster — he added SMARTER. Pairing first-with-last works because in an AP, as one partner grows by d, the other shrinks by d, so every pair-sum is identical.*

07 Detailed Solved Examples

8D • SOLVE — EVERY NCERT QUESTION, EVERY STEP

This section solves **every question** of NCERT Exercises 5.1, 5.2 and 5.3 (plus optional Exercise 5.4 as a bonus), followed by real **CBSE Previous Year Questions**. No step is skipped: substitutions are shown in full, and each step carries its reason.

EXERCISE 5.1 — RECOGNISING AN AP

EX 5.1 Q1

In which of the following situations does the list of numbers involved make an AP, and why? (i) Taxi fare: ₹15 for the first km, ₹8 for each additional km. (ii) Air in a cylinder when a vacuum pump removes 1/4 of the air REMAINING each time. (iii) Cost of digging a well: ₹150 for the first metre, rising by ₹50 for each subsequent metre. (iv) Money in an account when ₹10,000 is deposited at compound interest at 8% per annum.

THINK FIRST: For each situation, write the actual list of numbers, then test whether consecutive differences are all equal.

1. **(i)** Total fare after 1, 2, 3, ... km: 15, 15 + 8, 15 + 2(8), ... = 15, 23, 31, ... Differences: 23 − 15 = 8 and 31 − 23 = 8 — equal.
   Why: Each extra km adds the same ₹8 to the total fare.

2. **(ii)** Let the cylinder hold V units of air. After each pumping, 3/4 remains: V, 3V/4, 9V/16, ... Differences: 3V/4 − V = −V/4, but 9V/16 − 3V/4 = −3V/16. −V/4 ≠ −3V/16.
   Why: The pump removes 1/4 of what REMAINS, so the removed amount shrinks each time — the steps are not equal.

3. **(iii)** Cost after 1, 2, 3, ... metres: 150, 150 + 50, 150 + 2(50), ... = 150, 200, 250, ... Differences: 50 and 50 — equal.
   Why: Each extra metre adds the same ₹50.

4. **(iv)** Amount after each year: 10000, 10000(1.08) = 10800, 10800(1.08) = 11664, ... Differences: 10800 − 10000 = 800, but 11664 − 10800 = 864. 800 ≠ 864.
   Why: Compound interest is calculated on a GROWING balance, so the interest itself grows — steps not equal.

ANSWER: (i) AP (d = 8) • (ii) Not an AP • (iii) AP (d = 50) • (iv) Not an AP

CBSE MARKING SCHEME: 1 mark for each correct list • 1 mark for each difference test and conclusion (4 marks total).

EXERCISE 5.1 — CONTINUED

EX 5.1 Q2

Write the first four terms of the AP when a and d are given: (i) a = 10, d = 10 (ii) a = −2, d = 0 (iii) a = 4, d = −3 (iv) a = −1, d = 1/2 (v) a = −1.25, d = −0.25

THINK FIRST: Keep adding d to the previous term: $a_1 = a$, $a_2 = a + d$, $a_3 = a + 2d$, $a_4 = a + 3d$.

1. **(i)** 10, 10 + 10, 10 + 2(10), 10 + 3(10) = **10, 20, 30, 40**.
   Why: Add 10 at each step.

2. **(ii)** −2, −2 + 0, −2 + 0, −2 + 0 = **−2, −2, −2, −2**.
   Why: d = 0 gives a constant AP.

3. **(iii)** 4, 4 + (−3), 4 + 2(−3), 4 + 3(−3) = **4, 1, −2, −5**.
   Why: Adding −3 means subtracting 3.

4. **(iv)** −1, −1 + 1/2, −1 + 2(1/2), −1 + 3(1/2) = **−1, −1/2, 0, 1/2**.
   Why: Work in fractions to stay exact.

5. **(v)** −1.25, −1.25 − 0.25, −1.25 − 0.50, −1.25 − 0.75 = **−1.25, −1.50, −1.75, −2.00**.
   Why: Each step subtracts 0.25.

ANSWER: (i) 10, 20, 30, 40 (ii) −2, −2, −2, −2 (iii) 4, 1, −2, −5 (iv) −1, −1/2, 0, 1/2 (v) −1.25, −1.50, −1.75, −2.00

CBSE MARKING SCHEME: 1 mark for the method a, a+d, a+2d, a+3d • 1 mark for each two correct lists (3 marks total).

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EXERCISE 5.1 — CONTINUED

EX 5.1 Q3

For the following APs, write the first term and the common difference: (i) 3, 1, −1, −3, … (ii) −5, −1, 3, 7, … (iii) 1/3, 5/3, 9/3, 13/3, … (iv) 0.6, 1.7, 2.8, 3.9, …

THINK FIRST: a is simply the first number. d = second term − first term (then quickly check with the next pair).

1. **(i)** a = 3. d = 1 − 3 = **−2**. Check: −1 − 1 = −2. Consistent.
   Why: Always verify d with a second pair of terms.

2. **(ii)** a = −5. d = −1 − (−5) = −1 + 5 = **4**. Check: 3 − (−1) = 4. Consistent.
   Why: Subtracting a negative becomes adding.

3. **(iii)** a = 1/3. d = 5/3 − 1/3 = **4/3**. Check: 9/3 − 5/3 = 4/3. Consistent.
   Why: Same denominator — subtract numerators only.

4. **(iv)** a = 0.6. d = 1.7 − 0.6 = **1.1**. Check: 2.8 − 1.7 = 1.1. Consistent.
   Why: Decimal subtraction, digit by digit.

ANSWER: (i) a = 3, d = −2 (ii) a = −5, d = 4 (iii) a = 1/3, d = 4/3 (iv) a = 0.6, d = 1.1

CBSE MARKING SCHEME: 1/2 mark for each correct a and each correct d (4 parts × 1 mark).

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EXERCISE 5.1 — CONTINUED

EX 5.1 Q4 (i)–(v)

Which of the following are APs? If an AP, find d and write three more terms. (i) 2, 4, 8, 16, … (ii) 2, 5/2, 3, 7/2, … (iii) −1.2, −3.2, −5.2, −7.2, … (iv) −10, −6, −2, 2, … (v) 3, 3 + √2, 3 + 2√2, 3 + 3√2, …

THINK FIRST: Compute ALL consecutive differences. Only if every difference is the same is it an AP; then keep adding d for three more terms.

1. **(i)** Differences: 4 − 2 = 2, 8 − 4 = 4, 16 − 8 = 8. Not all equal → **not an AP**.
   Why: This list doubles — doubling is not equal steps.

2. **(ii)** Differences: 5/2 − 2 = 1/2, 3 − 5/2 = 1/2, 7/2 − 3 = 1/2. All equal → **AP, d = 1/2**. Next terms: 7/2 + 1/2 = 4, 4 + 1/2 = 9/2, 9/2 + 1/2 = 5.
   Why: Keep adding d = 1/2.

3. **(iii)** Differences: −3.2 − (−1.2) = −2, −5.2 − (−3.2) = −2, −7.2 − (−5.2) = −2. All equal → **AP, d = −2**. Next: −9.2, −11.2, −13.2.
   Why: Keep adding d = −2.

4. **(iv)** Differences: −6 − (−10) = 4, −2 − (−6) = 4, 2 − (−2) = 4. All equal → **AP, d = 4**. Next: 6, 10, 14.
   Why: Keep adding d = 4.

5. **(v)** Differences: (3 + √2) − 3 = √2, (3 + 2√2) − (3 + √2) = √2, (3 + 3√2) − (3 + 2√2) = √2. All equal → **AP, d = √2**. Next: 3 + 4√2, 3 + 5√2, 3 + 6√2.
   Why: Only the √2 part changes — the 3 stays fixed.

ANSWER: (i) Not an AP (ii) AP, d = 1/2; 4, 9/2, 5 (iii) AP, d = −2; −9.2, −11.2, −13.2 (iv) AP, d = 4; 6, 10, 14 (v) AP, d = √2; 3 + 4√2, 3 + 5√2, 3 + 6√2

CBSE MARKING SCHEME: 1 mark for each correct difference test • 1 mark for d and the three terms where it is an AP.

EXERCISE 5.1 — CONTINUED

EX 5.1 Q4 (vi)–(x)

Which of the following are APs? If an AP, find d and write three more terms. (vi) 0.2, 0.22, 0.222, 0.2222, … (vii) 0, −4, −8, −12, … (viii) −1/2, −1/2, −1/2, −1/2, … (ix) 1, 3, 9, 27, … (x) a, 2a, 3a, 4a, …

THINK FIRST: Same test: every consecutive difference must be equal.

1. **(vi)** Differences: 0.22 − 0.2 = 0.02, 0.222 − 0.22 = 0.002, 0.2222 − 0.222 = 0.0002. Not equal → **not an AP**.
   Why: The jumps shrink by a factor of 10 each time.

2. **(vii)** Differences: −4 − 0 = −4, −8 − (−4) = −4, −12 − (−8) = −4. All equal → **AP, d = −4**. Next: −16, −20, −24.
   Why: Keep adding −4.

3. **(viii)** Differences: 0, 0, 0. All equal → **AP, d = 0**. Next: −1/2, −1/2, −1/2.
   Why: A constant list is a perfectly good AP.

4. **(ix)** Differences: 3 − 1 = 2, 9 − 3 = 6, 27 − 9 = 18. Not equal → **not an AP**.
   Why: This list multiplies by 3 — a geometric pattern, not an AP.

5. **(x)** Differences: 2a − a = a, 3a − 2a = a, 4a − 3a = a. All equal → **AP, d = a**. Next: 5a, 6a, 7a.
   Why: Letters obey the same rules as numbers.

ANSWER: (vi) Not an AP (vii) AP, d = −4; −16, −20, −24 (viii) AP, d = 0; −1/2, −1/2, −1/2 (ix) Not an AP (x) AP, d = a; 5a, 6a, 7a

CBSE MARKING SCHEME: 1 mark for each correct difference test • 1 mark for d and the three terms where it is an AP.

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EXERCISE 5.1 — CONTINUED

EX 5.1 Q4 (xi)–(xv)

Which of the following are APs? If an AP, find d and write three more terms. (xi) a, a^2, a^3, a^4, … (xii) √2, √8, √18, √32, … (xiii) √3, √6, √9, √12, … (xiv) 1^2, 3^2, 5^2, 7^2, … (xv) 1^2, 5^2, 7^2, 73, …

THINK FIRST: For root lists, first simplify each root (e.g. √8 = 2√2) — the AP often hides behind the roots.

1. **(xi)** Differences: $a^2 - a$ and $a^3 - a^2 = a^2(a - 1)$ vs $a(a - 1)$ — not equal (for $a \neq 0, 1$) → **not an AP**.
   Why: Powers grow faster and faster — differences keep changing.

2. **(xii)** Simplify: √2, 2√2, 3√2, 4√2. Differences: √2, √2, √2. All equal → **AP, d = √2**. Next: 5√2 = √50, 6√2 = √72, 7√2 = √98.
   Why: Because √8 = √(4×2) = 2√2, √18 = 3√2, √32 = 4√2.

3. **(xiii)** Differences: $\sqrt{6} - \sqrt{3} \approx 0.717$, but $\sqrt{9} - \sqrt{6} = 3 - \sqrt{6} \approx 0.551$. Not equal → **not an AP**.
   Why: Here the roots do NOT simplify into multiples of one root.

4. **(xiv)** Values: 1, 9, 25, 49. Differences: 8, 16, 24. Not equal → **not an AP**.
   Why: Squares of odd numbers spread further and further apart.

5. **(xv)** Values: 1, 25, 49, 73. Differences: 25 − 1 = 24, 49 − 25 = 24, 73 − 49 = 24. All equal → **AP, d = 24**. Next: 97, 121, 145.
   Why: A surprise AP — always compute the actual values first!

ANSWER: (xi) Not an AP (xii) AP, d = √2; √50, √72, √98 (xiii) Not an AP (xiv) Not an AP (xv) AP, d = 24; 97, 121, 145

CBSE MARKING SCHEME: 1 mark for each correct simplification/difference test • 1 mark for d and the three terms where it is an AP.

EXERCISE 5.2 — THE N-TH TERM

EX 5.2 Q1

Fill in the blanks in the table, given a is the first term, d the common difference and a_n the n-th term: (i) a = 7, d = 3, n = 8, a_n = … (ii) a = −18, d = …, n = 10, a_n = 0 (iii) a = …, d = −3, n = 18, a_n = −5 (iv) a = −18.9, d = 2.5, n = …, a_n = 3.6 (v) a = 3.5, d = 0, n = 105, a_n = …

THINK FIRST: One formula does all five: $a_n = a + (n - 1)d$. In each part, exactly one quantity is unknown — substitute the other three and solve.

1. **(i)** $a_8 = 7 + (8 - 1)(3) = 7 + 21 = \mathbf{28}$.
   Why: Direct substitution.

2. **(ii)** $0 = -18 + (10 - 1)d \rightarrow 0 = -18 + 9d \rightarrow 9d = 18 \rightarrow d = \mathbf{2}$.
   Why: Move −18 to the other side, then divide by 9.

3. **(iii)** $-5 = a + (18 - 1)(-3) \rightarrow -5 = a - 51 \rightarrow a = -5 + 51 = \mathbf{46}$.
   Why: Work backwards from the last term to the first.

4. **(iv)** $3.6 = -18.9 + (n - 1)(2.5) \rightarrow (n - 1)(2.5) = 3.6 + 18.9 = 22.5 \rightarrow n - 1 = 22.5/2.5 = 9 \rightarrow n = \mathbf{10}$.
   Why: Isolate (n − 1) first, then add 1.

5. **(v)** $a_{105} = 3.5 + (105 - 1)(0) = 3.5 + 0 = \mathbf{3.5}$.
   Why: With d = 0, no term ever changes.

ANSWER: (i) 28 (ii) d = 2 (iii) a = 46 (iv) n = 10 (v) 3.5

CBSE MARKING SCHEME: 1/2 mark for each correct substitution • 1/2 mark for each correct value (5 marks total).

EXERCISE 5.2 — CONTINUED

EX 5.2 Q2

Choose the correct choice and justify: (i) 30th term of the AP 10, 7, 4, … is (A) 97 (B) 77 (C) −77 (D) −87. (ii) 11th term of the AP −3, −1/2, 2, … is (A) 28 (B) 22 (C) −38 (D) −48½.

THINK FIRST: Read a and d from the list, then apply $a_n = a + (n - 1)d$. The options are built from common errors — compute honestly and match.

1

(i) a = 10, d = 7 − 10 = −3. $a_{30} = 10 + (30 - 1)(-3) = 10 - 87 =$ **−77** → option **(C)**.

Why: d is negative — options (A) and (B) are traps for students who take d = +3 or forget the sign.

2

(ii) a = −3, d = −1/2 − (−3) = 5/2. $a_{11} = -3 + (11 - 1)(5/2) = -3 + 25 =$ **22** → option **(B)**.

Why: −1/2 + 3 = 5/2 — careful with the fraction and the double negative.

ANSWER: (i) (C) −77 (ii) (B) 22

CBSE MARKING SCHEME: 1 mark for correct a and d • 1 mark for the correct option with justification, in each part.

EXERCISE 5.2 — CONTINUED

EX 5.2 Q3 (i)–(iii)

Find the missing terms in the boxes: (i) 2, [ ], 26 (ii) [ ], 13, [ ], 3 (iii) 5, [ ], [ ], 9½

THINK FIRST: Key fact: in an AP, terms equally spaced from the ends have a steady step. Find d from the two known terms by dividing their gap by the number of jumps between them.

1. **(i)** From 2 to 26 there are 2 jumps: d = (26 − 2)/2 = 12. Missing term = 2 + 12 = **14**.
   Why: The middle term of three in AP is the average: (2 + 26)/2 = 14.

2. **(ii)** $a_2 = 13$ and $a_4 = 3$ are 2 jumps apart: d = (3 − 13)/2 = −5. First box: $a_1 = 13 - d = 13 - (-5) =$ **18**. Second box: $a_3 = 13 + d = 13 - 5 =$ **8**.
   Why: Step backwards for the first box, forwards for the second.

3. **(iii)** From 5 to 9½ there are 3 jumps: d = (9½ − 5)/3 = (9/2)/3 = 3/2. Boxes: 5 + 3/2 = **6½** and 6½ + 3/2 = **8**.
   Why: Count the jumps between known terms — here 3 jumps, not 4 terms.

ANSWER: (i) 14 (ii) 18, 8 (iii) 6½, 8

CBSE MARKING SCHEME: 1 mark for each correct d • 1 mark for each completed box set.

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EXERCISE 5.2 — CONTINUED

EX 5.2 Q3 (iv)–(v)

Find the missing terms in the boxes: (iv) −4, [ ], [ ], [ ], [ ], 6 (v) [ ], 38, [ ], [ ], [ ], −22

THINK FIRST: Same method: divide the total gap by the number of jumps between the two known terms.

1. **(iv)** From −4 to 6 there are 5 jumps: d = (6 − (−4))/5 = 10/5 = 2. Boxes: −4 + 2 = **−2**, then **0**, then **2**, then **4**. Check: 4 + 2 = 6. Correct.
   Why: Six terms means FIVE jumps — the fence-post idea again.

2. **(v)** a_2 = 38 and a_6 = −22 are 4 jumps apart: d = (−22 − 38)/4 = −60/4 = −15. First box: a_1 = 38 − (−15) = **53**. Remaining boxes: 38 − 15 = **23**, 23 − 15 = **8**, 8 − 15 = **−7**. Check: −7 − 15 = −22. Correct.
   Why: Work out d once, then walk forwards and backwards.

ANSWER: (iv) −2, 0, 2, 4 (v) 53, 23, 8, −7

CBSE MARKING SCHEME: 1 mark for each correct d • 1 mark for the completed boxes in each part.

PIN POINT: Missing-term questions: d = (gap between known terms) ÷ (number of jumps between them). Jumps = difference of positions.

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EXERCISE 5.2 — CONTINUED

EX 5.2 Q4

Which term of the AP 3, 8, 13, 18, ... is 78?

THINK FIRST: Suppose 78 is the n-th term. Put $a_n = 78$ in the formula and solve for n.

1. Read a and d: a = 3, d = 8 − 3 = 5.
   Why: Always write a and d first.
2. Set up: $a_n = a + (n - 1)d \rightarrow 78 = 3 + (n - 1)(5)$.
   Why: The unknown is the POSITION n.
3. Solve: $78 - 3 = 5(n - 1) \rightarrow 75 = 5(n - 1) \rightarrow n - 1 = 15 \rightarrow n = 16$.
   Why: n came out a whole number, so 78 really is a term.
4. Check: $a_{16} = 3 + 15(5) = 78$. Correct.
   Why: A 5-second check catches most errors.

ANSWER: 78 is the 16th term of the AP.

CBSE MARKING SCHEME: 1 mark for a and d • 1 mark for the equation • 1 mark for n = 16.

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EXERCISE 5.2 — CONTINUED

EX 5.2 Q5

Find the number of terms in each AP: (i) 7, 13, 19, ..., 205 (ii) 18, 15½, 13, ..., −47

THINK FIRST: The last term is a_n. Use the formula with the last term's value and solve for n.

1. **(i)** a = 7, d = 6, a_n = 205. 205 = 7 + (n − 1)(6) → 198 = 6(n − 1) → n − 1 = 33 → n = **34**.
   Why: Subtract a first, then divide by d, then add 1.

2. **(ii)** a = 18, d = 15½ − 18 = −5/2, a_n = −47. −47 = 18 + (n − 1)(−5/2) → −65 = −(5/2)(n − 1) → n − 1 = 65 × 2/5 = 26 → n = **27**.
   Why: Both sides negative — the negatives cancel; keep calm and continue.

ANSWER: (i) 34 terms (ii) 27 terms

CBSE MARKING SCHEME: 1 mark for a, d in each part • 1 mark for the equation • 1 mark for the correct n.

EX 5.2 Q6

Check whether −150 is a term of the AP 11, 8, 5, 2, ...

THINK FIRST: Assume −150 is the n-th term and solve. The verdict depends on whether n is a whole number.

1. a = 11, d = 8 − 11 = −3.
   Why: Write a and d.

2. Set a_n = −150: −150 = 11 + (n − 1)(−3).
   Why: Same set-up as Q4.

3. Solve: −150 − 11 = −3(n − 1) → −161 = −3(n − 1) → n − 1 = 161/3 → n = 164/3 = 54⅔.
   Why: n is a fraction.

4. Conclude: a position number must be a positive whole number. Since n = 164/3 is not a whole number, −150 is **not** a term of this AP.
   Why: The formula never lies: a fractional n means 'this value is skipped by the AP'.

ANSWER: −150 is NOT a term of the AP.

CBSE MARKING SCHEME: 1 mark for the equation • 1 mark for n = 164/3 • 1 mark for the correct conclusion with reason.

EXERCISE 5.2 — CONTINUED

EX 5.2 Q7

Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

THINK FIRST: Two given terms give two equations in a and d. But there is a shortcut: the gap between the 16th and 11th terms is 5 jumps of d.

1. From a_{11} to a_{16} there are 5 jumps: $5d = 73 - 38 = 35$, so $d = 7$.
   Why: Terms 5 positions apart differ by 5d.
2. Find a: $a_{11} = a + 10d \rightarrow 38 = a + 10(7) \rightarrow a = 38 - 70 = -32$.
   Why: Walk back 10 jumps from the 11th term.
3. Now $a_{31} = a + 30d = -32 + 30(7) = -32 + 210 = \mathbf{178}$.
   Why: Or walk forward: $a_{31} = a_{16} + 15d = 73 + 105 = 178$. Same answer.

ANSWER: The 31st term is 178.

CBSE MARKING SCHEME: 1 mark for d • 1 mark for a • 1 mark for $a_{31} = 178$.

EX 5.2 Q8

An AP consists of 50 terms, of which the 3rd term is 12 and the last term is 106. Find the 29th term.

THINK FIRST: The last term is the 50th term. a_3 and a_{50} are 47 jumps apart.

1. 47 jumps take us from 12 to 106: $47d = 106 - 12 = 94$, so $d = 2$.
   Why: Jumps between positions 3 and 50 = 50 − 3 = 47.
2. Find a: $a_3 = a + 2d \rightarrow 12 = a + 4 \rightarrow a = 8$.
   Why: Walk back 2 jumps.
3. $a_{29} = a + 28d = 8 + 28(2) = 8 + 56 = \mathbf{64}$.
   Why: Apply the n-th term formula with n = 29.

ANSWER: The 29th term is 64.

CBSE MARKING SCHEME: 1 mark for d • 1 mark for a • 1 mark for $a_{29} = 64$.

EXERCISE 5.2 — CONTINUED

EX 5.2 Q9

If the 3rd and the 9th terms of an AP are 4 and −8 respectively, which term of this AP is zero?

THINK FIRST: Write a_3 and a_9 with the formula; their difference gives d directly.

1. $a_9 - a_3$ spans 6 jumps: $6d = -8 - 4 = -12$, so $d = -2$.
   Why: Positions 3 and 9 are 6 jumps apart.
2. Find a: $a_3 = a + 2d \rightarrow 4 = a + 2(-2) \rightarrow a = 4 + 4 = 8$.
   Why: Walk back 2 jumps.
3. Let the n-th term be 0: $0 = 8 + (n - 1)(-2) \rightarrow 2(n - 1) = 8 \rightarrow n - 1 = 4 \rightarrow n = 5$.
   Why: Solve $a_n = 0$ for n.
4. Check: terms are 8, 6, 4, 2, 0 — yes, the 5th term is 0.
   Why: Listing five terms confirms it.

ANSWER: The 5th term of the AP is zero.

CBSE MARKING SCHEME: 1 mark for d • 1 mark for a • 1 mark for n = 5.

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EXERCISE 5.2 — CONTINUED

EX 5.2 Q10

The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

THINK FIRST: Translate the sentence into algebra: $a_{17} - a_{10} = 7$.

1. $a_{17} = a + 16d$ and $a_{10} = a + 9d$.
   Why: Write both terms with the formula.
2. Subtract: $(a + 16d) - (a + 9d) = 7d$.
   Why: The a cancels — only the jumps survive: 16 − 9 = 7 jumps.
3. Given this difference is 7: $7d = 7$, so $d = \mathbf{1}$.
   Why: One line of algebra.

ANSWER: The common difference is d = 1.

CBSE MARKING SCHEME: 1 mark for writing both terms • 1 mark for d = 1.

TOPPER TIP: *In general, $a_m - a_k = (m - k)d$. Terms far apart differ by many jumps — this one fact solves Q7, Q9 and Q10 type questions in seconds.*

EXERCISE 5.2 — CONTINUED

EX 5.2 Q11

Which term of the AP 3, 15, 27, 39, … will be 132 more than its 54th term?

THINK FIRST: Find the 54th term first; add 132; then find the position of that value.

1. $a = 3$, $d = 15 - 3 = 12$. $a_{54} = 3 + 53(12) = 3 + 636 = 639$.
   Why: The 54th term needs 53 jumps.
2. Target value = 639 + 132 = 771.
   Why: Read the question literally: 132 MORE than the 54th term.
3. Let 771 be the n-th term: $771 = 3 + (n - 1)(12) \rightarrow 768 = 12(n - 1) \rightarrow n - 1 = 64 \rightarrow n = 65$.
   Why: Solve for the position.

ANSWER: The 65th term will be 132 more than the 54th term.

CBSE MARKING SCHEME: 1 mark for a_{54} • 1 mark for the target 771 • 1 mark for n = 65.

EX 5.2 Q12

Two APs have the same common difference. The difference between their 100th terms is 100. What is the difference between their 1000th terms?

THINK FIRST: Let the first terms be a and A. Watch what happens to the difference at any position n.

1. 100th terms: (a + 99d) and (A + 99d). Their difference = a − A (the 99d cancels). Given a − A = 100.
   Why: Same d means both APs climb in lock-step.
2. 1000th terms: (a + 999d) and (A + 999d). Difference = a − A = 100 again.
   Why: The difference between the two APs NEVER changes — it is fixed at the very first term.

ANSWER: The difference between their 1000th terms is also 100.

CBSE MARKING SCHEME: 1 mark for setting up the two 100th terms • 1 mark for the conclusion with reason.

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EXERCISE 5.2 — CONTINUED

EX 5.2 Q13

How many three-digit numbers are divisible by 7?

THINK FIRST: The three-digit multiples of 7 form an AP: 105, 112, ..., 994. Find how many terms it has.

1. Smallest three-digit multiple of 7: 7 × 15 = 105. Largest: 7 × 142 = 994. So the AP is 105, 112, ..., 994 with a = 105, d = 7.
   Why: 100 is not divisible by 7 (7 × 14 = 98, 7 × 15 = 105); 999 is not either (7 × 142 = 994).
2. 994 = 105 + (n − 1)(7) → 889 = 7(n − 1) → n − 1 = 127 → n = 128.
   Why: Solve for the number of terms.

ANSWER: There are 128 three-digit numbers divisible by 7.

CBSE MARKING SCHEME: 1 mark for the first and last multiples • 1 mark for the equation • 1 mark for 128.

EX 5.2 Q14

How many multiples of 4 lie between 10 and 250?

THINK FIRST: 'Between 10 and 250' excludes both ends. The multiples form the AP 12, 16, ..., 248.

1. First multiple of 4 after 10 is 12; last one before 250 is 248 (since 250 is not a multiple of 4). a = 12, d = 4, last term 248.
   Why: Careful reading of 'between' — 250 itself is excluded anyway.
2. 248 = 12 + (n − 1)(4) → 236 = 4(n − 1) → n − 1 = 59 → n = 60.
   Why: Solve for n.

ANSWER: 60 multiples of 4 lie between 10 and 250.

CBSE MARKING SCHEME: 1 mark for the correct first/last multiples • 1 mark for the equation • 1 mark for 60.

EXERCISE 5.2 — CONTINUED

EX 5.2 Q15

For what value of n are the n-th terms of the two APs 63, 65, 67, ... and 3, 10, 17, ... equal?

THINK FIRST: Write the n-th term of each AP and set them equal.

1. First AP: a = 63, d = 2, so its n-th term = 63 + (n − 1)(2) = 2n + 61.
   Why: Simplify the expression fully.
2. Second AP: a = 3, d = 7, so its n-th term = 3 + (n − 1)(7) = 7n − 4.
   Why: Same routine.
3. Set equal: 2n + 61 = 7n − 4 → 61 + 4 = 7n − 2n → 65 = 5n → n = 13.
   Why: Solve the linear equation.
4. Check: first AP's 13th term = 63 + 24 = 87; second AP's 13th term = 3 + 84 = 87. Equal. Correct.
   Why: The second AP climbs faster (d = 7) and catches up at n = 13.

ANSWER: For n = 13, the n-th terms of the two APs are equal (both 87).

CBSE MARKING SCHEME: 1 mark for each n-th term expression • 1 mark for n = 13.

EX 5.2 Q16

Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

THINK FIRST: '7th exceeds 5th by 12' means $a_7 - a_5 = 12$ — that is 2 jumps.

1. $a_7 - a_5 = 2d = 12$, so d = 6.
   Why: Positions 5 and 7 are 2 jumps apart.
2. $a_3 = a + 2d \rightarrow 16 = a + 12 \rightarrow a = 4$.
   Why: Walk back 2 jumps from the 3rd term.
3. Build the AP: 4, 4 + 6, 4 + 12, 4 + 18, ... = 4, 10, 16, 22, ...
   Why: Check: 3rd term is 16. Correct.

ANSWER: The AP is 4, 10, 16, 22, ... (a = 4, d = 6).

CBSE MARKING SCHEME: 1 mark for d • 1 mark for a • 1 mark for the AP.

EXERCISE 5.2 — CONTINUED

EX 5.2 Q17

Find the 20th term from the last term of the AP 3, 8, 13, ..., 253.

THINK FIRST: Counting from the end is counting backwards: the AP reversed has first term 253 and difference −5.

1. Method (reverse AP): last term l = 253, and going backwards d becomes −5. 20th term from the end = 253 + (20 − 1)(−5) = 253 − 95 = **158**.
   Why: From the end, each step back subtracts d.

2. Check by counting terms: total terms n: 253 = 3 + (n − 1)(5) → n = 51. The 20th from the end is the (51 − 20 + 1) = 32nd from the start: a_{32} = 3 + 31(5) = 158. Same.
   Why: Two routes, one answer — the reverse method is faster.

ANSWER: The 20th term from the last is 158.

CBSE MARKING SCHEME: 1 mark for the reverse-AP idea (or term count) • 1 mark for 158.

EXERCISE 5.2 — CONTINUED

EX 5.2 Q18

The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

THINK FIRST: Write both conditions with a and d; subtract the equations to kill a.

1. $a_4 + a_8 = (a + 3d) + (a + 7d) = 2a + 10d = 24$. ...(1)
   Why: Expand each term with the formula.

2. $a_6 + a_{10} = (a + 5d) + (a + 9d) = 2a + 14d = 44$. ...(2)
   Why: Same routine.

3. (2) − (1): $4d = 20$, so $d = 5$.
   Why: Subtracting removes a completely.

4. From (1): $2a + 10(5) = 24 \rightarrow 2a = 24 - 50 = -26 \rightarrow a = -13$.
   Why: Substitute d back.

5. First three terms: -13, $-13 + 5$, $-13 + 10$ = **$-13, -8, -3$**.
   Why: Check: $a_4 + a_8 = 2 + 22 = 24$. Correct.

ANSWER: The first three terms are $-13, -8, -3$.

CBSE MARKING SCHEME: 1 mark for each equation • 1 mark for d • 1 mark for a and the three terms.

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EXERCISE 5.2 — CONTINUED

EX 5.2 Q19

Subba Rao started work in 1995 at an annual salary of ₹5000 and received an increment of ₹200 each year. In which year did his income reach ₹7000?

THINK FIRST: Salaries form an AP with a = 5000 and d = 200. Find n for a_n = 7000, then convert the term number into a calendar year.

1. 7000 = 5000 + (n − 1)(200).
   Why: Set the n-th term equal to the target salary.
2. 2000 = 200(n − 1) → n − 1 = 10 → n = 11.
   Why: It is the 11th salary.
3. The 1st salary was in 1995, so the 11th salary is in 1995 + 10 = **2005**.
   Why: Add (n − 1) years to the starting year — not n years.

ANSWER: His income reached ₹7000 in the year 2005.

CBSE MARKING SCHEME: 1 mark for the equation • 1 mark for n = 11 • 1 mark for the year 2005.

EX 5.2 Q20

Ramkali saved ₹5 in the first week of a year and then increased her weekly savings by ₹1.75. If in the n-th week her weekly savings become ₹20.75, find n.

THINK FIRST: Weekly savings form an AP: a = 5, d = 1.75, a_n = 20.75.

1. 20.75 = 5 + (n − 1)(1.75).
   Why: Substitute into the n-th term formula.
2. 15.75 = 1.75(n − 1) → n − 1 = 15.75 / 1.75 = 9.
   Why: Divide carefully: 1.75 × 9 = 15.75 exactly.
3. n = 9 + 1 = **10**.
   Why: Do not forget the final +1.

ANSWER: n = 10 — her weekly savings reach ₹20.75 in the 10th week.

CBSE MARKING SCHEME: 1 mark for the equation • 1 mark for n − 1 = 9 • 1 mark for n = 10.

TOPPER TIP: *The pattern of Ex 5.2: almost every question is $a_n = a + (n - 1)d$ wearing a different costume. Write a and d first, ALWAYS — the costume falls off immediately.*

EXERCISE 5.3 — THE SUM OF N TERMS

EX 5.3 Q1

Find the sum of the following APs: (i) 2, 7, 12, ..., to 10 terms (ii) −37, −33, −29, ..., to 12 terms (iii) 0.6, 1.7, 2.8, ..., to 100 terms (iv) 1/15, 1/12, 1/10, ..., to 11 terms

THINK FIRST: Use $S_n = n/2\,[2a + (n - 1)d]$. Read a and d, substitute n, and simplify in small steps.

1. **(i)** a = 2, d = 5, n = 10. $S_{10} = 10/2\,[2(2) + 9(5)] = 5\,[4 + 45] = 5 \times 49 = \mathbf{245}$.
   Why: Simplify inside the bracket before multiplying.

2. **(ii)** a = −37, d = −33 − (−37) = 4, n = 12. $S_{12} = 12/2\,[2(-37) + 11(4)] = 6\,[-74 + 44] = 6 \times (-30) = \mathbf{-180}$.
   Why: A negative sum is fine — the terms are mostly negative.

3. **(iii)** a = 0.6, d = 1.1, n = 100. $S_{100} = 100/2\,[2(0.6) + 99(1.1)] = 50\,[1.2 + 108.9] = 50 \times 110.1 = \mathbf{5505}$.
   Why: 99 × 1.1 = 108.9 — do it as 99 × 11 = 1089, then place the decimal.

4. **(iv)** a = 1/15, d = 1/12 − 1/15 = (5 − 4)/60 = 1/60, n = 11. $S_{11} = 11/2\,[2/15 + 10(1/60)] = 11/2\,[2/15 + 1/6] = 11/2 \times (4/30 + 5/30) = 11/2 \times 9/30 = 99/60 = \mathbf{33/20}$.
   Why: LCM 60 for d; LCM 30 inside the bracket.

ANSWER: (i) 245 (ii) −180 (iii) 5505 (iv) 33/20 (= 1.65)

CBSE MARKING SCHEME: 1 mark for each correct a, d • 1 mark for each correct sum (4 parts).

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EXERCISE 5.3 — CONTINUED

EX 5.3 Q2

Find the sums given below: (i) 7 + 10½ + 14 + … + 84 (ii) 34 + 32 + 30 + … + 10 (iii) −5 + (−8) + (−11) + … + (−230)

THINK FIRST: Here the LAST term is given, not n. First find n from $a_n = l$, then use $S_n = n/2\ (a + l)$.

1 **(i)** a = 7, d = 3½, l = 84. 84 = 7 + (n − 1)(7/2) → 77 = (7/2)(n − 1) → n − 1 = 22 → n = 23. S = 23/2 (7 + 84) = 23/2 × 91 = **1046½**.

Why: Two-stage question: n first, sum second.

2 **(ii)** a = 34, d = −2, l = 10. 10 = 34 + (n − 1)(−2) → −24 = −2(n − 1) → n − 1 = 12 → n = 13. S = 13/2 (34 + 10) = 13 × 22 = **286**.

Why: The (a + l) form is fastest once n is known.

3 **(iii)** a = −5, d = −3, l = −230. −230 = −5 + (n − 1)(−3) → −225 = −3(n − 1) → n − 1 = 75 → n = 76. S = 76/2 (−5 − 230) = 38 × (−235) = **−8930**.

Why: Watch the signs: −225 ÷ −3 = +75.

ANSWER: (i) 1046½ (ii) 286 (iii) −8930

CBSE MARKING SCHEME: 1 mark for each n • 1 mark for each sum (3 parts).

EXERCISE 5.3 — CONTINUED

EX 5.3 Q3 (i)–(v)

In an AP: (i) a = 5, d = 3, a_n = 50, find n and S_n. (ii) a = 7, a_{13} = 35, find d and S_{13}. (iii) a_{12} = 37, d = 3, find a and S_{12}. (iv) a_3 = 15, S_{10} = 125, find d and a_{10}. (v) d = 5, S_9 = 75, find a and a_9.

THINK FIRST: Each part hides one or two unknowns. Use the term formula and the sum formula as a pair of tools — whichever gives the missing piece first.

1. **(i)** 50 = 5 + (n − 1)(3) → 45 = 3(n − 1) → n = 16. S_{16} = 16/2 (5 + 50) = 8 × 55 = **440**.
   Why: n from the term formula; sum from the (a + l) form.

2. **(ii)** 35 = 7 + 12d → 12d = 28 → d = **7/3**. S_{13} = 13/2 (7 + 35) = 13 × 21 = **273**.
   Why: A fractional d is perfectly acceptable.

3. **(iii)** a = a_{12} − 11d = 37 − 33 = **4**. S_{12} = 12/2 (4 + 37) = 6 × 41 = **246**.
   Why: Walk back 11 jumps to reach a.

4. **(iv)** a_3 = a + 2d = 15 ...(1). S_{10} = 10/2 [2a + 9d] = 125 → 2a + 9d = 25 ...(2). From (1), a = 15 − 2d; substitute in (2): 2(15 − 2d) + 9d = 25 → 30 + 5d = 25 → d = **−1**, so a = 17. a_{10} = 17 + 9(−1) = **8**.
   Why: Two conditions, two unknowns — solve simultaneously.

5. **(v)** S_9 = 9/2 [2a + 8(5)] = 75 → 2a + 40 = 75 × 2/9 = 50/3 → 2a = 50/3 − 40 = −70/3 → a = **−35/3**. a_9 = a + 8d = −35/3 + 40 = **85/3**.
   Why: Multiply both sides by 2/9 early to keep numbers small.

ANSWER: (i) n = 16, S_n = 440 (ii) d = 7/3, S_{13} = 273 (iii) a = 4, S_{12} = 246 (iv) d = −1, a_{10} = 8 (v) a = −35/3, a_9 = 85/3

CBSE MARKING SCHEME: 1 mark for each unknown found (2 per part, 5 parts).

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EXERCISE 5.3 — CONTINUED

EX 5.3 Q3 (vi)–(x)

In an AP: (vi) a = 2, d = 8, S_n = 90, find n and a_n. (vii) a = 8, a_n = 62, S_n = 210, find n and d. (viii) a_n = 4, d = 2, S_n = −14, find n and a. (ix) a = 3, n = 8, S = 192, find d. (x) l = 28, S = 144, total 9 terms, find a.

THINK FIRST: Same two tools. When S_n and the term formula both contain n, expect a quadratic — and remember n must be a positive whole number.

1. **(vi)** 90 = n/2 [2(2) + (n − 1)(8)] → 180 = n(8n − 4) → $2n^2 - n - 45 = 0$ → (2n + 9)(n − 5) = 0 → n = **5** (n = −9/2 rejected). a_5 = 2 + 4(8) = **34**.
   Why: Factorise the quadratic; reject the negative/fraction root.

2. **(vii)** S_n = n/2 (a + l): 210 = n/2 (8 + 62) = 35n → n = **6**. Then 62 = 8 + 5d → d = 54/5 = **10.8**.
   Why: The (a + l) form gives n in one step here.

3. **(viii)** a = a_n − (n − 1)d = 4 − 2(n − 1) = 6 − 2n. S_n = n/2 (a + 4) = n/2 (10 − 2n) = n(5 − n) = −14 → $n^2 - 5n - 14 = 0$ → (n − 7)(n + 2) = 0 → n = **7**. a = 6 − 14 = **−8**.
   Why: Express a in terms of n first, then use the sum.

4. **(ix)** 192 = 8/2 [2(3) + 7d] → 192 = 4(6 + 7d) → 48 = 6 + 7d → d = **6**.
   Why: Divide by 4 first.

5. **(x)** 144 = 9/2 (a + 28) → a + 28 = 144 × 2/9 = 32 → a = **4**.
   Why: l means the last term — use S = n/2 (a + l).

ANSWER: (vi) n = 5, a_n = 34 (vii) n = 6, d = 54/5 (viii) n = 7, a = −8 (ix) d = 6 (x) a = 4

CBSE MARKING SCHEME: 1 mark for the set-up equation • 1 mark for each unknown found (5 parts).

EXERCISE 5.3 — CONTINUED

EX 5.3 Q4

How many terms of the AP 9, 17, 25, ... must be taken to give a sum of 636?

THINK FIRST: a = 9, d = 8, and S_n = 636 with n unknown — the sum formula becomes a quadratic in n.

1. Substitute: $636 = n/2\,[2(9) + (n - 1)(8)] = n/2\,(8n + 10)$.
   Why: Write the sum formula with the given values.

2. Multiply out: $1272 = 8n^2 + 10n$ → divide by 2: $4n^2 + 5n - 636 = 0$.
   Why: Bring everything to one side.

3. Factorise: $4n^2 - 48n + 53n - 636 = 4n(n - 12) + 53(n - 12) = (4n + 53)(n - 12) = 0$.
   Why: Split the middle term: $-48 + 53 = 5$ and $-48 \times 53 = 4 \times (-636)$.

4. So n = 12 or n = −53/4. Reject the fraction — n counts terms.
   Why: n must be a positive whole number.

ANSWER: 12 terms must be taken.

CBSE MARKING SCHEME: 1 mark for the sum equation • 1 mark for factorising • 1 mark for n = 12 with rejection of the invalid root.

EX 5.3 Q5

The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

THINK FIRST: a and l are both known, so $S_n = n/2\,(a + l)$ gives n immediately; then the term formula gives d.

1. $400 = n/2\,(5 + 45) = 25n \rightarrow n = 400/25 =$ **16**.
   Why: One step with the (a + l) form.

2. $45 = 5 + (16 - 1)d \rightarrow 40 = 15d \rightarrow d = 40/15 =$ **8/3**.
   Why: The last term is the 16th term.

ANSWER: Number of terms = 16, common difference = 8/3.

CBSE MARKING SCHEME: 1 mark for n • 1 mark for setting up the term equation • 1 mark for d = 8/3.

EXERCISE 5.3 — CONTINUED

EX 5.3 Q6

The first and last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

THINK FIRST: Find n from the term formula first (a, d, l known), then the sum.

1. $350 = 17 + (n - 1)(9) \rightarrow 333 = 9(n - 1) \rightarrow n - 1 = 37 \rightarrow n = \mathbf{38}$.
   Why: Subtract a, divide by d, add 1.

2. $S_{38} = 38/2\ (17 + 350) = 19 \times 367 = \mathbf{6973}$.
   Why: Use S = n/2 (a + l) — no need for the long form.

ANSWER: There are 38 terms and their sum is 6973.

CBSE MARKING SCHEME: 1 mark for n • 1 mark for the sum formula choice • 1 mark for 6973.

EX 5.3 Q7

Find the sum of the first 22 terms of an AP in which d = 7 and the 22nd term is 149.

THINK FIRST: The 22nd term is the last term here: l = 149. First recover a by walking back 21 jumps.

1. $a = a_{22} - 21d = 149 - 21(7) = 149 - 147 = 2$.
   Why: $a_{22} = a + 21d$, so $a = 149 - 147$.

2. $S_{22} = 22/2\ (2 + 149) = 11 \times 151 = \mathbf{1661}$.
   Why: Now the (a + l) form finishes it.

ANSWER: The sum of the first 22 terms is 1661.

CBSE MARKING SCHEME: 1 mark for a • 1 mark for the sum • 1 mark for 1661.

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EXERCISE 5.3 — CONTINUED

EX 5.3 Q8

Find the sum of the first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

THINK FIRST: Neighbouring terms give d at once; then walk back one jump for a.

1. $d = a_3 - a_2 = 18 - 14 = 4$.
   Why: Consecutive terms differ by exactly d.

2. $a = a_2 - d = 14 - 4 = 10$.
   Why: One jump back.

3. $S_{51} = 51/2\ [2(10) + 50(4)] = 51/2 \times 220 = 51 \times 110 = \mathbf{5610}$.
   Why: Substitute a, d, n into the sum formula.

ANSWER: The sum of the first 51 terms is 5610.

CBSE MARKING SCHEME: 1 mark for d • 1 mark for a • 1 mark for 5610.

EXERCISE 5.3 — CONTINUED

EX 5.3 Q9

If the sum of the first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of the first n terms.

THINK FIRST: Write S_7 and S_{17} with the formula; the two equations give a and d; then write S_n in general.

1. $S_7 = 7/2\ [2a + 6d] = 49 \rightarrow 2a + 6d = 14 \rightarrow a + 3d = 7$. ...(1)
   Why: Divide through by 7/2, then simplify.

2. $S_{17} = 17/2\ [2a + 16d] = 289 \rightarrow 2a + 16d = 34 \rightarrow a + 8d = 17$. ...(2)
   Why: Same routine.

3. (2) − (1): $5d = 10 \rightarrow d = 2$. Then $a = 7 - 3(2) = 1$.
   Why: Subtract to eliminate a.

4. $S_n = n/2\ [2(1) + (n - 1)(2)] = n/2 \times 2n = \mathbf{n^2}$.
   Why: Beautiful check: $S_7 = 49 = 7^2$ and $S_{17} = 289 = 17^2$. Consistent!

ANSWER: $S_n = n^2$ (the AP is 1, 3, 5, 7, ... — the odd numbers!).

CBSE MARKING SCHEME: 1 mark for each equation • 1 mark for a and d • 1 mark for $S_n = n^2$.

EXERCISE 5.3 — CONTINUED

EX 5.3 Q10

Show that $a_1, a_2, \ldots$ form an AP where a_n is defined as: (i) $a_n = 3 + 4n$ (ii) $a_n = 9 - 5n$. Also find the sum of the first 15 terms in each case.

THINK FIRST: Compute $a_{n+1} - a_n$: if it does not depend on n, the list is an AP and that constant is d.

1. **(i)** $a_{n+1} - a_n = [3 + 4(n + 1)] - [3 + 4n] = 4$ — constant. So it is an AP with d = 4. a = a_1 = 3 + 4 = 7.
   Why: The n terms cancel — that is exactly what 'constant difference' means.

2. **(i)** $S_{15} = 15/2\ [2(7) + 14(4)] = 15/2 \times 70 =$ **525**.
   Why: Now use the sum formula.

3. **(ii)** $a_{n+1} - a_n = [9 - 5(n + 1)] - [9 - 5n] = -5$ — constant. AP with d = −5. a = a_1 = 9 − 5 = 4.
   Why: Same proof, negative d.

4. **(ii)** $S_{15} = 15/2\ [2(4) + 14(-5)] = 15/2 \times (-62) =$ **−465**.
   Why: The AP falls fast: 4, −1, −6, … so a negative sum is expected.

ANSWER: (i) AP with d = 4; $S_{15} = 525$ (ii) AP with d = −5; $S_{15} = -465$

CBSE MARKING SCHEME: 1 mark for each difference proof • 1 mark for each S_{15}.

EXERCISE 5.3 — CONTINUED

EX 5.3 Q11

If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is, S_1)? What is the sum of the first two terms? What is the 2nd term? Similarly find the 3rd, the 10th and the n-th terms.

THINK FIRST: Terms hide inside sums: $a_1 = S_1$, and for $k \geq 2$, $a_k = S_k - S_{k-1}$ (the k-th term is what the sum gains at step k).

1. $S_1 = 4(1) - 1^2 = 3$. So the first term $a_1 = \mathbf{3}$.
   Why: S_1 IS the first term — a sum of one term.

2. $S_2 = 4(2) - 2^2 = 8 - 4 = \mathbf{4}$ (sum of first two terms). $a_2 = S_2 - S_1 = 4 - 3 = \mathbf{1}$.
   Why: The 2nd term is the increase from S_1 to S_2.

3. $a_3 = S_3 - S_2 = (12 - 9) - 4 = 3 - 4 = \mathbf{-1}$.
   Why: Same move: $S_3 = 4(3) - 9 = 3$.

4. $a_{10} = S_{10} - S_9 = (40 - 100) - (36 - 81) = (-60) - (-45) = \mathbf{-15}$.
   Why: Substitute $n = 10$ and $n = 9$.

5. General term: $a_n = S_n - S_{n-1} = (4n - n^2) - [4(n - 1) - (n - 1)^2] = 4n - n^2 - (-n^2 + 6n - 5) = \mathbf{5 - 2n}$.
   Why: Expand carefully: $4(n-1) - (n-1)^2 = 4n - 4 - n^2 + 2n - 1 = -n^2 + 6n - 5$.
   Check: $n = 1$ gives 3. Correct.

ANSWER: $a_1 = 3$; $S_2 = 4$; $a_2 = 1$; $a_3 = -1$; $a_{10} = -15$; $a_n = 5 - 2n$.

CBSE MARKING SCHEME: 1 mark for S_1 and S_2 • 1 mark for a_2, a_3 • 1 mark for a_{10} • 1 mark for $a_n = 5 - 2n$.

PIN POINT: $a_n = S_n - S_{n-1}$. Whenever a question hands you S_n as a formula, this one line unlocks every term.

EXERCISE 5.3 — CONTINUED

EX 5.3 Q12

Find the sum of the first 40 positive integers divisible by 6.

THINK FIRST: The numbers are 6, 12, 18, ..., 240 — an AP with a = 6, d = 6, n = 40.

1. The 40th such number is 6 × 40 = 240, so l = 240.
   Why: The k-th positive integer divisible by 6 is simply 6k.

2. S_{40} = 40/2 (6 + 240) = 20 × 246 = **4920**.
   Why: Use the (a + l) form.

ANSWER: The sum is 4920.

CBSE MARKING SCHEME: 1 mark for identifying the AP and l • 1 mark for 4920.

EX 5.3 Q13

Find the sum of the first 15 multiples of 8.

THINK FIRST: Multiples of 8: 8, 16, ..., 120 — AP with a = 8, d = 8, n = 15.

1. 15th multiple = 8 × 15 = 120, so l = 120.
   Why: Same idea as Q12.

2. S_{15} = 15/2 (8 + 120) = 15 × 64 = **960**.
   Why: 15/2 × 128 = 15 × 64.

ANSWER: The sum is 960.

CBSE MARKING SCHEME: 1 mark for l = 120 • 1 mark for 960.

EXERCISE 5.3 — CONTINUED

EX 5.3 Q14

Find the sum of the odd numbers between 0 and 50.

THINK FIRST: The odd numbers between 0 and 50 form an AP: 1, 3, 5, ..., 49. Count the terms first, then add them with $S_n = n/2\ (a + l)$.

1. The list is 1, 3, 5, ..., 49, so a = 1, d = 2 and the last term l = 49.
   Why: Odd numbers step by 2, starting just after 0 and ending just before 50.

2. Count the terms: $49 = 1 + (n - 1)(2) \rightarrow 48 = 2(n - 1) \rightarrow n - 1 = 24 \rightarrow n = 25$.
   Why: Exactly half of the numbers from 1 to 50 are odd — 25 of them.

3. $S_{25} = 25/2\ (a + l) = 25/2\ (1 + 49) = 25/2 \times 50 =$ **625**.
   Why: Use the (a + l) form of the sum formula.

4. Shortcut check: the sum of the first n odd numbers is n^2 (proved in Section 12, Reasoning 3), and $25^2 = 625$. Same answer.
   Why: Two independent routes agreeing — full confidence.

ANSWER: The sum of the odd numbers between 0 and 50 is 625.

CBSE MARKING SCHEME: 1 mark for a, d, l • 1 mark for n = 25 • 1 mark for 625.

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EXERCISE 5.3 — CONTINUED

EX 5.3 Q15

A contract on a construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹200 for the first day, ₹250 for the second day, ₹300 for the third day, etc., the penalty for each succeeding day being ₹50 more than for the preceding day. How much money does the contractor have to pay as penalty if he has delayed the work by 30 days?

THINK FIRST: The daily penalties form an AP: 200, 250, 300, … 'Total penalty for 30 days' means S_{30}.

1. Read off the AP: a = 200, d = 250 − 200 = 50, n = 30.
   Why: Each day's penalty is ₹50 more than the previous day's.

2. Substitute: S_{30} = 30/2 [2(200) + (30 − 1)(50)] = 15 [400 + 29 × 50].
   Why: Use $S_n = n/2\ [2a + (n - 1)d]$ — n is known, l is not.

3. 29 × 50 = 1450, so S_{30} = 15 × (400 + 1450) = 15 × 1850 = **27750**.
   Why: Simplify inside the bracket before multiplying.

ANSWER: The contractor has to pay a total penalty of ₹27,750.

CBSE MARKING SCHEME: 1 mark for a, d, n • 1 mark for the substitution • 1 mark for ₹27,750.

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EXERCISE 5.3 — CONTINUED

EX 5.3 Q16

A sum of ₹700 is to be used to give seven cash prizes to students for their overall academic performance. If each prize is ₹20 less than its preceding prize, find the value of each of the prizes.

THINK FIRST: Prizes form a falling AP: n = 7, d = −20, S_7 = 700. Find the first prize a.

1. 700 = 7/2 [2a + 6(−20)] → 700 = 7/2 (2a − 120).
   Why: Substitute into the sum formula.

2. Multiply both sides by 2/7: 200 = 2a − 120 → 2a = 320 → a = 160.
   Why: The largest (first) prize is ₹160.

3. List the prizes, subtracting 20 each time: 160, 140, 120, 100, 80, 60, 40. Check the total: 160 + 140 + 120 + 100 + 80 + 60 + 40 = 700. Correct.
   Why: Always run the total check on prize questions.

ANSWER: The prizes are ₹160, ₹140, ₹120, ₹100, ₹80, ₹60, ₹40.

CBSE MARKING SCHEME: 1 mark for the sum equation • 1 mark for a = 160 • 1 mark for the full list of prizes.

EXERCISE 5.3 — CONTINUED

EX 5.3 Q17

In a school, students plant trees to reduce air pollution: each section of Class I plants 1 tree, Class II plants 2 trees, and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

THINK FIRST: One section of each class plants 1 + 2 + ... + 12 trees; there are 3 sections per class, so multiply by 3.

1. Trees by one section-line: $1 + 2 + \ldots + 12 = S_{12}$ with a = 1, d = 1: = 12/2 (1 + 12) = 6 × 13 = 78.
   Why: Sum of the first 12 natural numbers.

2. Three sections of every class plant 3 times as many: 3 × 78 = **234**.
   Why: Do not forget the three sections — the classic trap of this question.

ANSWER: 234 trees will be planted.

CBSE MARKING SCHEME: 1 mark for $S_{12} = 78$ • 1 mark for multiplying by 3 • 1 mark for 234.

EXERCISE 5.3 — CONTINUED

EX 5.3 Q18

A spiral is made up of successive semicircles with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take π = 22/7)

THINK FIRST: The length of a semicircle of radius r is πr. So the lengths are π(0.5), π(1.0), π(1.5), ... — an AP in disguise.

1. Lengths: 0.5π, 1.0π, 1.5π, ..., up to 13 terms. The radii 0.5, 1.0, ..., 6.5 form an AP with a = 0.5, d = 0.5, n = 13.
   Why: Factor π out: total length = π × (sum of radii).

2. Sum of radii = 13/2 [2(0.5) + 12(0.5)] = 13/2 × 7 = 45.5 cm.
   Why: Apply the sum formula to the radii.

3. Total length = π × 45.5 = (22/7) × 45.5 = 22 × 6.5 = **143 cm**.
   Why: 45.5/7 = 6.5 exactly — the numbers are designed to cancel.

ANSWER: The total length of the spiral is 143 cm.

CBSE MARKING SCHEME: 1 mark for the semicircle lengths • 1 mark for the sum of radii • 1 mark for 143 cm.

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EXERCISE 5.3 — CONTINUED

EX 5.3 Q19

200 logs are stacked: 20 logs in the bottom row, 19 in the next, 18 in the next, and so on. In how many rows are the 200 logs placed and how many logs are in the top row?

THINK FIRST: Rows hold 20, 19, 18, … logs — an AP with a = 20, d = −1, and total $S_n = 200$. Find n, then the top row is a_n.

1. $200 = n/2\,[2(20) + (n - 1)(-1)] = n/2\,(41 - n)$.
   Why: Substitute into the sum formula.

2. $400 = 41n - n^2 \rightarrow n^2 - 41n + 400 = 0 \rightarrow (n - 16)(n - 25) = 0 \rightarrow n = 16$ or $n = 25$.
   Why: Factorise: 16 × 25 = 400 and 16 + 25 = 41.

3. Reject n = 25: the 21st row would need $a_{21} = 20 - 20 = 0$ logs and later rows would need NEGATIVE logs — impossible. So n = 16.
   Why: Both roots satisfy the equation, but only one satisfies reality. This rejection step carries a mark!

4. Top row = $a_{16} = 20 + 15(-1) =$ **5 logs**.
   Why: The 16th row from the bottom.

ANSWER: The logs are placed in 16 rows, and the top row has 5 logs.

CBSE MARKING SCHEME: 1 mark for the quadratic • 1 mark for both roots • 1 mark for rejecting 25 with reason • 1 mark for 5 logs.

EXERCISE 5.3 — CONTINUED

EX 5.3 Q20

In a potato race, a bucket is at the starting point, 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, and continues until all potatoes are in the bucket. What is the total distance the competitor has to run?

THINK FIRST: For each potato she runs out AND back: distance = 2 × (distance of that potato). Potato k is 5 + 3(k − 1) metres away.

1. Potato positions: 5, 8, 11, ..., so running distances (out and back) are: 2(5), 2(8), 2(11), ... = 10, 16, 22, ... — an AP with a = 10, d = 6.
   Why: The doubling is the key insight of this question.

2. There are 10 potatoes, so n = 10. Total = S_{10} = 10/2 [2(10) + 9(6)] = 5 × 74 = **370 m**.
   Why: Sum the 10 running distances.

ANSWER: The competitor runs a total of 370 m.

CBSE MARKING SCHEME: 1 mark for the running-distance AP • 1 mark for the sum formula • 1 mark for 370 m.

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EXERCISE 5.4 — OPTIONAL BONUS SET (NOT IN THE RATIONALISED EXERCISE LIST, BUT LOVED BY EXAMINERS)

EX 5.4 Q1

Which term of the AP 121, 117, 113, ... is its first negative term?

THINK FIRST: We need the smallest n for which $a_n < 0$, with a = 121, d = −4.

1. $a_n = 121 + (n - 1)(-4) = 125 - 4n$.
   Why: Simplify the general term.

2. Require $a_n < 0$: $125 - 4n < 0 \rightarrow 4n > 125 \rightarrow n > 31.25$.
   Why: Solve the inequality.

3. The smallest whole number with n > 31.25 is n = 32. Check: $a_{31} = 125 - 124 = 1$ (still positive); $a_{32} = 125 - 128 = -3$ (negative).
   Why: Verify the boundary terms on both sides.

ANSWER: The 32nd term is the first negative term (it is −3).

CBSE MARKING SCHEME: 1 mark for a_n • 1 mark for the inequality • 1 mark for the 32nd term with check.

EXERCISE 5.4 — CONTINUED

EX 5.4 Q2

The sum of the third and seventh terms of an AP is 6 and their product is 8. Find the sum of the first sixteen terms.

THINK FIRST: $a_3 + a_7 = 2(a + 4d)$ — notice both terms are symmetric around $a_5 = a + 4d$. Let me set $u = a + 4d$.

1. $a_3 + a_7 = (a + 2d) + (a + 6d) = 2a + 8d = 6 \rightarrow a + 4d = 3$.
   Why: Add the two term expressions.

2. Write $a_3 = (a + 4d) - 2d = 3 - 2d$ and $a_7 = (a + 4d) + 2d = 3 + 2d$. Product: $(3 - 2d)(3 + 2d) = 9 - 4d^2 = 8 \rightarrow 4d^2 = 1 \rightarrow d = 1/2$ or $d = -1/2$.
   Why: Difference of squares: $(3)^2 - (2d)^2$.

3. Case $d = 1/2$: $a = 3 - 4(1/2) = 1$. $S_{16} = 16/2\,[2(1) + 15(1/2)] = 8 \times 9.5 = 76$.
   Why: First possible AP: 1, 1.5, 2, ...

4. Case $d = -1/2$: $a = 3 - 4(-1/2) = 5$. $S_{16} = 16/2\,[2(5) + 15(-1/2)] = 8 \times 2.5 = 20$.
   Why: Second possible AP: 5, 4.5, 4, ... Both satisfy the given conditions.

ANSWER: $S_{16} = 76$ (if $d = 1/2$) or 20 (if $d = -1/2$).

CBSE MARKING SCHEME: 1 mark for $a + 4d = 3$ • 1 mark for $d = \pm 1/2$ • 1 mark for each value of S_{16}.

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EXERCISE 5.4 — CONTINUED

EX 5.4 Q3

A ladder has rungs 25 cm apart. The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and bottom rungs are 2½ m apart, what is the length of wood required for the rungs?

THINK FIRST: First count the rungs: distance 250 cm with a rung every 25 cm — and do not forget the rung at BOTH ends.

1. Number of rungs = 250/25 + 1 = 10 + 1 = 11.
   Why: Gaps + 1 = posts — the fence-post idea from Section 06.

2. Rung lengths form an AP: 45, ..., 25 with n = 11. Total wood = S_{11} = 11/2 (45 + 25) = 11 × 35 = **385 cm**.
   Why: Use the (a + l) form — d is not needed.

ANSWER: The length of wood required is 385 cm (3.85 m).

CBSE MARKING SCHEME: 1 mark for 11 rungs (the +1 is the trap) • 1 mark for the sum • 1 mark for 385 cm.

EXERCISE 5.4 — CONTINUED

EX 5.4 Q4

The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding house number x equals the sum of the numbers following it. Find x.

THINK FIRST: Sum before x is $S_{x-1} = (x - 1)x/2$. Sum after x is (total S_{49}) − S_x. Set them equal.

1. Total of all house numbers: $S_{49} = 49 \times 50/2 = 1225$.
   Why: Sum of the first 49 natural numbers.

2. Sum before x: $(x - 1)x/2$. Sum up to and including x: $S_x = x(x + 1)/2$, so sum after x = $1225 - x(x + 1)/2$.
   Why: Use $S_k = k(k + 1)/2$ for natural numbers.

3. Set equal: $(x - 1)x/2 = 1225 - x(x + 1)/2$ → multiply by 2: $x^2 - x = 2450 - x^2 - x \rightarrow 2x^2 = 2450 \rightarrow x^2 = 1225$.
   Why: The −x cancels on both sides — elegant!

4. $x = \sqrt{1225} = 35$ ($x > 0$). Check: sum before 35 = $34 \times 35/2 = 595$; sum after = $1225 - 35 \times 36/2 = 1225 - 630 = 595$. Equal. Shown.
   Why: Always verify a 'show that' answer numerically.

ANSWER: Such a value exists: x = 35.

CBSE MARKING SCHEME: 1 mark for S_{49} • 1 mark for the two sums • 1 mark for the equation • 1 mark for x = 35 with verification.

EXERCISE 5.4 — CONTINUED

EX 5.4 Q5

A small terrace at a football ground has 15 steps, each 50 m long and built of solid concrete. Each step has a rise of 1/4 m and a tread of 1/2 m. Calculate the total volume of concrete required to build the terrace.

THINK FIRST: Step 1 is a slab 50 m long, 1/4 m high and 1/2 m deep. Step 2 sits behind it and is twice as deep, step 3 three times — the depths grow in AP.

1. Volume of step 1 = 50 × (1/4) × (1/2) = 25/4 m^3. Step 2 = 50 × (1/4) × 1 = 25/2 m^3. Step 3 = 50 × (1/4) × (3/2) = 75/4 m^3.
   Why: Each step k has tread-depth k × (1/2) m.

2. Volumes form an AP: 25/4, 25/2, 75/4, ... with a = 25/4, d = 25/4, n = 15.
   Why: Each volume grows by 25/4.

3. Total = S_{15} = 15/2 [2(25/4) + 14(25/4)] = 15/2 × (25/4)(16) = 15/2 × 100 = **750 m^3**.
   Why: Factor 25/4 out of the bracket: 2 + 14 = 16.

ANSWER: The total volume of concrete required is 750 m^3.

CBSE MARKING SCHEME: 1 mark for one step's volume • 1 mark for the AP of volumes • 1 mark for 750 m^3.

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PREVIOUS YEAR QUESTIONS — REAL CBSE BOARD QUESTIONS (YEAR-VERIFIED)

PYQ 1 • CBSE 2024

The next (4th) term of the AP √18, √50, √98, ... is: (A) √128 (B) √140 (C) √162 (D) √200

THINK FIRST: Simplify the roots first: each is a multiple of √2.

1. √18 = 3√2, √50 = 5√2, √98 = 7√2. So the AP is 3√2, 5√2, 7√2, ... with d = 2√2.
   Why: Because √18 = √(9×2), √50 = √(25×2), √98 = √(49×2).
2. 4th term = 7√2 + 2√2 = 9√2 = √(81 × 2) = √162.
   Why: Convert back: 9√2 = √(9² × 2).

ANSWER: Option (C) √162.

CBSE-STYLE MARKING SCHEME: 1 mark — correct option.

PYQ 2 • CBSE 2024

The common difference of an AP in which $a_{15} - a_{11} = 48$, is: (A) 12 (B) 16 (C) −12 (D) −16

THINK FIRST: Terms 4 positions apart differ by 4d.

1. $a_{15} - a_{11} = (a + 14d) - (a + 10d) = 4d$.
   Why: Count the jumps: 15 − 11 = 4.
2. 4d = 48 → d = 12.
   Why: Divide by 4.

ANSWER: Option (A) 12.

CBSE-STYLE MARKING SCHEME: 1 mark — correct option.

PREVIOUS YEAR QUESTIONS — CONTINUED

PYQ 3 • CBSE 2024

Which term of the AP −29, −26, −23, ..., 61 is 16? (A) 11th (B) 16th (C) 10th (D) 31st

THINK FIRST: a = −29, d = 3. Set $a_n = 16$.

1. 16 = −29 + (n − 1)(3) → 45 = 3(n − 1) → n − 1 = 15 → n = 16.
   Why: −26 − (−29) = 3, so d = 3.

ANSWER: Option (B) 16th.

CBSE-STYLE MARKING SCHEME: 1 mark — correct option.

PYQ 4 • CBSE 2023

The sum of the first 15 terms of an AP is 750 and its first term is 15. Find its 20th term.

THINK FIRST: $S_{15} = 750$ gives d; then the term formula gives a_{20}.

1. 750 = 15/2 [2(15) + 14d] → 750 × 2/15 = 100 = 30 + 14d → 14d = 70 → d = 5.
   Why: Divide 750 by 15/2 first: 750 × 2/15 = 100.
2. a_{20} = 15 + 19(5) = 15 + 95 = **110**.
   Why: 19 jumps from the first term.

ANSWER: The 20th term is 110.

CBSE-STYLE MARKING SCHEME: 1 mark for the sum equation • 1 mark for d = 5 • 1 mark for $a_{20} = 110$.

PREVIOUS YEAR QUESTIONS — CONTINUED

PYQ 5 • CBSE 2023

Rohan repays his total loan of ₹1,18,000 by paying every month, starting with the first instalment of ₹1,000. If he increases the instalment by ₹100 every month, what amount will be paid by him in the 30th instalment? What amount of loan has he paid after the 30th instalment?

THINK FIRST: Instalments form an AP: a = 1000, d = 100. The first part asks for a TERM (a_{30}); the second asks for a SUM (S_{30}).

1. $a_{30} = 1000 + (30 - 1)(100) = 1000 + 2900 =$ **₹3,900**.
   Why: 29 jumps of ₹100 added to the first instalment.

2. $S_{30} = 30/2\ [2(1000) + (30 - 1)(100)] = 15 \times (2000 + 2900) = 15 \times 4900 =$ **₹73,500**.
   Why: Sum of the first 30 instalments — use the sum formula, not the term formula.

3. Reality check: loan still left = 1,18,000 − 73,500 = ₹44,500. He has NOT cleared the loan in 30 months.
   Why: The board loves this trap — do not claim the loan is finished.

ANSWER: 30th instalment = ₹3,900; total loan paid after 30 instalments = ₹73,500.

CBSE-STYLE MARKING SCHEME: 1 mark for $a_{30} = 3900$ • 1 mark for the sum formula • 1 mark for $S_{30} = 73,500$.

PREVIOUS YEAR QUESTIONS — CONTINUED

PYQ 6 • NCERT EXEMPLAR

Solve the equation: −4 + (−1) + 2 + ... + x = 437.

THINK FIRST: The left side is an AP with a = −4, d = 3, last term x and sum 437. First find HOW MANY terms there are, then x is the last term.

1. Let there be n terms. 437 = n/2 [2(−4) + (n − 1)(3)] = n/2 (3n − 11).
   Why: Substitute into the sum formula.

2. $874 = 3n^2 - 11n \rightarrow 3n^2 - 11n - 874 = 0$. Discriminant: 121 + 12(874) = 121 + 10488 = 10609 = 103^2. n = (11 + 103)/6 = 19.
   Why: Reject the negative root (11 − 103)/6.

3. $x = a_{19} = -4 + 18(3) = -4 + 54 = \mathbf{50}$.
   Why: x is the 19th term.

ANSWER: x = 50 (the series has 19 terms).

CBSE-STYLE MARKING SCHEME: 1 mark for the sum equation • 1 mark for n = 19 • 1 mark for x = 50.

PYQ 7 • CBSE 2020

If 4 times the 4th term of an AP is equal to 18 times the 18th term, find the 22nd term of the AP.

THINK FIRST: Translate: $4a_4 = 18a_{18}$. Expand and watch a and d collapse into a single relation.

1. 4(a + 3d) = 18(a + 17d) → 4a + 12d = 18a + 306d.
   Why: Expand both sides.

2. Bring terms together: 0 = 14a + 294d → divide by 14: 0 = a + 21d.
   Why: 294/14 = 21 exactly.

3. But a + 21d is precisely a_{22}. So $a_{22} = \mathbf{0}$.
   Why: The question was hiding the 22nd term inside the algebra all along!

ANSWER: The 22nd term is 0.

CBSE-STYLE MARKING SCHEME: 1 mark for the equa[illegible] • 1 mark for a + 21d = 0 • 1 mark for recognising a[illegible]

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PREVIOUS YEAR QUESTIONS — CONTINUED

PYQ 8 • CBSE 2020

How many terms of the AP 24, 21, 18, ... must be taken so that their sum is 78?

THINK FIRST: a = 24, d = −3, S_n = 78. This quadratic has TWO valid answers — be ready to explain both.

1. $78 = n/2\,[2(24) + (n - 1)(-3)] = n/2\,(51 - 3n)$.
   Why: Substitute: 48 − 3n + 3 = 51 − 3n.

2. $156 = 51n - 3n^2 \rightarrow n^2 - 17n + 52 = 0 \rightarrow (n - 4)(n - 13) = 0 \rightarrow n = 4$ or $n = 13$.
   Why: Both roots are positive whole numbers this time!

3. Why both work: terms 5 to 13 are 12, 9, 6, 3, 0, −3, −6, −9, −12 — they cancel in pairs around 0, adding nothing. So $S_{13} = S_4 = 78$.
   Why: Unlike Ex 5.3 Q19, no root is impossible here — the AP passes through 0 and into negatives quite legally.

ANSWER: n = 4 or n = 13 — both give a sum of 78.

CBSE-STYLE MARKING SCHEME: 1 mark for the quadratic • 1 mark for both roots • 1 mark for explaining why both are valid.

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PREVIOUS YEAR QUESTIONS — CONTINUED

PYQ 9 • CBSE 2019

How many two-digit numbers are divisible by 3?

THINK FIRST: Two-digit multiples of 3: 12, 15, ..., 99 — an AP with a = 12, d = 3.

1. First two-digit multiple of 3 is 12 (3 × 4); last is 99 (3 × 33).
   Why: 10 and 11 are not multiples of 3; 100 is three-digit.

2. 99 = 12 + (n − 1)(3) → 87 = 3(n − 1) → n − 1 = 29 → n = **30**.
   Why: Solve for the count.

ANSWER: There are 30 two-digit numbers divisible by 3.

CBSE-STYLE MARKING SCHEME: 1 mark for the first/last multiples • 1 mark for n = 30.

PYQ 10 • CBSE BOARD EXAM QUESTION

How many terms of the AP 18, 16, 14, ... be taken so that their sum is zero?

THINK FIRST: a = 18, d = −2, $S_n = 0$.

1. 0 = n/2 [2(18) + (n − 1)(−2)] = n/2 (38 − 2n) = n(19 − n).
   Why: Simplify inside: 36 − 2n + 2 = 38 − 2n.

2. So n = 0 or n = 19. Reject n = 0 (no terms, no sum). n = **19**.
   Why: The AP falls through 0 at the 10th term; the positive and negative halves cancel by term 19.

ANSWER: 19 terms must be taken.

CBSE-STYLE MARKING SCHEME: 1 mark for the equation • 1 mark for rejecting n = 0 • 1 mark for 19.

TOPPER TIP: *Board pattern spotted: CBSE returns to the same five ideas every year — find n from a sum, find a term, two-digit/three-digit counting, instalment stories, and 'which term'. Master the NCERT exercises and the PYQs become old friends.*

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08 Multiple Solution Methods

8D • CHALLENGE — ONE PROBLEM, MANY ROADS

Toppers are not faster at ONE method — they know TWO methods and pick the shorter one, or use the second to check the first. Practise both roads for these two model problems.

METHOD DUEL 1

Find the 15th term of the AP 7, 11, 15, 19, ...

THINK FIRST: The formula is the highway; building the list is the country road. Know both.

1. **Method 1 — formula (direct).** a = 7, d = 4. $a_{15} = 7 + (15 - 1)(4) = 7 + 56 = 63$.

   Why: One substitution — best when the term is far away.

2. **Method 2 — build the list in jumps.** Count in 4s from a known term: $a_5 = 23$, $a_{10} = 23 + 5(4) = 43$, $a_{15} = 43 + 5(4) = 63$.

   Why: Hop 5 terms at a time (+20 each hop). Useful when no formula is allowed or to double-check.

3. Both roads agree: $a_{15} = 63$.

   Why: Agreement between two methods is the strongest check in mathematics.

ANSWER: $a_{15} = 63$ (by both methods).

METHOD DUELS — CONTINUED

METHOD DUEL 2

Find the sum of the first 12 terms of the AP 5, 8, 11, 14, ...

1. **Method 1 — sum formula.** a = 5, d = 3, n = 12. S_{12} = 12/2 [2(5) + 11(3)] = 6 × 43 = 258.
   Why: The standard road.

2. **Method 2 — Gauss pairing.** Last term a_{12} = 5 + 11(3) = 38. Pair the ends: (5 + 38) + (8 + 35) + ... — 6 pairs, each totalling 43. Sum = 6 × 43 = 258.
   Why: Pairing shows WHY the formula's (a + l) appears — and needs no memorised formula at all.

3. Both roads agree: S_{12} = 258.
   Why: In the exam, use Method 1 to write and Method 2 (roughly, in the margin) to verify.

ANSWER: S_{12} = 258 (by both methods).

PIN POINT: Stuck on a sum? Pair first + last. Stuck on a term? Count the jumps. Every AP problem is one of these two moves.

09 Interactive Simulations & Hands-on Activities

8D • VISUALISE — LEARN BY DOING

Lab 1: The Matchstick Squares Pattern

Build squares in a row with matchsticks, sharing one side between neighbours: 1 square needs 4 sticks, 2 squares need 7 sticks, 3 squares need 10 sticks.

Squares	1	2	3	4	5	10	20
Sticks	4	7	10	?	?	?	?

The stick counts 4, 7, 10, ... form an AP with a = 4, d = 3 (each new square shares a side, so it adds only 3 sticks). Fill the table with $a_n = 4 + (n - 1)(3) = 3n + 1$: 4 squares → 13, 5 squares → 16, 10 squares → 31, 20 squares → **61 sticks**. Now really build it with sticks or straws — your hands will remember the +3 forever.

Lab 2: My Own Savings Plan

Plan: save ₹100 in week 1, and increase the weekly saving by ₹50 every week.

Week	1	2	3	4	5	10
Saving that week (₹)	100	150	200	250	300	550
Total saved so far (₹)	100	250	450	700	1000	3250

Check the last column with the formula: $S_{10} = 10/2\ [2(100) + 9(50)] = 5 \times 650 = 3250$. Correct! Now design YOUR plan: pick your own a and d, and use S_n to find when you cross ₹10,000.

Lab 3: Staircase Sum (do it on real stairs)

Write the numbers 1 to 20 on slips of paper. Place slip 1 on step 1, slip 2 on step 2, and so on. Now pair step 1 with step 20, step 2 with step 19 — call out each pair's total. Every pair says 21! Ten pairs × 21 = 210 = S_{20}. You have just walked through Gauss's derivation.

DIGITAL COMPANION: QR code and link activate with the KalviYogi app/website at print. Use it for the video lesson, the interactive AP simulator, online tests and learning analytics for this chapter.

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10 Common Mistakes

8D • SOLVE — WHERE MARKS ARE LOST

COMMON MISTAKE: Using n jumps instead of (n − 1). **Wrong: $a_{10} = a + 10d$. Right: $a_{10} = a + 9d$** — the first term needs no jump. Count gaps, not posts.

COMMON MISTAKE: Losing the sign of d in a falling AP. For 20, 17, 14, ..., **$d = 3$. $d = 17 - 20 = -3$**. Always compute d = (later term) − (earlier term), in that order.

COMMON MISTAKE: Confusing S_n with a_n. $S_{10} = 55$ for 1, 2, 3, ... is the SUM of ten terms; $a_{10} = 10$ is just the tenth term. Read the question: 'sum' → S_n; 'term' → a_n.

COMMON MISTAKE: Declaring a list an AP after checking only ONE difference. 1, 2, 4, 7 has differences 1, 2, 3 — the first two terms always 'work'. Check EVERY consecutive pair.

COMMON MISTAKE: Finding a term from S_n wrongly: given S_n, students write $a_2 = S_2$. **Right: $a_2 = S_2 - S_1$** — the second term is only the INCREASE in the sum.

COMMON MISTAKE: Accepting a fractional n in 'which term' questions. If n = 54⅔, the value is NOT a term of the AP (see Ex 5.2 Q6). n must be a positive whole number — say so in your answer.

COMMON MISTAKE: In counting questions (logs, prizes, rows), accepting both quadratic roots blindly. One root may demand negative logs or negative prizes (Ex 5.3 Q19). Always test roots against reality.

TOPPER TIP: *Examiners say the same thing every year: most AP marks are lost not on hard steps, but on (n − 1), the sign of d, and unread questions. Slow down for exactly those three.*

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FIND THE ERROR — 10 REAL STUDENT SLIPS, CORRECTED

ERROR HUNT 1

Find the 10th term of the AP 3, 7, 11, ...

STUDENT'S WORKING: $a_{10} = a + 10d = 3 + 10(4) = 43$.

1. The error: the student used **10 jumps** to reach the 10th term. But the 1st term is the starting post — reaching term 10 takes only 9 jumps.
   Why: The (n − 1) rule from Section 06.

2. Correct: $a_{10} = 3 + 9(4) = 3 + 36 = 39$.
   Why: Substitute n − 1 = 9.

CORRECT ANSWER: $a_{10} = 39$ (not 43).

ERROR HUNT 2

Find the sum 5 + 10 + 15 + ... + 50.

STUDENT'S WORKING: $S = n/2\ (a + l) = 50/2\ (5 + 50) = 25 \times 55 = 1375$.

1. The error: the student put **n = 50**, confusing the LAST TERM with the NUMBER OF TERMS. The terms are 5, 10, ..., 50 — only 10 terms.
   Why: n counts terms; l is the last term. They are different numbers!

2. Correct: n = 10, so $S = 10/2\ (5 + 50) = 5 \times 55 = 275$.
   Why: Count first: (50 − 5)/5 + 1 = 10 terms.

CORRECT ANSWER: S = 275 (not 1375).

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FIND THE ERROR — CONTINUED

ERROR HUNT 3

Find the common difference of the AP 20, 17, 14, 11, ...

STUDENT'S WORKING: d = 20 − 17 = 3.

1. The error: the subtraction is backwards. d = (second term) − (first term) = 17 − 20 = **−3**.
   Why: This is a FALLING AP — d must be negative.

2. With d = 3 the AP would rise: 20, 23, 26 — clearly not the given list. The sign check takes two seconds.
   Why: Always sanity-check d against the list's direction.

CORRECT ANSWER: d = −3 (not 3).

FIND THE ERROR — CONTINUED

ERROR HUNT 4

The sum of the first n terms of an AP is $S_n = 2n^2 + n$. Find the 2nd term.

STUDENT'S WORKING: $a_2 = S_2 = 2(2^2) + 2 = 10$.

1. The error: S_2 is the sum of the first TWO terms, not the 2nd term itself.
   Why: $a_n = S_n - S_{n-1}$ — the term is the increase in the sum.

2. Correct: $a_2 = S_2 - S_1 = 10 - (2 + 1) = 10 - 3 = 7$.
   Why: Check with the AP: $a_1 = 3$, $a_2 = 7$, $d = 4$; indeed $S_n = n/2\ [6 + (n-1)4] = 2n^2 + n$. Correct.

CORRECT ANSWER: $a_2 = 7$ (not 10).

ERROR HUNT 5

Which term of the AP 6, 10, 14, ... is 62?

STUDENT'S WORKING: $6 + 4n = 62 \rightarrow 4n = 56 \rightarrow n = 14$. The 14th term.

1. The error: the student wrote a + nd instead of a + (n − 1)d.
   Why: Same fence-post slip as Error Hunt 1, hiding inside an equation.

2. Correct: $6 + (n - 1)(4) = 62 \rightarrow (n - 1) = 14 \rightarrow n = 15$. Check: $a_{15} = 6 + 56 = 62$. Correct.
   Why: And $a_{14} = 58$ — the student's answer was one term too early.

CORRECT ANSWER: 62 is the 15th term (not the 14th).

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FIND THE ERROR — CONTINUED

ERROR HUNT 6

Find the sum of the first 20 even numbers: 2 + 4 + ... + 40.

STUDENT'S WORKING: $S_{20} = 20 \times (2 + 40) = 840$.

1. The error: the student forgot the **/2** in $S_n = n/2\ (a + l)$.

   Why: The formula averages the first and last terms — n × (a + l) counts every pair twice.

2. Correct: $S_{20} = 20/2 \times (2 + 40) = 10 \times 42 = 420$.

   Why: Quick check by pairing: 10 pairs each totalling 42 → 420.

CORRECT ANSWER: $S_{20} = 420$ (not 840).

FIND THE ERROR — CONTINUED

ERROR HUNT 7

Is 2, 3, 5, 8, 12 an AP? A student writes: 'Differences are 1, 2, 3, 4 — they increase by 1 each time, a perfect pattern, so YES, it is an AP.'

STUDENT'S WORKING: — (the claim itself is the error)

1. The error: an AP needs the differences THEMSELVES to be equal — not to follow their own pretty pattern.
   Why: 1, 2, 3, 4 are not equal, so the list is not an AP.

2. This list is actually the 'add one more each time' pattern (triangular-style). Interesting — but not an AP.
   Why: A pattern is not automatically an AP; only EQUAL steps qualify.

CORRECT ANSWER: Not an AP — differences 1, 2, 3, 4 are not constant.

FIND THE ERROR — CONTINUED

ERROR HUNT 8

Find the 6th term of an AP with a = −5 and d = 3.

STUDENT'S WORKING: $a_6 = -5 + 6 \times 3 = 13$.

1. The error: 6 jumps used instead of 5 — the (n − 1) slip again, the most common AP error in India.
   Why: It appears in Error Hunts 1, 5 and 8 because examiners meet it every single year.
2. Correct: $a_6 = -5 + 5(3) = -5 + 15 = 10$.
   Why: List to double-check: −5, −2, 1, 4, 7, 10. Yes, 6th term is 10.

CORRECT ANSWER: a_6 = 10 (not 13).

ERROR HUNT 9

Find the sum of the first 100 natural numbers.

STUDENT'S WORKING: $S_{100} = 100/2 \times (2(1) + 99(1)) = 50 \times 101 = 5050$. — wait, the student actually wrote: $S = 100 \times (1 + 100) = 10100$.

1. The error (in the student's final line): the /2 of the pairing formula was dropped — same family as Error Hunt 6.
   Why: n/2 (a + l): the 2 in the denominator is not decoration.
2. Correct: $S_{100} = 100/2 \times (1 + 100) = 50 \times 101 = 5050$ — young Gauss's famous answer.
   Why: 50 pairs, each summing to 101.

CORRECT ANSWER: S_{100} = 5050 (not 10100).

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FIND THE ERROR — CONTINUED

ERROR HUNT 10

Find the missing term: 4, [ ], 16 (an AP).

STUDENT'S WORKING: The gap is 16 − 4 = 12, so d = 12 and the missing term is 4 + 12 = 16.

1. The error: the total gap of 12 spans TWO jumps (4 → box → 16), so d = 12/2 = 6, not 12.
   Why: Divide the gap by the number of jumps between the known terms.

2. Correct: missing term = 4 + 6 = 10. Check: 4, 10, 16 has d = 6 throughout. (Also: the middle term is the average (4 + 16)/2 = 10.)
   Why: Notice the student's method even produced 16 — the term that was already given!

CORRECT ANSWER: The missing term is 10 (not 16).

TOPPER TIP: If you can spot all 10 errors before reading the corrections, your AP basics are exam-proof. Revisit this page on Day 3 of the Smart Memory cycle.

11 Graded Practice Problems

8D • SOLVE — THE FIVE-LEVEL SYSTEM (ANSWERS IN THE ANSWER KEY)

Work level by level. Do not climb to the next level until the current one feels easy — that is exactly how R.D. Sharma's gradation and the Singapore CPA ladder work. Full answers are in the Answer Key at the end of the chapter.

LEVEL 1 • FOUNDATION — DIRECT CONCEPTS

1.1 For the AP 5, 9, 13, 17, ..., write the common difference and the 6th term.

1.2 Write the first five terms of the AP with a = 3 and d = −2.

1.3 Is 4, 10, 16, 22, ... an AP? If yes, write its common difference.

1.4 Find the 20th term of the AP 2, 5, 8, 11, ...

1.5 Find the sum of the AP 1, 2, 3, ..., 10 (all ten terms).

1.6 If a = 12 and d = −4, find a_8.

LEVEL 2 • SKILL BUILDER — TEXTBOOK STANDARD

2.1 Which term of the AP 7, 13, 19, ... is 205?

2.2 Is 301 a term of the AP 5, 11, 17, 23, ...? Justify your answer.

2.3 Find the sum of the first 25 terms of an AP with a = −3 and d = 2.

2.4 The 3rd term of an AP is 12 and the 8th term is 37. Find a and d.

2.5 How many terms of the AP 3, 6, 9, ... must be added to give a sum of 165?

2.6 Find the 15th term from the last term of the AP 10, 17, 24, ..., 276.

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LEVEL 3 • APPLICATION — REAL LIFE

3.1 A clerk's monthly salary starts at ₹8,000. Every year, her monthly salary rises by ₹400. What will her monthly salary be in the 10th year?

3.2 A factory produced 800 cars in its first year. Production rises by 50 cars every year. Find the total production in 8 years.

3.3 A loan is repaid in monthly instalments of ₹1,000 in the first month, increasing by ₹100 every month. How much is paid in total in 12 months?

3.4 A hall has 20 rows of seats. The first row has 15 seats and each next row has 2 seats more than the previous row. How many seats does the hall have?

LEVEL 4 • ADVANCED REASONING — MULTI-CONCEPT

4.1 The sum of the first 9 terms of an AP is 144 and the sum of the first 15 terms is 330. Find the 20th term.

4.2 How many terms of the AP 24, 21, 18, ... should be taken so that their sum is 78? Explain why there are TWO answers.

4.3 The sum of the first n terms of an AP is $S_n = 3n^2 + 5n$. Find the n-th term and the common difference.

LEVEL 5 • MATHEMATICAL GENIUS — OLYMPIAD STYLE

5.1 Find the sum of all two-digit numbers that are divisible by 6.

5.2 The sums of the first n terms of two APs are in the ratio (7n + 1) : (4n + 27). Find the ratio of their 11th terms. (Hint: the ratio of the m-th terms equals the ratio of sums with n replaced by 2m − 1.)

> ***TOPPER TIP:*** *Level 5 is not 'hard Level 2'. It needs a trick you must DISCOVER. Spend 10 honest minutes before opening the key — the discovery is the training.*

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CONCEPT MCQs — 25 RAPID-FIRE QUESTIONS (KEY WITH EXPLANATIONS AT THE END)

M1. The common difference of the AP 7, 5, 3, 1, … is

(A) 2 (B) −2 (C) 5 (D) −5

M2. Which of the following lists is an AP?

(A) 2, 4, 8, 16 (B) 5, 8, 11, 14 (C) 1, 4, 9, 16 (D) 3, 6, 12, 24

M3. If a = 5 and d = 4, then a_6 is

(A) 21 (B) 25 (C) 29 (D) 9

M4. The n-th term of the AP 3, 7, 11, 15, … is

(A) 4n + 1 (B) 4n − 1 (C) 3n + 4 (D) n + 4

M5. If the 3rd term of an AP is 12 and d = 3, then the first term is

(A) 3 (B) 6 (C) 9 (D) 18

M6. The sum of the first 10 natural numbers is

(A) 45 (B) 50 (C) 55 (D) 100

M7. An AP has a = 9 and d = 0. Its 50th term is

(A) 0 (B) 9 (C) 450 (D) 50

M8. The number of terms in the AP 5, 10, 15, …, 100 is

(A) 19 (B) 20 (C) 21 (D) 95

M9. The 20th term of the AP 1, 4, 7, 10, … is

(A) 57 (B) 58 (C) 60 (D) 61

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CONCEPT MCQs — CONTINUED

M10. If $S_n = n^2$ for an AP, then its common difference is

(A) 1 (B) 2 (C) 0 (D) 4

M11. Which term of the AP 2, 5, 8, … is 50?

(A) 15th (B) 16th (C) 17th (D) 18th

M12. The sum of the first 20 terms of the AP 2, 4, 6, … is

(A) 400 (B) 420 (C) 440 (D) 210

M13. If a = 2, d = 3 and $a_n = 59$, then n is

(A) 19 (B) 20 (C) 21 (D) 18

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CONCEPT MCQs — CONTINUED

M14. The 10th term from the end of the AP 4, 9, 14, …, 254 is

(A) 204 (B) 209 (C) 214 (D) 199

M15. If $a_5 = 19$ and $a_9 = 35$ in an AP, then d is

(A) 2 (B) 3 (C) 4 (D) 16

M16. The sum of the first 15 multiples of 5 is

(A) 300 (B) 450 (C) 600 (D) 750

M17. Which of the following is NOT an AP?

(A) $-3, -1, 1, 3$ (B) 0.5, 1, 1.5, 2 (C) 4, 4, 4, 4 (D) 1, 3, 6, 10

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CONCEPT MCQs — CONTINUED

M18. If $a_n = 3n + 2$, then the common difference is

(A) 2 (B) 3 (C) 5 (D) 1

M19. The sum 10 + 20 + 30 + … + 100 equals

(A) 500 (B) 550 (C) 5050 (D) 450

M20. In an AP, $a = 7$, $a_n = 55$ and $n = 13$. Then d is

(A) 3 (B) 4 (C) 5 (D) 48

M21. Two APs have the same d. Their first terms differ by 12. Their 50th terms differ by

(A) 12 (B) 600 (C) 0 (D) 62

M22. If $S_1 = 5$ and $S_2 = 12$ for an AP, then a_2 is

(A) 5 (B) 12 (C) 7 (D) 17

M23. The AP x, x + 3, x + 6, … has 10th term 40. Then x is

(A) 10 (B) 13 (C) 37 (D) 7

M24. The sum of the first 12 terms of the AP 1, 3, 5, … is

(A) 121 (B) 144 (C) 169 (D) 100

M25. An AP has $a = 20$ and $d = -2$. Its first negative term is the

(A) 10th term (B) 11th term (C) 12th term (D) 13th term

TOPPER TIP: Exam technique: in MCQs, eliminate first. In M25, the 11th term is 0 — and 0 is NOT negative. Options are built on exactly such traps.

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12 Mathematical Reasoning

8D • CHALLENGE — PROVE IT, DON'T JUST COMPUTE IT

REASONING 1

Prove that the list defined by $a_n = 5n + 2$ (n = 1, 2, 3, ...) is an AP, and find its common difference.

THINK FIRST: Proving is showing it for ALL n at once. Three examples would only be evidence, not proof.

1. Compute the difference between consecutive terms: $a_{n+1} - a_n = [5(n + 1) + 2] - [5n + 2] = 5n + 7 - 5n - 2 = 5$.
   Why: Work with the general n, not with examples — that is what makes it a PROOF.
2. The difference is 5 for EVERY n — it does not depend on n. Hence the list is an AP with d = 5.
   Why: Constant difference is the definition of an AP.
3. Terms: 7, 12, 17, 22, ... — indeed steps of 5.
   Why: A quick listing confirms the proof.

ANSWER: $a_n = 5n + 2$ always gives an AP, with d = 5. (General truth: $a_n = pn + q$ is an AP with d = p.)

REASONING 2

If a, b, c are in AP, prove that b = (a + c)/2 (the middle term is the average of its neighbours).

THINK FIRST: Start from the definition, not from the conclusion. One equality, two rearrangements, done.

1. Since a, b, c are in AP, the two steps are equal: b − a = c − b.
   Why: This equality IS the definition of three terms in AP.
2. Rearrange: b + b = a + c → 2b = a + c.
   Why: Move b to the left and a to the right.
3. Divide by 2: b = (a + c)/2. Proved.
   Why: This is why b is called the ARITHMETIC MEAN of a and c.

ANSWER: b = (a + c)/2 — proved. (So the middle term of 2, [ ], 26 is (2 + 26)/2 = 14 — remember Ex 5.2 Q3!)

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MATHEMATICAL REASONING — CONTINUED

REASONING 3

Prove that the sum of the first n odd numbers is n^2: $1 + 3 + 5 + \ldots + (2n - 1) = n^2$.

THINK FIRST: A sum formula plus one observation (the last odd number) completes the proof.

1. The odd numbers form an AP with a = 1, d = 2, and last term l = 2n − 1.
   Why: The n-th odd number is 2n − 1.

2. Apply $S_n = n/2\ (a + l)$: sum = $n/2\ [1 + (2n - 1)] = n/2 \times 2n = n^2$. Proved.
   Why: The bracket collapses beautifully to 2n.

3. Picture check: $1 + 3 = 4 = 2^2$, $1 + 3 + 5 = 9 = 3^2$, $1 + 3 + 5 + 7 = 16 = 4^2$ — each new odd number wraps the old square into a bigger square.
   Why: Draw 1 dot, then an L-shaped border of 3 dots, then 5 — you will SEE the squares grow.

ANSWER: $1 + 3 + 5 + \ldots + (2n - 1) = n^2$ — proved.

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ASSERTION – REASON — 10 QUESTIONS (KEY AT THE END)

A-R 1

Assertion (A): The 10th term of the AP 2, 5, 8, … is 29.

Reason (R): For an AP, $a_n = a + (n - 1)d$.

(A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true but R is NOT the correct explanation of A (C) A is true but R is false (D) A is false but R is true

A-R 2

Assertion (A): The sum of the first 20 natural numbers is 200.

Reason (R): The sum of the first n natural numbers is n(n + 1)/2.

(A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true but R is NOT the correct explanation of A (C) A is true but R is false (D) A is false but R is true

A-R 3

Assertion (A): The list 1, 4, 9, 16, … is an AP.

Reason (R): In an AP, consecutive differences are all equal.

(A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true but R is NOT the correct explanation of A (C) A is true but R is false (D) A is false but R is true

A-R 4

Assertion (A): If d = 0 in an AP, all terms are equal.

Reason (R): $a_n = a + (n - 1)(0) = a$ for every n.

(A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true but R is NOT the correct explanation of A (C) A is true but R is false (D) A is false but R is true

A-R 5

Assertion (A): The sum of the first 10 terms of the AP 3, 6, 9, … is 165.

Reason (R): $S_n = n/2\,[2a + (n - 1)d]$.

(A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true but R is NOT the correct explanation of A (C) A is true but R is false (D) A is false but R is true

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ASSERTION – REASON — CONTINUED

A-R 6

Assertion (A): −150 is a term of the AP 11, 8, 5, 2, ...
Reason (R): A value is a term of an AP only if its position n comes out a positive whole number.

(A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true but R is NOT the correct explanation of A (C) A is true but R is false (D) A is false but R is true

A-R 7

Assertion (A): The 5th term of the AP 10, 7, 4, ... is −2.
Reason (R): The common difference of 10, 7, 4, ... is 3.

(A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true but R is NOT the correct explanation of A (C) A is true but R is false (D) A is false but R is true

A-R 8

Assertion (A): For $n \geq 2$, the n-th term of an AP can be found as $a_n = S_n - S_{n-1}$.
Reason (R): $S_n - S_{n-1}$ removes the first $(n - 1)$ terms from the sum of n terms, leaving only the n-th term.

(A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true but R is NOT the correct explanation of A (C) A is true but R is false (D) A is false but R is true

A-R 9

Assertion (A): The AP 100, 90, 80, ... has common difference 10.
Reason (R): $d = a_2 - a_1$.

(A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true but R is NOT the correct explanation of A (C) A is true but R is false (D) A is false but R is true

A-R 10

Assertion (A): If a, b, c are in AP, then $2b = a + c$.
Reason (R): For a, b, c in AP, $b - a = c - b$.

(A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true but R is NOT the correct explanation of A (C) A is true but R is false (D) A is false but R is true

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13 Real-Life Applications

8D • APPLY — THE FORMULAS AT WORK

APPLICATION 1

Priya joins a company at a monthly salary of ₹25,000. Her monthly salary increases by ₹1,200 every year. (a) What is her monthly salary in the 8th year? (b) How much does she earn in total over the 8 years?

THINK FIRST: Two APs are nested here: monthly salary within each year, and yearly totals across years. Separate them cleanly.

1. (a) Monthly salaries form an AP with a = 25000, d = 1200. 8th year monthly salary = 25000 + 7(1200) = 25000 + 8400 = **₹33,400**.
   Why: 7 jumps to reach year 8.

2. (b) Yearly earnings: year 1 = 12 × 25000 = ₹3,00,000; each year the yearly total rises by 12 × 1200 = ₹14,400. So yearly totals form an AP with a = 300000, d = 14400.
   Why: Convert monthly to yearly first — mixing the two is the classic trap here.

3. Total in 8 years = S_8 = 8/2 [2(300000) + 7(14400)] = 4 × (600000 + 100800) = 4 × 700800 = **₹28,03,200**.
   Why: Sum the eight yearly totals.

ANSWER: (a) ₹33,400 per month (b) Total earnings = ₹28,03,200.

APPLICATION 2

A factory produced 500 toys in the first week. It increases production by 40 toys every week. How many toys does it produce in total in 8 weeks?

1. Weekly production: 500, 540, 580, ... — AP with a = 500, d = 40, n = 8.
   Why: Identify the AP inside the story.

2. Total = S_8 = 8/2 [2(500) + 7(40)] = 4 × (1000 + 280) = 4 × 1280 = **5120 toys**.
   Why: Apply the sum formula.

ANSWER: Total production in 8 weeks = 5120 toys.

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REAL-LIFE APPLICATIONS — CONTINUED

APPLICATION 3

Meera buys a television on instalments: ₹1,500 in the first month, increasing by ₹100 every month. How much does she pay in total in 10 months?

1. Instalments: a = 1500, d = 100, n = 10.
   Why: Read the AP out of the story.

2. Total = S_{10} = 10/2 [2(1500) + 9(100)] = 5 × 3900 = **₹19,500**.
   Why: Sum formula, substituted step by step.

ANSWER: Total paid in 10 months = ₹19,500.

REAL-LIFE APPLICATIONS — CONTINUED

APPLICATION 4

Raju saves ₹200 in January and increases his monthly saving by ₹50 every month. (a) In which month does his monthly saving reach ₹650? (b) How much does he save in the whole year?

1. (a) $650 = 200 + (n - 1)(50) \rightarrow 450 = 50(n - 1) \rightarrow n - 1 = 9 \rightarrow n = 10 \rightarrow$ the 10th month = **October**.

   Why: Term number 10 starting from January is October.

2. (b) Total in 12 months = $S_{12} = 12/2\ [2(200) + 11(50)] = 6 \times 950 =$ **₹5,700**.

   Why: Sum over the full year.

ANSWER: (a) October (b) ₹5,700 saved in the year.

14 Competency-Based Questions

8D • APPLY — FIVE CASE STUDIES, TWENTY QUESTIONS

CBSE now asks nearly half its paper through real-world cases like these. Read the case, pull out a and d, and answer all four sub-questions.

CASE 1 — The School Auditorium

A school auditorium has 18 rows of seats. The first row has 24 seats, and each successive row has 4 seats more than the previous row.

CASE 1 QUESTIONS

(i) How many seats are in the 18th row? (ii) How many seats are there in the auditorium in total? (iii) If 5 more rows are added in the same pattern, how many seats will the new last (23rd) row have? (iv) How many seats are there in the first 10 rows?

1. **(i)** a = 24, d = 4. a_{18} = 24 + 17(4) = 24 + 68 = **92 seats**.
   Why: 17 jumps to row 18.

2. **(ii)** Total = S_{18} = 18/2 (24 + 92) = 9 × 116 = **1044 seats**.
   Why: The (a + l) form, since the last row is known.

3. **(iii)** a_{23} = 24 + 22(4) = 24 + 88 = **112 seats**.
   Why: The pattern simply continues.

4. **(iv)** S_{10} = 10/2 [2(24) + 9(4)] = 5 × 84 = **420 seats**.
   Why: Sum formula with n = 10.

ANSWER: (i) 92 (ii) 1044 (iii) 112 (iv) 420

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COMPETENCY CASES — CONTINUED

CASE 2 — Divya's Savings Challenge

Divya saves ₹150 in the first week of a 20-week challenge. She increases her weekly saving by ₹25 every week.

CASE 2 QUESTIONS

(i) How much does she save in the 20th week? (ii) How much does she save in total in 20 weeks? (iii) How much does she save in the 8th week? (iv) In which week does her weekly saving first reach ₹500?

1. **(i)** a = 150, d = 25. a_{20} = 150 + 19(25) = 150 + 475 = **₹625**.
   Why: 19 jumps.

2. **(ii)** Total = S_{20} = 20/2 (150 + 625) = 10 × 775 = **₹7,750**.
   Why: Pair first and last weeks.

3. **(iii)** a_8 = 150 + 7(25) = **₹325**.
   Why: 7 jumps.

4. **(iv)** 500 = 150 + (n − 1)(25) → 350 = 25(n − 1) → n − 1 = 14 → n = **15th week**.
   Why: Solve for the position.

ANSWER: (i) ₹625 (ii) ₹7,750 (iii) ₹325 (iv) 15th week

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COMPETENCY CASES — CONTINUED

CASE 3 — The Toy Factory

A factory makes 200 toys on the first day. It increases production by 15 toys every day and works for 30 days.

CASE 3 QUESTIONS

(i) How many toys are made on the 30th day? (ii) How many toys are made in all 30 days? (iii) How many toys are made on the 12th day? (iv) How many toys are made in the first 10 days?

1. **(i)** a = 200, d = 15. a_{30} = 200 + 29(15) = 200 + 435 = **635 toys**.
   Why: 29 jumps.

2. **(ii)** Total = S_{30} = 30/2 (200 + 635) = 15 × 835 = **12,525 toys**.
   Why: Use (a + l) with l = 635.

3. **(iii)** a_{12} = 200 + 11(15) = **365 toys**.
   Why: 11 jumps.

4. **(iv)** S_{10} = 10/2 [2(200) + 9(15)] = 5 × 535 = **2,675 toys**.
   Why: Sum formula for 10 days.

ANSWER: (i) 635 (ii) 12,525 (iii) 365 (iv) 2,675

COMPETENCY CASES — CONTINUED

CASE 4 — The Brick Wall

A garden wall is built in 12 horizontal layers of bricks. The bottom layer has 40 bricks, and each higher layer has 3 bricks fewer than the layer below it.

CASE 4 QUESTIONS

(i) How many bricks are in the top (12th) layer? (ii) How many bricks are used in the whole wall? (iii) How many bricks are in the 7th layer? (iv) How many bricks are used in the first 6 layers?

1. **(i)** $a = 40$, $d = -3$. $a_{12} = 40 + 11(-3) = 40 - 33 =$ **7 bricks**.
   Why: A falling AP — d is negative.

2. **(ii)** Total $= S_{12} = 12/2\ (40 + 7) = 6 \times 47 =$ **282 bricks**.
   Why: Pair bottom and top layers.

3. **(iii)** $a_7 = 40 + 6(-3) =$ **22 bricks**.
   Why: 6 jumps.

4. **(iv)** $S_6 = 6/2\ [2(40) + 5(-3)] = 3 \times 65 =$ **195 bricks**.
   Why: Sum formula with n = 6.

ANSWER: (i) 7 (ii) 282 (iii) 22 (iv) 195

COMPETENCY CASES — CONTINUED

CASE 5 — The Marathon Runner

A runner trains for 21 days. She runs 2 km on the first day and increases her daily distance by 0.5 km every day.

CASE 5 QUESTIONS

(i) How far does she run on the 21st day? (ii) What total distance does she cover in 21 days? (iii) How far does she run on the 10th day? (iv) What total distance does she cover in the first 7 days?

1. **(i)** $a = 2$, $d = 0.5$. $a_{21} = 2 + 20(0.5) =$ **12 km**.
   Why: 20 jumps of half a kilometre.

2. **(ii)** Total $= S_{21} = 21/2\ (2 + 12) = 21 \times 7 =$ **147 km**.
   Why: The (a + l) form is cleanest here.

3. **(iii)** $a_{10} = 2 + 9(0.5) =$ **6.5 km**.
   Why: 9 jumps.

4. **(iv)** $S_7 = 7/2\ [2(2) + 6(0.5)] = 7/2 \times 7 =$ **24.5 km**.
   Why: Sum formula with n = 7.

ANSWER: (i) 12 km (ii) 147 km (iii) 6.5 km (iv) 24.5 km

TOPPER TIP: *In every case study the story changes, but the skeleton never does: find a, find d, decide 'term or sum?', and finish. Underline a and d in the case text before touching any formula.*

15 Olympiad & Foundation Challenges

8D • CHALLENGE — TEN PROBLEMS FOR MATHEMATICAL THINKERS

OLYMPIAD 1

Find the sum of all integers from 1 to 100 that are divisible by 2 or by 3.

THINK FIRST: Split the problem into three APs and fix the double counting.

1. Sum of multiples of 2: 2 + 4 + ... + 100 = 50/2 (2 + 100) = 2550.
   Why: 50 multiples of 2.
2. Sum of multiples of 3: 3 + 6 + ... + 99 = 33/2 (3 + 99) = 1683.
   Why: 33 multiples of 3.
3. Multiples of BOTH 2 and 3 (i.e. of 6) were counted twice: 6 + 12 + ... + 96 = 16/2 (6 + 96) = 816. Subtract the double count once.
   Why: This is the inclusion–exclusion idea.
4. Total = 2550 + 1683 − 816 = **3417**.
   Why: Add both lists, remove the overlap.

ANSWER: Sum = 3417.

OLYMPIAD 2

The 5th term of an AP is 19 and the 9th term is 35. Find the sum of the first 25 terms.

1. $a_9 - a_5 = 4d = 35 - 19 = 16$, so $d = 4$.
   Why: Four jumps between positions 5 and 9.
2. $a = a_5 - 4d = 19 - 16 = 3$.
   Why: Walk back 4 jumps.
3. $S_{25} = 25/2\ [2(3) + 24(4)] = 25/2 \times 102 =$ **1275**.
   Why: Substitute into the sum formula.

ANSWER: $S_{25} = 1275$.

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OLYMPIAD — CONTINUED

OLYMPIAD 3

Find the sum of all three-digit numbers divisible by 8.

1. First: 104 (8 × 13). Last: 992 (8 × 124). Count: n = (992 − 104)/8 + 1 = 111 + 1 = 112.
   Why: Count = (last − first)/d + 1 — the +1 is the fence-post again.

2. Sum = 112/2 (104 + 992) = 56 × 1096 = **61,376**.
   Why: Apply S = n/2 (a + l).

ANSWER: Sum = 61,376.

OLYMPIAD — CONTINUED

OLYMPIAD 4

Insert five arithmetic means between 8 and 26 (i.e. build an AP with 8 first, 26 last, and five terms between them).

1. With 5 means inserted, the AP has 7 terms: 8 = a_1 and 26 = a_7. So 26 = 8 + 6d → d = 3.
   Why: The gap of 18 is split into 6 equal jumps.

2. The means are: 8 + 3, 8 + 6, 8 + 9, 8 + 12, 8 + 15 = **11, 14, 17, 20, 23**.
   Why: Walk forward in steps of 3.

ANSWER: The five arithmetic means are 11, 14, 17, 20, 23 (AP: 8, 11, 14, 17, 20, 23, 26).

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OLYMPIAD — CONTINUED

OLYMPIAD 5

The sums of the first n terms of two APs are in the ratio (7n + 1) : (4n + 27) for every n. Find the ratio of their 11th terms.

1. Key fact: the m-th term of an AP is the 'middle' of the first (2m − 1) terms — $a_m = S_{2m-1} / (2m - 1)$... more precisely, the ratio of the m-th terms equals the ratio of the sums of the first (2m − 1) terms.
   Why: Because S_{2m-1} = (2m − 1) × (middle term) = (2m − 1) × a_m for each AP, and the (2m − 1) cancels in the ratio.

2. For m = 11: use n = 2(11) − 1 = 21 in the sum ratio: (7(21) + 1) : (4(21) + 27) = 148 : 111.
   Why: Substitute n = 21.

3. Simplify: 148/111 = 4/3 (both divisible by 37). So the 11th terms are in the ratio **4 : 3**.
   Why: 148 = 4 × 37 and 111 = 3 × 37.

ANSWER: The 11th terms are in the ratio 4 : 3.

OLYMPIAD 6

A clock strikes once at 1 o'clock, twice at 2 o'clock, and so on, up to 12 times at 12 o'clock. How many times does it strike in a full day?

1. In 12 hours it strikes 1 + 2 + ... + 12 = 12/2 (1 + 12) = 78 times.
   Why: Sum of the first 12 natural numbers.

2. A full day has two 12-hour cycles: 2 × 78 = **156 strikes**.
   Why: Do not forget the second half of the day.

ANSWER: The clock strikes 156 times in a day.

OLYMPIAD — CONTINUED

OLYMPIAD 7

Three numbers in AP have sum 24 and product 440. Find the numbers.

1. Take the numbers as a − d, a, a + d (the symmetric choice). Sum = 3a = 24 → a = 8.
   Why: Choosing symmetric forms makes the d terms cancel in the sum.

2. Product: $(8 - d)(8)(8 + d) = 8(64 - d^2) = 440 \rightarrow 64 - d^2 = 55 \rightarrow d^2 = 9 \rightarrow d = 3$ (taking d > 0).
   Why: Difference of squares again.

3. Numbers: 8 − 3, 8, 8 + 3 = **5, 8, 11**. Check product: 5 × 8 × 11 = 440. Correct.
   Why: d = −3 would give the same three numbers in reverse order.

ANSWER: The numbers are 5, 8, 11.

OLYMPIAD — CONTINUED

OLYMPIAD 8

How many multiples of 5 must be added, starting from 5, so that their sum is 330? (5 + 10 + 15 + ... = 330)

1. The sum of the first n multiples of 5 is $5(1 + 2 + ... + n) = 5n(n + 1)/2$.
   Why: Factor out 5 and use the natural-number sum.

2. Set $5n(n + 1)/2 = 330 \rightarrow n(n + 1) = 132 \rightarrow n = 11$ (since $11 \times 12 = 132$).
   Why: Look for two consecutive integers whose product is 132.

ANSWER: 11 multiples of 5 (5 + 10 + ... + 55 = 330).

OLYMPIAD — CONTINUED

OLYMPIAD 9

An AP has 20 terms. Its first term is 5 and its 20th term is 62. Find (a) the sum of all 20 terms, (b) the 10th term.

1. (a) $S_{20} = 20/2\ (5 + 62) = 10 \times 67 = \mathbf{670}$.
   Why: The (a + l) form needs no d at all.

2. (b) First find d: $62 = 5 + 19d \rightarrow d = 57/19 = 3$. Then $a_{10} = 5 + 9(3) = \mathbf{32}$.
   Why: 19 jumps separate the 1st and 20th terms.

3. Elegant check: the 10th term from the start pairs with the 11th from the start around the centre; $a_{10} + a_{11} = a + l = 67$, and indeed $32 + 35 = 67$.
   Why: Pairs equidistant from the ends always sum to a + l.

ANSWER: (a) 670 (b) $a_{10} = 32$.

OLYMPIAD 10

The sum of the first n terms of an AP is $S_n = 3n^2 + n$. Find the AP and its 15th term.

1. $a_1 = S_1 = 3 + 1 = 4$.
   Why: The sum of one term is the term.

2. $a_n = S_n - S_{n-1} = (3n^2 + n) - [3(n - 1)^2 + (n - 1)] = 3n^2 + n - (3n^2 - 5n + 2) = 6n - 2$.
   Why: Expand $3(n-1)^2 + (n-1) = 3n^2 - 6n + 3 + n - 1 = 3n^2 - 5n + 2$.

3. So the AP is 4, 10, 16, 22, ... (d = 6). $a_{15} = 6(15) - 2 = \mathbf{88}$. Check by formula: $4 + 14(6) = 88$. Correct.
   Why: Both routes agree.

ANSWER: The AP is 4, 10, 16, ... and $a_{15} = 88$.

TOPPER TIP: *Olympiad secret: every one of these ten problems is still just a, d, a_n and S_n — the genius part is choosing WHICH symmetric form (a − d, a, a + d) or which pairing makes the algebra collapse.*

16

Chapter Mind Map

8D • RECALL — THE WHOLE CHAPTER ON ONE PAGE

ARITHMETIC PROGRESSIONS

A list where every step adds the SAME number d — a staircase with equal risers.

RECOGNISE

Compute ALL consecutive differences. All equal → AP. d = $a_2 - a_1$; d may be +, −, 0 or a fraction.

THE TERM

$a_n = a + (n - 1)d$ • only (n − 1) gaps • from the end: l − (k − 1)d • from sums: $a_n = S_n - S_{n-1}$.

THE SUM

$S_n = n/2\,[2a + (n - 1)d] = n/2\,(a + l)$ • Gauss pairing • naturals: n(n + 1)/2 • odds: n^2.

SOLVE STORIES

Seats, salary, savings, instalments, production → read off a and d → ask: ONE term or a SUM? → pick the formula.

TRAPS

n vs (n − 1) • sign of d • S_n vs a_n • fractional n = not a term • reject impossible roots • +1 in counting.

BOARD FAVOURITES

Counting multiples (2019) • sum-zero AP (2016) • loan instalments (2023) • √-AP and which-term MCQs (2024).

Cover this page and redraw the map from memory — that is your Day 3 recall test.

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PIN Pin-Point Flash Cards

CUT OUT • CARRY • REVISE IN 5 MINUTES A DAY

How to use: Cut along the dashed borders to make 16 pocket cards. Revise them on your Smart Memory days (Day 1, 3, 7, 14, 21, 30): read the title, say the fact aloud from memory, then flip and check. Any card you miss goes back into tomorrow's pile.

1 · The n-th Term

$a_n = a + (n - 1)d$. First term a, common difference d, position n.

2 · Count Gaps, Not Posts

Term n is reached in (n − 1) jumps. The first term needs ZERO jumps.

3 · Sum — Form 1

$S_n = n/2\,[2a + (n - 1)d]$. Use when you know a, d and n.

4 · Sum — Form 2

$S_n = n/2\,(a + l)$. Use when you know the first and last terms.

5 · Term Hidden in Sums

$a_n = S_n - S_{n-1}$ $(n \geq 2)$, and $a_1 = S_1$.

6 · d From Any Two Terms

$d = (a_m - a_k) \div (m - k)$. Divide by the gap in positions.

7 · Three Numbers in AP

Write them as a − d, a, a + d. Their sum is simply 3a.

8 · Arithmetic Mean

If a, b, c are in AP, then $b = (a + c)/2$. The middle is the average.

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PIN-POINT FLASH CARDS — CARDS 9 TO 16

9 · First n Naturals

$1 + 2 + \ldots + n = n(n + 1)/2$.

10 · First n Odd Numbers

$1 + 3 + \ldots + (2n - 1) = n^2$. A perfect square!

11 · First n Even Numbers

$2 + 4 + \ldots + 2n = n(n + 1)$.

12 · Is It an AP?

Check EVERY consecutive difference. One unequal difference kills the AP.

13 · Which-Term Check

n must be a positive whole number. A fraction means the value is NOT a term.

14 · Term From the End

k-th term from the end = $l - (k - 1)d$. Walk backwards with $-d$.

15 · Middle-Term Trick

For odd n: $S_n = n \times$ (middle term).

Example: $S_9 = 9 \times a_5$.

16 · Constant AP

If $d = 0$, every term equals a, and $S_n = n \times a$.

Stick cards 2, 5 and 13 on your wall — they guard the three most expensive traps in this chapter.

17 Active Recall

8D • RECALL — CLOSE THE BOOK AND ANSWER

No peeking! Answer aloud or on rough paper, then check with the Answer Key. Struggling to remember is not failure — the struggle is exactly what builds the memory.

10 rapid questions

R1 What is d in 14, 9, 4, −1, ...?

R2 Find a_7 of 3, 6, 9, 12, ...

R3 Is 2, 4, 8, 16 an AP? Why?

R4 Find 2 + 4 + 6 + 8 + 10.

R5 If a = −2 and d = 3, find a_{12}.

R6 Find 1 + 2 + ... + 20 without adding one by one.

R7 Fill the box: 7, [ ], 19 (an AP).

R8 Write a_n for the AP 10, 13, 16, ...

R9 How many terms are in 6, 12, 18, ..., 96?

R10 Find the 20th term from the end of the AP 5, 9, 13, ..., 161.

5 formula recall prompts

F1 Say the n-th term formula, and explain the (n − 1) using the fence-post picture.

F2 Say BOTH forms of S_n, and when each form is the better choice.

F3 How do you recover a_n if you are given only S_n?

F4 How do you find d if you know a_5 and a_{12}?

F5 Say the sums of the first n naturals, first n odds, and first n evens.

> **TOPPER TIP:** *If you can answer all 15 without the book, this chapter is yours. Mark the ones you missed in the Error Tracker on the next page.*

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18 Self-Assessment & Mastery Tracker

8D • RECALL — THE KALVIYOGI SMART MEMORY SYSTEM

Your Smart Memory schedule for this chapter

Day	Activity	Done?
Day 0	Pre-learning check: attempt the Level 1 questions cold. Note your score honestly.	☐
Day 1	Learn the chapter fully: Sections 03–08, all NCERT solutions, Flash Cards 1–8.	☐
Day 3	Without-book recall: Active Recall (Section 17) + redraw the Mind Map from memory + Find-the-Error page.	☐
Day 7	Independent solving: **Day 7 Mini-Test** (below) + Level 2 practice.	☐
Day 14	Mixed practice: Level 3 + Level 4 + Competency Cases 1–3 again.	☐
Day 21	Long-term recall: **Day 21 Mini-Test** + Flash Cards 9–16 + Model Test B.	☐
Day 30	Chapter mastery: **Day 30 Mini-Test** + Model Test C + Olympiad retry.	☐

Concept Mastery Score (rate yourself 1–5 after each test)

Concept	Day 1	Day 7	Day 21	Day 30
Recognising an AP and finding d				
n-th term problems				
Sum of n terms				
Terms from S_n ($a_n = S_n - S_{n-1}$)				
Story / competency problems				

Error Tracker (copy every mistake here — errors are data, not shame)

Date	Question	My error	The fix (Pin-Point no.)

Formula Recall Tracker

☐ $a_n = a + (n - 1)d$ ☐ $S_n = n/2\,[2a + (n - 1)d]$ ☐ $S_n = n/2\,(a + l)$ ☐ $a_n = S_n - S_{n-1}$ ☐ $n(n + 1)/2$ ☐ n^2 (odds)

Learning Time Tracker: Day 0: ___ min • Day 1: ___ min • Day 3: ___ min • Day 7: ___ min • Day 14: ___ min • Day 21: ___ min • Day 30: ___ min

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SMART MEMORY MINI-TESTS — DAY 7 • DAY 21 • DAY 30 (KEYS AT THE END)

Day 7 Mini-Test — Terms (10 questions)

1 Find d for 12, 7, 2, −3, … **2** Find a_9 of 4, 9, 14, … **3** Write the first four terms for a = 0, d = −1/2. **4** Is 100 a term of 9, 13, 17, …? **5** Find a_{15} of −2, −5, −8, …

6 Fill the box: 8, [ ], 20. **7** Write a_n for 6, 11, 16, … **8** If a_3 = 15 and d = −2, find a. **9** How many terms are in 3, 6, 9, …, 99? **10** Find the 20th term from the end of 2, 5, 8, …, 152.

Day 21 Mini-Test — Sums (10 questions)

1 Find 1 + 2 + … + 12. **2** Find S_{20} of 5, 10, 15, … **3** Sum of the first 30 odd numbers. **4** Find S_{15} of −10, −5, 0, … **5** If S_{10} = 200 and a = 5, find the last term.

6 Sum of the multiples of 3 up to 60. **7** If $S_n = n(2n + 1)$, find S_5. **8** How many terms of 2, 4, 6, … give a sum of 240? **9** Find S_{25} for a = 4, d = 4. **10** If $S_n = 5n - n^2$, find a_1.

Day 30 Mini-Test — Mixed Mastery (10 questions)

1 If a_2 = 8 and a_5 = 23, find a_7. **2** Which term of 11, 8, 5, … is −34? **3** Find 7 + 11 + 15 + … + 99. **4** If a_4 = 0 and d = −2, find a_{10}. **5** Three numbers in AP have sum 21 and d = 2. Find them.

6 If a = 3 and a_{18} = 37, find S_{18}. **7** If $a_n = 7 - 2n$, write d and a_1. **8** Find 11 + 22 + … + 110. **9** If S_9 = 81, find a_5. **10** Find the sum of the first 12 terms of 3, 8, 13, …

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TEST Chapter Model Tests

THREE PAPERS • 20 MARKS EACH • KEYS AT THE END • TIME: 40 MINUTES EACH

MODEL TEST A — BASICS

A1 (1) The common difference of the AP 9, 6, 3, 0, … is: (A) 3 (B) −3 (C) 9 (D) 0

A2 (1) The 5th term of the AP 2, 6, 10, … is: (A) 14 (B) 18 (C) 22 (D) 10

A3 (2) Write the first four terms of the AP with $a = -3$, $d = 3$.

A4 (2) Find a_{12} of the AP 5, 8, 11, …

A5 (2) Is 1/2, 1, 3/2, 2, … an AP? Justify.

A6 (3) Find the sum of the first 16 terms of the AP 3, 6, 9, …

A7 (3) Which term of the AP 7, 11, 15, … is 103?

A8 (6) Arun saves ₹500 in January and increases his monthly saving by ₹100 every month. (a) How much does he save in December? (b) How much does he save in the whole year?

MODEL TEST B — STANDARD

B1 (1) If $a_n = 4n - 3$, then a_1 is: (A) 1 (B) 4 (C) −3 (D) 7

B2 (1) The sum 2 + 4 + 6 + … + 40 is: (A) 400 (B) 420 (C) 440 (D) 840

B3 (2) Fill the boxes: [ ], 15, [ ], 39 (an AP).

B4 (2) If $a_3 = 7$ and $a_7 = 23$ in an AP, find a and d.

B5 (2) Is 73 a term of the AP 4, 9, 14, …? Justify.

B6 (3) How many terms of the AP 4, 7, 10, … give a sum of 375?

B7 (3) Find the sum of the first 18 terms of the AP −5, −2, 1, …

B8 (6) The sum of the 4th and 8th terms of an AP is 44 and the sum of the 6th and 10th terms is 64. Find the AP and the sum of its first 15 terms.

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MODEL TEST C — ADVANCED

C1 (1) If $S_n = 3n^2 - n$ for an AP, then a_{10} is: (A) 54 (B) 56 (C) 58 (D) 290

C2 (1) The number of two-digit multiples of 3 is: (A) 28 (B) 29 (C) 30 (D) 33

C3 (2) Insert three arithmetic means between 6 and 30.

C4 (2) If $a_8 = 31$ and $a_{15} = 52$ in an AP, find a and d.

C5 (2) If $a_5 = 25$ and $a_{11} = 49$ in an AP, find a and d.

C6 (3) Find the sum of all integers between 100 and 500 that are divisible by 5.

C7 (3) Find the sum of the first 24 terms of the list of numbers whose n-th term is $a_n = 3 + 2n$.

C8 (6) A sum of ₹2,400 is to be given as 8 cash prizes. Each prize is ₹50 less than the preceding prize. Find the value of each prize.

> ***TOPPER TIP:*** *Sit these tests like the real exam: no book, no phone, full working shown. Then mark yourself honestly with the key — and log every lost mark in the Error Tracker.*

Answer Key

Five-Level Practice (Section 11)

L1: 1.1 d = 4, a_6 = 25. 1.2 3, 1, −1, −3, −5. 1.3 Yes, d = 6. 1.4 59. 1.5 55. 1.6 −16.

L2: 2.1 34th term (7 + 33×6 = 205). 2.2 No: n − 1 = 296/6 is not a whole number. 2.3 S_{25} = 25/2 [−6 + 48] = 525. 2.4 a = 2, d = 5. 2.5 10 terms (3n(n+1)/2 = 165 → n = 10). 2.6 276 − 14×7 = 178.

L3: 3.1 ₹8,000 + 9×400 = ₹11,600. 3.2 S_8 = 4 × (1600 + 350) = 7,800 cars. 3.3 S_{12} = 6 × (2000 + 1100) = ₹18,600. 3.4 S_{20} = 10 × (30 + 38) = 680 seats.

L4: 4.1 From 2a + 8d = 32 and 2a + 14d = 44: d = 2, a = 8, so a_{20} = 8 + 38 = 46. 4.2 n = 4 or n = 13: terms 5 to 13 (12, 9, 6, 3, 0, −3, −6, −9, −12) cancel in pairs around 0, so both sums are 78. 4.3 $a_n = S_n - S_{n-1}$ = 6n + 2; d = 6.

L5: 5.1 Two-digit multiples of 6: 12, ..., 96 — 15 numbers; sum = 15/2 × 108 = 810. 5.2 Use n = 2(11) − 1 = 21: (7×21 + 1) : (4×21 + 27) = 148 : 111 = 4 : 3.

Active Recall (Section 17)

R1 −5. **R2** 21. **R3** No — differences 2, 4 are not equal. **R4** 30. **R5** −2 + 11×3 = 31. **R6** 20×21/2 = 210. **R7** 13 (the average of 7 and 19). **R8** a_n = 3n + 7. **R9** 16 terms. **R10** 161 − 19×4 = 85.

F1 a_n = a + (n − 1)d; (n − 1) because reaching post n crosses n − 1 gaps. **F2** S_n = n/2 [2a + (n − 1)d] (know a, d, n) and S_n = n/2 (a + l) (know first & last). **F3** $a_n = S_n - S_{n-1}$. **F4** d = $(a_{12} - a_5)/7$. **F5** n(n + 1)/2; n^2; n(n + 1).

Answer Key — Concept MCQs

M1 (B) −2: 5 − 7 = −2. **M2 (B)** 5, 8, 11, 14 — steady +3. **M3 (B)** 25: 5 + 5×4. **M4 (B)** 4n − 1: a = 3, d = 4. **M5 (B)** 6: 12 − 2×3.

M6 (C) 55: 10×11/2. **M7 (B)** 9: d = 0 never changes a term. **M8 (B)** 20: (100 − 5)/5 + 1. **M9 (B)** 58: 1 + 19×3. **M10 (B)** 2: $a_2 - a_1 = (4 - 1) - 1$... precisely: $a_1 = 1$, $a_2 = S_2 - S_1 = 3$, so d = 2.

M11 (C) 17th: (50 − 2)/3 + 1. **M12 (B)** 420: 10 pairs × 42. **M13 (B)** 20: (59 − 2)/3 + 1. **M14 (B)** 209: 254 − 9×5. **M15 (C)** 4: (35 − 19)/4.

M16 (C) 600: 15/2 × (5 + 75). **M17 (D)** 1, 3, 6, 10 — differences 2, 3, 4. **M18 (B)** 3: the coefficient of n. **M19 (B)** 550: 10/2 × 110. **M20 (B)** 4: (55 − 7)/12.

M21 (A) 12: equal d means the gap never changes. **M22 (C)** 7: $S_2 - S_1$. **M23 (B)** 13: 40 − 9×3.

M24 (B) 144 = 12^2 (sum of first n odds = n^2). **M25 (C)** 12th: $a_{11} = 0$ is not negative; $a_{12} = -2$ is the first negative term.

Assertion – Reason (Section 12)

AR1 (A) $a_{10} = 2 + 9\times3 = 29$, and R is the very formula used. **AR2 (D)** A is false: 20×21/2 = 210, not 200; R is true. **AR3 (D)** A is false (differences 3, 5, 7); R is true. **AR4 (A)** R shows exactly why A holds. **AR5 (A)** $S_{10} = 5 \times (6 + 27) = 165$ by the formula in R.

AR6 (D) A is false (n = 164/3, Ex 5.2 Q6); R is true and is the reason A fails. **AR7 (C)** A is true: 10 + 4(−3) = −2; R is false: d = −3, not 3. **AR8 (A)** R correctly explains A. **AR9 (D)** A is false: d = 90 − 100 = −10; R is true. **AR10 (A)** From R: 2b = a + c follows at once.

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Answer Key — Mini-Tests & Model Tests

Smart Memory Mini-Tests (Section 18)

Day 7: 1) −5. 2) 44. 3) 0, −1/2, −1, −3/2. 4) No (n = 91/4). 5) −44. 6) 14. 7) $a_n = 5n + 1$. 8) a = 19. 9) 33. 10) 152 − 19×3 = 95.

Day 21: 1) 78. 2) 1050. 3) 900 = 30^2. 4) 375. 5) l = 35 (200 = 5 × (5 + l)). 6) 630. 7) 55. 8) 15 terms (n(n + 1) = 240). 9) 1300. 10) $a_1 = S_1 = 4$.

Day 30: 1) d = 5, $a_7 = 33$. 2) 16th term. 3) 1272 (24 terms, 12 pairs of 106). 4) −12. 5) 5, 7, 9. 6) $S_{18} = 9 \times 40 = 360$. 7) d = −2, $a_1 = 5$. 8) 605. 9) $a_5 = 81/9 = 9$ (middle-term trick). 10) 366.

Model Test A — Basics

A1 (B) −3. A2 (B) 18. A3 −3, 0, 3, 6. A4 5 + 11×3 = 38. A5 Yes — differences all 1/2. A6 $S_{16} = 8 \times (6 + 45) = 408$. A7 25th term (7 + 24×4 = 103). A8 (a) 500 + 11×100 = ₹1,600. (b) $S_{12} = 6 \times (1000 + 1100)$ = ₹12,600.

Model Test B — Standard

B1 (A) 1. B2 (B) 420. B3 3 and 27 (d = 12). B4 d = 4, a = −1. B5 No: n − 1 = 69/5, not a whole number. B6 15 terms. B7 $S_{18} = 9 \times (-10 + 51) = 369$. B8 2a + 10d = 44, 2a + 14d = 64 → d = 5, a = −3; AP: −3, 2, 7, ...; $S_{15} = 15/2 \times (-6 + 70) = 480$.

Model Test C — Advanced

C1 (B) 56: $a_n = S_n - S_{n-1} = 6n - 4$, so $a_{10} = 56$. C2 (C) 30. C3 12, 18, 24 (d = 6). C4 d = 3, a = 10. C5 d = 4, a = 9. C6 23,700 (79 numbers: 105 to 495). C7 672 (a = 5, d = 2; $S_{24} = 12 \times 56$). C8 First prize ₹475; prizes: 475, 425, 375, 325, 275, 225, 175, 125 (check: total = 4 × 600 = 2,400).

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Revision Card — Chapter 5 on One Page

The two formulas (both derived in Section 06)

n-th term: $a_n = a + (n - 1)d$ — the first term plus (n − 1) jumps.

Sum: $S_n = n/2\,[2a + (n - 1)d] = n/2\,(a + l)$ — number of terms × average of first and last (Gauss pairing).

The rescue kit

$a_n = S_n - S_{n-1}$ • $d = (a_m - a_k)/(m - k)$ • term from end = $l - (k - 1)d$ • 3 numbers in AP: $a - d$, a, $a + d$ • naturals $n(n+1)/2$, odds n^2, evens $n(n+1)$.

Story signals

Term question: 'in the 8th year / 20th row / nth week' → a_n. **Sum question:** 'total / altogether / in all' → S_n. **Counting question:** first & last multiples, then $n = (l - a)/d + 1$.

The five traps

(n − 1), never n jumps • sign of d in falling APs • S_n is not a_n • fractional n means NOT a term • reject roots that break reality (but keep both when terms merely cancel, PYQ 8!).

Exam-eve checklist

☐ a and d written first in every solution ☐ Substitution shown before simplifying ☐ Answer restated in the story's words ☐ Impossible roots rejected WITH a reason ☐ Flash Cards 2, 5, 13 revised

Chapter 5 complete. You did not memorise Arithmetic Progressions — you discovered, visualised, derived and mastered them.

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CBSE · CLASS 10 · MATHEMATICS

MATHEMATICS BEYOND THE FORMULA

CHAPTER 6 · TRIANGLES

Don't Memorize Mathematics. Understand, Visualise and Discover It.

The KalviYogi 8D Way: Discover · Visualise · Understand · Derive · Solve · Apply · Challenge · Recall

Every solution with COMPLETE steps — one by one, nothing skipped

Similar Figures · Thales (BPT) · AA, SSS, SAS Criteria · Areas of Similar Triangles · Pythagoras Theorem

Five-Level Practice · Smart Memory System · Mastery Tracker

Every theorem in this chapter is PROVED before it is used.

Every figure is drawn for you — read the picture, then read the proof.

369 TESLA × KALVIYOGI · THE 8D MATHEMATICAL LEARNING SYSTEM

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01 Chapter Introduction

8D • DISCOVER — WHY SHOULD I LEARN THIS?

Look around you. A photograph enlarged on a banner. A map of your city folded in a pocket. The shadow of a tree at 4 p.m. A ladder resting against a wall. Every one of these everyday sights is secretly a question about **triangles** — and this chapter teaches you to answer all of them without climbing, without measuring the un-measurable, and without guessing.

The hero idea of the chapter is **similarity**: two figures can have the *same shape* but *different sizes*. Once two triangles are known to be similar, every unknown length, height and area inside them becomes a simple proportion. That single idea measures the height of a flagpole from its shadow, the width of a river without crossing it, and the distance between two aeroplanes in the sky.

What you will master in this chapter

- **Similar figures** — the two conditions (equal angles, proportional sides) and how to test them.
- **Basic Proportionality Theorem (Thales)** and its converse — with full proofs.
- **Similarity criteria AA, SSS, SAS** — why each one works, not just how to quote it.
- **Areas of similar triangles** — why areas scale with the *square* of the side ratio.
- **Pythagoras theorem** and its converse — proved from similarity itself.

How this chapter is organised — the 8D route

Discover (01–03) → **Visualise** (04) → **Understand** (05) → **Derive** (06: every theorem proved) → **Solve** (07: all NCERT exercises, step by step) → **Apply** (08, 09, 13) → **Challenge** (11, 12, 14, 15) → **Recall** (16–19 + flash cards, tests and trackers). Work through it in order the first time; afterwards, jump straight to the section your tracker tells you to revisit.

TOPPER TIP: *Triangles is the most figure-heavy chapter of Class 10. Keep a pencil in your hand from page one: copy every figure into your notebook as you read. In geometry, the hand learns faster than the eye.*

KALVIYOGI DIGITAL COMPANION: Chapter 6 video lessons, interactive similarity simulations and online tests are linked from the KalviYogi app / website. The QR code and link for this chapter activate with the app / website at print. Until then, every simulation in Section 09 is designed to be done fully on paper.

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02 Prerequisite Knowledge

FROM CLASS 9 — THE TOOLS YOU ALREADY OWN

Nothing in this chapter is truly new machinery — it is Class 9 geometry, sharpened. Check that these five tools are in your hand before you move on.

Tool 1 — Congruent triangles (Class 9)

Two triangles are **congruent** if one can be placed exactly on the other: same shape *and* same size. Tests: **SSS, SAS, ASA, RHS**. This chapter relaxes "same size" — and gets **similarity**.

Tool 2 — Parallel lines and a transversal

When a transversal cuts two parallel lines: **corresponding angles are equal**, **alternate interior angles are equal**, and **co-interior angles are supplementary** (they add to 180°). Half the proofs in this chapter are just these three facts wearing a disguise.

Tool 3 — Ratio and proportion

a : b is the fraction a/b. Four quantities are in proportion when a/b = c/d, and then **cross-multiplication** gives ad = bc. Almost every numerical answer in this chapter is one cross-multiplication away.

Tool 4 — Area of a triangle

Area = ½ × base × height. Two facts we will use again and again: triangles on the **same base** and between the **same parallels** have **equal areas**; and triangles sharing the **same height** have areas in the ratio of their bases. The proof of the Basic Proportionality Theorem is built entirely from these two facts.

Tool 5 — Squares and square roots

You must be fluent with squares up to $25^2 = 625$ and with simplifying roots such as $\sqrt{252} = 6\sqrt{7}$ ($252 = 36 \times 7$). Pythagoras questions are lost on arithmetic, not on geometry.

Quick check — five questions, two minutes

QC1. In $\triangle ABC$ and $\triangle DEF$, AB = DE, BC = EF and CA = FD. The triangles are congruent by which test?

QC2. Two angles of a triangle are 35° and 65°. Find the third angle.

QC3. If x/4 = 6/8, find x.

QC4. Find the area of a triangle with base 10 cm and height 6 cm.

QC5. A transversal cuts two parallel lines. One angle is 110°. Write (a) the corresponding angle at the other line, (b) the co-interior angle on the same side.

Answers are in the Answer Key (Prerequisite Quick Check). If you missed more than one, revise the tool above it before starting Section 03.

If a Quick Check Went Wrong — Your 10-Minute Repair

QC1 or QC2 (angles & similarity): redraw the two triangles side by side, mark equal angles with matching arcs, and list the angle sum in each. **QC3 (proportions):** practise cross-multiplication on three fractions of your own until it is automatic — BPT is 90% cross-multiplication. **QC4 (area):** remember area always needs TWO lengths multiplied and halved for a triangle. **QC5 (parallel lines):** sketch the transversal, label all eight angles, and find the corresponding and co-interior pairs with your finger. Ten honest minutes here will save an hour of confusion later in the chapter.

03 A Real-Life Mathematical Story

DISCOVER — THE ANNUAL DAY PROBLEM

It is the week before Annual Day at Meera's school, and the ground is full of jobs nobody knows how to finish.

Job 1 — the flagpole. *The chief guest will hoist the flag, and the compere must announce the height of the flagpole. Nobody may climb it — the pole is freshly painted. At 3 p.m. Meera stands beside it. Her height is 1.5 m and her shadow is 2 m long. The flagpole's shadow, measured with a tape along the ground, is 16 m. "The pole and I are both standing straight, and the sun's rays are the same for both of us," Meera says slowly. "Our two triangles must be the same shape. So the pole should be..." She writes 1.5/2 = h/16 and whispers, "12 metres?" She is right — but why is she allowed to write that proportion? That is the first question of this chapter.*

Job 2 — the backdrop. *The stage backdrop is a giant triangular cloth. The art teacher wants a straight border strip stitched across it, parallel to the bottom edge, exactly one-third of the way down from the top — and she wants the strip's length before she cuts the cloth. The full base is 9 m. "If the cut is parallel," says Arjun, "the small top triangle is a scale model of the whole. The strip should be 3 m." A parallel line, a proportion, an answer — that is the* ***Basic Proportionality Theorem*** *hiding in a sewing job.*

Job 3 — the lights. *The electrician leans a 5 m ladder against the stage wall to fix a light 4 m above the ground. "How far is the foot of my ladder from the wall?" he asks. "If it is too close I will slip; too far and I cannot reach." Meera, who has just finished Job 1, smiles: "That is not similarity, sir. That is a right angle, a 3–4–5 triangle. Your ladder's foot is exactly 3 m from the wall."*

Job 4 — the banner. *The office photograph of the school is 10 cm × 15 cm. The banner printer will blow it up so that the 10 cm side becomes 60 cm. "And the other side?" asks the printer. "Ninety centimetres," says Meera at once, "and the banner will need 36 times as much cloth area, not 6 times — areas grow with the square."*

Four jobs, four ideas: **similarity from shadows**, **proportion from parallels**, **Pythagoras from right angles**, and **areas scaling with squares**. By the end of this chapter you will not just repeat Meera's answers — you will be able to *prove* every one of them.

> ***TOPPER TIP:*** *Whenever a board question feels abstract, ask: is this a shadow job, a parallel-cut job, a ladder job, or a photo job? Naming the story tells you which theorem to reach for.*

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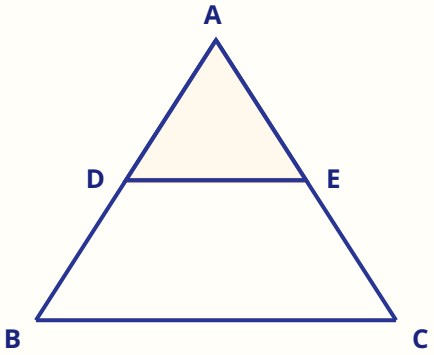
04 Visual Concept Explanation

VISUALISE — CONCRETE, PICTORIAL, ABSTRACT (SINGAPORE CPA)

Step 1 — Concrete: same shape, different size

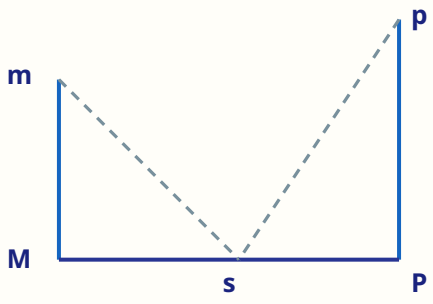
Take any photograph and print it at two sizes. Nothing about the *shape* changes — every angle in the picture stays exactly the same, and every length is multiplied by the *same* number (the scale factor). That is the whole of similarity, seen with the eyes before it is written with symbols.

Step 2 — Pictorial: one triangle living inside another



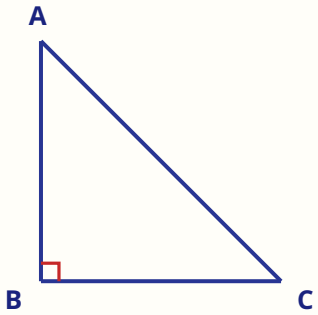
In the picture, DE is drawn parallel to BC. The small triangle ADE and the whole triangle ABC have the **same shape**: the angle at A is common, and the angles at D and E equal the angles at B and C (corresponding angles, DE ∥ BC). Same shape means the sides keep one ratio: AD/AB = AE/AC = DE/BC. This nested picture is the most important picture of the chapter — BPT, the AA criterion and half of Exercise 6.2 all live inside it.

Step 3 — Pictorial: why shadows make similar triangles



Meera (Mm) and the pole (Pp) both stand vertically, so both make a right angle with the ground. The sun's rays hit both at the same angle. Two angles equal ⇒ the triangles are the same shape ⇒ heights and shadows stay in proportion: height/shadow is one fixed number for everything standing in the same sunlight.

Step 4 — Pictorial: Pythagoras you can see



For the 3–4–5 right triangle: draw a square on each side. The squares hold 9, 16 and 25 unit squares — and 9 + 16 = 25. The square on the longest side (the **hypotenuse**) exactly swallows the other two squares. Pythagoras theorem says this happens for *every* right triangle: $(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$. In Section 06 we prove it — using similarity itself.

Similar is NOT "looks alike"

A square and a rhombus both have four equal sides, yet they are **not** similar — their angles differ. A rectangle 2 × 1 and a rectangle 4 × 3 "look alike" but are **not** similar — their side ratios 2/1 and 4/3 differ. Similarity is a promise about **every** angle and **every** side ratio, never a first impression.

Step 5 — Abstract: the language

We write △ABC ~ △PQR (read: "triangle ABC is similar to triangle PQR"). The **order of the letters is the meaning**: A matches P, B matches Q, C matches R. From that one line you may read off three angle equalities and three side ratios — but only if the order was written honestly.

PIN POINT: Same shape = every angle equal AND every side multiplied by one fixed scale factor. Miss either condition and the figures are NOT similar — no matter how alike they look.

05 Mathematical Foundations

UNDERSTAND — EVERY DEFINITION AND THEOREM, STATED CLEANLY

Similar figures

Two figures are **similar** if (i) their corresponding angles are **equal**, and (ii) their corresponding sides are in the **same ratio** (proportional). Both conditions are needed. All circles are similar; all squares are similar; all equilateral triangles are similar — for these, one condition automatically drags the other along. For general polygons, both must be checked (remember the square and the rhombus).

Similar triangles — the working definition

△ABC ~ △PQR means, in one breath: ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R, and AB/PQ = BC/QR = CA/RP. The common value of the ratios is the **scale factor**. Scale factor 1 means the triangles are not just similar but **congruent** — congruence is similarity at full size.

Idea	Congruent (≅)	Similar (~)
Shape	Same	Same
Size	Same	May differ
Angles	Equal	Equal
Sides	Equal	Proportional
Symbol test	SSS, SAS, ASA, RHS	AA, SSS, SAS (Section 06)

The theorems of this chapter (proved in Section 06)

Theorem 6.1 — Basic Proportionality Theorem (Thales)

If a line is drawn **parallel** to one side of a triangle, intersecting the other two sides in distinct points, then it divides those two sides in the **same ratio**: DE ∥ BC ⇒ AD/DB = AE/EC.

Theorem 6.2 — Converse of BPT

If a line **divides** any two sides of a triangle in the **same ratio**, then the line is **parallel** to the third side: AD/DB = AE/EC ⇒ DE ∥ BC.

The three similarity criteria

AA: two angles of one equal to two angles of the other. **SSS:** all three side-ratios equal. **SAS:** two side-ratios equal *and the angle between those sides* equal. Each is a shortcut: prove a little, get the full definition free.

Areas of similar triangles

The ratio of the areas of two similar triangles equals the **square** of the ratio of their corresponding sides: $ar(ABC)/ar(PQR) = (AB/PQ)^2 = (BC/QR)^2 = (CA/RP)^2$.

Pythagoras theorem — and its converse

In a right triangle, $(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$. Conversely, if a triangle's sides satisfy $a^2 + b^2 = c^2$, the angle opposite the side c is a right angle.

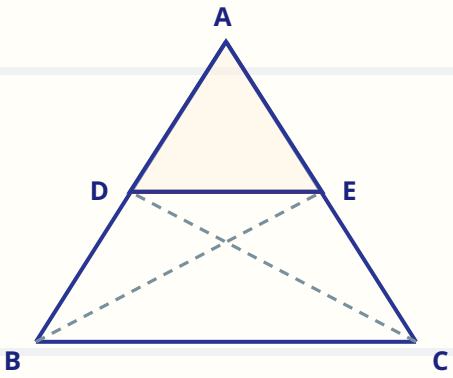
TOPPER TIP: *Learn the statements word-perfect. In a 3-mark board question, the statement itself, written correctly with the figure named, is often the first mark.*

06 Formula Derivations — The Proofs

8D • DERIVE — NO THEOREM IS USED BEFORE IT IS PROVED

Derivation 1 — Basic Proportionality Theorem (Thales), full proof

Given: △ABC with DE ∥ BC, D on AB, E on AC. **To prove:** AD/DB = AE/EC. **Construction:** join BE and CD.



1. Look at △ADE and △BDE. They share the same vertex E and their bases AD and DB lie on the same straight line AB, so they have the **same height**. Areas with equal heights are in the ratio of their bases: ar(ADE)/ar(BDE) = AD/DB … (1)

   Why: Same height ⇒ area ratio = base ratio. This is Prerequisite Tool 4 doing the work.

2. Now look at △ADE and △CDE. They share the vertex D and their bases AE and EC lie on AC, so again the heights are equal: ar(ADE)/ar(CDE) = AE/EC … (2)

   Why: Same pair of ideas, second side of the triangle.

3. △BDE and △CDE stand on the **same base DE** and lie between the **same parallels** DE and BC. So ar(BDE) = ar(CDE) … (3)

   Why: Same base + same parallels ⇒ equal areas. The parallel condition enters exactly here — nowhere else.

4. From (1) and (2), the left sides differ only in the denominator; by (3) the denominators are equal. Hence AD/DB = AE/EC. **Proved.**

   Why: Two ratios with equal numerators and equal denominators are equal.

5. **Useful twin form:** add 1 to both sides: AD/DB + 1 = AE/EC + 1 ⇒ (AD + DB)/DB = (AE + EC)/EC, and the same trick on the reciprocals gives **AD/AB = AE/AC** — the 'part-to-whole' form used in most numerical questions.

   Why: Board questions switch freely between part-to-part (AD/DB) and part-to-whole (AD/AB). Learn to convert in one line.

PIN POINT: BPT is an AREA theorem in disguise: parallels force two areas to be equal, and equal areas force the sides into proportion.

Derivation 2 — Converse of BPT, and why AA is enough

To prove: if AD/DB = AE/EC in △ABC (D on AB, E on AC), then DE ∥ BC.

1. Suppose, for contradiction, that DE is **not** parallel to BC.
   Why: Proof by contradiction: assume the opposite and watch it collapse.
2. Draw the line through D parallel to BC, meeting AC at a new point E′. By BPT (Derivation 1), AD/DB = AE′/E′C … (1)
   Why: There is exactly one line through D parallel to BC, so E′ is well defined.
3. But we are given AD/DB = AE/EC … (2). From (1) and (2): AE/EC = AE′/E′C.
   Why: The same ratio AD/DB cannot equal two different ratios.
4. Add 1 to both sides: (AE + EC)/EC = (AE′ + E′C)/E′C ⇒ AC/EC = AC/E′C ⇒ **EC = E′C**.
   Why: Both E and E′ lie on the segment AC, at the same distance from C.
5. So E′ is the same point as E — the line we drew *is* DE, and DE ∥ BC. The supposition was false. **Proved.**
   Why: This is why checking a ratio is enough to claim a parallel — Exercise 6.2 Q2 runs on this proof.

Derivation 3 — Why two angles are enough (AA)

Suppose ∠A = ∠P and ∠B = ∠Q in △ABC and △PQR. The angles of any triangle add to 180°, so ∠C = 180° − ∠A − ∠B = 180° − ∠P − ∠Q = ∠R. The third pair is equal *for free* — that is why the criterion is called **AA**, not AAA. Equal angles alone do not yet give proportional sides; that debt is paid by Derivations 4 and 5 below, where the criteria are proved from BPT.

Derivation 4 — The SSS and SAS criteria (proof idea, NCERT pattern)

Given for SSS: AB/PQ = BC/QR = CA/RP in △ABC and △PQR. **Idea:** cut a copy of the smaller triangle out of the larger one and let BPT finish the job. (If needed, swap the names so that PQ ≥ AB.)

1. On PQ mark S with PS = AB, and on PR mark T with PT = AC. Join ST.
   Why: We build a physical copy of △ABC inside △PQR.
2. PS/PQ = AB/PQ. By the given proportion AB/PQ = AC/PR = PT/PR. So PS/PQ = PT/PR ⇒ by the **converse of BPT**, ST ∥ QR.
   Why: Equal ratios on the two sides of ∠P force the parallel.
3. Hence ∠PST = ∠PQR and ∠PTS = ∠PRQ (corresponding angles), so △PST ~ △PQR by AA, giving PS/PQ = ST/QR.
   Why: The small cut-off triangle is a scale model of the big one.
4. But PS/PQ = AB/PQ = BC/QR (given). Therefore ST = BC.
   Why: Two quantities equal to the same ratio BC/QR — comparing gives ST = BC.
5. Now △ABC ≅ △PST by SSS congruence (AB = PS, AC = PT, BC = ST). So their angles match those of △PQR, and the side ratios were given. Hence △ABC ~ △PQR. **Proved.**
   Why: Congruence transfers the angles; the given ratios supply the sides. Both halves of the definition are satisfied.

For SAS the same film runs with one change: we are given AB/PQ = AC/PR with the *included* angles ∠A = ∠P equal. Cut PS = AB, PT = AC on PQ, PR. Then △ABC ≅ △PST immediately (SAS congruence, using the included angle), the ratio PS/PQ = PT/PR gives ST ∥ QR by converse-BPT, and AA finishes it. Notice where the word **included** was used: the equal angle must be the one *between* the two proportional sides, or Step 1 collapses — remember this in Exercise 6.3 Q1(v).

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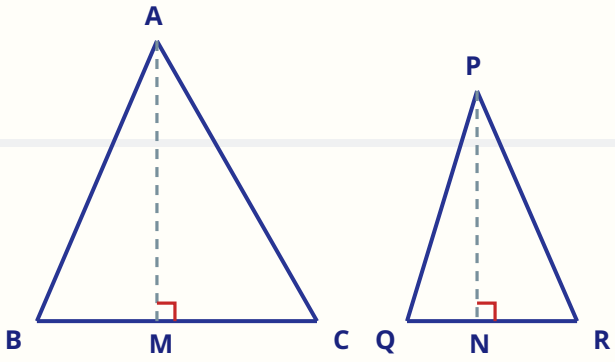
Derivation 5 — Areas of similar triangles (full proof)

Given: △ABC ~ △PQR. **To prove:** ar(ABC)/ar(PQR) = $(BC/QR)^2$. **Construction:** draw altitudes AM ⊥ BC and PN ⊥ QR.



1 Write both areas with the formula ½ × base × height: ar(ABC)/ar(PQR) = (½ · BC · AM)/(½ · QR · PN) = (BC/QR) · (AM/PN) … (1)

Why: The ½ cancels. The area ratio splits into a base ratio times a height ratio.

2 Compare △ABM and △PQN: ∠B = ∠Q (similar triangles) and ∠AMB = ∠PNQ = 90°. So △ABM ~ △PQN by AA.

Why: The altitude builds a smaller copy of the same similarity inside the figure.

3 Therefore AM/PN = AB/PQ. And AB/PQ = BC/QR (given similarity). So AM/PN = BC/QR … (2)

Why: Heights of similar triangles are in the same ratio as the sides — a fact worth remembering by itself.

4 Substitute (2) in (1): ar(ABC)/ar(PQR) = (BC/QR) · (BC/QR) = **$(BC/QR)^2$. Proved.**

Why: Areas scale with the square because BOTH the base and the height scale with the side ratio.

PIN POINT: Area ratio = $(\text{side ratio})^2$ — because a similar figure grows in two directions at once. The same argument shows corresponding medians, altitudes and angle bisectors all keep the side ratio (Exercise 6.3 Q16, Exercise 6.4 Q6).

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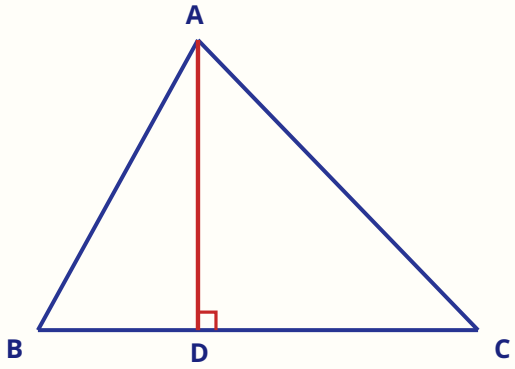
Derivation 6 — Pythagoras theorem, proved by similarity

Given: △ABC, right-angled at A. **To prove:** $BC^2 = AB^2 + AC^2$. **Construction:** draw AD ⊥ BC, with D on BC.



1. In △ABD and △CBA: ∠ADB = ∠BAC = 90° and ∠B is common. So △ABD ~ △CBA by AA.
   Why: The altitude splits the big right triangle into two smaller copies of itself.
2. Corresponding sides: AB/BC = BD/AB. Cross-multiply: **$AB^2 = BD \cdot BC$** … (1)
   Why: Each leg is the geometric mean of the hypotenuse and its own shadow segment.
3. In △ACD and △BCA: ∠ADC = ∠BAC = 90° and ∠C is common. So △ACD ~ △BCA by AA, giving AC/BC = CD/AC, i.e. **$AC^2 = CD \cdot BC$** … (2)
   Why: The same argument on the other side of the altitude.
4. Add (1) and (2): $AB^2 + AC^2 = BD \cdot BC + CD \cdot BC = BC \cdot (BD + CD) = BC \cdot BC =$ **BC^2**. **Proved.**
   Why: BD + CD is exactly BC — the two pieces reassemble into the whole hypotenuse.
5. **Bonus (the altitude rule):** △ABD ~ △CAD as well (both are similar to △CBA). Hence AD/BD = CD/AD, so **$AD^2 = BD \cdot DC$**.
   Why: Three relations from one altitude — Exercises 6.5 Q2 and Q3 are nothing but these three lines.

Derivation 7 — Converse of Pythagoras (proof sketch)

Given: a triangle ABC with $AC^2 = AB^2 + BC^2$. **To prove:** ∠B = 90°. **Construction:** build △PQR, right-angled at Q, with PQ = AB and QR = BC. By Pythagoras (Derivation 6), $PR^2 = PQ^2 + QR^2 = AB^2 + BC^2 = AC^2$, so PR = AC. Then △ABC ≅ △PQR by SSS, and therefore ∠B = ∠Q = 90°. **Proved.** — This converse is the surveyor's friend: it lets you *create* a right angle from three lengths, as the 3–4–5 rope in Section 09 shows.

TOPPER TIP: *Examiners award the Pythagoras proof (Derivation 6) step marks for: the construction line, the two AA similarities, the two product relations, and the final addition. Write all four, every time.*

Derivation Checkpoint — Prove You Own the Proofs

Close the book and answer these aloud. Each one is a derivation from this section, compressed into a single question.

1. In the BPT proof, why do △ADE and △BDE have equal heights? *(Both have apex E, and their bases AD and DB lie on the same line AB — so the perpendicular distance from E to AB serves both.)*

2. In the converse of BPT, where exactly does the contradiction bite? *(The parallel through D is unique, so the assumed second intersection point E′ must coincide with E.)*

3. In the areas proof, why does the altitude ratio equal the side ratio? *(The two altitude triangles are similar by AA: one equal angle from the similarity, one right angle each.)*

4. In the Pythagoras proof, what single addition turns two product relations into the theorem? *($AB^2 + AC^2 = BD \cdot BC + CD \cdot BC = BC(BD + CD) = BC^2$.)*

If any answer would not come, re-read that derivation now — the NCERT solutions in the next section use all seven as working tools.

07 Detailed Solved Examples — NCERT Exercises

8D • SOLVE — EVERY STEP, ONE BY ONE, WITH ITS REASON

These are the complete NCERT Class 10 Chapter 6 exercises (rationalised edition): 6.1, 6.2, 6.3, 6.4 and 6.5 — all 55 questions, solved in the KalviYogi format. Read the **THINK FIRST**, follow each numbered step, and notice that the marking line tells you where the examiner's marks live.

Exercise 6.1 — Similar figures

EX 6.1 Q1

Fill in the blanks using the correct word given in brackets: (i) All circles are _____ (congruent / similar). (ii) All squares are _____ (similar / congruent). (iii) All _____ triangles are similar (isosceles / equilateral). (iv) Two polygons of the same number of sides are similar if (a) their corresponding angles are _____ and (b) their corresponding sides are _____ (equal / proportional).

THINK FIRST: Each blank is decided by the two-condition test of Section 05: equal angles AND proportional sides.

1. (i) Any two circles differ only in radius — every angle (the full 360° round) matches and every length scales with the radius. So all circles are **similar**.

   Why: Congruent would demand equal radii too, which is not given.

2. (ii) Every square has four 90° angles, and its sides are trivially in proportion (all sides equal within each square). So all squares are **similar**.

   Why: Same reason as circles: shape is fixed, only size can change.

3. (iii) Every equilateral triangle has angles 60°, 60°, 60°, so any two are equiangular and hence similar: **equilateral**. Isosceles triangles can have very different vertex angles, so they need not be similar.

   Why: Isosceles only fixes two sides; the angles are free to change.

4. (iv) This is the definition itself: (a) **equal**, (b) **proportional**.

   Why: Both conditions, always — one alone never suffices for polygons.

ANSWER: (i) similar (ii) similar (iii) equilateral (iv) (a) equal, (b) proportional.

CBSE-STYLE MARKING SCHEME: Four blanks [1] each — no working is needed, but the two-condition definition must be the reason in your mind.

EX 6.1 Q2

Give two different examples of a pair of (i) similar figures, (ii) non-similar figures.

THINK FIRST: Pick examples where the two-condition test is easy to state.

1. (i) Similar: **any two squares** (say of sides 2 cm and 5 cm) — angles all 90°, sides in ratio 2 : 5 throughout. Also **any two circles**, e.g. a 10-rupee coin and a car wheel's rim circle.

   Why: Fixed-shape families are always safe examples.

2. (ii) Non-similar: a **square and a rhombus** (sides proportional, angles not equal) and a **2 × 1 rectangle with a 4 × 3 rectangle** (angles equal, but side ratios 2/1 ≠ 4/3).

   Why: One example failing each condition shows you understand both conditions.

ANSWER: (i) Two squares; two circles. (ii) A square and a rhombus; rectangles 2 × 1 and 4 × 3.

CBSE-STYLE MARKING SCHEME: [1] for a correct similar pair, [1] for a correct non-similar pair.

EX 6.1 Q3

State whether the following quadrilaterals are similar or not: a square ABCD of side 3 cm and a rhombus PQRS of side 1.5 cm (as in the NCERT figure).

1. Compare the sides first: AB/PQ = BC/QR = CD/RS = DA/SP = 3/1.5 = 2. The corresponding sides ARE in the same ratio.

   Why: One condition passes — do not stop here.

2. Compare the angles: every angle of the square is 90°, but the rhombus in the figure has angles different from 90° (it is slanted). The corresponding angles are NOT equal.

   Why: A rhombus that is not a square has two acute and two obtuse angles.

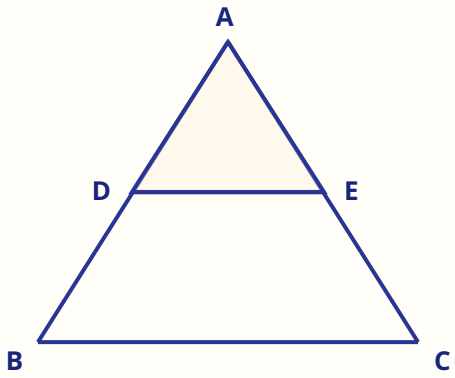
3. Similarity of polygons needs BOTH conditions. One has failed, so the quadrilaterals are **not similar**.

   Why: This is the standard trap: equal side ratios alone do not make polygons similar.

ANSWER: Not similar — sides are proportional (ratio 2) but corresponding angles are not equal.

Exercise 6.2 — The Basic Proportionality Theorem

Every question in this exercise is Derivation 1 or Derivation 2 at work. Say the theorem's name in your first step — it is worth a mark by itself.



EX 6.2 Q1

In the figure, DE ∥ BC. (i) AD = 1.5 cm, DB = 3 cm, AE = 1 cm. Find EC. (ii) AD is unknown, DB = 7.2 cm, AE = 1.8 cm, EC = 5.4 cm. Find AD.

THINK FIRST: DE ∥ BC is given, so BPT applies directly: AD/DB = AE/EC. One unknown in each part.

1. (i) By BPT: 1.5/3 = 1/EC.
   Why: Substitute the three known lengths into AD/DB = AE/EC.
2. Cross-multiply: 1.5 · EC = 3 · 1 = 3, so EC = 3/1.5 = **2 cm**.
   Why: Cross-multiplication turns the proportion into one division.
3. (ii) By BPT: AD/7.2 = 1.8/5.4. Note 1.8/5.4 = 1/3.
   Why: Simplify the known ratio first — it keeps the arithmetic clean.
4. So AD = 7.2 × (1/3) = **2.4 cm**.
   Why: AD is one-third of DB here.

ANSWER: (i) EC = 2 cm (ii) AD = 2.4 cm.

CBSE-STYLE MARKING SCHEME: Each part: BPT statement [1], substitution [½], answer [½].

EX 6.2 Q2

E and F are points on sides PQ and PR of △PQR. For each case, state whether EF ∥ QR: (i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm, FR = 2.4 cm. (ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm, FR = 9 cm. (iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm.

THINK FIRST: This is the CONVERSE of BPT: compute PE/EQ and PF/FR and compare. In (iii) you must first find EQ and FR by subtraction.

1. (i) PE/EQ = 3.9/3 = 1.3. PF/FR = 3.6/2.4 = 1.5. Since 1.3 ≠ 1.5, the ratios differ, so EF is **not parallel** to QR.
   Why: By the converse of BPT, parallelism needs the two ratios to be exactly equal.
2. (ii) PE/QE = 4/4.5 = 8/9. PF/FR = 8/9. The ratios are equal, so EF **is parallel** to QR.
   Why: Equal ratios ⇒ parallel, by Theorem 6.2.
3. (iii) EQ = PQ − PE = 1.28 − 0.18 = 1.10 cm; FR = PR − PF = 2.56 − 0.36 = 2.20 cm.
   Why: The exercise gives whole sides here, so the lower pieces come from subtraction.
4. PE/EQ = 0.18/1.10 = 9/55 and PF/FR = 0.36/2.20 = 9/55. Equal ⇒ EF **is parallel** to QR.
   Why: Both ratios reduce to the same fraction 9/55 — always reduce before comparing.

ANSWER: (i) Not parallel (ii) Parallel (iii) Parallel.

CBSE-STYLE MARKING SCHEME: Each case: both ratios computed [1], correct conclusion with Theorem 6.2 named [1].

EX 6.2 Q3

In the figure, LM ‖ CB and LN ‖ CD, where L is on AC, M is on AB and N is on AD (diagonal AC of quadrilateral ABCD). Prove that AM/AB = AN/AD.

THINK FIRST: Apply BPT twice — once in each of the two triangles that share the side AC — and notice both ratios pass through AL/AC.

1. In △ABC, LM ‖ CB, so by BPT in its whole-side form: AM/AB = AL/AC ... (1)
   Why: LM cuts AB and AC, so the ratio along AB equals the ratio along AC.
2. In △ADC, LN ‖ CD, so by BPT: AN/AD = AL/AC ... (2)
   Why: Same move in the second triangle.
3. The right sides of (1) and (2) are the same quantity AL/AC. Hence AM/AB = AN/AD. **Proved.**
   Why: Two quantities equal to a third are equal to each other.

ANSWER: AM/AB = AN/AD, by applying BPT in △ABC and △ADC and comparing through AL/AC.

CBSE-STYLE MARKING SCHEME: BPT in first triangle [1], BPT in second triangle [1], comparison and conclusion [1].

EX 6.2 Q4

In the figure, DE ‖ AC and DF ‖ AE, where D is on AB, E and F are on BC (F between B

Exercise 6.2 — Questions 5 to 10

EX 6.2 Q5

In the figure, DE ∥ OQ and DF ∥ OR, where P is the apex, D is on PO, E is on PQ and F is on PR. Show that EF ∥ QR.

THINK FIRST: Use BPT twice to manufacture equal ratios PE/EQ and PF/FR, then finish with the CONVERSE of BPT.

1. In △POQ, DE ∥ OQ, so by BPT: PD/DO = PE/EQ ... (1)
   Why: First parallel pair, first triangle.
2. In △POR, DF ∥ OR, so by BPT: PD/DO = PF/FR ... (2)
   Why: Second parallel pair, second triangle.
3. From (1) and (2): PE/EQ = PF/FR in △PQR.
   Why: Both ratios equal PD/DO.
4. By the converse of BPT (Theorem 6.2), EF ∥ QR. **Proved.**
   Why: Equal division ratios on PQ and PR force the joining line to be parallel.

ANSWER: EF ∥ QR, by BPT twice followed by the converse of BPT.

CBSE-STYLE MARKING SCHEME: Each BPT application [1], converse correctly invoked [1].

EX 6.2 Q6

In the figure, A, B and C are points on OP, OQ and OR respectively such that AB ∥ PQ and AC ∥ PR. Show that BC ∥ QR.

THINK FIRST: Exactly the same architecture as Q5 — two BPT ratios through OA/AP, then the converse.

1. In △OPQ, AB ∥ PQ ⇒ OA/AP = OB/BQ ... (1) (BPT)
   Why: A and B cut OP and OQ proportionally.
2. In △OPR, AC ∥ PR ⇒ OA/AP = OC/CR ... (2) (BPT)
   Why: A and C cut OP and OR proportionally.
3. Hence OB/BQ = OC/CR in △OQR.
   Why: Both equal OA/AP.
4. By the converse of BPT, BC ∥ QR. **Proved.**

Exercise 6.3 — The Similarity Criteria

The game in every question: choose the right criterion (AA, SSS or SAS), verify its conditions *exactly*, and write the similarity with vertices in matching order.

EX 6.3 Q1

State which pairs of triangles in the NCERT figure are similar. Write the similarity criterion used and the pairs in symbolic form. (i) △ABC and △PQR with ∠A = ∠P = 60°, ∠B = ∠Q = 80°, ∠C = ∠R = 40°. (ii) △ABC with sides AB = 2.5, BC = 2, CA = 3 and △PQR with PQ = 6, QR = 5, RP = 4. (iii) △LMP with LM = 2.7, MP = 2, LP = 3 and △DEF with DE = 4, EF = 5, DF = 6. (iv) △MNL and △QPR where MN = 2.5, ML = 5, QP = 5, QR = 10 and the included angles ∠M = ∠Q = 70°. (v) △ABC with AB = 2.5, BC = 3, ∠A = 80° and △DEF with DF = 5, EF = 6, ∠F = 80°. (vi) △DEF with ∠D = 70°, ∠E = 80° and △PQR with ∠Q = 80°, ∠R = 30°.

THINK FIRST: Test the criteria in a fixed order: angles first (AA), then all three side ratios (SSS), then two ratios with the included angle (SAS).

1. (i) All three angle pairs are equal, so by AAA (AA): **△ABC ~ △PQR**.

   Why: Matching angles in order A↔P, B↔Q, C↔R fixes the symbolic form.

2. (ii) Match longest with longest: AB/QR = 2.5/5 = 1/2, BC/RP = 2/4 = 1/2, CA/PQ = 3/6 = 1/2. All three ratios equal, so by SSS: **△ABC ~ △QRP**.

   Why: Note the order QRP: A matches Q (2.5 ↔ 5), B matches R (2 ↔ 4), C matches P (3 ↔ 6). Writing △PQR here would be wrong.

3. (iii) MP/DE = 2/4 = 1/2 and LP/DF = 3/6 = 1/2, but LM/EF = 2.7/5 = 27/50 ≠ 1/2. One ratio breaks away, so the triangles are **not similar**.

   Why: SSS is an all-or-nothing test — two equal ratios out of three prove nothing.

4. (iv) MN/QP = 2.5/5 = 1/2 and ML/QR = 5/10 = 1/2, and the angle between these sides is ∠M = ∠Q = 70°. By SAS: **△MNL ~ △QPR**.

   Why: The equal angle is the INCLUDED angle — it sits between the two proportional sides in both triangles.

5. (v) AB/DF = 2.5/5 = 1/2 and BC/EF = 3/6 = 1/2, and an 80° angle appears in both. BUT in △ABC the angle between sides AB and BC is ∠B, not the given ∠A = 80°; the 80° angle is not the included angle of the proportional sides. SAS does **not** apply — the triangles are **not similar** (as far as the given data can show).

   Why: This is the single most-tested trap of the chapter: SAS with a non-included angle is not SAS at all.

6. (vi) In △DEF: ∠F = 180° − 70° − 80° = 30°. In △PQR: ∠P = 180° − 80° − 30° = 70°. So ∠D = ∠P = 70°, ∠E = ∠Q = 80°, ∠F = ∠R = 30°, and by AA: **△DEF ~ △PQR**.

Exercise 6.3 — Questions 5 to 8

EX 6.3 Q5

S and T are points on sides PR and QR of △PQR such that ∠P = ∠RTS. Show that △RPQ ~ △RTS.

THINK FIRST: Two angles are already in hand: the given equality and the angle at R, which belongs to both triangles.

1. In △RPQ and △RTS: ∠RPQ = ∠RTS (given: ∠P = ∠RTS).
   Why: First pair, straight from the question.
2. ∠PRQ = ∠TRS — this is the same angle at R, common to both triangles.
   Why: Second pair, free of charge.
3. By AA, **△RPQ ~ △RTS. Proved.**
   Why: Note the order: R↔R, P↔T, Q↔S — written so the equal angles sit in matching positions.

ANSWER: △RPQ ~ △RTS, by AA (given angle + common angle at R).

CBSE-STYLE MARKING SCHEME: Given angle identified [1], common angle [1], AA conclusion in correct order [1].

EX 6.3 Q6

In the figure, if △ABE ≅ △ACD, show that △ADE ~ △ABC. (D is on AB and E is on AC of the larger configuration.)

THINK FIRST: Congruence gives equal sides (CPCT). Turn those equalities into one proportion, add the common angle at A, and SAS similarity finishes it.

1. From △ABE ≅ △ACD, corresponding parts are equal: AB = AC and AD = AE … (1)
   Why: CPCT: congruent triangles have all corresponding sides equal.
2. Divide the equalities in (1): AD/AB = AE/AC … (2)
   Why: Equal numerators over equal denominators keep the ratio equal.
3. In △ADE and △ABC, ∠A is common, and the sides about it are proportional by (2).
   Why: The two SAS ingredients are ready.
4. By SAS, **△ADE ~ △ABC. Proved.**
   Why: Congruence (Class 9) feeding similarity (Class 10) — a favourite examiner bridge.

ANSWER: △ADE ~ △ABC, by SAS, using AD/AB = AE/AC from the congruent pair.

CBSE-STYLE MARKING SCHEME: CPCT equalities [1], proportion formed [1], SAS with common angle [1].

EX 6.3 Q7

In the figure, altitudes AD and CE of △ABC intersect each other at P. Show that: (i) △AEP ~ △CDP (ii) △ABD ~ △CBE (iii) △AEP ~ △ADB (iv) △PDC ~ △BEC.

THINK FIRST: Every pair hides one right angle (from an altitude) plus either a common angle or a pair of vertically opposite angles. AA four times.

1. (i) In △AEP and △CDP: ∠AEP = ∠CDP = 90° (altitudes CE and AD) and ∠APE = ∠CPD (vertically opposite). So △AEP ~ △CDP by AA.

   Why: The two altitude feet give the right angles; the crossing at P gives the vertical pair.

2. (ii) In △ABD and △CBE: ∠ADB = ∠CEB = 90° and ∠B is common. So △ABD ~ △CBE by AA.

   Why: Same altitude right angles, now with the shared angle at B.

3. (iii) In △AEP and △ADB: ∠AEP = ∠ADB = 90° and ∠A is common. So △AEP ~ △ADB by AA.

   Why: Shared angle at A this time.

4. (iv) In △PDC and △BEC: ∠PDC = ∠BEC = 90° and ∠C is common. So △PDC ~ △BEC by AA. **All four proved.**

   Why: Once you see the pattern — right angle + common/vertical angle — all four parts are one line each.

ANSWER: (i), (ii), (iii), (iv) all hold by AA — right angle from an altitude plus a common or vertically opposite angle.

CBSE-STYLE MARKING SCHEME: Each part [1]: right angles identified [½], second angle pair and AA conclusion [½].

EX 6.3 Q8

E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that △ABE ~ △CFB.

THINK FIRST: Two angle pairs: opposite angles of the parallelogram, and alternate interior angles from AD ∥ BC.

1. ABCD is a parallelogram, so its opposite angles are equal: ∠A = ∠C ... (1). In our triangles these are ∠BAE and ∠FCB.

   Why: The angle at A of △ABE and the angle at C of △CFB are just the parallelogram's opposite corners.

2. AD ∥ BC (opposite sides of a parallelogram), and BE is a transversal, so ∠AEB = ∠CBF (alternate interior angles) ... (2)

   Why: E sits on AD produced, so the line AE is the same line as AD — the parallel still applies.

3. From (1) and (2), by AA: **△ABE ~ △CFB. Proved.**

   Why: A↔C, E↔B, B↔F — the order in the answer matches the angle pairs found.

Exercise 6.3 — Questions 9 to 12

EX 6.3 Q9

In the figure, ABC and AMP are two right triangles, right-angled at B and M respectively (M lies on BC and P is positioned so that △AMP sits inside the angle at A). Prove that: (i) △ABC ~ △AMP (ii) CA/PA = BC/MP.

THINK FIRST: One common angle and one right angle in each triangle — AA immediately; part (ii) is just reading off the proportion.

1. (i) In △ABC and △AMP: ∠CAB = ∠MAP (the same angle at A, common to both) and ∠ABC = ∠AMP = 90° (given right angles).

   Why: The two AA ingredients.

2. By AA, **△ABC ~ △AMP**.

   Why: A↔A, B↔M, C↔P.

3. (ii) Corresponding sides of similar triangles are proportional: CA/PA = AB/AM = BC/MP. In particular, **CA/PA = BC/MP**. **Proved.**

   Why: Read the pair the question asks for straight off the similarity statement — in matching positions.

ANSWER: (i) △ABC ~ △AMP by AA. (ii) CA/PA = BC/MP by proportionality of corresponding sides.

CBSE-STYLE MARKING SCHEME: Common angle + right angles [1], AA [1], proportion read correctly [1].

EX 6.3 Q10

CD and GH are respectively the bisectors of ∠ACB and ∠EGF, with D on AB and H on FE, and △ABC ~ △FEG. Show that: (i) CD/GH = AC/FG (ii) △DCB ~ △HGE (iii) △DCA ~ △HGF.

THINK FIRST: Strategy: first prove the small bisector triangles similar (△ACD ~ △FGH), which hands us (i) and (iii); then (ii) follows from the leftover angle halves.

1. From △ABC ~ △FEG: ∠A = ∠F, ∠B = ∠E and ∠ACB = ∠FGE … (1)

   Why: Write out the correspondence A↔F, B↔E, C↔G before touching the bisectors.

2. CD bisects ∠ACB and GH bisects ∠FGE; since the whole angles are equal by (1), their halves are equal: ∠ACD = ∠FGH and ∠DCB = ∠HGE … (2)

   Why: Halves of equal angles are equal — the one line that powers the whole question.

3. (iii) In △DCA and △HGF: ∠A = ∠F (by (1)) and ∠ACD = ∠FGH (by (2)). By AA, **△DCA ~ △HGF**.

   Why: This is part (iii) — proved first because (i) falls out of it.

Exercise 6.3 — Questions 13 to 16

EX 6.3 Q13

D is a point on the side BC of △ABC such that ∠ADC = ∠BAC. Show that $CA^2 = CB \cdot CD$.

THINK FIRST: The given angle plus the common angle at C gives AA similarity between △ADC and △BAC; the required product is one cross-multiplication of its proportion.

1. In △ADC and △BAC: ∠ADC = ∠BAC (given).
   Why: First pair, from the question.
2. ∠ACD = ∠BCA — the angle at C is common to both triangles.
   Why: Second pair.
3. By AA, △ADC ~ △BAC.
   Why: Order check: A↔B, D↔A, C↔C.
4. Corresponding sides: CA/CB = CD/CA.
   Why: CA sits opposite ∠ADC's partner... read carefully: in △ADC the side CA corresponds to CB in △BAC, and CD corresponds to CA.
5. Cross-multiply: **$CA^2 = CB \cdot CD$. Proved.**
   Why: Products like $CA^2 = CB \cdot CD$ are always cross-multiplied proportions in disguise.

ANSWER: $CA^2 = CB \cdot CD$, from △ADC ~ △BAC (AA) and cross-multiplication.

CBSE-STYLE MARKING SCHEME: Given angle + common angle [1], AA [1], correct proportion [1], cross-multiplication [1].

EX 6.3 Q14

Sides AB and AC and median AD of △ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that △ABC ~ △PQR.

THINK FIRST: Here the proportional sides meet at A and P, so halving (Q12's trick) does not directly work. The classic rescue: DOUBLE the median to build a parallelogram, turning the median into a full side of a new triangle.

1. **Construction:** produce AD to E with AD = DE, and join CE. Produce PM to N with PM = MN, and join RN.
   Why: Doubling the median is the standard construction for median problems — learn it as a move.
2.

Exercise 6.4 — Areas of Similar Triangles

One theorem runs this whole exercise (Derivation 5): area ratio = (corresponding side ratio)². Most errors come from forgetting the square — or from applying it to triangles never proved similar.

EX 6.4 Q1

Let △ABC ~ △DEF and their areas be respectively 64 cm² and 121 cm². If EF = 15.4 cm, find BC.

THINK FIRST: Area ratio 64 : 121 must be unsquared first: it is the square of the side ratio BC/EF.

1. By the areas theorem: ar(ABC)/ar(DEF) = (BC/EF)².
   Why: BC and EF are corresponding sides (B↔E, C↔F).
2. Substitute: 64/121 = (BC/15.4)².
   Why: Areas in the same order as the triangles in the similarity.
3. Take square roots (lengths are positive): BC/15.4 = 8/11.
   Why: 64 = 8² and 121 = 11² — the numbers were chosen to root cleanly.
4. BC = 15.4 × 8/11 = 1.4 × 8 = **11.2 cm**.
   Why: 15.4/11 = 1.4 first, then multiply.

ANSWER: BC = 11.2 cm.

CBSE-STYLE MARKING SCHEME: Theorem applied [1], substitution [1], square root step [1], answer [1].

EX 6.4 Q2

Diagonals of a trapezium ABCD with AB ∥ DC intersect each other at the point O. If AB = 2 CD, find the ratio of the areas of triangles AOB and COD.

THINK FIRST: First prove △AOB ~ △COD (the trapezium pair from Exercise 6.3 Q3), then square the side ratio AB/CD = 2.

1. In △AOB and △COD: ∠OAB = ∠OCD and ∠OBA = ∠ODC (alternate interior angles, AB ∥ DC), and ∠AOB = ∠COD (vertically opposite). So △AOB ~ △COD by AA.
   Why: No area theorem may be used until similarity is established.
2. By the areas theorem: ar(AOB)/ar(COD) = (AB/CD)².
   Why: AB and CD are corresponding sides of this pair.

Exercise 6.4 — Questions 6 to 9

EX 6.4 Q6

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

THINK FIRST: Two-storey proof again: first show the half-triangles (cut by the medians) are similar by SAS, which says medians keep the side ratio; then quote the areas theorem.

1. Let △ABC ~ △DEF with medians AM and DN (M on BC, N on EF). From similarity: AB/DE = BC/EF and ∠B = ∠E ... (1)

   Why: Set up notation: medians land on BC and EF.

2. M and N are mid-points, so BM = BC/2 and EN = EF/2, giving BC/EF = BM/EN. With (1): AB/DE = BM/EN ... (2)

   Why: Halving preserves the ratio, as in Exercise 6.3 Q16.

3. In △ABM and △DEN: AB/DE = BM/EN (by (2)) with included angle ∠B = ∠E. By SAS, △ABM ~ △DEN, so AM/DN = AB/DE ... (3)

   Why: Medians of similar triangles are proportional to the sides — proved exactly like Exercise 6.3 Q16.

4. By the areas theorem, ar(ABC)/ar(DEF) = $(AB/DE)^2$. Replace AB/DE by AM/DN using (3): ar(ABC)/ar(DEF) = **$(AM/DN)^2$. Proved.**

   Why: Any corresponding linear part — side, median, altitude, bisector — can stand inside the square.

ANSWER: ar(ABC)/ar(DEF) = (corresponding median ratio)², proved via △ABM ~ △DEN (SAS) and the areas theorem.

CBSE-STYLE MARKING SCHEME: Median halving [1], SAS on half-triangles [1], median ratio extracted [1], areas theorem substitution [1].

EX 6.4 Q7

Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

THINK FIRST: Both triangles are equilateral, hence similar to each other. Their side ratio is diagonal : side = √2 : 1, and the areas theorem squares it.

1. Let the square have side a. Its diagonal is a√2 (Pythagoras: $\sqrt{(a^2 + a^2)} = a\sqrt{2}$).

   Why: The diagonal computation is itself a one-line Pythagoras.

2.

Exercise 6.5 — Pythagoras Theorem

The routine never changes: find the longest side, square all three, and let the converse of Pythagoras (Derivation 7) deliver the verdict. For proofs, the altitude-on-hypotenuse similarities of Derivation 6 do all the heavy lifting.

EX 6.5 Q1

Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse. (i) 7 cm, 24 cm, 25 cm (ii) 3 cm, 8 cm, 6 cm (iii) 50 cm, 80 cm, 100 cm (iv) 13 cm, 12 cm, 5 cm.

THINK FIRST: In each part: square the longest side; square and add the other two; compare.

1. (i) Longest side 25: $25^2 = 625$. Others: $7^2 + 24^2 = 49 + 576 = 625$. Equal ⇒ **right triangle, hypotenuse 25 cm**.

   Why: 625 = 625 — the converse of Pythagoras is satisfied exactly.

2. (ii) Longest side 8: $8^2 = 64$. Others: $3^2 + 6^2 = 9 + 36 = 45$. $45 \neq 64$ ⇒ **not** a right triangle.

   Why: The trap: 3-6-8 'looks like' a scaled triple but is not one.

3. (iii) Longest side 100: $100^2 = 10000$. Others: $50^2 + 80^2 = 2500 + 6400 = 8900$. $8900 \neq 10000$ ⇒ **not** a right triangle.

   Why: Round numbers are not evidence — only the squares speak.

4. (iv) Longest side 13: $13^2 = 169$. Others: $12^2 + 5^2 = 144 + 25 = 169$. Equal ⇒ **right triangle, hypotenuse 13 cm**.

   Why: The 5-12-13 triple — memorise it along with 3-4-5 and 7-24-25.

ANSWER: (i) Right, hypotenuse 25 cm (ii) Not right (iii) Not right (iv) Right, hypotenuse 13 cm.

CBSE-STYLE MARKING SCHEME: Each part [1]: longest side squared [⅓], sum of squares [⅓], verdict (+ hypotenuse) [⅓].

EX 6.5 Q2

PQR is a triangle right-angled at P and M is a point on QR such that PM ⊥ QR. Show that $PM^2 = QM \cdot MR$.

THINK FIRST: This is the altitude rule of Derivation 6 wearing different letters: show △PMQ ~ △RMP by AA, then cross-multiply.

1.

Exercise 6.5 — Questions 7 to 9

EX 6.5 Q7

Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

THINK FIRST: Use the rhombus's special gift: its diagonals bisect each other at right angles. One small right triangle at the centre contains a quarter of the whole story.

1. Let the rhombus be ABCD with diagonals AC and BD meeting at O. The diagonals of a rhombus bisect each other at right angles, so AO = AC/2, BO = BD/2 and ∠AOB = 90°.
   Why: State the rhombus diagonal property before using it — it is the first mark.
2. In right △AOB, by Pythagoras: $AB^2 = AO^2 + BO^2 = (AC/2)^2 + (BD/2)^2 = (AC^2 + BD^2)/4$.
   Why: One side of the rhombus against the two half-diagonals.
3. Hence $4\,AB^2 = AC^2 + BD^2$ ... (1)
   Why: Clear the fraction.
4. All sides of a rhombus are equal, so $AB^2 + BC^2 + CD^2 + DA^2 = 4\,AB^2$. By (1) this equals **$AC^2 + BD^2$**. **Proved.**
   Why: The sum over four equal sides collapses into the single relation (1).

ANSWER: Sum of squares of sides = $AC^2 + BD^2$ = sum of squares of diagonals.

CBSE-STYLE MARKING SCHEME: Diagonal property stated [1], Pythagoras in △AOB [1], four equal sides summed [1].

EX 6.5 Q8

In the figure, O is a point in the interior of △ABC, with OD ⊥ BC, OE ⊥ CA and OF ⊥ AB. Show that: (i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$ (ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$.

THINK FIRST: Six right triangles meet at O. Write Pythagoras in all six, then add or subtract with intent: (i) uses the three triangles OFA, ODB, OEC; (ii) compares the two triples.

1. In right △OFA (right angle at F): $OA^2 = AF^2 + OF^2$ ... (1). In right △ODB: $OB^2 = BD^2 + OD^2$ ... (2). In right △OEC: $OC^2 = CE^2 + OE^2$ ... (3)
   Why: Each vertex of the big triangle contributes one equation linking OA, OB or OC to a foot segment.
2. Add (1), (2), (3): $OA^2 + OB^2 + OC^2 = AF^2 + BD^2 + CE^2 + OD^2 + OE^2 + OF^2$. Rearranged, this is exactly (i). **Proved.**

Exercise 6.5 — Questions 10 to 13

EX 6.5 Q10

A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire is taut?

THINK FIRST: Same right-triangle skeleton as Q9: wire = hypotenuse 24, pole = leg 18, ground distance unknown. This time the root will not be a whole number — simplify it.

1. Let the stake be x m from the base. Pole ⊥ ground gives a right triangle with hypotenuse 24 m.

   Why: Taut wire = straight line = hypotenuse.

2. By Pythagoras: $x^2 + 18^2 = 24^2$, so $x^2 = 576 - 324 = 252$.

   Why: Leg unknown ⇒ subtract the squares.

3. $x = \sqrt{252} = \sqrt{(36 \times 7)} =$ **$6\sqrt{7}$ m** (about 15.9 m).

   Why: Pull the largest square factor (36) out of the root — board answers must be left in simplest surd form.

ANSWER: The stake should be driven $6\sqrt{7}$ m (approx. 15.9 m) from the base of the pole.

CBSE-STYLE MARKING SCHEME: Right triangle + Pythagoras [1], $x^2 = 252$ [1], surd simplified [1].

EX 6.5 Q11

An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will the two planes be after 1½ hours?

THINK FIRST: North and west are perpendicular, so after 1.5 hours the two flight paths are the legs of a right triangle and the separation is its hypotenuse.

1. Distance flown north = 1000 × 1.5 = 1500 km. Distance flown west = 1200 × 1.5 = 1800 km.

   Why: Distance = speed × time, for each plane separately.

2. North ⊥ west, so by Pythagoras the separation d satisfies $d^2 = 1500^2 + 1800^2$.

   Why: The line joining the planes is the hypotenuse.

3. $d^2 = 2250000 + 3240000 = 5490000$.

   Why: Add the squares.

Exercise 6.5 — Questions 14 to 17

EX 6.5 Q14

The perpendicular from A on side BC of △ABC intersects BC at D such that DB = 3 CD. Prove that $2\,AB^2 = 2\,AC^2 + BC^2$.

THINK FIRST: Write AB^2 and AC^2 from the two right triangles ABD and ACD; their difference kills AD^2. Then express DB and CD as fractions of BC.

1. DB = 3 CD and DB + CD = BC, so 4 CD = BC: CD = BC/4 and DB = 3BC/4 ... (1)
   Why: Convert the ratio condition into fractions of the whole base first.
2. In right △ABD: $AB^2 = AD^2 + DB^2$ ... (2). In right △ACD: $AC^2 = AD^2 + CD^2$ ... (3)
   Why: Pythagoras in each half of the figure.
3. Subtract (3) from (2): $AB^2 - AC^2 = DB^2 - CD^2$ (AD^2 cancels).
   Why: Subtracting is the move whenever the same altitude appears in two equations.
4. Using (1): $DB^2 - CD^2 = (3BC/4)^2 - (BC/4)^2 = (9/16 - 1/16)\,BC^2 = BC^2/2$.
   Why: 9/16 − 1/16 = 8/16 = 1/2.
5. So $AB^2 - AC^2 = BC^2/2$. Multiply by 2: **$2\,AB^2 = 2\,AC^2 + BC^2$. Proved.**
   Why: Multiply through to clear the fraction, as the target demands.

ANSWER: $2\,AB^2 = 2\,AC^2 + BC^2$, by subtracting the two Pythagoras equations and using DB = 3BC/4, CD = BC/4.

CBSE-STYLE MARKING SCHEME: Fractions of BC found [1], both Pythagoras equations [1], subtraction [1], final form [1].

EX 6.5 Q15

In an equilateral triangle ABC, D is a point on side BC such that BD = (1/3) BC. Prove that $9\,AD^2 = 7\,AB^2$.

THINK FIRST: Drop the altitude AE: it lands at the midpoint E, so DE is the small gap between BD = a/3 and BE = a/2. Then Pythagoras in △AED, using the altitude AE = a√3/2 from Q6's result.

1. Let each side be a: AB = BC = CA = a. Then BD = a/3.
   Why: One letter for the side; everything else will be a fraction of it.
2. **Construction:** draw AE ⊥ BC. E is the midpoint of BC, so BE = a/2, and DE = BE − BD = a/2 − a/3 = a/6.

08 Multiple Solution Methods

APPLY — THE SAME ANSWER, BY DIFFERENT ROADS

A topper owns at least two roads to every standard answer. When one road is blocked in the exam hall, the other is already open.

Duel 1 — A BPT length: AD = 3, DB = 6, AE = 5, DE ∥ BC. Find EC.

Method A — BPT directly (part-to-part)

AD/DB = AE/EC ⇒ 3/6 = 5/EC ⇒ EC = 6 × 5/3 = **10 cm**. Two lines. Use this when the question hands you the two pieces of each side.

Method B — Whole-triangle similarity (part-to-whole)

DE ∥ BC makes △ADE ~ △ABC (AA). AB = AD + DB = 9, so AD/AB = 3/9 = 1/3 = AE/AC ⇒ AC = 15, and EC = AC − AE = 15 − 5 = **10 cm**. Use this when the question gives whole sides, or asks for DE/BC as well.

Duel 2 — An area: △ABC ~ △DEF, AB = 6 cm, DE = 8 cm, ar(ABC) = 27 cm². Find ar(DEF).

Method A — Areas theorem

ar(DEF)/ar(ABC) = $(DE/AB)^2$ = $(8/6)^2$ = 16/9. So ar(DEF) = 27 × 16/9 = **48 cm²**.

Method B — Rebuild with base and height

Take AB as base: ar(ABC) = ½ × 6 × h = 27 gives the altitude h = 9 cm. Altitudes scale like sides (Derivation 5), so the altitude of △DEF on DE is 9 × (8/6) = 12 cm. Hence ar(DEF) = ½ × 8 × 12 = **48 cm²**. Slower — but it proves the theorem is honest arithmetic, not magic.

Duel 3 — Proving a parallel: AD = 2, AB = 6, AE = 3, AC = 9. Show DE ∥ BC.

Method A — Converse of BPT

DB = 6 − 2 = 4 and EC = 9 − 3 = 6. AD/DB = 2/4 = 1/2 and AE/EC = 3/6 = 1/2. Equal ⇒ DE ∥ BC by Theorem 6.2.

Method B — SAS similarity, then angles

AD/AB = 2/6 = 1/3 and AE/AC = 3/9 = 1/3, with ∠A common. So △ADE ~ △ABC (SAS), giving ∠ADE = ∠ABC. These are corresponding angles for lines DE and BC with transversal AB — hence DE ∥ BC.

TOPPER TIP: *In the exam, choose Method A roads for speed. Learn Method B roads for the questions where A is blocked — for example when DB is not given but AB is.*

09 Interactive Simulations & Hands-on Activities

APPLY — DO IT WITH YOUR HANDS BEFORE YOU TRUST IT

Activity 1 — The Shadow Lab (similarity in sunlight)

On a sunny day, plant a 1 m stick vertically. Measure its shadow. Then measure the shadow of a tree, a pole or your school building — anything too tall to climb. Record: stick height / stick shadow / object shadow / object height = (object shadow) × (stick height)/(stick shadow). Do it at two different times of day and check that your two answers for the same object nearly agree. *Why it works:* right angle + sun's angle ⇒ AA ⇒ one fixed height-to-shadow ratio (Section 04).

Activity 2 — The BPT Paper Lab

Draw any large triangle ABC. Mark D on AB so that AD : DB = 2 : 3. Through D, draw the line parallel to BC (use set squares), meeting AC at E. Now measure AE and EC with a ruler. You will find AE : EC = 2 : 3, every time, for every triangle. Then reverse it: mark D and E so that AD/DB = AE/EC = 1 : 2 *without* drawing a parallel, join DE, and test DE against BC with set squares — it is parallel. You have just run Theorem 6.1 and Theorem 6.2 on paper.

Activity 3 — The 3-4-5 Rope (converse of Pythagoras)

Tie 13 knots in a rope at equal gaps, making 12 equal pieces. Peg the rope into a triangle of sides 3, 4 and 5 pieces. The angle opposite the 5-side is a perfect right angle — gardeners and masons have laid out rectangular corners this way for thousands of years. *Why it works:* $3^2 + 4^2 = 5^2$, so by the converse of Pythagoras the angle must be 90° (Derivation 7).

Activity 4 — The Enlargement Grid (discovering the square law)

On squared paper, draw a small right triangle with legs 2 and 3 units (area 3 square units). Redraw it with every length doubled (legs 4 and 6, area 12) and tripled (legs 6 and 9, area 27). Measure the angles: unchanged. Compute the areas: 3, 12 = 3 × 4, 27 = 3 × 9. Lengths ×2 and ×3 became areas ×4 and ×9 — you have discovered Derivation 5 with a pencil.

KALVIYOGI DIGITAL COMPANION: the same four activities run as interactive simulations (drag the sun, slide the parallel line, stretch the enlargement) in the KalviYogi app / website chapter pack. The QR code and link activate with the app / website at print. The paper versions above are complete on their own.

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10 Common Mistakes

CHALLENGE — THE SIX WAYS THIS CHAPTER STEALS MARKS

Each mistake below is one real students make in board exams every year. Read the wrong way, feel why it is wrong, and lock in the right way.

COMMON MISTAKE: Wrong way: From ∠A = ∠D, ∠B = ∠E, writing "△ABC ~ △DEF, so AB/EF = BC/DF" with mismatched pairs.
Why wrong: The symbolic form is a contract: position 1 matches position 1. If A matches D and B matches E, then C matches F, and the correct statement is △ABC ~ △DEF only when the correspondence really is A↔D, B↔E, C↔F — here it is, but then AB corresponds to DE, not EF. A careless order poisons every ratio that follows.
Right way: Before writing any ratio, write the angle matches in a row (A↔D, B↔E, C↔F), build the similarity statement from them, and read ratios only from matching positions.

COMMON MISTAKE: Wrong way: Claiming SAS similarity because two side-ratios are equal and *some* angle is equal — the trap of Exercise 6.3 Q1(v).
Why wrong: SAS demands the angle *between* the two proportional sides. An equal angle elsewhere proves nothing; non-included-angle data can fit two genuinely different triangles.
Right way: For SAS, physically point at the angle and check that the two proportional sides are the ones forming it.

COMMON MISTAKE: Wrong way: "Sides are in ratio 2 : 1, so areas are in ratio 2 : 1."
Why wrong: Areas scale with the SQUARE of the side ratio (Derivation 5). Ratio 2 : 1 in sides means 4 : 1 in areas — this exact slip is the planted trap in Exercise 6.4 Q8.
Right way: The moment the word 'area' appears with similar triangles, square the ratio before doing anything else.

COMMON MISTAKE: Wrong way: Using BPT in a figure where the line is not stated or shown to be parallel — or using Pythagoras in a triangle never shown to be right-angled.
Why wrong: Every theorem has a licence condition. BPT's licence is the parallel; Pythagoras's licence is the right angle. No licence, no theorem.
Right way: In your first step, write the licence out loud: "DE ∥ BC (given), so by BPT..." or "∠B = 90°, so by Pythagoras...". If you cannot write it, prove it first (often by the converse).

COMMON MISTAKE: Wrong way: In a right triangle with legs 6 and 8 and hypotenuse 10, writing $6^2 = 8^2 + 10^2$ or hunting for 'the hypotenuse' as whichever side is drawn slanted.
Why wrong: The hypotenuse is defined by position, not by looks: it is the side opposite the right angle, and it is always the longest side.
Right way: Find the right angle first; the side facing it is the hypotenuse. Sanity check: it must be longer than each leg.

COMMON MISTAKE: Wrong way: Mixing the two BPT forms: writing AD/AB = AE/EC when the data gives pieces (AD, DB, AE), or AD/DB = AE/AC.

11 Graded Practice Problems

FIVE LEVELS — CLIMB ONE LEVEL AT A TIME • ANSWERS IN THE ANSWER KEY

Rules of the climb: finish a level with at most one error before moving up. From Level 2 onwards, full written steps — in the board exam an unsupported answer earns nothing.

LEVEL 1 — FOUNDATION (direct concepts)

L1.1 In △ABC, DE ∥ BC with D on AB and E on AC. If AD = 2 cm, DB = 4 cm and AE = 3 cm, find EC.

L1.2 State whether the triangles with sides 4 cm, 6 cm, 8 cm and 6 cm, 9 cm, 12 cm are similar. Name the criterion.

L1.3 △ABC ~ △PQR. If AB = 6 cm, PQ = 9 cm and BC = 8 cm, find QR.

L1.4 The corresponding sides of two similar triangles are in the ratio 3 : 5. Find the ratio of their areas.

L1.5 Determine whether a triangle with sides 9 cm, 12 cm and 15 cm is right-angled. If yes, write its hypotenuse.

L1.6 In △ABC, DE ∥ BC and AD/AB = 1/3. If DE = 5 cm, find BC.

LEVEL 2 — SKILL BUILDER (textbook standard)

L2.1 In △ABC, D and E are points on AB and AC with AD = 3 cm, DB = 6 cm, AE = 4.5 cm and EC = 9 cm. Is DE ∥ BC? Justify your answer.

L2.2 △ABC ~ △DEF and ar(ABC) = 81 cm^2, ar(DEF) = 49 cm^2. If AB = 13.5 cm, find DE.

L2.3 A ladder 25 m long reaches a window 24 m above the ground. Find the distance of the foot of the ladder from the wall.

L2.4 In △ABC, ∠A = 50° and ∠B = 70°. In △PQR, ∠Q = 70° and ∠R = 60°. Are the triangles similar? If yes, write the similarity in symbolic form.

L2.5 Two poles of heights 8 m and 14 m stand on level ground, with their feet 8 m apart. Find the distance between their tops.

L2.6 Find the altitude of an equilateral triangle of side 10 cm.

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Graded Practice — Levels 3, 4 and 5

LEVEL 3 — APPLICATION (real-life problems)

L3.1 A girl 1.5 m tall casts a shadow 3 m long. At the same time, a tree casts a shadow 24 m long. Find the height of the tree.

L3.2 A student 1.6 m tall uses a mirror lying flat on the ground to measure a building. She stands 4 m from the mirror, and the mirror is 20 m from the building's base, positioned so that she sees the top of the building in the mirror (the angles of incidence and reflection are equal). Find the height of the building.

L3.3 A guy wire 20 m long is attached to the top of a vertical pole and staked to the ground 12 m from the pole's base. Find the height of the pole.

L3.4 A map is drawn to the scale 1 : 1000. A triangular park measures 6 cm^2 in area on the map. Find the actual area of the park in square metres.

LEVEL 4 — ADVANCED REASONING (multi-concept)

L4.1 In $\triangle ABC$, DE ∥ BC with AD : DB = 3 : 2. If ar($\triangle ADE$) = 54 cm^2, find ar($\triangle ABC$) and the area of trapezium DBCE.

L4.2 $\triangle ABC$ is right-angled at A with AB = 6 cm, AC = 8 cm and BC = 10 cm. AD is drawn perpendicular to BC. Find AD, BD and DC.

L4.3 $\triangle ABC \sim \triangle PQR$ with corresponding sides in the ratio 2 : 3. The perimeter of $\triangle ABC$ is 36 cm and its area is 48 cm^2. Find the perimeter and the area of $\triangle PQR$.

LEVEL 5 — MATHEMATICAL GENIUS (olympiad flavour)

L5.1 In $\triangle ABC$, DE ∥ BC. The area of trapezium DBCE is 7 times the area of $\triangle ADE$, and DE = 6 cm. Find BC.

L5.2 In $\triangle ABC$, AB = 5 cm, AC = 7 cm and BC = 8 cm. Find the length of the median drawn from A to BC. (Hint: coordinates, or Apollonius's relation $2AB^2$... first find a route of your own.)

> **TOPPER TIP:** *Level 5 is not about speed. L5.1 hides a square root inside an area ratio; L5.2 rewards any honest method — coordinate geometry from Chapter 7 works beautifully.*

12 Mathematical Reasoning

CHALLENGE — THREE PROOFS EVERY TOPPER SHOULD OWN

REASONING 1 — The Angle Bisector Theorem

The bisector of ∠A of △ABC meets BC at D. Prove that BD/DC = AB/AC. (This theorem is NOT in your NCERT exercise list — it is the most useful 'extra' of the chapter.)

THINK FIRST: Bisectors suggest equal angles; equal angles suggest an isosceles triangle; a parallel line lets BPT carry the ratio across.

1. **Construction:** through C, draw CE ∥ DA, meeting BA produced at E.
   Why: Manufacture a parallel so that BPT can enter; the bisector's equal angles will travel along it.
2. ∠BAD = ∠AEC (corresponding angles, CE ∥ DA, transversal BE) and ∠DAC = ∠ACE (alternate interior angles, transversal AC).
   Why: The two halves of ∠A reappear inside △AEC.
3. But ∠BAD = ∠DAC (AD bisects ∠A). Hence ∠AEC = ∠ACE, so △AEC is isosceles with AE = AC … (1)
   Why: Equal angles ⇒ equal opposite sides.
4. In △BEC, DA ∥ EC, so by BPT: BD/DC = AB/AE … (2)
   Why: D divides BC; A divides BE; the parallel DA links them.
5. Replace AE by AC using (1): **BD/DC = AB/AC. Proved.**
   Why: The bisector divides the opposite side in the ratio of the adjacent sides.

ANSWER: BD/DC = AB/AC — the internal bisector of an angle divides the opposite side in the ratio of the sides forming the angle.

REASONING 2 — The Median to the Hypotenuse

In △ABC, right-angled at A, D is the mid-point of the hypotenuse BC. Prove that AD = BD = DC = BC/2.

THINK FIRST: The claim smells like a rectangle hiding inside the triangle. Doubling the median builds it.

1. **Construction:** extend AD to E so that AD = DE, and join BE and CE.
   Why: Double the median — the same construction as Exercise 6.3 Q14.
2. The diagonals of quadrilateral ABEC bisect each other at D (BD = DC since D is the mid-point of BC, and AD = DE by construction). So ABEC is a parallelogram.
   Why: A quadrilateral whose diagonals bisect each other is a parallelogram.

13 Real-Life Applications

APPLY — FOUR JOBS, FOUR THEOREMS

APPLICATION 1 — The Building's Height (shadows)

At 4 p.m., a 2 m stick planted vertically casts a shadow 1.6 m long. At the same moment, a building casts a shadow 24 m long. Find the height of the building.

THINK FIRST: Any time two shadows are measured at the same moment, the sun has built two similar triangles for you.

1. Both objects stand vertically, so both triangles are right-angled at the ground; the sun's angle is common. The triangles are similar by AA.

   Why: Same sunlight, same shape — Section 04's picture with new numbers.

2. Height/shadow is therefore one ratio: h/24 = 2/1.6.

   Why: Corresponding sides in proportion.

3. h = 24 × 2/1.6 = 48/1.6 = **30 m**.

   Why: Divide 48 by 1.6: multiply top and bottom by 10, 480/16 = 30.

ANSWER: The building is 30 m tall.

APPLICATION 2 — The Width of a River (sighting triangles)

A surveyor must measure the width of a river without crossing it. A tree T stands on the far bank, directly opposite point A on her bank. She walks 40 m along the bank to B, then 10 m further to C. From C she walks directly inland (away from the river, perpendicular to the bank) to a point D from which the tree T, the point B and she herself are in one straight line of sight. She measures CD = 8 m. Find the width of the river.

THINK FIRST: When you cannot measure a length, build a similar triangle on land you CAN measure, and let the proportion carry the number across.

1. Let the width TA be w. The two right triangles are △TAB (right-angled at A) and △DCB (right-angled at C, since she walked perpendicular to the bank).

   Why: Name the unknown and locate the right angles first.

2. ∠ABT = ∠CBD: they are the angles the same sighting line TB makes with the bank line on its two sides (vertically opposite / alternate angles along the straight bank).

   Why: The single line of sight T–B–D is what locks the two triangles together.

3. So △TAB ~ △DCB by AA, giving TA/CB = AB/CD, i.e. w/10 = 40/8.

14 Competency-Based Questions

CHALLENGE — FIVE CASE STUDIES, TWENTY QUESTIONS, ALL SOLVED

Case-based questions are the board's favourite modern format: one real situation, four linked questions. Read the situation once, carefully — all four answers live inside it.

Case Study 1 — The School Garden

A triangular school garden ABC is divided by a straight path DE parallel to the side BC, with D on AB and E on AC. AD = 4 m, DB = 6 m and AE = 6 m.

CS 1.1

Find EC.

1. By BPT: AD/DB = AE/EC ⇒ 4/6 = 6/EC.
   Why: Parallel path ⇒ proportional pieces.
2. EC = 6 × 6/4 = **9 m**.
   Why: Cross-multiply.

ANSWER: EC = 9 m.

CS 1.2

Find the full length AC.

1. AC = AE + EC = 6 + 9 = **15 m**.
   Why: The whole is the sum of its pieces — the step students most often forget to write.

ANSWER: AC = 15 m.

CS 1.3

If DE = 5 m, find BC.

1. △ADE ~ △ABC (AA, DE ∥ BC), so DE/BC = AD/AB = 4/(4 + 6) = 2/5.
   Why: Part-to-whole form — AB = AD + DB = 10 m.
2. BC = 5 × 5/2 = **12.5 m**.
   Why: Scale the small triangle up by 5/2.

ANSWER: BC = 12.5 m.

CS 1.4

Case Studies 3, 4 and 5

Case Study 3 — The Kite Festival

At a kite festival, Ravi's kite flies on a taut string 41 m long. The kite is 40 m vertically above his hand.

CS 3.1

How far is the kite's position on the ground (the point directly below it) from Ravi, horizontally?

1 String, height and ground distance form a right triangle with hypotenuse 41 m. Ground distance = $\sqrt{(41^2 - 40^2)} = \sqrt{(1681 - 1600)} = \sqrt{81} =$ **9 m**.

Why: The unknown is a leg — subtract the squares.

ANSWER: 9 m.

CS 3.2

Meena's kite has a 25 m string and flies 24 m above her hand. Find her kite's horizontal distance.

1 $\sqrt{(25^2 - 24^2)} = \sqrt{(625 - 576)} = \sqrt{49} =$ **7 m**.

Why: The 7-24-25 triple.

ANSWER: 7 m.

CS 3.3

Are the two kite triangles similar? Justify.

1 Ravi's triangle has sides 9, 40, 41; Meena's has 7, 24, 25. Ratios: 9/7 ≠ 40/24 (1.29 vs 1.67). The side ratios are not all equal, so the triangles are **not similar**.

Why: Right-angled alone does not mean similar — the acute angles differ.

ANSWER: No — the side ratios differ.

CS 3.4

Verify that 9, 40, 41 can be the sides of a right triangle.

1 $9^2 + 40^2 = 81 + 1600 = 1681 = 41^2$. By the converse of Pythagoras, yes — **a right triangle** with hypotenuse 41.

15 Olympiad Challenges

CHALLENGE — TEN PROBLEMS BEYOND THE TEXTBOOK, FULLY SOLVED

OLYMPIAD 1

In △ABC, DE ∥ BC with AD : DB = 2 : 3. If ar(△ADE) = 36 cm², find ar(△ABC).

1. AB = AD + DB, so AD/AB = 2/(2 + 3) = 2/5.
   Why: Convert the part-to-part ratio into part-to-whole before using areas.
2. △ADE ~ △ABC (AA), so ar(ADE)/ar(ABC) = $(2/5)^2$ = 4/25.
   Why: Areas theorem.
3. ar(ABC) = 36 × 25/4 = **225 cm²**.
   Why: Scale up by the reciprocal 25/4.

ANSWER: ar(△ABC) = 225 cm².

OLYMPIAD 2

In a right triangle, the altitude drawn to the hypotenuse divides the hypotenuse into segments of 4 cm and 9 cm. Find the altitude, and the two legs.

1. By the altitude rule (Derivation 6): $AD^2 = BD \times DC = 4 \times 9 = 36$, so AD = **6 cm**.
   Why: The altitude is the geometric mean of the two segments.
2. Leg rules: $AB^2 = BD \times BC = 4 \times 13 = 52 \Rightarrow AB = 2\sqrt{13}$ cm. $AC^2 = DC \times BC = 9 \times 13 = 117 \Rightarrow AC = 3\sqrt{13}$ cm.
   Why: Each leg is the geometric mean of the hypotenuse and its own segment.
3. Check by Pythagoras: $52 + 117 = 169 = 13^2$, exactly as required.
   Why: Always close a surd answer with a numeric check.

ANSWER: Altitude = 6 cm; legs = 2√13 cm and 3√13 cm.

OLYMPIAD 3

In △ABC, AB = 8 cm, AC = 6 cm and BC = 7 cm. The bisector of ∠A meets BC at D. Find BD and DC.

1. By the Angle Bisector Theorem (Reasoning 1): BD/DC = AB/AC = 8/6 = 4/3.
   Why: The bisector splits the opposite side in the ratio of the adjacent sides.
2. BD + DC = 7 cm, split in the ratio 4 : 3: BD = 7 × 4/7 = **4 cm**, DC = 7 × 3/7 = **3 cm**.
   Why: Divide the whole in the ratio — total parts 4 + 3 = 7, a gift of numbers.

ANSWER: BD = 4 cm and DC = 3 cm.

OLYMPIAD 4

Three parallel lines are cut by two transversals. On the first transversal the intercepts are AB = 3 cm and BC = 4.5 cm; on the second, the first intercept is DE = 5 cm. Find the second intercept EF.

1. Three parallel lines divide any two transversals proportionally (BPT applied twice, through the middle line): AB/BC = DE/EF.

   Why: Each pair of parallels creates a BPT triangle with the intersection point of the transversals... if the transversals are parallel too, the intercepts are simply equal — here they are not parallel, so proportions rule.

2. 3/4.5 = 5/EF ⇒ EF = 5 × 4.5/3 = **7.5 cm**.

   Why: Cross-multiply.

ANSWER: EF = 7.5 cm.

OLYMPIAD 5

An isosceles triangle has sides 10 cm, 10 cm and 12 cm. D is a point on the base (the 12 cm side) with BD = 3 cm, where B is one base corner. Find AD^2.

1. Let the equal sides meet at A, base BC = 12. Drop the altitude AM: M is the mid-point of BC, so BM = 6 and $AM^2 = AB^2 - BM^2 = 100 - 36 = 64$.

   Why: In an isosceles triangle the altitude to the base bisects the base.

2. DM = BM − BD = 6 − 3 = 3 cm.

   Why: Locate D relative to the altitude foot.

3. In right △AMD: $AD^2 = AM^2 + DM^2 = 64 + 9 =$ **73** (so AD = √73 ≈ 8.54 cm).

   Why: Pythagoras finishes what the altitude started.

ANSWER: $AD^2 = 73$, i.e. AD = √73 cm.

OLYMPIAD 6

In △ABC, AB = 7 cm, AC = 9 cm and BC = 10 cm. Find the length of the median from A to the mid-point of BC.

1. Use Apollonius's relation (proved in Olympiad 10): $AB^2 + AC^2 = 2\,AD^2 + BC^2/2$, where D is the mid-point of BC.

   Why: The median is the one length Pythagoras cannot reach directly — Apollonius is the tool built for it.

2. $49 + 81 = 2\,AD^2 + 100/2 \Rightarrow 130 = 2\,AD^2 + 50 \Rightarrow 2\,AD^2 = 80 \Rightarrow AD^2 = 40$.

   Why: Substitute and solve.

Olympiad Challenges — 8 to 10

OLYMPIAD 8

D is a point on side BC of △ABC such that BD : DC = 2 : 1. Find ar(△ABD) : ar(△ADC), and explain why no similarity is involved.

1. Both triangles share the same vertex A and their bases BD, DC lie on the same line BC, so they have the SAME altitude from A.

   Why: Areas with a common height compare by bases alone.

2. ar(ABD)/ar(ADC) = (½ × BD × h)/(½ × DC × h) = BD/DC = **2 : 1**.

   Why: The ½ and h cancel. This is the 'same-altitude' area rule — the engine inside the BPT proof of Section 06.

ANSWER: 2 : 1 — triangles with the same altitude have areas in the ratio of their bases.

OLYMPIAD 9

A triangular plot has sides 20 m, 21 m and 29 m. Show that the plot is right-angled, and find its area.

1. $20^2 + 21^2 = 400 + 441 = 841 = 29^2$. By the converse of Pythagoras, the plot is right-angled, with hypotenuse 29 m.

   Why: Test the largest side as the hypotenuse candidate.

2. The legs 20 m and 21 m are perpendicular, so area = ½ × 20 × 21 = **210 m²**.

   Why: In a right triangle, the two legs serve as base and height — no altitude construction needed.

ANSWER: Right-angled ($20^2 + 21^2 = 29^2$); area = 210 m².

OLYMPIAD 10 — Apollonius's Theorem

In △ABC, D is the mid-point of BC. Prove that $AB^2 + AC^2 = 2AD^2 + BC^2/2$.

1. Place BC on a number line: B at 0, D at a, C at 2a (so BD = DC = a and BC = 2a). Let the foot of the perpendicular from A to BC be H at position h, and let AH = p.

   Why: Coordinates turn a geometry claim into algebra — a legitimate and powerful proof style.

2. By Pythagoras three times: $AB^2 = p^2 + h^2$; $AC^2 = p^2 + (2a - h)^2$; $AD^2 = p^2 + (h - a)^2$.

   Why: Each length is a hypotenuse in its own right triangle with height p.

3. Add the first two: $AB^2 + AC^2 = 2p^2 + h^2 + (2a - h)^2 = 2p^2 + 2h^2 - 4ah + 4a^2$.

16 Mind Map

RECALL — THE WHOLE CHAPTER ON ONE PAGE

TRIANGLES — SIMILARITY & PYTHAGORAS

SIMILARITY
Same shape, any size
AA • SSS • SAS criteria
Correspondence order is law
Congruent = ratio 1 case

BASIC PROPORTIONALITY
Thm 6.1: DE ∥ BC ⇒ AD/DB = AE/EC
Thm 6.2 (converse): equal ratios ⇒ parallel
Mid-points ⇒ DE = BC/2

AREAS
Ratio of areas = (ratio of sides)^2
Same altitude ⇒ areas ∝ bases
Perimeters, altitudes, medians scale with k

PYTHAGORAS
$a^2 + b^2 = c^2$ (c = hypotenuse)
Converse tests right angles
Triples: 3-4-5, 5-12-13, 7-24-25, 8-15-17

ALTITUDE RULES
Right △, altitude to hypotenuse:
$AD^2 = BD \cdot DC$
$AB^2 = BD \cdot BC$ • $AC^2 = CD \cdot BC$

APPLICATIONS
Shadows & heights (AA)
Maps & scale (areas square!)
Ladders, poles, wires (Pythagoras)
Median to hypotenuse = BC/2

Cover this page and redraw the map from memory. If a box comes out blank, that box is tonight's revision target.

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17 Pin-Point Flash Cards

RECALL — CUT ALONG THE DASHED LINES • QUESTION ON THE FRONT, ANSWER ON THE BACK

PIN POINT — BPT: DE ∥ BC in △ABC. What equals what?
AD/DB = AE/EC — and also AD/AB = AE/AC. Pieces with pieces, or pieces with wholes.
PIN POINT — How do you PROVE a line is parallel to BC?
Show AD/DB = AE/EC. Then the converse of BPT (Thm 6.2) gives DE ∥ BC.
PIN POINT — AA criterion in one line?
Two angles of one triangle equal two angles of the other ⇒ similar. The third pair follows from the angle sum.
PIN POINT — SSS similarity in one line?
All three ratios of corresponding sides equal ⇒ similar (and then the angles are equal too).
PIN POINT — SAS similarity — what is the trap?
The equal angle must be the INCLUDED angle, between the two proportional sides. Any other angle proves nothing.
PIN POINT — △ABC ~ △PQR actually means what?
A↔P, B↔Q, C↔R. Ratios are read only between matching positions — order is law.
PIN POINT — Side ratio k. Area ratio?
k^2. Always square it. (Sides 2:1 ⇒ areas 4:1.)
PIN POINT — Side ratio k. Perimeter, altitude, median ratios?
All equal k — they are lengths. Only area squares the ratio.
PIN POINT — Pythagoras, stated safely?
In a RIGHT triangle: $(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$. The hypotenuse is opposite the right angle and is the longest side.
PIN POINT — $a^2 + b^2 = c^2$ holds for a triangle's sides. Conclusion?
By the converse of Pythagoras, the triangle is right-angled — the right angle is opposite the side c.
PIN POINT — Pythagorean triples to know cold?
3-4-5 • 5-12-13 • 7-24-25 • 8-15-17 • 20-21-29 — and every multiple of each.
PIN POINT — Altitude to the hypotenuse: the three rules?
$AD^2 = BD \cdot DC$ • $AB^2 = BD \cdot BC$ • $AC^2 = CD \cdot BC$.
PIN POINT — Two objects, same sunlight. What is equal?
height ÷ shadow length — the same ratio for both, by AA similarity.
PIN POINT — Altitude of an equilateral triangle of side a?
$a\sqrt{3}/2$. Equivalently, $3a^2 = 4 \times (\text{altitude})^2$.
PIN POINT — DE joins the mid-points of AB and AC. Two facts?
DE ∥ BC and DE = BC/2 — BPT's converse plus similarity with ratio 1/2.
PIN POINT — Median drawn to the hypotenuse equals?
Half the hypotenuse. The mid-point of the hypotenuse is equidistant from all three vertices.

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18 Active Recall

RECALL — CLOSE THE BOOK, ANSWER ALOUD, THEN CHECK THE KEY

Recall is the workout; reading is only the warm-up. Say each answer out loud (or write it) BEFORE checking. Answers are in the Answer Key.

Rapid Fire — 10 questions, 60 seconds each

R1 State the Basic Proportionality Theorem.

R2 In $\triangle ABC$, DE ∥ BC, AD = 5 cm, DB = 10 cm, AE = 4 cm. EC = ?

R3 Which criterion: all three pairs of corresponding sides proportional?

R4 Two similar triangles have areas 25 cm^2 and 100 cm^2. The smaller triangle has a side of 3 cm. Find the corresponding side of the larger.

R5 A right triangle has legs 5 cm and 12 cm. Hypotenuse = ?

R6 Is a triangle with sides 8, 15, 17 right-angled? Which side would be the hypotenuse?

R7 $\triangle ABC \sim \triangle DEF$ with $\angle A = 70°$ and $\angle B = 60°$. Find $\angle F$.

R8 In right $\triangle PQR$ ($\angle Q = 90°$), the altitude from Q to PR divides PR into 9 cm and 16 cm. Find the altitude.

R9 Corresponding sides of two similar triangles are in ratio 4 : 7. Ratio of perimeters = ? Ratio of areas = ?

R10 State the converse of the Basic Proportionality Theorem.

Say It In Your Own Words

W1 Explain to a Class 9 student why measuring a shadow can give you a building's height.

W2 Why must area ratios be squared while perimeter ratios are not?

W3 Your friend says: 'Two sides proportional and one angle equal — the triangles must be similar.' When is your friend right, and when wrong?

W4 Why does the 3-4-5 rope trick guarantee a right angle? Name the theorem.

W5 Explain the difference between congruent and similar using one example of each from daily life.

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19 Self-Assessment & Mastery Tracker

RECALL — THE SMART MEMORY SYSTEM FOR THIS CHAPTER

The Smart Memory Schedule

Similarity fades fast if you only read it. Follow the KalviYogi recall rhythm — short, spaced, and always from memory first.

Day	Your Mission	Done?
Day 0 (today)	Finish the chapter. Redo 3 NCERT questions you found hardest, with full steps.	☐
Day 1	Flash cards 1–16 from memory. Redraw the Mind Map blank-page style.	☐
Day 3	Re-derive BPT and the areas theorem aloud, start to finish, without the book.	☐
Day 7	Attempt the **Day 7 Mini-Test** (Annex). Target: 8/10 or more.	☐
Day 14	Redo every question in your Error Tracker below, from a blank page.	☐
Day 21	Attempt the **Day 21 Mini-Test** (Annex). Target: 9/10 or more.	☐
Day 30	Attempt the **Day 30 Mini-Test** (Annex) + one Model Test under exam timing.	☐

My Error Tracker

Every error you make is data. Log it here the same day — an error written down rarely repeats.

Question	What went wrong	Mistake type	Fixed on (date)
		Concept / Calculation / Steps / Licence	
		Concept / Calculation / Steps / Licence	
		Concept / Calculation / Steps / Licence	
		Concept / Calculation / Steps / Licence	
		Concept / Calculation / Steps / Licence	

Theorem & Formula Recall Tracker

Result	Can state it	Can prove it	Can apply it
Basic Proportionality Theorem + converse	☐	☐	☐
AA / SSS / SAS criteria	☐	☐	☐
Areas of similar triangles (square law)	☐	☐	☐

Mastery Tracker — Score Yourself Honestly

Concept Mastery Score

Tick a box only when you can do it from a blank page, without peeking. Your honest score today is your study plan for tomorrow.

- ☐ I can state and prove BPT, and use its converse to prove lines parallel.
- ☐ I write similarity statements in the correct correspondence order, every time.
- ☐ I can choose correctly between AA, SSS and SAS — and I never fall for the non-included-angle trap.
- ☐ I square the ratio for areas automatically, and never for perimeters, altitudes or medians.
- ☐ I can prove the areas-of-similar-triangles theorem.
- ☐ I apply Pythagoras only in right triangles, with the hypotenuse opposite the right angle.
- ☐ I use the converse of Pythagoras to test right angles, and I know the standard triples.
- ☐ I know the altitude-to-hypotenuse rules ($AD^2 = BD \cdot DC$ and friends).
- ☐ I can solve shadow, map-scale, ladder and pole problems by first naming the theorem.
- ☐ I write every solution step by step, with a reason for each step.

Score guide: 9–10 ticks: chapter mastered — move on, keep the Day 7/21/30 rhythm. 6–8: redo the unticked topics from Sections 06–07. 3–5: rebuild from Section 05 with the flash cards beside you. 0–2: restart from the Story (Section 03) — the picture first, the symbols after.

Learning Time Tracker

Date	Section / Test	Minutes	Score or Result

TOPPER TIP: The tracker is not homework for your teacher — it is a mirror for you. Ten honest minutes with it saves ten hours of blind re-reading before the board exam.

Practice Plus — 25 Concept MCQs (1–13)

One mark each. Choose the single best option, then check the Answer Key — the tricky ones carry a one-line explanation there.

Q1. In △ABC, DE ∥ BC with AD = 1.5 cm, DB = 3 cm and AE = 1 cm. Then EC is
(a) 1 cm (b) 2 cm (c) 3 cm (d) 4.5 cm

Q2. Which of the following pairs of figures is ALWAYS similar?
(a) All isosceles triangles (b) All equilateral triangles (c) All rectangles (d) All rhombuses

Q3. △ABC ~ △DEF with AB/DE = 3/4. The ratio ar(ABC) : ar(DEF) is
(a) 3 : 4 (b) 4 : 3 (c) 9 : 16 (d) 16 : 9

Q4. A right triangle has hypotenuse 13 cm and one leg 5 cm. The other leg is
(a) 8 cm (b) 12 cm (c) √194 cm (d) 18 cm

Q5. Which set of lengths forms a right triangle?
(a) 4, 5, 6 (b) 6, 8, 10 (c) 5, 6, 7 (d) 2, 3, 4

Q6. △ABC ~ △PQR with ∠A = 55° and ∠C = 65°. Then ∠Q is
(a) 55° (b) 60° (c) 65° (d) 70°

Q7. In △ABC, D and E are the mid-points of AB and AC. If DE = 6 cm, then BC is
(a) 3 cm (b) 6 cm (c) 9 cm (d) 12 cm

Q8. A 6 m pole casts a 4 m shadow. At the same time, a tower's shadow is 10 m. The tower is
(a) 12 m (b) 15 m (c) 18 m (d) 20 m

Q9. The areas of two similar triangles are in the ratio 49 : 81. The ratio of their corresponding sides is
(a) 49 : 81 (b) 7 : 9 (c) 9 : 7 (d) √7 : √9

Q10. The altitude of an equilateral triangle of side 6 cm is
(a) 3 cm (b) 3√3 cm (c) 6√3 cm (d) √3 cm

Q11. A ladder 10 m long leans against a wall with its foot 6 m from the wall. It reaches a height of
(a) 4 m (b) 8 m (c) √136 m (d) 16 m

Q12. In △PQR, right-angled at Q, PQ = 3 cm and QR = 4 cm. Then PR is
(a) 5 cm (b) 7 cm (c) √7 cm (d) 25 cm

Q13. Which of the following does NOT prove that two triangles are similar?
(a) Two pairs of equal angles (b) Three pairs of proportional sides (c) Two pairs of proportional sides with the included angle equal (d) Two pairs of proportional sides with a non-included angle equal

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Practice Plus — 25 Concept MCQs (14–25)

Q14. In △ABC, DE ∥ BC and AD/AB = 2/5. If BC = 15 cm, then DE is
(a) 4 cm (b) 6 cm (c) 7.5 cm (d) 10 cm

Q15. A map has scale 1 : 5000. A distance of 2 cm on the map is, in reality,
(a) 10 m (b) 100 m (c) 1000 m (d) 10 km

Q16. △ABC ~ △DEF, ar(ABC) = 36 cm² and ar(DEF) = 64 cm². If AB = 6 cm, then DE is
(a) 8 cm (b) 9 cm (c) 10.67 cm (d) 4.5 cm

Q17. A right triangle has hypotenuse 25 cm and one leg 7 cm. The other leg is
(a) 18 cm (b) 24 cm (c) √674 cm (d) 32 cm

Q18. △ABC ~ △PQR with AB : PQ = 5 : 3. If the perimeter of △PQR is 45 cm, the perimeter of △ABC is
(a) 27 cm (b) 75 cm (c) 15 cm (d) 135 cm

Q19. Two poles of heights 6 m and 14 m stand with their feet 6 m apart. The distance between their tops is
(a) 8 m (b) 10 m (c) √180 m (d) 14 m

Q20. The converse of Pythagoras' theorem is used to
(a) find the hypotenuse of a right triangle (b) test whether a triangle is right-angled (c) prove two triangles similar (d) find the area of a triangle

Q21. In a right triangle, the altitude to the hypotenuse divides it into segments of 2 cm and 8 cm. The altitude is
(a) 4 cm (b) 5 cm (c) 10 cm (d) 16 cm

Q22. If △ABC ~ △ZYX, then vertex B corresponds to vertex
(a) Z (b) Y (c) X (d) none of these

Q23. A guy wire runs from the top of a 12 m pole to a stake 5 m from its base. The wire's length is
(a) 13 m (b) 17 m (c) √119 m (d) 7 m

Q24. In △ABC, DE ∥ BC with AD : DB = 3 : 1. Then ar(△ADE) : ar(△ABC) is
(a) 3 : 4 (b) 9 : 16 (c) 3 : 1 (d) 9 : 1

Q25. A triangle with sides 1.5 cm, 2 cm and 2.5 cm is
(a) acute-angled (b) obtuse-angled (c) right-angled (d) not possible

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Assertion – Reason Questions (1–5)

For each question, choose (a), (b), (c) or (d) using the standard code printed with every question. Answers are in the Answer Key.

A–R 1

Assertion (A): If a line DE is drawn parallel to side BC of △ABC, then AD/DB = AE/EC.
Reason (R): A line drawn parallel to one side of a triangle divides the other two sides in the same ratio (Basic Proportionality Theorem).
(a) Both A and R are true and R is the correct explanation of A (b) Both A and R are true but R is NOT the correct explanation of A (c) A is true but R is false (d) A is false but R is true

A–R 2

Assertion (A): A triangle with sides 6 cm, 8 cm and 10 cm is right-angled.
Reason (R): $6^2 + 8^2 = 36 + 64 = 100 = 10^2$.
(a) Both A and R are true and R is the correct explanation of A (b) Both A and R are true but R is NOT the correct explanation of A (c) A is true but R is false (d) A is false but R is true

A–R 3

Assertion (A): Two triangles are similar whenever two pairs of sides are proportional and one pair of angles is equal.
Reason (R): The SAS similarity criterion requires the equal angle to be the angle included between the proportional sides.
(a) Both A and R are true and R is the correct explanation of A (b) Both A and R are true but R is NOT the correct explanation of A (c) A is true but R is false (d) A is false but R is true

A–R 4

Assertion (A): A rope marked in lengths 3, 4 and 5 units, folded into a triangle, always forms a right angle.
Reason (R): Knotted ropes were used in ancient times to lay out fields and foundations.
(a) Both A and R are true and R is the correct explanation of A (b) Both A and R are true but R is NOT the correct explanation of A (c) A is true but R is false (d) A is false but R is true

A–R 5

Assertion (A): If in △ABC, $AB^2 + AC^2 = BC^2$, then ∠A = 90°.
Reason (R): In any right triangle, the hypotenuse is the shortest side.

Assertion – Reason Questions (6–10)

A–R 6

Assertion (A): If D and E are the mid-points of AB and AC of △ABC, then DE = BC/2.
Reason (R): Equal division ratios give DE ∥ BC by the converse of BPT, and then △ADE ~ △ABC with ratio 1/2, so DE/BC = 1/2.
(a) Both A and R are true and R is the correct explanation of A (b) Both A and R are true but R is NOT the correct explanation of A (c) A is true but R is false (d) A is false but R is true

A–R 7

Assertion (A): If △ABC ~ △PQR, then AB/QR = AC/PR.
Reason (R): Corresponding sides of similar triangles are in the same ratio.
(a) Both A and R are true and R is the correct explanation of A (b) Both A and R are true but R is NOT the correct explanation of A (c) A is true but R is false (d) A is false but R is true

A–R 8

Assertion (A): The altitude of an equilateral triangle of side a is $a\sqrt{3}/2$.
Reason (R): By Pythagoras in half the triangle, $(\text{altitude})^2 = a^2 - (a/2)^2 = 3a^2/4$.
(a) Both A and R are true and R is the correct explanation of A (b) Both A and R are true but R is NOT the correct explanation of A (c) A is true but R is false (d) A is false but R is true

A–R 9

Assertion (A): On a map of scale 1 : 1000, areas shrink by a factor of 10&sup6;.
Reason (R): On this map, 1 cm represents 10 m in reality.
(a) Both A and R are true and R is the correct explanation of A (b) Both A and R are true but R is NOT the correct explanation of A (c) A is true but R is false (d) A is false but R is true

A–R 10

Assertion (A): If a line divides two sides of a triangle in the same ratio, then it is parallel to the third side.
Reason (R): The converse of every theorem is automatically true, so the converse of BPT needs no separate proof.
(a) Both A and R are true and R is the correct explanation of A (b) Both A and R are true but R is NOT the correct explanation of A (c) A is true but R is false (d) A is false but R is true

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Find the Error — Questions 1–5

Each piece of working below contains exactly one planted mistake — the kind real students make in board exams. Spot it before you check the key.

ERROR 1

"In △ABC, DE ∥ BC, AD = 2 cm, DB = 4 cm, AE = 3 cm. By BPT: AD/AB = AE/EC, so 2/6 = 3/EC, giving EC = 9 cm."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 2

"The sides of two similar triangles are in the ratio 3 : 5. Therefore their areas are in the ratio 3 : 5."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 3

"In △ABC and △DEF, AB/DE = AC/DF and ∠B = ∠E. Hence △ABC ~ △DEF by the SAS criterion."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 4

"A triangle has sides 4, 5 and 6. Since $4^2 + 5^2 = 41$, which is greater than $6^2 = 36$, by Pythagoras the triangle is right-angled."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 5

"In two triangles, ∠A = ∠D = 50° and ∠B = ∠F = 60°. Hence △ABC ~ △DEF by AA."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

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Find the Error — Questions 6–10

ERROR 6

"D is the mid-point of AB in △ABC. A line through D meets AC at E. By BPT, E must be the mid-point of AC."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 7

"To test whether a triangle with sides 6, 8, 10 is right-angled: $6^2 + 10^2 = 136 \neq 64 = 8^2$. So the triangle is not right-angled."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 8

"In △ABC and △PQR, AB/PQ = AC/PR and ∠B = ∠Q. Hence the triangles are similar by SAS."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 9

"The altitude to the hypotenuse divides it into segments of 2 cm and 8 cm. The altitude is the average, (2 + 8)/2 = 5 cm."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 10

"A 1 m stick casts a 2 m shadow. A tree casts a 30 m shadow. So the tree's height = 30 × (2/1) = 60 m."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

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Previous Years' Board Questions (PYQs) — Solved

Real CBSE board questions, verified against official papers. Solve them the way the board expects: theorem named, steps in order, answer boxed.

PYQ — CBSE 2024 (Set 30/5/1)

In △ABC, DE ‖ BC. If AD = 2.4 cm, DB = 4 cm and AE = 2 cm, find the length of AC. (MCQ in the paper; here we show the working.)

1. By BPT: AD/DB = AE/EC ⇒ 2.4/4 = 2/EC.
   Why: DE ‖ BC is given — the licence for BPT.
2. EC = 2 × 4/2.4 = 8/2.4 = 10/3 cm.
   Why: Cross-multiply; keep the fraction exact.
3. AC = AE + EC = 2 + 10/3 = **16/3 cm** (about 5.33 cm).
   Why: The question asks for the whole side, not the piece — the final addition is where this mark hides.

ANSWER: AC = 16/3 cm.

PYQ — CBSE 2024 (Set 30/4/1)

In △ABC, DE ‖ BC. If AD = 4 cm, AB = 9 cm and AC = 13.5 cm, find EC. (MCQ in the paper.)

1. Whole-side form of BPT: AD/AB = AE/AC ⇒ 4/9 = AE/13.5.
   Why: AB and AC are given whole — use the part-to-whole form directly.
2. AE = 13.5 × 4/9 = 6 cm.
   Why: 13.5 × 4 = 54; 54/9 = 6.
3. EC = AC − AE = 13.5 − 6 = **7.5 cm**.
   Why: Subtract the piece from the whole.

ANSWER: EC = 7.5 cm.

PYQ — CBSE 2023 (Set 30/6/1)

In △ABC, DE ‖ BC. If AD = x, DB = x − 2, AE = x + 2 and EC = x − 1 (all in cm), find the value of x.

1. By BPT: AD/DB = AE/EC ⇒ x/(x − 2) = (x + 2)/(x − 1).
   Why: Substitute the four expressions into the proportion.

PYQs — Continued

PYQ — CBSE 2024 Compartment, Basic (Set 430/S/1)

In △ABC, DE ∥ BC. Which of these equalities is correct? (a) AD/DB = AE/AC (b) AD/AB = AE/AC (c) AB/AD = AC/EC (d) AD/AB = EC/AC. (MCQ in the paper.)

1. BPT pairs pieces with pieces: AD/DB = AE/EC — not among the options in this form.
   Why: Eliminate options that cross a piece with a whole on one side only.
2. The part-to-whole form pairs each piece with its own whole: AD/AB = AE/AC. That is option **(b)**.
   Why: Option (a) mixes piece/whole wrongly; (c) and (d) scramble the pairing.

ANSWER: Option (b): AD/AB = AE/AC.

PYQ — CBSE 2024 Basic (Set 430/4/1)

In Fig., △ABC ~ △PQR. BD and QM are medians of △ABC and △PQR respectively, where D lies on AC and M lies on PR. Prove that AB/PQ = BD/QM.

1. D and M are mid-points, so AD = AC/2 and PM = PR/2. Hence AD/PM = AC/PR = AB/PQ (similarity of ABC and PQR) ... (1)
   Why: Halving corresponding sides preserves their ratio.
2. Also ∠A = ∠P (corresponding angles of similar triangles) ... (2)
   Why: The angle at A is included between AB and AD; the angle at P is included between PQ and PM.
3. From (1) and (2), △ABD ~ △PQM by the SAS criterion.
   Why: Two sides around an equal included angle, in proportion.
4. Therefore AB/PQ = BD/QM (corresponding sides of similar triangles). **Proved.**
   Why: Medians of similar triangles stand in the same ratio as the sides – the same result as Olympiad 7, straight from a board paper.

ANSWER: AB/PQ = BD/QM, proved via △ABD ~ △PQM (SAS).

Chapter Model Test A — Basics (20 Marks)

Time: 40 minutes. Attempt all questions. Show every step — the marking key in the Answer Key shows where each mark lives.

Q1. In △ABC, DE ∥ BC. If AD = 3 cm, DB = 9 cm and AE = 2 cm, find EC. **[2 marks]**

Q2. State whether the triangles with sides 3 cm, 4 cm, 5 cm and 6 cm, 8 cm, 10 cm are similar. Name the criterion and write one conclusion about their angles. **[3 marks]**

Q3. The corresponding sides of two similar triangles are in the ratio 2 : 3. If the area of the smaller triangle is 20 cm^2, find the area of the larger. **[3 marks]**

Q4. A ladder 13 m long reaches a window 12 m above the ground. Find the distance of the foot of the ladder from the building. **[3 marks]**

Q5. In △ABC, points D and E on AB and AC satisfy AD = 2 cm, DB = 4 cm, AE = 3 cm and EC = 6 cm. Prove that DE ∥ BC. **[4 marks]**

Q6. A stick 1.2 m tall casts a shadow of 0.8 m. At the same time, a tower casts a shadow of 20 m. Find the height of the tower, justifying each step. **[5 marks]**

Chapter Model Test B — Standard (20 Marks)

Time: 40 minutes. Board-pattern questions at full Class 10 level.

Q1. In △ABC, DE ∥ BC. If AD = 4 cm, AB = 10 cm and AE = 6 cm, find EC. **[4 marks]**

Q2. △ABC ~ △PQR with ar(ABC) = 81 cm^2 and ar(PQR) = 144 cm^2. If AB = 9 cm, find PQ. **[4 marks]**

Q3. Two poles of heights 9 m and 15 m stand on level ground with their feet 8 m apart. Find the distance between their tops. **[4 marks]**

Q4. The hypotenuse of a right triangle is 26 cm. Find the length of the median drawn to the hypotenuse, with justification. **[3 marks]**

Q5. In △ABC, DE ∥ BC with AD : DB = 1 : 2. If ar(△ABC) = 72 cm^2, find ar(△ADE) and the area of trapezium DBCE. **[5 marks]**

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Chapter Model Test C — Challenge (20 Marks)

Time: 45 minutes. Proof-heavy, topper-level. Every claim needs its reason.

Q1. In △ABC, right-angled at A, AD is perpendicular to BC. Prove that $AD^2 = BD \times DC$. **[5 marks]**

Q2. △ABC is right-angled at A with AB = 6 cm, AC = 8 cm and BC = 10 cm. Find the median from A to BC in two different ways, and state the theorem each way uses. **[5 marks]**

Q3. In △ABC, AB = 10 cm, AC = 6 cm and BC = 8 cm. The bisector of ∠A meets BC at D. Find BD and DC, naming the theorem used. **[4 marks]**

Q4. A map is drawn to the scale 1 : 25000. A lake covers 3 cm^2 on the map. Find its actual area in square metres. **[3 marks]**

Q5. In △ABC, DE ∥ BC with AD = 3x cm, DB = (2x + 4) cm, AE = 6 cm and EC = 8 cm. Find x and AD. **[3 marks]**

Smart Memory Mini-Tests — Day 7 and Day 21

Ten quick questions each, closed book, five minutes. Answers are in the Answer Key. These are the Day 7 and Day 21 stops of your Smart Memory schedule (Section 19).

DAY 7 TEST

1. State the Basic Proportionality Theorem.

2. DE ∥ BC in △ABC with AD = 6 cm, DB = 3 cm and AE = 8 cm. Find EC.

3. By which criterion are the triangles of sides 2, 3, 4 and 4, 6, 8 similar?

4. Find the hypotenuse of a right triangle with legs 9 cm and 12 cm.

5. Corresponding sides of similar triangles are in the ratio 5 : 6. Find the ratio of their areas.

6. △ABC ~ △DEF with ∠A = 80° and ∠B = 45°. Find ∠F.

7. Complete the Pythagorean triple: 5, 12, __.

8. D and E are the mid-points of AB and AC. If BC = 14 cm, find DE.

9. Find the altitude of an equilateral triangle of side 4 cm.

10. A 2 m pole casts a 1 m shadow. A tree casts a 9 m shadow at the same time. Find the tree's height.

DAY 21 TEST

1. State the converse of the Basic Proportionality Theorem.

2. DE ∥ BC with AD/AB = 3/7 and DE = 9 cm. Find BC.

3. The areas of two similar triangles are in the ratio 16 : 25. Find the ratio of their corresponding sides.

4. Is a triangle with sides 10 cm, 24 cm and 26 cm right-angled? If yes, name the hypotenuse.

5. The altitude to the hypotenuse divides it into segments of 3 cm and 12 cm. Find the altitude.

6. A ladder 20 m long reaches a window 16 m high. Find the distance of its foot from the wall.

7. △ABC ~ △PQR with corresponding sides in the ratio 3 : 5. Find the ratio of their perimeters.

8. The bisector of ∠A in △ABC meets BC at D. If AB = 9 cm, AC = 12 cm and BC = 14 cm, find BD and DC.

9. On a map of scale 1 : 100, a length of 5 cm represents what real length?

10. In one line: why does the AA criterion need only two pairs of equal angles?

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Smart Memory Mini-Test — Day 30 Mastery Test

The final checkpoint. Ten questions, closed book — if you score 9 or 10, this chapter is yours for the whole year.

DAY 30 MASTERY TEST

1. DE ∥ BC in △ABC with AD = 5 cm, AE = 10 cm and EC = 8 cm. Find DB.

2. DE ∥ BC with AD : DB = 2 : 1. If ar(△ABC) = 90 cm^2, find ar(△ADE).

3. Two poles of heights 7 m and 13 m have their feet 8 m apart. Find the distance between their tops.

4. The median to the hypotenuse of a right triangle is 18 cm. Find the hypotenuse.

5. In △ABC, AB = AC = 5 cm and BC = 6 cm. Find the median from A to BC.

6. State, with the calculation, whether 9, 40, 41 is a Pythagorean triple.

7. Two pairs of sides of two triangles are proportional and a non-included angle is equal. Must the triangles be similar? Answer yes or no, with one line of reason.

8. A photo of area 20 cm^2 is enlarged by a linear factor of 3. Find the new area.

9. Find the altitude of an equilateral triangle of side 8 cm.

10. State the theorem about areas of two similar triangles.

> **TOPPER TIP:** *Missed any Day 30 question? That question's Flash Card goes back into your pocket box for one more week.*

Answer Key — Prerequisites & Graded Practice

Prerequisites Quick Check (Section 02)

QC1 Similar, by SSS: 5/10 = 7/14 = 9/18 = 1/2. **QC2** 80°. **QC3** x = 3 (4x = x + 9). **QC4** 30 cm^2. **QC5** (a) 110° (b) 70° (co-interior angles are supplementary).

Level 1

L1.1 EC = 6 cm (BPT: 2/4 = 3/EC). **L1.2** Yes, similar by SSS: 4/6 = 6/9 = 8/12 = 2/3. **L1.3** QR = 12 cm (scale factor 9/6 = 3/2). **L1.4** 9 : 25 (square the side ratio). **L1.5** Yes: $9^2 + 12^2 = 225 = 15^2$; hypotenuse 15 cm. **L1.6** BC = 15 cm (DE/BC = AD/AB = 1/3).

Level 2

L2.1 Yes: AD/DB = 3/6 = 1/2 and AE/EC = 4.5/9 = 1/2, so DE ∥ BC by the converse of BPT. **L2.2** DE = 10.5 cm (side ratio = √(49/81) = 7/9; 13.5 × 7/9). **L2.3** 7 m (√($25^2 - 24^2$) = √49). **L2.4** Yes: ∠C = 180° − 50° − 70° = 60° = ∠R and ∠B = ∠Q, so △ABC ~ △PQR by AA. **L2.5** 10 m (legs 8 and 14 − 8 = 6; √(64 + 36)). **L2.6** 5√3 cm (10√3/2).

Level 3

L3.1 12 m (height/shadow = 1.5/3 = 1/2; 24 × 1/2). **L3.2** 8 m (equal angles at the mirror give AA similarity; height/1.6 = 20/4 = 5). **L3.3** 16 m (√($20^2 - 12^2$) = √256). **L3.4** 600 m^2 (area scale = 1000^2 = 10&sup6;; 6 cm^2 × 10&sup6; = 6 × 10&sup6; cm^2 = 600 m^2).

Level 4

L4.1 ar(ABC) = 150 cm^2 (AD/AB = 3/5, areas ratio 9/25, 54 × 25/9); trapezium DBCE = 150 − 54 = 96 cm^2. **L4.2** AD = 4.8 cm (AB·AC/BC = 48/10); BD = 3.6 cm (AB^2/BC = 36/10); DC = 6.4 cm (AC^2/BC = 64/10). **L4.3** Perimeter of PQR = 54 cm (36 × 3/2); area of PQR = 108 cm^2 (48 × 9/4).

Level 5

L5.1 BC = 12√2 cm: ar(ABC) = ar(ADE) + 7 ar(ADE) = 8 ar(ADE), so $(BC/DE)^2 = 8$, BC = 6 × 2√2. **L5.2** √21 cm: by Apollonius, $25 + 49 = 2m^2 + 64/2 \Rightarrow 2m^2 = 42 \Rightarrow m^2 = 21$. (Coordinate geometry gives the same: place B = (0,0), C = (8,0), A from the two distances, mid-point of BC = (4,0).)

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Answer Key — Active Recall, MCQs & Assertion-Reason

Active Recall (Section 18)

R1 A line parallel to one side of a triangle divides the other two sides in the same ratio. **R2** 8 cm. **R3** SSS criterion. **R4** 6 cm (area ratio 1 : 4 ⇒ side ratio 1 : 2). **R5** 13 cm. **R6** Yes; hypotenuse 17 cm. **R7** 50° (∠C = 50° and C corresponds to F). **R8** 12 cm (√(9 × 16)). **R9** Perimeters 4 : 7; areas 16 : 49. **R10** If a line divides two sides of a triangle in the same ratio, it is parallel to the third side.

W1–W5 (key points): W1: same sun angle + right angles ⇒ AA ⇒ height/shadow is one fixed ratio. W2: perimeter is a sum of lengths (scales with k); area is a product of two lengths (scales with k^2). W3: right only when the equal angle is the included angle between the proportional sides. W4: the converse of Pythagoras: $3^2 + 4^2 = 5^2$ forces the right angle. W5: congruent = same shape AND size (two coins of the same minting); similar = same shape, any size (a photo and its enlargement).

25 Concept MCQs

1 (b) 2 (b) 3 (c) 4 (b) 5 (b) 6 (b) 7 (d) 8 (b) 9 (b) 10 (b) 11 (b) 12 (a) 13 (d) 14 (b) 15 (b) 16 (a) 17 (b) 18 (b) 19 (b) 20 (b) 21 (a) 22 (b) 23 (a) 24 (b) 25 (c)

Explanations for the tricky ones: Q9 — take square roots of the area ratio: √49 : √81 = 7 : 9. Q13 — SAS demands the INCLUDED angle; (d) is the classic trap. Q16 — side ratio = √(36/64) = 6/8, so DE = 6 × 8/6 = 8 cm. Q18 — perimeters scale linearly: 45 × 5/3 = 75 cm. Q21 — altitude = √(2 × 8) = 4 cm (geometric mean, not the average 5). Q24 — AD/AB = 3/(3 + 1) = 3/4; square it: 9/16.

Assertion – Reason

1 (a) 2 (a) 3 (d) 4 (b) 5 (c) 6 (a) 7 (d) 8 (a) 9 (b) 10 (c)

Why: Q3 — A is false: without the included angle, similarity does not follow; R is true. Q4 — both true, but the rope works because of the converse of Pythagoras, not because of its history. Q5 — A is true by the converse of Pythagoras; R is false (the hypotenuse is the LONGEST side). Q7 — A mis-pairs the correspondence (AB matches PQ, not QR); R is true. Q9 — both true, but the factor 10&sup6; comes from SQUARING the linear scale 1000; R alone does not explain it. Q10 — A is true (it is the converse of BPT); R is false — converses are never automatic; each needs its own proof.

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Answer Key — Find the Error & Model Tests

Find the Error — Corrections

E1 The pairing AD/AB = AE/EC crosses a piece with a whole. Correct: AD/DB = AE/EC ⇒ 2/4 = 3/EC ⇒ EC = 6 cm. **E2** Areas take the SQUARE of the ratio: 9 : 25, not 3 : 5. **E3** ∠B and ∠E are not the included angles (the included pair would be ∠A and ∠D, between the proportional sides). SAS does not apply. **E4** Pythagoras needs EQUALITY: 41 ≠ 36, so the converse gives no right angle (in fact $4^2 + 5^2 > 6^2$ means the angle opposite the side 6 is acute — the triangle is acute-angled). **E5** Correspondence: B matches F, so the correct statement is △ABC ~ △DFE (A↔D, B↔F, C↔E). **E6** BPT's licence is missing: nothing says the line through D is parallel to BC, so no conclusion about E is possible. **E7** The hypotenuse is the longest side, 10: $6^2 + 8^2 = 100 = 10^2$, so the triangle IS right-angled. **E8** ∠B and ∠Q are not included between the sides AB, AC and PQ, PR (the included angles are ∠A and ∠P). **E9** The altitude is the geometric mean of the segments: √(2 × 8) = 4 cm, not the arithmetic average 5 cm. **E10** The ratio is inverted: height/shadow = 1/2 for the stick, so the tree's height = 30 × 1/2 = 15 m, not 60 m.

Model Test A — Key

Q1 EC = 6 cm [BPT proportion 1, answer 1]. **Q2** Similar by SSS (ratios all 1/2); corresponding angles are equal [criterion 1, ratio check 1, conclusion 1]. **Q3** 45 cm^2 [square law stated 1, ratio 9/4 1, answer 1]. **Q4** 5 m [Pythagoras set up 1, subtraction 169 − 144 1, answer 1]. **Q5** AD/DB = 2/4 = 1/2 and AE/EC = 3/6 = 1/2, hence DE ∥ BC by the converse of BPT [both ratios 2, theorem named 1, conclusion 1]. **Q6** 30 m [AA similarity justified 2, proportion h/20 = 1.2/0.8 1, computation 1, answer 1].

Model Test B — Key

Q1 EC = 9 cm [part-to-whole form 1, AC = 15 found 2, EC = AC − AE 1]. **Q2** PQ = 12 cm [side ratio = √(81/144) = 9/12 2, proportion 1, answer 1]. **Q3** 10 m [levelling construction 1, legs 6 and 8 identified 1, Pythagoras 1, answer 1]. **Q4** 13 cm [median-to-hypotenuse theorem stated 2, half of 26 1]. **Q5** ar(ADE) = 8 cm^2, trapezium = 64 cm^2 [AD/AB = 1/3 1, squared ratio 1/9 1, ADE 1, trapezium by subtraction 2].

Model Test C — Key

Q1 △DBA ~ △DAC by AA, giving BD/AD = AD/DC, hence AD^2 = BD·DC [similarity set up 2, ratio written 2, conclusion 1]. **Q2** 5 cm, two ways: (i) median to the hypotenuse = BC/2 = 5 (Reasoning 2); (ii) Apollonius: $36 + 64 = 2m^2 + 50 \Rightarrow m = 5$ [each method 2, theorems named 1]. **Q3** BD = 5 cm, DC = 3 cm, by the Angle Bisector Theorem: BD/DC = 10/6 = 5/3 with BD + DC = 8 [theorem named 1, ratio 1, split of 8 cm 2]. **Q4** 187500 m^2 [area scale $(25000)^2$ = 6.25 × 10&sup8; 1, × 3 =

Answer Key — Smart Memory Mini-Tests

Day 7 Test

1. A line parallel to one side of a triangle divides the other two sides in the same ratio. 2. EC = 4 cm. 3. SSS. 4. 15 cm. 5. 25 : 36. 6. ∠F = 55°. 7. 13. 8. DE = 7 cm. 9. $2\sqrt{3}$ cm. 10. 18 m.

Day 21 Test

1. If a line divides two sides of a triangle in the same ratio, it is parallel to the third side. 2. BC = 21 cm. 3. 4 : 5. 4. Yes: $10^2 + 24^2 = 676 = 26^2$; hypotenuse 26 cm. 5. 6 cm ($\sqrt{(3 \times 12)}$). 6. 12 m. 7. 3 : 5. 8. BD = 6 cm, DC = 8 cm. 9. 5 m. 10. The third pair of angles is forced equal by the angle-sum property (180°).

Day 30 Mastery Test

1. DB = 4 cm. 2. 40 cm^2 (AD/AB = 2/3, squared 4/9, 90 × 4/9). 3. 10 m (legs 8 and 6). 4. 36 cm. 5. 4 cm. 6. Yes: $9^2 + 40^2 = 81 + 1600 = 1681 = 41^2$. 7. No — the equal angle must be the included angle between the proportional sides. 8. 180 cm^2. 9. $4\sqrt{3}$ cm. 10. The ratio of the areas of two similar triangles equals the square of the ratio of their corresponding sides.

TOPPER TIP: *Checked everything? Then close the book, take a blank page, and write the five theorem statements of this chapter from memory. That page is your real certificate.*

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Revision Card — Chapter 6: Triangles (One-Page Summary)

The Five Engines

1. BPT (Thm 6.1): DE ∥ BC ⇒ AD/DB = AE/EC = AD/AB = AE/AC pattern. **Converse (Thm 6.2):** equal ratios ⇒ DE ∥ BC.
2. Similarity criteria: AA (two angles) • SSS (three side ratios) • SAS (two side ratios + INCLUDED angle). Correspondence order is law.
3. Areas theorem: areas ratio = (sides ratio)^2. Perimeters, altitudes, medians, bisectors scale with the plain ratio.
4. Pythagoras: right triangle ⇒ $a^2 + b^2 = c^2$. **Converse:** $a^2 + b^2 = c^2$ ⇒ right angle opposite c. Triples: 3-4-5, 5-12-13, 7-24-25, 8-15-17, 20-21-29.
5. Altitude to hypotenuse: $AD^2 = BD \cdot DC$ • $AB^2 = BD \cdot BC$ • $AC^2 = CD \cdot BC$.

Golden Rules

• Name the licence before the theorem: parallel for BPT, right angle for Pythagoras.
• Pieces with pieces, wholes with wholes — never cross them.
• The word ‘area’ with similar triangles means: square the ratio.
• The hypotenuse is opposite the right angle and is the longest side.
• Equilateral altitude = $a\sqrt{3}/2$ • Median to hypotenuse = half the hypotenuse • Angle bisector splits the opposite side as AB : AC.

Applications Map

Shadows & heights → AA similarity • Maps & enlargements → scale factor (square it for areas) • Ladders, poles, wires, kites → Pythagoras • Parallel cuts & mid-points → BPT and its converse.

You now own the geometry of shape itself: how to prove two triangles are the same shape, how parallels cut sides fairly, how areas grow with the square, and how the right angle turns two legs into one hypotenuse. In **Chapter 7 — Coordinate Geometry**, these same triangles move onto the number grid, and you will measure them with coordinates instead of rulers. The distance formula waiting for you there is Pythagoras in disguise — you have already met it.

— KalviYogi Mathematics Team, Edunivra Pvt Ltd

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CBSE · CLASS 10 · MATHEMATICS

MATHEMATICS BEYOND THE FORMULA

CHAPTER 7 · COORDINATE GEOMETRY

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01 Chapter Introduction

8D • DISCOVER — WHY SHOULD I LEARN THIS?

Open any map app on a phone. The moment you type a place name, two numbers appear — and the app instantly tells you how far the place is, which spot lies halfway, and where you are standing right now. Those two numbers are **coordinates**, and the mathematics behind that instant answer is exactly this chapter.

Coordinate Geometry is the art of describing position with numbers. Once a point becomes a pair of numbers (x, y), geometry turns into arithmetic: distances become a formula, midpoints become an average, and questions like "are these three points in one straight line?" get answered by calculation — not by guessing from a drawing.

What you will master in this chapter

- **The distance formula** — born from Pythagoras, derived step by step, never memorised blindly.
- **The section formula** — finding a point that divides a segment in any ratio, including trisection.
- **The midpoint formula** — the section formula's simplest child, and the key to parallelograms and circles.
- **Shape detective work** — proving squares, rhombuses and parallelograms using only coordinates.
- **Area of a triangle from coordinates** — a powerful extension tool (see the honest note in Section 05).

This chapter carries about **6 marks** in the board paper almost every year, and its formulas return inside Circles, Trigonometry applications and case-study questions. Master it once — it pays you back four times.

> **TOPPER TIP:** *Coordinate Geometry is the most 'formula-friendly' chapter of Class 10: only three formulas, but a hundred disguises. In this book you will DERIVE all three, so even if memory fails in the exam hall, logic will rebuild them.*

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Your Chapter Map — 19 Stops, One Journey

Stop	Section	What happens there
01	Chapter Introduction	Why coordinates run the modern world
02	Prerequisite Knowledge	Class 9 tools: plotting, Pythagoras, square roots
03	Real-Life Story	Meena's GPS ride to school
04	Visual Concepts (CPA)	The plane, quadrants, distance as a hypotenuse
05	Mathematical Foundations	Every term defined precisely
06	Formula Derivations	All three formulas DERIVED, step by step
07	Detailed Solved Examples	NCERT Exercises 7.1 & 7.2 — every step shown
08	Multiple Solution Methods	One problem, several roads
09	Simulations & Activities	Learn by plotting, measuring, dividing
10	Common Mistakes	The errors to never make
11	Graded Practice	Five levels, Foundation to Genius
12	Mathematical Reasoning	Prove it — coordinate proof style
13	Real-Life Applications	Ships, drones, parks and towers
14	Competency-Based Questions	5 case studies + 10 assertion–reason
15	Olympiad Challenges	10 problems for deep thinkers
16	Chapter Mind Map	The whole chapter on one page
17	Pin-Point Flash Cards	16 yellow cards for your pocket
18	Active Recall	Close the book. Answer. Then check.
19	Mastery Tracker	Smart Memory: Day 0 to Day 30 plan

After Section 19 you will find **Practice Plus**: 25 Concept MCQs, Find-the-Error questions, verified Previous Year Questions, three Model Tests, the Smart Memory mini-tests, the full Answer Key and a one-page Revision Card.

KALVIYOGI DIGITAL COMPANION — CHAPTER 7: COORDINATE GEOMETRY
Video lessons, the Coordinate Plane Simulator, plotting games and online tests for this chapter live in the KalviYogi Digital Ecosystem.
QR code and link activate with the KalviYogi app/website at launch.

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02 Prerequisite Knowledge

CLASS 9 RECAP — THE TOOLS YOU ALREADY OWN

Nothing in this chapter is truly new — it is Class 9 ideas wearing a new coat. Check that these four tools are sharp before you move on.

Tool 1 — Reading and plotting points (Class 9)

A point is written (x, y): first walk along the **x-axis** (right if positive, left if negative), then climb the **y-axis** (up if positive, down if negative). The first number is the **abscissa**, the second is the **ordinate**. The axes cross at the **origin** O(0, 0) and cut the plane into four **quadrants**.

Tool 2 — Pythagoras' theorem (Class 9–10)

In a right triangle, $(\text{hypotenuse})^2 = (\text{base})^2 + (\text{height})^2$. The distance formula of this chapter is nothing but Pythagoras hiding inside a grid — you will see the derivation in Section 06.

Tool 3 — Square roots of sums of squares

You must be fluent with results like $\sqrt{(3^2 + 4^2)} = \sqrt{25} = 5$, $\sqrt{(6^2 + 8^2)} = 10$ and $\sqrt{(5^2 + 12^2)} = 13$. These "Pythagorean triples" (3-4-5, 6-8-10, 5-12-13, 9-12-15) appear again and again — recognising them saves minutes.

Tool 4 — Averages and simple equations

The midpoint of two numbers is their average: the midpoint of 4 and 10 is $(4 + 10)/2 = 7$. The midpoint formula is this same idea applied to x and y separately. You will also solve small equations like $(x + 2)/2 = 5$, giving $x = 8$.

PIN POINT: Every formula in this chapter is built from just two school friends: ***Pythagoras*** and the ***average***. If both are strong, this whole chapter is yours.

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Quick Check — Are You Ready? (5 Questions)

Answer on rough paper before looking at the answers below. If you score 4 or 5, start Section 03. If not, revise the tool above each missed question first.

Q1. A point is 3 units to the RIGHT of the origin and 4 units UP. Write its coordinates.

Q2. A right triangle has legs 5 cm and 12 cm. Find its hypotenuse.

Q3. Simplify: √(9 + 16). (Careful — this is a trap for the sleepy.)

Q4. Solve for x: (x + 2)/2 = 5.

Q5. On a number line, what is the distance between −3 and 4?

Quick Check — Answers

Q1. (3, 4). Right gives +3 on x, up gives +4 on y.

Q2. √(25 + 144) = √169 = **13 cm**.

Q3. √25 = **5** — add INSIDE the root first. (√9 + √16 = 3 + 4 = 7 would be a different question!)

Q4. x + 2 = 10, so **x = 8**.

Q5. 4 − (−3) = **7 units**. Distance is a difference — this same idea, in two directions at once, becomes the distance formula.

TOPPER TIP: Quick Check Q3's trap returns in Section 10 as Common Mistake 2. Roots never split over addition: √(a + b) is NOT √a + √b.

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03 Real-Life Mathematical Story

DISCOVER • VISUALISE — MEENA AND THE HONEST MAP

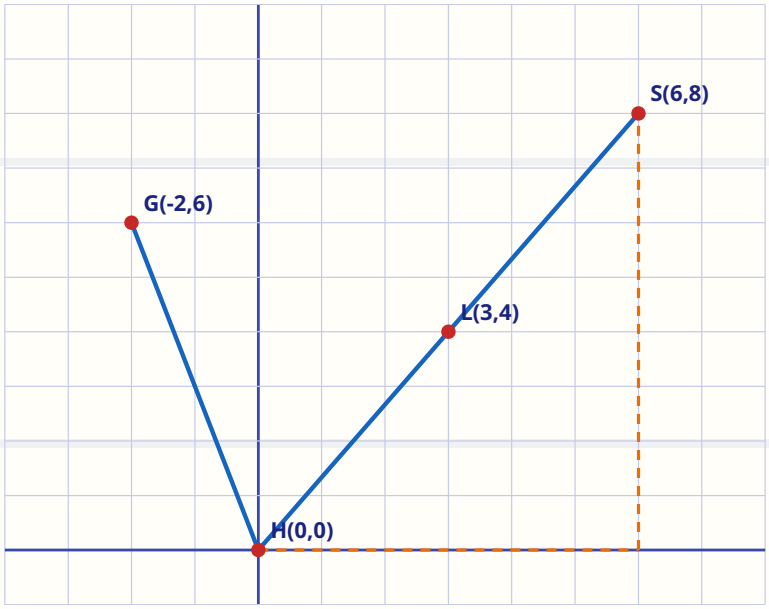
Meena's new bicycle had one problem: the road to school twisted through three streets, and the odometer said 12 km. "But the school is just across the lake!" she complained. Her grandfather, a retired surveyor, smiled and opened an old survey map of the town.

"Roads bend, Meena. But distance does not. Put your home at the origin of this grid — H(0, 0). Every square on the map is 1 km. Your school S sits at (6, 8). Now tell me: how far is the school as the crow flies?"

Meena drew a horizontal line from home to below the school, and a vertical line up to the school. A right triangle appeared on the map, with legs 6 km and 8 km. "Grandfather... this is just Pythagoras wearing a map's clothes!"

"Exactly. And the library where you stop every day? It sits exactly halfway, at (3, 4). Even your grandmother's house G at (−2, 6) — west and north of home — is only a formula away. Surveyors, pilots, ship captains and game designers all live inside this one idea: ***two numbers can pin down any place on Earth, and Pythagoras can measure between any two of them.***"

Three questions hide inside this story. Let us pull them out and solve them properly — the way you will solve every problem in this chapter.



Solving the Story — Step by Step

STORY PROBLEM 1

Meena's home is H(0, 0) and her school is S(6, 8) on the survey map (1 square = 1 km). Find the straight-line distance from home to school.

1. Draw the right triangle: from H go 6 km east to (6, 0), then 8 km north to S(6, 8). The straight line HS is the hypotenuse.
   Why: Every distance problem secretly contains this right triangle.
2. Horizontal leg = 6 − 0 = 6 km. Vertical leg = 8 − 0 = 8 km.
   Why: The legs are just the DIFFERENCES of the coordinates.
3. By Pythagoras: $HS^2 = 6^2 + 8^2 = 36 + 64 = 100$.
4. HS = √100 = **10 km**. (Distance is never negative, so we take the positive root.)

ANSWER: The school is 10 km from home as the crow flies — 2 km shorter than the bending road.

STORY PROBLEM 2

The library L is exactly halfway between home H(0, 0) and school S(6, 8). Find its coordinates, and verify your answer.

1. Halfway means: average the x-coordinates and average the y-coordinates, separately.
   Why: Midpoint = average, applied twice.
2. x-coordinate of L = (0 + 6)/2 = 3. y-coordinate of L = (0 + 8)/2 = 4. So L = (3, 4).
3. Verify: $HL = \sqrt{(3^2 + 4^2)} = \sqrt{25} = 5$, and $LS = \sqrt{((6-3)^2 + (8-4)^2)} = \sqrt{(9 + 16)} = 5$.
   Why: A midpoint must be at EQUAL distances from both ends.
4. HL = LS = 5 km and L lies on segment HS (5 + 5 = 10 = HS). Verified.

ANSWER: The library is at L(3, 4), exactly 5 km from both home and school.

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Solving the Story — Problem 3: Grandmother's House

STORY PROBLEM 3

Grandmother's house is G(−2, 6). Find (a) the distance from home H(0, 0) to G, and (b) the distance from G to the school S(6, 8).

1. (a) Horizontal difference = −2 − 0 = −2. Vertical difference = 6 − 0 = 6.

   Why: Negative coordinates are welcome — squaring will handle the sign.

2. HG = $\sqrt{((-2)^2 + 6^2)} = \sqrt{(4 + 36)} = \sqrt{40} =$ **$2\sqrt{10} \approx 6.32$ km**.

   Why: $(-2)^2 = +4$. A square never keeps a minus sign.

3. (b) From G(−2, 6) to S(6, 8): horizontal difference = 6 − (−2) = 8. Vertical difference = 8 − 6 = 2.

   Why: Subtract carefully: minus a negative becomes a plus.

4. GS = $\sqrt{(8^2 + 2^2)} = \sqrt{(64 + 4)} = \sqrt{68} =$ **$2\sqrt{17} \approx 8.25$ km**.

ANSWER: (a) HG = $2\sqrt{10}$ km ≈ 6.32 km. (b) GS = $2\sqrt{17}$ km ≈ 8.25 km.

TOPPER TIP: *Notice what the story just taught you: you never needed the map after the coordinates were known. The numbers (0, 0), (6, 8), (3, 4), (−2, 6) carried the whole geography inside them. That is the superpower of this chapter.*

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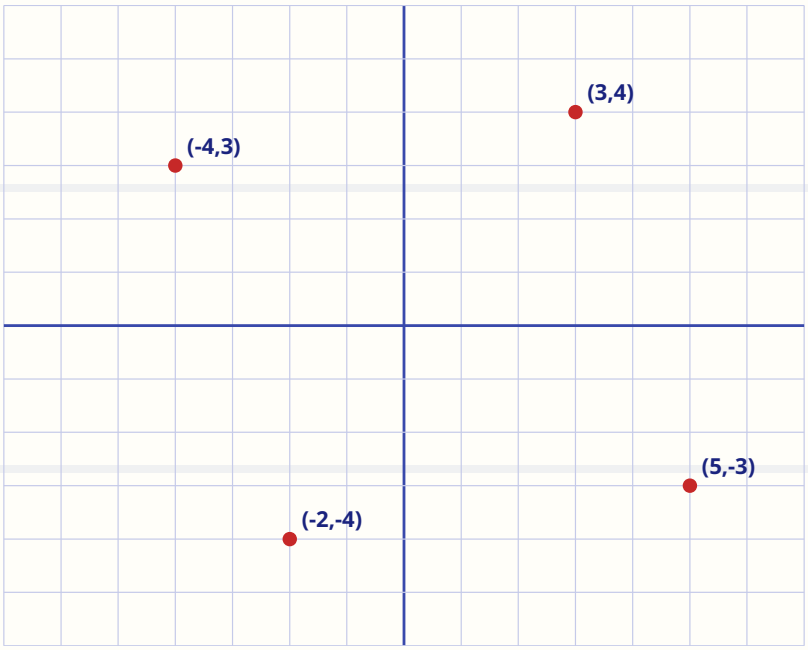
04 Visual Concept Explanation

SINGAPORE CPA — CONCRETE → PICTORIAL → ABSTRACT

Concrete — the floor of your classroom

Imagine the classroom floor tiled in squares. Stand at the door: that tile is (0, 0). Walking 4 tiles east and 3 tiles north lands you on tile (4, 3). Every tile has exactly one address, and every address names exactly one tile. A chessboard, a cinema hall and a cricket field's wagon-wheel are all the same idea.

Pictorial — the plane and its four quadrants

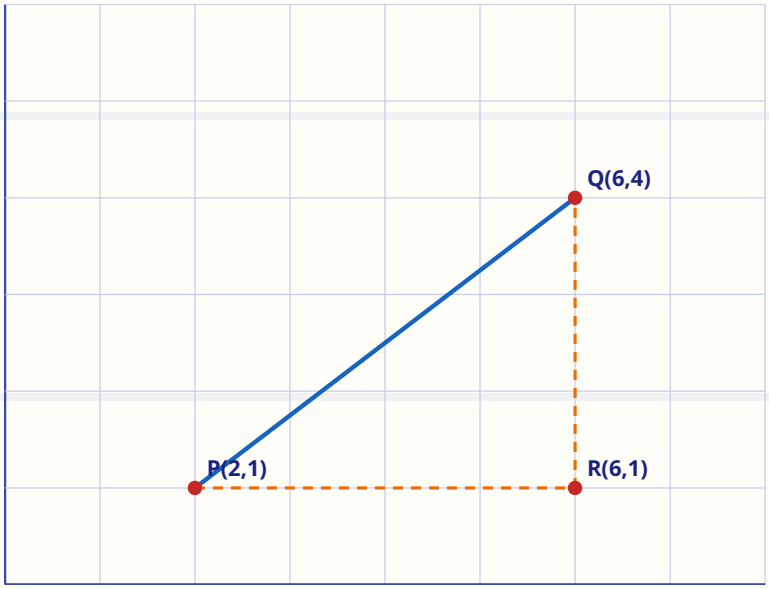


The two axes divide the plane into four **quadrants**, counted anti-clockwise from the top right: **Quadrant I** (+, +), **Quadrant II** (−, +), **Quadrant III** (−, −), **Quadrant IV** (+, −). The four points plotted above sit one in each quadrant. Points ON an axis belong to no quadrant.

> **PIN POINT:** Read a point as *(walk, climb)*: x first, y second — always. Swapping them is the single most common plotting error in this chapter.

Visual Concepts — Distance Is a Hypotenuse

Pictorial: Join P(2, 1) and Q(6, 4). Complete the right triangle with the corner R(6, 1). The horizontal leg PR... wait, look carefully: the horizontal leg runs from (2, 1) to (6, 1) — length 4. The vertical leg runs from (6, 1) to (6, 4) — length 3. The segment PQ is the hypotenuse.



Abstract: PQ = $\sqrt{(4^2 + 3^2)} = \sqrt{25} = 5$. The "4" was 6 − 2 (difference of x's); the "3" was 4 − 1 (difference of y's). Replace the numbers with letters and the picture becomes the **distance formula** — derived formally in Section 06.

Pictorial — dividing a rope in the ratio 2 : 3

A rope from A to B is 5 equal parts long. A point P placed after 2 parts (counting from A) divides the rope in the ratio **AP : PB = 2 : 3**. The section formula does exactly this on a grid: P's coordinates are the weighted averages of A's and B's coordinates, where each end is weighted by the length of the *opposite* piece. Hold this rope picture in mind — it prevents the famous weight-swap mistake of Section 10.

> **PIN POINT:** Distance = *hypotenuse of the coordinate differences*. Division = *weighted average, weights crossed*. Two pictures; the whole chapter.

05 Mathematical Foundations

UNDERSTAND — THE PRECISE IDEAS, WORD BY WORD

The coordinate plane

The **x-axis** (horizontal) and **y-axis** (vertical) meet at the **origin O(0, 0)**. For a point P(x, y): x is the **abscissa** (signed distance from the y-axis), y is the **ordinate** (signed distance from the x-axis). Every point on the x-axis has the form **(x, 0)**; every point on the y-axis has the form **(0, y)**. The distance of P(x, y) from the x-axis is $|y|$; from the y-axis is $|x|$.

The three master formulas

Distance: between $P(x_1, y_1)$ and $Q(x_2, y_2)$: $PQ = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$. From the origin: $OP = \sqrt{(x^2 + y^2)}$.

Section formula: the point dividing PQ internally in the ratio m : n is $((m\,x_2 + n\,x_1)/(m + n),\ (m\,y_2 + n\,y_1)/(m + n))$.

Midpoint: $((x_1 + x_2)/2,\ (y_1 + y_2)/2)$ — the section formula with m = n = 1.

Words the examiner loves

Collinear: three points are collinear if they lie on one straight line — test: the largest of the three distances equals the sum of the other two.
Equidistant: a point is equidistant from A and B if its distances to A and B are equal — solve by setting the two squared distances equal.
Trisection: two points that cut a segment into three equal parts — they divide it in the ratios 1 : 2 and 2 : 1.

PIN POINT: A point on the *x-axis* always looks like (x, 0) and a point on the *y-axis* always looks like (0, y). Half of all 'find the point' problems begin with this one line.

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Foundations — An Honest Note About the Area Formula

Extension: Area of a Triangle (beyond the current NCERT exercises)

The rationalised NCERT textbook removed the old Exercise 7.3 (Area of a Triangle), so CBSE will not set exercise questions from it. But the formula remains a favourite **collinearity test** in board-style and Olympiad questions, and Exemplar/HOTS books still use it. We therefore teach it honestly labelled as an **extension**:

Area of △ with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3)

$$= \tfrac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

Three points are collinear exactly when this area is **zero**. Its logic is derived in Section 06 (Derivation D), and it appears in this book only inside clearly-marked extension questions.

The centroid (extension, one line)

The **centroid** of a triangle is the meeting point of its medians; it divides each median in the ratio 2 : 1 and has coordinates $((x_1 + x_2 + x_3)/3, (y_1 + y_2 + y_3)/3)$ — simply the average of the three vertices. Olympiad Challenge 5 shows where this comes from.

How to recognise which formula a problem wants:

The problem says…	Reach for…
“distance between”, “how far”, side lengths of a figure	Distance formula
“divides in the ratio”, “point of division”, trisection	Section formula
“midpoint”, “centre of the circle”, “diagonals bisect”	Midpoint formula
“equidistant from”	Set the two squared distances equal
“collinear” / “name the quadrilateral”	Distances (and diagonals) — or the area test

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06 Formula Derivations

8D • DERIVE — NEVER MEMORISE WHAT YOU CAN DERIVE

Derivation A — The Distance Formula, from Pythagoras

Take two points P(2, 1) and Q(6, 4) — first with numbers, so the logic is visible; then with letters, so it becomes a formula forever.

1. Complete the right triangle: draw a horizontal line from P and a vertical line from Q. They meet at R(6, 1).
   Why: The corner R takes its x from Q and its y from P.
2. PR = 6 − 2 = 4 (horizontal leg) and QR = 4 − 1 = 3 (vertical leg).
   Why: Legs are coordinate DIFFERENCES.
3. Pythagoras in $\triangle PQR$: $PQ^2 = PR^2 + QR^2 = 4^2 + 3^2 = 25$, so PQ = 5.
4. Now replace the numbers by letters: for $P(x_1, y_1)$ and $Q(x_2, y_2)$, the legs are $(x_2 - x_1)$ and $(y_2 - y_1)$.
   Why: The same picture, drawn once, works for every pair of points in the universe.
5. Therefore **$PQ = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$**. Special case: distance from the origin O(0, 0) is $\sqrt{(x^2 + y^2)}$. ∎

PIN POINT: Forgot the distance formula in the exam? Draw the little right triangle, write Pythagoras, and read the formula off your own picture. Thirty seconds — full recovery.

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Derivation B — The Section Formula, from Similar Triangles

Let P(x, y) divide the segment joining A(x_1, y_1) and B(x_2, y_2) internally in the ratio **m : n**, so AP : PB = m : n. We hunt for x and y.

1. Drop perpendiculars from A, P and B to the x-axis, meeting it at points with x-coordinates x_1, x and x_2. The horizontal distances are $(x - x_1)$ and $(x_2 - x)$.
   Why: We project the slanted segment onto the x-axis.
2. The two small right triangles formed are similar (both share the slope of AB), so horizontal pieces are in the same ratio as the pieces of the segment: $(x - x_1) : (x_2 - x) = m : n$.
   Why: Similar triangles — Chapter 6 doing quiet work inside Chapter 7.
3. Cross-multiply: $n(x - x_1) = m(x_2 - x) \Rightarrow nx - n x_1 = m x_2 - m x \Rightarrow (m + n)x = m x_2 + n x_1$.
4. So $\mathbf{x = (m x_2 + n x_1)/(m + n)}$. Read it: x_2 (the FAR end, B) is multiplied by m — the piece near A. The weights CROSS OVER, exactly like the rope of Section 04.
   Why: If P is close to B (m large), x should be close to x_2 — and indeed m multiplies x_2. The formula agrees with common sense.
5. Repeat the same projection on the y-axis: $\mathbf{y = (m y_2 + n y_1)/(m + n)}$. Together, these two lines ARE the section formula. ∎
6. Check with numbers: divide A(1, 2) to B(7, 8) in the ratio 2 : 1. $x = (2\times7 + 1\times1)/3 = 15/3 = 5$; $y = (2\times8 + 1\times2)/3 = 18/3 = 6$. Point: (5, 6) — two-thirds of the way, closer to B, as it should be.

Derivation C — Midpoint: the One-Line Corollary

1. A midpoint divides the segment in the ratio 1 : 1, so put m = n = 1 in the section formula.
2. $x = (1\cdot x_2 + 1\cdot x_1)/(1 + 1) = (x_1 + x_2)/2$ and $y = (y_1 + y_2)/2$.
   Why: The section formula collapses into a plain average.
3. So the midpoint formula is not a third thing to memorise — it is the section formula at m = n. ∎

Derivation D (Extension) — Where the Area Formula Comes From

Take the triangle A(0, 0), B(4, 3), C(2, 5). We find its area with nothing but rectangles and right triangles — then watch the general formula fall out.

1. Trap the triangle inside the smallest axis-parallel rectangle: corners (0, 0), (4, 0), (4, 5), (0, 5). Its area = 4 × 5 = 20.
2. The rectangle is split into our triangle plus three right triangles in the corners.
3. Corner triangle 1, with vertices (0, 0), (4, 0), (4, 3): area = ½ × 4 × 3 = 6.
4. Corner triangle 2, with vertices (4, 3), (4, 5), (2, 5): area = ½ × 2 × 2 = 2.
5. Corner triangle 3, with vertices (2, 5), (0, 5), (0, 0): area = ½ × 2 × 5 = 5.
6. Area of △ABC = 20 − (6 + 2 + 5) = **7 square units**.
7. Now do the same subtraction with letters (x_1, y_1), (x_2, y_2), (x_3, y_3) and simplify: every rectangle term cancels except $\mathbf{\frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|}$. Check it on our numbers: ½ | 0(3 − 5) + 4(5 − 0) + 2(0 − 3) | = ½ |0 + 20 − 6| = 7. Same answer. ∎

   Why: The | | bars matter: vertices listed in the other order would give −7, and area is never negative.

TOPPER TIP: *The cyclic pattern is the memory hook: x_1 pairs with $(y_2 - y_3)$, then everything shifts one place forward: 1→2→3→1. Write the pattern once on your rough sheet and every area question becomes copying.*

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07 Detailed Solved Examples

8D • SOLVE — NCERT EXERCISES 7.1 & 7.2, EVERY STEP SHOWN

Every NCERT question of the rationalised textbook, solved the KalviYogi way: **Think First**, then numbered steps — one by one, nothing skipped — then the answer and the marking scheme. Exercise 7.1 builds the distance formula; Exercise 7.2 builds the section formula.

Exercise 7.1 — The Distance Formula

EXERCISE 7.1 Q1

Find the distance between the following pairs of points: (i) (2, 3), (4, 1) (ii) (−5, 7), (−1, 3) (iii) (a, b), (−a, −b).

THINK FIRST: Three warm-up distances. In each, write the two differences first, THEN square — never square in your head while subtracting.

1. (i) Differences: $x_2 - x_1 = 4 - 2 = 2$; $y_2 - y_1 = 1 - 3 = -2$.
2. (i) Distance = $\sqrt{(2^2 + (-2)^2)} = \sqrt{(4 + 4)} = \sqrt{8} = \mathbf{2\sqrt{2}}$ **units**.
   Why: $(-2)^2 = +4$. The square quietly removes the minus.
3. (ii) Differences: $-1 - (-5) = 4$; $3 - 7 = -4$.
   Why: Minus a negative: $-1 + 5 = 4$.
4. (ii) Distance = $\sqrt{(4^2 + (-4)^2)} = \sqrt{(16 + 16)} = \sqrt{32} = \mathbf{4\sqrt{2}}$ **units**.
5. (iii) Differences: $-a - a = -2a$; $-b - b = -2b$.
6. (iii) Distance = $\sqrt{((-2a)^2 + (-2b)^2)} = \sqrt{(4a^2 + 4b^2)} = \mathbf{2\sqrt{(a^2 + b^2)}}$ **units**.
   Why: Take 4 out of the root as 2: $\sqrt{(4 \times (a^2 + b^2))} = 2\sqrt{(a^2 + b^2)}$.

ANSWER: (i) $2\sqrt{2}$ units (ii) $4\sqrt{2}$ units (iii) $2\sqrt{(a^2 + b^2)}$ units.

CBSE-STYLE MARKING SCHEME: 1 mark each part: ½ for correct differences, ½ for the simplified distance.

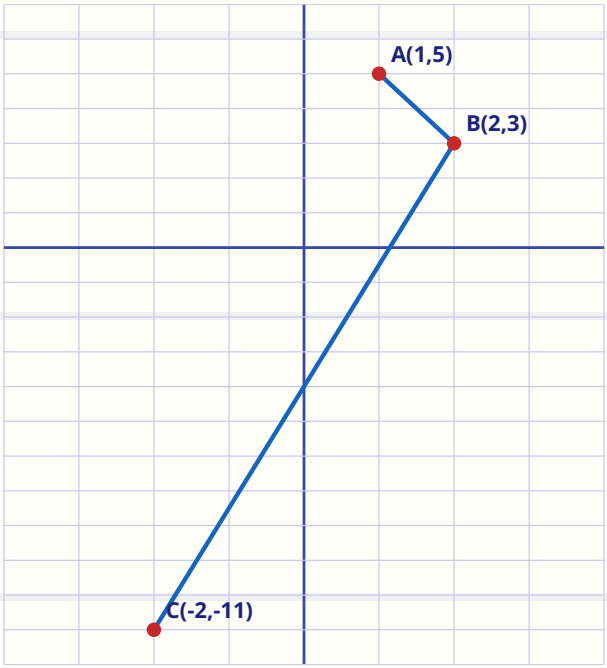
EXERCISE 7.1 Q2

Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in the NCERT introduction?

THINK FIRST: One point is the origin, so the special form $\sqrt{(x^2 + y^2)}$ applies directly.

1. Distance = $\sqrt{(36^2 + 15^2)} = \sqrt{(1296 + 225)} = \sqrt{1521}$.
2. $1521 = 39^2$ (since $40^2 = 1600$ and $39^2 = 1521$), so distance = **39 units**.

Exercise 7.1 Q3 — Are They Collinear?



EXERCISE 7.1 Q3

Determine if the points (1, 5), (2, 3) and (−2, −11) are collinear.

THINK FIRST: Collinear test: compute all three distances. The points are collinear ONLY if the largest distance equals the sum of the other two, exactly.

1. AB = $\sqrt{((2-1)^2 + (3-5)^2)} = \sqrt{(1 + 4)} = \mathbf{\sqrt{5}} \approx 2.24$.

2. BC = $\sqrt{((-2-2)^2 + (-11-3)^2)} = \sqrt{(16 + 196)} = \sqrt{212} = \mathbf{2\sqrt{53}} \approx 14.56$.

3. AC = $\sqrt{((-2-1)^2 + (-11-5)^2)} = \sqrt{(9 + 256)} = \mathbf{\sqrt{265}} \approx 16.28$.

4. Check the largest: AB + BC ≈ 2.24 + 14.56 = 16.80, but AC ≈ 16.28. They are NOT equal.
   Why: Close is not equal. In collinearity, 'almost' is a NO.

5. (Cross-check with the extension area test: area = ½|1(3+11) + 2(−11−5) + (−2)(5−3)| = ½|14 − 32 − 4| = 11 ≠ 0. Same conclusion.)

ANSWER: The points are NOT collinear.

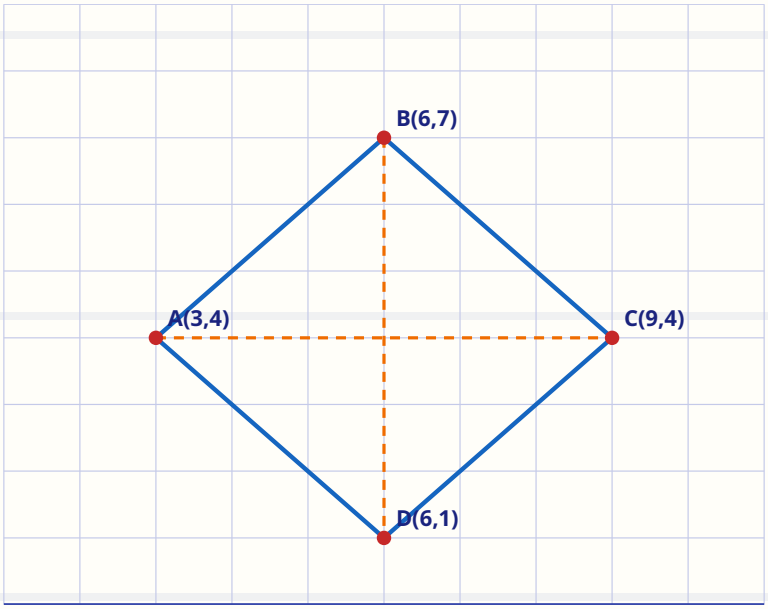
CBSE-STYLE MARKING SCHEME: 2 marks: 1 for the three distances, 1 for the correct comparison and conclusion.

EXERCISE 7.1 Q4

Check whether (5, −2), (6, 4) and (7, −2) are the vertices of an isosceles triangle.

THINK FIRST: An isosceles triangle needs TWO equal sides. Compute all three sides and compare.

Exercise 7.1 Q5 — The Classroom Square



EXERCISE 7.1 Q5

In a classroom, 4 friends are seated at the points A(3, 4), B(6, 7), C(9, 4) and D(6, 1). Champa says ABCD is a square; Chameli disagrees. Who is correct, and why?

THINK FIRST: A square needs BOTH: all four sides equal AND both diagonals equal. Sides alone could be a rhombus; diagonals alone could be a rectangle. Check everything.

1. AB = $\sqrt{((6-3)^2 + (7-4)^2)} = \sqrt{(9 + 9)} = \mathbf{\sqrt{18} = 3\sqrt{2}}$.

2. BC = $\sqrt{((9-6)^2 + (4-7)^2)} = \sqrt{(9 + 9)} = \mathbf{3\sqrt{2}}$.

3. CD = $\sqrt{((6-9)^2 + (1-4)^2)} = \sqrt{(9 + 9)} = \mathbf{3\sqrt{2}}$; DA = $\sqrt{((3-6)^2 + (4-1)^2)} = \sqrt{(9 + 9)} = \mathbf{3\sqrt{2}}$.
   Why: All four sides equal — so far it is at least a rhombus.

4. Diagonal AC = $\sqrt{((9-3)^2 + (4-4)^2)} = \sqrt{36} = \mathbf{6}$.

5. Diagonal BD = $\sqrt{((6-6)^2 + (1-7)^2)} = \sqrt{36} = \mathbf{6}$.
   Why: A rhombus with EQUAL diagonals is a square — this is the deciding test.

ANSWER: Champa is correct: all sides are 3√2 and both diagonals are 6, so ABCD is a square.

CBSE-STYLE MARKING SCHEME: 3 marks: 1 for the four sides, 1 for the two diagonals, 1 for the conclusion with the reason.

PIN POINT: Shape tests by coordinates: *equal sides + equal diagonals = square*; equal sides + unequal diagonals = rhombus; equal diagonals + unequal adjacent sides = rectangle; only opposite sides equal = parallelogram.

Exercise 7.1 Q6 — Name the Quadrilateral

EXERCISE 7.1 Q6 (i)

Name the type of quadrilateral formed by (−1, −2), (1, 0), (−1, 2), (−3, 0), taken in order. Give reasons.

THINK FIRST: Same detective kit as Q5: four sides, then two diagonals.

1. AB = $\sqrt{((1+1)^2 + (0+2)^2)} = \sqrt{(4 + 4)} = \mathbf{\sqrt{8} = 2\sqrt{2}}$.
2. BC = $\sqrt{((-1-1)^2 + (2-0)^2)} = \sqrt{(4 + 4)} = \mathbf{2\sqrt{2}}$.
3. CD = $\sqrt{((-3+1)^2 + (0-2)^2)} = \sqrt{(4 + 4)} = \mathbf{2\sqrt{2}}$; DA = $\sqrt{((-1+3)^2 + (-2-0)^2)} = \sqrt{(4 + 4)} = \mathbf{2\sqrt{2}}$.
4. Diagonal AC = $\sqrt{(0 + 16)} = \mathbf{4}$; diagonal BD = $\sqrt{(16 + 0)} = \mathbf{4}$.
5. All sides equal AND diagonals equal.

ANSWER: (i) A square.

CBSE-STYLE MARKING SCHEME: 2 marks: 1 for sides and diagonals, 1 for the name with reason.

EXERCISE 7.1 Q6 (ii)

Name the type of quadrilateral formed by (−3, 5), (3, 1), (0, 3), (−1, −4), taken in order. Give reasons.

THINK FIRST: The point (0, 3) looks suspiciously close to the line from (−3, 5) to (3, 1). Test that first.

1. AC (first to third point) = $\sqrt{((0+3)^2 + (3-5)^2)} = \sqrt{(9 + 4)} = \mathbf{\sqrt{13}}$.
   Why: Labels: A(−3, 5), B(3, 1), C(0, 3), D(−1, −4).
2. AB = $\sqrt{((3+3)^2 + (1-5)^2)} = \sqrt{(36 + 16)} = \mathbf{\sqrt{52} = 2\sqrt{13}}$.
3. CB = $\sqrt{((3-0)^2 + (1-3)^2)} = \sqrt{(9 + 4)} = \mathbf{\sqrt{13}}$.
4. AC + CB = $\sqrt{13} + \sqrt{13} = 2\sqrt{13}$ = AB. So C lies ON the segment AB — the points A, C, B are collinear.
   Why: Three points in one line — no four-sided figure can close.

Exercise 7.1 Q6 (iii) and Q7

EXERCISE 7.1 Q6 (iii)

Name the type of quadrilateral formed by (4, 5), (7, 6), (4, 3), (1, 2), taken in order. Give reasons.

THINK FIRST: Sides first, diagonals as the tie-breaker.

1. AB = $\sqrt{((7-4)^2 + (6-5)^2)} = \sqrt{(9 + 1)} = \mathbf{\sqrt{10}}$.
2. BC = $\sqrt{((4-7)^2 + (3-6)^2)} = \sqrt{(9 + 9)} = \mathbf{\sqrt{18} = 3\sqrt{2}}$.
3. CD = $\sqrt{((1-4)^2 + (2-3)^2)} = \sqrt{(9 + 1)} = \mathbf{\sqrt{10}}$; DA = $\sqrt{((4-1)^2 + (5-2)^2)} = \sqrt{(9 + 9)} = \mathbf{3\sqrt{2}}$.
   Why: Opposite sides are equal in pairs.
4. Diagonal AC = $\sqrt{(0 + 4)} = \mathbf{2}$; diagonal BD = $\sqrt{(36 + 16)} = \mathbf{\sqrt{52} = 2\sqrt{13}}$. The diagonals are NOT equal.
   Why: Unequal diagonals rule out rectangle and square.

ANSWER: (iii) A parallelogram (opposite sides equal, diagonals unequal).

CBSE-STYLE MARKING SCHEME: 2 marks: 1 for the side/diagonal evidence, 1 for the name with reason.

EXERCISE 7.1 Q7

Find the point on the x-axis which is equidistant from (2, −5) and (−2, 9).

THINK FIRST: A point on the x-axis has the form (x, 0). Equidistant means: distance to the first point EQUALS distance to the second.

1. Let the point be P(x, 0). $PA^2 = (x - 2)^2 + (0 - (-5))^2 = (x - 2)^2 + 25$.
   Why: Set up with squares from the start – cleaner algebra, no roots.
2. $PB^2 = (x - (-2))^2 + (0 - 9)^2 = (x + 2)^2 + 81$.
3. Equidistant: $PA^2 = PB^2 \Rightarrow (x - 2)^2 + 25 = (x + 2)^2 + 81$.
4. Expand and cancel x^2 and 4: $-4x + 25 = 4x + 81$.
   Why: The equation is secretly LINEAR – equidistant equations always are.
5. $-8x = 56$, so $x = \mathbf{-7}$.

Exercise 7.1 Q8 and Q9

EXERCISE 7.1 Q8

Find the values of y for which the distance between the points P(2, −3) and Q(10, y) is 10 units.

THINK FIRST: Distance is given; the coordinate is unknown. Write the formula, square both sides, and expect TWO answers — a point at a fixed distance can sit on either side.

1. $PQ^2 = (10 - 2)^2 + (y - (-3))^2 = 64 + (y + 3)^2$.
2. PQ = 10, so $PQ^2 = 100$: $64 + (y + 3)^2 = 100 \Rightarrow (y + 3)^2 = 36$.
3. $y + 3 = 6$ or $y + 3 = -6$, giving $y = 3$ or $y = -9$.

   Why: A square equation has two roots — reporting only one loses half the answer.

ANSWER: y = 3 or y = −9.

CBSE-STYLE MARKING SCHEME: 2 marks: 1 for $(y + 3)^2 = 36$, 1 for BOTH values of y.

EXERCISE 7.1 Q9

If Q(0, 1) is equidistant from P(5, −3) and R(x, 6), find the values of x. Also find the distances QR and PR.

THINK FIRST: Equidistant again — but watch out: x appears inside a square, so expect two positions for R, and PR will differ for each.

1. $QP^2 = (5 - 0)^2 + (-3 - 1)^2 = 25 + 16 = \mathbf{41}$.
2. $QR^2 = (x - 0)^2 + (6 - 1)^2 = x^2 + 25$.
3. Equidistant: $x^2 + 25 = 41 \Rightarrow x^2 = 16 \Rightarrow$ **x = 4 or x = −4**.
4. QR = √41 (for both positions, since $QR^2 = 41$).
5. If x = 4: R(4, 6), PR = $\sqrt{((5-4)^2 + (-3-6)^2)} = \sqrt{(1 + 81)} = \mathbf{\sqrt{82}}$.
6. If x = −4: R(−4, 6), PR = $\sqrt{((5+4)^2 + (-3-6)^2)} = \sqrt{(81 + 81)} = \sqrt{162} = \mathbf{9\sqrt{2}}$.

ANSWER: x = 4 or −4; QR = √41; PR = √82 (when x = 4) or 9√2 (when x = −4).

CBSE-STYLE MARKING SCHEME: 3 marks: 1 for x = ±4, 1 for QR, 1 for both values of PR.

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Exercise 7.1 Q10 — The Perpendicular Bisector in Disguise

EXERCISE 7.1 Q10

Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (−3, 4).

THINK FIRST: No specific point is asked — instead, ALL points (x, y) with the equidistance property. Their collective address is one equation: the relation.

1. Let P(x, y) be any point equidistant from A(3, 6) and B(−3, 4). Then $PA^2 = PB^2$.
2. $PA^2 = (x - 3)^2 + (y - 6)^2 = x^2 - 6x + 9 + y^2 - 12y + 36$.
3. $PB^2 = (x + 3)^2 + (y - 4)^2 = x^2 + 6x + 9 + y^2 - 8y + 16$.
4. Equate: $x^2 - 6x + y^2 - 12y + 45 = x^2 + 6x + y^2 - 8y + 25$. Cancel x^2, y^2: $-6x - 12y + 45 = 6x - 8y + 25$.
5. Collect: $-12x - 4y + 20 = 0$. Divide by −4: **$3x + y - 5 = 0$**.
6. (Meaning: every point on the line $3x + y = 5$ is equidistant from A and B — this line is the perpendicular bisector of AB. Its midpoint $((3-3)/2, (6+4)/2) = (0, 5)$ indeed satisfies $0 + 5 = 5$.)

   Why: A beautiful check: the midpoint of AB must lie on the equidistant line — and it does.

ANSWER: The relation is $3x + y - 5 = 0$ (i.e. $3x + y = 5$).

CBSE MARKING SCHEME: 2 marks: 1 for equating the squared distances, 1 for the simplified relation.

TOPPER TIP: *Q7, Q9 and Q10 are one family: 'equidistant' always becomes 'square both distances and equate'. The x^2 and y^2 terms always cancel, leaving a friendly linear equation. Recognise the family and all three questions take two minutes each.*

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Exercise 7.2 — The Section Formula

EXERCISE 7.2 Q1

Find the coordinates of the point which divides the join of (−1, 7) and (4, −3) in the ratio 2 : 3.

THINK FIRST: Section formula with m = 2, n = 3 — remember the cross-over: m multiplies the SECOND point.

1. Here $(x_1, y_1) = (-1, 7)$, $(x_2, y_2) = (4, -3)$, m = 2, n = 3.
   Why: Label the points BEFORE substituting.
2. $x = (m\,x_2 + n\,x_1)/(m + n) = (2\times4 + 3\times(-1))/5 = (8 - 3)/5 = \mathbf{1}$.
3. $y = (m\,y_2 + n\,y_1)/(m + n) = (2\times(-3) + 3\times7)/5 = (-6 + 21)/5 = \mathbf{3}$.
4. Sanity check: (1, 3) lies between the two points, closer to (−1, 7) — correct, since its share 2 is smaller than 3.
   Why: The point sits 2/5 of the way from the first point to the second.

ANSWER: The point is (1, 3).

CBSE MARKING SCHEME: 2 marks: 1 for correct substitution, 1 for (1, 3).

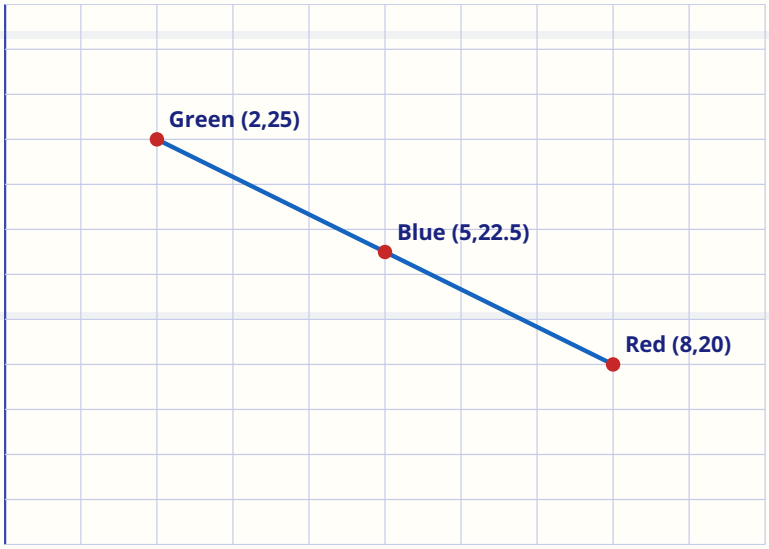
EXERCISE 7.2 Q2

Find the coordinates of the points of trisection of the line segment joining (4, −1) and (−2, −3).

THINK FIRST: Trisection = two points cutting the segment into three equal parts: ratios 1 : 2 and 2 : 1.

1. Let A(4, −1), B(−2, −3). First trisection point P divides AB in the ratio 1 : 2.
2. P: $x = (1\times(-2) + 2\times4)/3 = 6/3 = \mathbf{2}$; $y = (1\times(-3) + 2\times(-1))/3 = -5/3$.
3. Second trisection point Q divides AB in the ratio 2 : 1.
4. Q: $x = (2\times(-2) + 1\times4)/3 = 0/3 = \mathbf{0}$; $y = (2\times(-3) + 1\times(-1))/3 = -7/3$.

Exercise 7.2 Q3 — Sports Day Flags



EXERCISE 7.2 Q3

To conduct Sports Day activities, in your rectangular-shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. If AD is 100 m, Niharika runs 1/4 of the distance AD on the 2nd line and posts a green flag, and Preet runs 1/5 of AD on the 8th line and posts a red flag. (i) What is the distance between the two flags? (ii) If Rashmi has to post a blue flag exactly halfway between the two flags, where should she post it?

THINK FIRST: The ground is a coordinate grid in disguise: the LINE number is the x-coordinate, and the distance run along AD is the y-coordinate.

1. Green flag: 2nd line, and 1/4 of 100 m = 25 m along. Coordinates: **(2, 25)**.
2. Red flag: 8th line, and 1/5 of 100 m = 20 m along. Coordinates: **(8, 20)**.
3. (i) Distance between flags = $\sqrt{((8-2)^2 + (20-25)^2)} = \sqrt{(36 + 25)} = \mathbf{\sqrt{61}\ m} \approx 7.81$ m.
4. (ii) Halfway = midpoint: $((2 + 8)/2, (25 + 20)/2) = (5, 22.5)$.

   Why: Halfway between two points is ALWAYS the midpoint — no other formula is invited.

ANSWER: (i) The flags are √61 m (about 7.81 m) apart. (ii) Rashmi should post the blue flag on the 5th line, 22.5 m along AD — at (5, 22.5).

CBSE-STYLE MARKING SCHEME: 3 marks: 1 for both flag coordinates, 1 for √61, 1 for the midpoint with its line position.

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Exercise 7.2 Q4 and Q5

EXERCISE 7.2 Q4

Find the ratio in which the line segment joining the points (−3, 10) and (6, −8) is divided by (−1, 6).

THINK FIRST: The dividing point is GIVEN and the ratio is the unknown. Call the ratio k : 1 and let the x-coordinate do the work; then verify with y.

1. Let the ratio be k : 1. By the section formula, the x-coordinate of the dividing point is (6k + (−3))/(k + 1) = (6k − 3)/(k + 1).
   Why: With ratio k : 1, m = k and n = 1.
2. This must equal −1: (6k − 3)/(k + 1) = −1 ⇒ 6k − 3 = −k − 1 ⇒ 7k = 2 ⇒ k = 2/7.
3. So the ratio is k : 1 = **2 : 7**.
4. Verify with y: (−8k + 10)/(k + 1) with k = 2/7 gives (−16/7 + 10)/(9/7) = (54/7)/(9/7) = 6. Matches the given point (−1, 6).
   Why: Always verify a found ratio with the OTHER coordinate – it costs ten seconds and catches every slip.

ANSWER: The point (−1, 6) divides the segment in the ratio 2 : 7.

CBSE-STYLE MARKING SCHEME: 2 marks: 1 for the equation in k, 1 for the ratio 2 : 7.

EXERCISE 7.2 Q5

Find the ratio in which the line segment joining A(1, −5) and B(−4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

THINK FIRST: Any point on the x-axis has y-coordinate 0. That single fact creates the equation.

1. Let the x-axis divide AB in the ratio k : 1 at point P. P's y-coordinate = (5k + (−5))/(k + 1) = (5k − 5)/(k + 1).
2. P lies on the x-axis, so its y-coordinate is 0: 5k − 5 = 0 ⇒ k = 1. Ratio = **1 : 1**.
   Why: Ratio 1 : 1 means the x-axis cuts AB at its MIDPOINT.
3. Point of division = midpoint of AB = ((1 + (−4))/2, (−5 + 5)/2) = **(−3/2, 0)**.

ANSWER: The x-axis divides AB in the ratio 1 : 1, at the point (−3/2, 0).

CBSE MARKING SCHEME: 2 marks: 1 for the ratio via y = 0, 1 for the point.

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Exercise 7.2 Q6 and Q7

EXERCISE 7.2 Q6

If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

THINK FIRST: Parallelogram magic word: the DIAGONALS BISECT EACH OTHER. So the midpoint of one diagonal equals the midpoint of the other — two tiny equations.

1. Vertices in order: A(1, 2), B(4, y), C(x, 6), D(3, 5). The diagonals are AC and BD.
   Why: Taken in order matters: A pairs with C, B pairs with D.
2. Midpoint of AC = ((1 + x)/2, (2 + 6)/2) = ((1 + x)/2, 4).
3. Midpoint of BD = ((4 + 3)/2, (y + 5)/2) = (7/2, (y + 5)/2).
4. Equate x-parts: (1 + x)/2 = 7/2 ⇒ 1 + x = 7 ⇒ **x = 6**.
5. Equate y-parts: (y + 5)/2 = 4 ⇒ y + 5 = 8 ⇒ **y = 3**.

ANSWER: x = 6 and y = 3.

CBSE MARKING SCHEME: 2 marks: 1 for the bisect idea, ½ + ½ for x and y.

EXERCISE 7.2 Q7

Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, −3) and B is the point (1, 4).

THINK FIRST: The centre of a circle is the MIDPOINT of every diameter. So (2, −3) is the midpoint of A and B — run the midpoint formula backwards.

1. Let A = (x, y). Midpoint of AB = ((x + 1)/2, (y + 4)/2) = (2, −3).
2. x-part: (x + 1)/2 = 2 ⇒ x + 1 = 4 ⇒ **x = 3**.
3. y-part: (y + 4)/2 = −3 ⇒ y + 4 = −6 ⇒ **y = −10**.

ANSWER: A = (3, −10).

CBSE-STYLE MARKING SCHEME: 2 marks: 1 for the midpoint setup, 1 for A(3, −10).

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Exercise 7.2 Q8 and Q9

EXERCISE 7.2 Q8

If A and B are (−2, −2) and (2, −4) respectively, find the coordinates of P such that AP = (3/7) AB and P lies on the line segment AB.

THINK FIRST: AP is 3/7 of the whole segment, so the remaining PB is 4/7. That makes the ratio AP : PB = 3 : 4 — converting a fraction-of-length into a ratio is the whole trick of this question.

1. AP = (3/7) AB ⇒ PB = AB − AP = (4/7) AB. So P divides AB in the ratio AP : PB = **3 : 4**.
   Why: Fractions of a segment convert to ratios by subtracting from the whole: 3/7 taken, 4/7 left.
2. x-coordinate: (3×2 + 4×(−2))/(3 + 4) = (6 − 8)/7 = **−2/7**.
3. y-coordinate: (3×(−4) + 4×(−2))/7 = (−12 − 8)/7 = **−20/7**.

ANSWER: P = (−2/7, −20/7).

CBSE-STYLE MARKING SCHEME: 2 marks: 1 for converting 3/7 into the ratio 3 : 4, 1 for P.

EXERCISE 7.2 Q9

Find the coordinates of the points which divide the line segment joining A(−2, 2) and B(2, 8) into four equal parts.

THINK FIRST: Four equal parts need THREE points, dividing AB in the ratios 1 : 3, 1 : 1 (the midpoint) and 3 : 1. Three section-formula runs — or find the midpoint first, then halve each half.

1. Point 1 (ratio 1 : 3): x = (1×2 + 3×(−2))/4 = −4/4 = **−1**; y = (1×8 + 3×2)/4 = 14/4 = **7/2**. Point: (−1, 7/2).
2. Point 2 (ratio 1 : 1, the midpoint): ((−2 + 2)/2, (2 + 8)/2) = **(0, 5)**.
3. Point 3 (ratio 3 : 1): x = (3×2 + 1×(−2))/4 = 4/4 = **1**; y = (3×8 + 1×2)/4 = 26/4 = **13/2**. Point: (1, 13/2).
4. Check the even spacing: x goes −2, −1, 0, 1, 2 (steps of 1) and y goes 2, 7/2, 5, 13/2, 8 (steps of 3/2). Perfectly even — four equal parts confirmed.

ANSWER: The three points are (−1, 7/2), (0, 5) and (1, 13/2).

CBSE-STYLE MARKING SCHEME: 3 marks: 1 for each dividing point.

Exercise 7.2 Q10 — The Rhombus

EXERCISE 7.2 Q10

Find the area of a rhombus whose vertices are (3, 0), (4, 5), (−1, 4) and (−2, −1) taken in order. (Hint: area of a rhombus = ½ × product of its diagonals.)

THINK FIRST: A rhombus's area comes from its DIAGONALS, and diagonals are just distances between opposite vertices. Two distance-formula runs, then the hint's formula.

1. Vertices in order: A(3, 0), B(4, 5), C(−1, 4), D(−2, −1). Diagonal AC joins A to C.

2. AC = $\sqrt{((-1-3)^2 + (4-0)^2)} = \sqrt{(16 + 16)} = \sqrt{32} = \mathbf{4\sqrt{2}}$.

3. BD = $\sqrt{((-2-4)^2 + (-1-5)^2)} = \sqrt{(36 + 36)} = \sqrt{72} = \mathbf{6\sqrt{2}}$.

4. Area = ½ × AC × BD = ½ × $4\sqrt{2}$ × $6\sqrt{2}$ = ½ × 24 × 2 = **24 square units**.
   Why: $\sqrt{2} \times \sqrt{2} = 2$ — the roots dissolve, leaving a whole number.

ANSWER: The area of the rhombus is 24 square units.

CBSE MARKING SCHEME: 3 marks: 1 for each diagonal, 1 for the area.

TOPPER TIP: *End of the NCERT exercises. If you solved along, you have now used the distance formula 20+ times and the section formula 15+ times. The formulas are no longer formulas — they are habits. Sections 08–15 now stretch those habits.*

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08 Multiple Solution Methods

8D • CHALLENGE — ONE PROBLEM, SEVERAL ROADS

A topper is never trapped by one method. Here are this chapter's three classic problems, each solved two ways — learn to choose the faster road.

METHOD DUEL 1

Find the distance between (1, 2) and (4, 6).

1. **Method 1 — Formula:** $\sqrt{((4-1)^2 + (6-2)^2)} = \sqrt{(9 + 16)} = \sqrt{25} = 5$.
2. **Method 2 — Picture:** Plot the points, complete the right triangle: horizontal leg 3, vertical leg 4, hypotenuse 5 by the 3-4-5 triple — no formula written at all.

   Why: For friendly numbers, the picture IS the calculation.

ANSWER: Distance = 5 units, by both roads.

METHOD DUEL 2

Find the midpoint of (−3, 5) and (5, −1).

1. **Method 1 — Midpoint formula:** ((−3 + 5)/2, (5 + (−1))/2) = (2/2, 4/2) = (1, 2).
2. **Method 2 — Section formula with m = n = 1:** x = (1×5 + 1×(−3))/2 = 1; y = (1×(−1) + 1×5)/2 = 2. Same point (1, 2) — as Derivation C promised.

ANSWER: Midpoint = (1, 2), by both roads.

METHOD DUEL 3

Show that (1, 1), (4, 5) and (7, 9) are collinear.

1. **Method 1 — Distances:** AB = $\sqrt{(9 + 16)} = 5$; BC = $\sqrt{(9 + 16)} = 5$; AC = $\sqrt{(36 + 64)} = 10$. AB + BC = 10 = AC, so the points are collinear.
2. **Method 2 — Area test (extension):** area = ½|1(5−9) + 4(9−1) + 7(1−5)| = ½|−4 + 32 − 28| = 0. Zero area — no triangle exists — collinear.

   Why: The area test is faster when the coordinates are large and the distances are ugly roots.

ANSWER: The points are collinear, proved both ways.

TOPPER TIP: *Exam strategy: distances when numbers are small and friendly; area test when coordinates are big; midpoint/section whenever the word 'divide' appears. Choosing the road is worth as many marks as walking it.*

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09 Interactive Simulations & Hands-on Activities

TRY IT YOURSELF — LEARN BY DOING

Coordinates are learned by the hand, not by the eye. Do these four activities with graph paper, a ruler and a pencil before reading the answers.

Activity 1 — Measure it, then compute it

Plot A(1, 1) and B(5, 4) on graph paper. Measure AB with a ruler (in grid units). Then compute with the distance formula: $\sqrt{((5-1)^2 + (4-1)^2)} = \sqrt{(16 + 9)} = 5$. Your ruler and the formula must agree — if they don't, your plotting is wrong, not the formula.

Activity 2 — The treasure midpoint

A treasure is buried exactly halfway between the old well W(0, 0) and the banyan tree T(8, 6). Find the spot by the midpoint formula, plot all three points, and check with a ruler that both halves measure the same. (Answer in the key below.)

Activity 3 — The paper-strip section lab

Cut a paper strip 10 cm long; call its ends A and B. Mark the point that divides it in the ratio 3 : 7. Before measuring, predict: the point sits 3/(3+7) of the way from A, i.e. at 3 cm. Measure to confirm. Now repeat for 1 : 4 (predict 2 cm) and 4 : 1 (predict 8 cm). Feel in your fingers why the weights cross over in the formula.

Activity 4 (Extension) — Area by counting

Draw the triangle (0, 0), (4, 0), (0, 3) on squared paper. Count: it covers exactly half of a 4 × 3 rectangle, so its area is 6. Now the formula: $\frac{1}{2}|0(0-3) + 4(3-0) + 0(0-0)| = \frac{1}{2}(12) = 6$. Counting and formula agree.

DIGITAL SIMULATION: In the Coordinate Plane Simulator (KalviYogi Digital Companion, Section 01), drag point B around point A and watch the right triangle and the distance change live. Freeze B where the distance reads exactly 13 — how many such positions can you find? (Answer: infinitely many — a whole circle of them!)

Activity Answers — check after trying

Activity 2: Midpoint = ((0 + 8)/2, (0 + 6)/2) = **(4, 3)**. Both halves measure $\sqrt{(16 + 9)} = 5$ units.

Activity 3: Ratio m : n from A places the point at a fraction m/(m + n) of the strip: 3 : 7 → 3 cm; 1 : 4 → 2 cm; 4 : 1 → 8 cm.

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10 Common Mistakes

LEARN FROM OTHER STUDENTS' ERRORS — BEFORE THEY BECOME YOURS

COMMON MISTAKE: Mixing the pairs: writing distance as $\sqrt{[(x_2 - y_1)^2 + \ldots]}$. **Wrong:** for (2, 3) and (6, 7), computing $\sqrt{((6-3)^2 + \ldots)}$ — an x minus a y! **Right:** x always meets x, y always meets y: $\sqrt{((6-2)^2 + (7-3)^2)} = \sqrt{32} = 4\sqrt{2}$. **Why it matters:** the legs of the triangle are horizontal and vertical differences — never diagonal mixtures.

COMMON MISTAKE: Splitting the root over addition: **Wrong:** $\sqrt{(9 + 16)} = \sqrt{9} + \sqrt{16} = 3 + 4 = 7$. **Right:** add first, then root: $\sqrt{25} = 5$. **Why:** $\sqrt{}$ is not a bracket you can distribute — test with any numbers: $\sqrt{(9+16)} = 5$ but $3 + 4 = 7$.

COMMON MISTAKE: The vanishing minus: **Wrong:** $(-8)^2 = -64$, giving $\sqrt{(36 - 64)}$ and panic. **Right:** $(-8)^2 = +64$, always. A square of a negative is positive — this is exactly why the order of subtraction in the distance formula does not matter.

COMMON MISTAKE: Swapping the section-formula weights: to divide (0, 0)–(10, 0) in the ratio 2 : 3, **wrong:** $x = (2\times0 + 3\times10)/5 = 6$. **Right:** $x = (2\times10 + 3\times0)/5 = 4$ — m multiplies the FAR point. Sanity check: ratio 2 : 3 puts the point closer to the FIRST point (0), so 4 (not 6) is believable.

COMMON MISTAKE: Dividing only one coordinate by 2 in the midpoint: **Wrong:** midpoint of (2, 5) and (4, 9) written as (3, −2) by subtracting the y's, or as (6, 7) by forgetting to halve y. **Right:** average BOTH: ((2+4)/2, (5+9)/2) = (3, 7).

COMMON MISTAKE: The x-axis ratio trap: for the segment (2, −4)–(6, 8), **wrong:** announcing the x-axis divides it in the ratio 8 : 4 = 2 : 1 by reading the y's top-to-bottom. **Right:** the ratio is $(-y_1) : y_2 = 4 : 8 = 1 : 2$. The minus sign on the first y is part of the rule, not decoration.

COMMON MISTAKE: Declaring collinearity from 'close enough': in Ex 7.1 Q3 style questions, **wrong:** 'AB + BC ≈ 16.8 and AC ≈ 16.3 — nearly equal, so collinear'. **Right:** collinearity demands EXACT equality (or area exactly 0). 'Nearly collinear' is not collinear.

11 Graded Practice Problems

FIVE LEVELS — CLIMB ONE LEVEL AT A TIME • ANSWERS IN THE ANSWER KEY

Start at Level 1 and climb only when a level feels easy. Every question is original; the Answer Key waits at the end of the chapter.

LEVEL 1 — FOUNDATION

L1.1 Find the distance between (3, 4) and (0, 0).

L1.2 Find the distance of the point (−6, 8) from the origin.

L1.3 Find the midpoint of the segment joining (2, 4) and (6, 8).

L1.4 Find the distance between (1, 2) and (4, 6).

L1.5 Find the point which divides the join of (0, 0) and (6, 3) in the ratio 1 : 2.

L1.6 Find the distance between (5, 1) and (5, 7). (Look before you compute!)

LEVEL 2 — SKILL BUILDER

L2.1 Find the distance between (−2, 3) and (3, 15).

L2.2 Find the midpoint of (−5, 7) and (3, −1).

L2.3 Find the point which divides (2, 3)–(8, 9) in the ratio 2 : 1.

L2.4 Show that (1, 2), (3, 4) and (5, 6) are collinear.

L2.5 The distance between (3, −1) and (6, y) is 5 units. Find the values of y.

L2.6 Find the point on the y-axis which is equidistant from (5, 2) and (−3, 4).

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Graded Practice — Levels 3, 4 and 5

LEVEL 3 — APPLICATION

L3.1 On a town map (1 unit = 1 km), Asha's house is at (2, 3) and her office at (8, 11). How far is the office from her house, as the crow flies?

L3.2 A rectangular park has corners (0, 0), (10, 0), (10, 6) and (0, 6). A fountain is placed at the centre of the park. Find the fountain's coordinates and its distance from the corner (0, 0).

L3.3 A boat sails in a straight line from a jetty at (0, 0) to an island at (12, 5). How long is the sailing route? At what point is the boat exactly halfway?

L3.4 Ravi's school is at S(1, 1) and his home at H(7, 9). A bookshop stands exactly midway between them. Find its coordinates.

LEVEL 4 — ADVANCED REASONING

L4.1 Find k if the point (k, 2) is equidistant from (3, 4) and (5, 6).

L4.2 A(1, 1), B(4, 2) and C(5, 5) are three vertices of a parallelogram ABCD, taken in order. Find the fourth vertex D.

L4.3 Prove that A(2, 4), B(2, 1) and C(5, 1) form a right-angled isosceles triangle, and find the midpoint of its hypotenuse.

LEVEL 5 — MATHEMATICAL GENIUS

L5.1 Two opposite vertices of a square are (5, 4) and (−1, −2). Find the other two vertices. (Condition: all four vertices must form a genuine square — verify each side.)

L5.2 Find ALL points which are equidistant from A(3, 6) and B(−3, 4) AND lie at a distance of exactly 5 units from the origin. (Hint: you met the first condition in Ex 7.1 Q10.)

> *TOPPER TIP: Level 5 is not about speed — sit with these two problems for a full evening if needed. L5.1 needs the midpoint AND the 'turn the half-diagonal by 90°' idea; L5.2 marries this chapter to a circle.*

12 Mathematical Reasoning

JUSTIFY AND PROVE — FULL SOLUTIONS

REASONING 1

Using coordinates, prove that the diagonals of a rectangle are equal in length.

THINK FIRST: The trick of every coordinate proof: place the figure cleverly, then let the distance formula speak.

1. Place the rectangle at the friendliest possible address: O(0, 0), A(a, 0), B(a, b), C(0, b), where a and b are the side lengths.
   Why: Coordinate proof rule 1: YOU choose where the figure lives. Choose the origin and axes to make the algebra lazy.
2. Diagonal OB = $\sqrt{(a^2 + b^2)}$ (distance from (0, 0) to (a, b)).
3. Diagonal AC = $\sqrt{((0-a)^2 + (b-0)^2)} = \sqrt{(a^2 + b^2)}$.
4. OB = AC. Since a and b were ANY side lengths, every rectangle has equal diagonals. ∎
   Why: One proof with letters beats a thousand examples with numbers.

ANSWER: Diagonals of a rectangle are equal — proved for all rectangles at once.

REASONING 2

Prove that the midpoint of the hypotenuse of a right-angled triangle is equidistant from all three vertices.

THINK FIRST: Three distances, one surd — if all three computations land on $\sqrt{(a^2 + b^2)}$, the proof is complete.

1. Place the right angle at the origin: O(0, 0), A(2a, 0), B(0, 2b). (Writing 2a and 2b instead of a and b keeps the midpoint free of fractions.)
   Why: Rule 2: choose letters that make the coming arithmetic clean.
2. The hypotenuse is AB; its midpoint M = ((2a + 0)/2, (0 + 2b)/2) = (a, b).
3. MO = $\sqrt{(a^2 + b^2)}$.
4. MA = $\sqrt{((2a-a)^2 + (0-b)^2)} = \sqrt{(a^2 + b^2)}$.
5. MB = $\sqrt{((0-a)^2 + (2b-b)^2)} = \sqrt{(a^2 + b^2)}$.
6. MO = MA = MB, so M is equidistant from all three vertices. ∎ (This is why the midpoint of the hypotenuse is the circumcentre of a right triangle.)

Reasoning 3 — An Equilateral Triangle from Three Strange Points

REASONING 3

Prove that the points (a, a), (−a, −a) and (−√3 a, √3 a) are the vertices of an equilateral triangle (for a ≠ 0).

THINK FIRST: Ugly coordinates, beautiful cancellations. Trust the algebra and push through the expansions one line at a time.

1. Call the points P(a, a), Q(−a, −a), R(−√3 a, √3 a). An equilateral triangle needs all three squared distances equal — compare squares to avoid root clutter.

   Why: Compare SQUARED distances in proof questions: cleaner, and equality is unaffected.

2. $PQ^2 = (a + a)^2 + (a + a)^2 = 4a^2 + 4a^2 = \mathbf{8a^2}$.

3. $QR^2 = (-a + \sqrt{3}\,a)^2 + (-a - \sqrt{3}\,a)^2 = a^2(\sqrt{3} - 1)^2 + a^2(\sqrt{3} + 1)^2 = a^2[(4 - 2\sqrt{3}) + (4 + 2\sqrt{3})] = \mathbf{8a^2}$.

   Why: The √3 terms cancel in pairs — the problem was designed for this moment.

4. $RP^2 = (a + \sqrt{3}\,a)^2 + (a - \sqrt{3}\,a)^2 = a^2[(\sqrt{3} + 1)^2 + (\sqrt{3} - 1)^2] = \mathbf{8a^2}$.

5. $PQ^2 = QR^2 = RP^2 = 8a^2$, so $PQ = QR = RP = 2\sqrt{2}\,|a|$. All sides equal. ∎

ANSWER: The three points form an equilateral triangle of side $2\sqrt{2}\,|a|$.

TOPPER TIP: *Coordinate proofs follow one recipe: (1) place the figure at a friendly address, (2) compute with the distance/midpoint formulas, (3) compare. Master the recipe here and Chapter 6's theorem proofs feel twice as easy.*

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13 Real-Life Applications

8D • APPLY — THE CHAPTER AT WORK IN THE WORLD

APPLICATION 1

A lighthouse stands at L(0, 0) on the coast guard's chart. A fishing boat is at B(6, 8) and a cargo ship at C(−9, 12) (units in km). Which vessel is closer to the lighthouse, and by how much?

THINK FIRST: Radar screens are coordinate planes. Every blip is an (x, y); every range reading is this formula.

1. LB = $\sqrt{(6^2 + 8^2)} = \sqrt{100} =$ **10 km**.
   Why: Distance from the origin uses the short form $\sqrt{(x^2 + y^2)}$.
2. LC = $\sqrt{((-9)^2 + 12^2)} = \sqrt{(81 + 144)} = \sqrt{225} =$ **15 km**.
3. Difference: 15 − 10 = **5 km**.

ANSWER: The fishing boat is closer, by 5 km.

APPLICATION 2

A delivery drone leaves its hub at (0, 0) and must reach a house at (9, 12) (units in 100 m). The battery allows a flight path of at most 16 units. (a) Can the drone reach directly? (b) A charging pad sits exactly halfway — where is it?

THINK FIRST: Delivery companies solve this exact problem thousands of times a day — with your formula.

1. (a) Direct distance = $\sqrt{(9^2 + 12^2)} = \sqrt{(81 + 144)} = \sqrt{225} =$ **15 units** = 1500 m.
2. 15 < 16, so yes — the drone reaches with 1 unit (100 m) of range to spare.
3. (b) Charging pad = midpoint = ((0 + 9)/2, (0 + 12)/2) = **(4.5, 6)**.

ANSWER: (a) Yes, the direct flight is 15 units, within range. (b) The charging pad is at (4.5, 6).

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Real-Life Applications 3 and 4

APPLICATION 3

A town square park has corners P(1, 1), Q(1, 7), R(7, 7) and S(7, 1) (units in 10 m). (a) Show the park is square. (b) A lamp must be placed at the centre of the park — find its position. (c) How long is the diagonal footpath from P to R?

THINK FIRST: Town planners really do place lamps, fountains and gates with midpoints and diagonals.

1. (a) Side PQ = 7 − 1 = 6; QR = 7 − 1 = 6; RS = 6; SP = 6. All four sides are 6 units.
   Why: Sides parallel to the axes: just subtract.
2. (a) Diagonal PR = $\sqrt{((7-1)^2 + (7-1)^2)} = \sqrt{72} = 6\sqrt{2}$; diagonal QS = $\sqrt{((7-1)^2 + (1-7)^2)} = 6\sqrt{2}$. Equal diagonals too — the park is a square.
3. (b) Centre = midpoint of diagonal PR = ((1 + 7)/2, (1 + 7)/2) = **(4, 4)**.
   Why: The centre of any rectangle/square is the midpoint of a diagonal.
4. (c) The diagonal footpath PR = $6\sqrt{2}$ units ≈ 8.49 units ≈ **84.9 m**.

ANSWER: (a) Square (sides 6, diagonals $6\sqrt{2}$). (b) Lamp at (4, 4). (c) Diagonal path ≈ 84.9 m.

APPLICATION 4

Two mobile towers stand at $T_1(2, 3)$ and $T_2(8, 11)$ (units in km). A village V claims it receives equally strong signals from both towers, and surveyors find V is exactly midway between them. (a) Find V's coordinates. (b) How far is V from each tower?

THINK FIRST: 'Exactly midway' and 'equidistant' in one sentence: the midpoint formula answers both.

1. (a) V = midpoint of T_1T_2 = ((2 + 8)/2, (3 + 11)/2) = **(5, 7)**.
2. (b) $VT_1 = \sqrt{((5-2)^2 + (7-3)^2)} = \sqrt{(9 + 16)} =$ **5 km**.
3. Check: $VT_2 = \sqrt{((8-5)^2 + (11-7)^2)} = \sqrt{(9 + 16)} = 5$ km, and $T_1T_2 = \sqrt{(36 + 64)} = 10$ km = 5 + 5. V truly is the midpoint, equidistant from both.
   Why: A midpoint is equidistant from the ends BY CONSTRUCTION – the check just confirms the arithmetic.

ANSWER: (a) V = (5, 7). (b) V is 5 km from each tower.

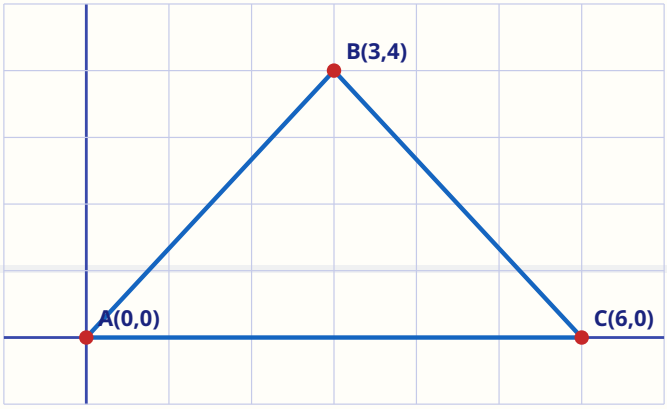
14 Competency-Based Questions

CASE STUDIES & ASSERTION–REASON — BOARD EXAM STYLE, FULLY SOLVED

Five case studies, four questions each — twenty competency questions, all solved step by step.

Case Study 1 — The Metro Map

A city metro map is drawn on a coordinate grid (1 unit = 1 km). Three stations are Central A(0, 0), Northgate B(3, 4) and Eastend C(6, 0).



CASE 1 (a)

Find the distance between Central and Northgate.

1. AB = $\sqrt{((3-0)^2 + (4-0)^2)} = \sqrt{(9 + 16)} =$ **5 km**.

ANSWER: AB = 5 km.

CASE 1 (b)

Find the distance between Northgate and Eastend.

1. BC = $\sqrt{((6-3)^2 + (0-4)^2)} = \sqrt{(9 + 16)} =$ **5 km**.

ANSWER: BC = 5 km.

CASE 1 (c)

A shuttle runs straight from Central to Eastend. Find AC, and the station point exactly midway on AC.

1. AC = $\sqrt{(36 + 0)} =$ **6 km**.
2. Midpoint of AC = $((0 + 6)/2, 0) =$ **(3, 0)**.

ANSWER: AC = 6 km; midway point (3, 0), directly below Northgate.

CASE 1 (d)

Case Study 2 — The School Garden

A school garden is a rectangle with corners O(0, 0), P(8, 0), Q(8, 6) and R(0, 6) (units in metres). A rose bed is planted at the centre, and a tap is fitted at corner O.

CASE 2 (a)

Find the coordinates of the rose bed at the centre of the garden.

1 Centre = midpoint of diagonal OQ = ((0 + 8)/2, (0 + 6)/2) = **(4, 3)**.

ANSWER: The rose bed is at (4, 3).

CASE 2 (b)

Find the length of the diagonal OQ.

1 OQ = $\sqrt{(8^2 + 6^2)} = \sqrt{100} =$ **10 m**.

Why: The 6-8-10 triple again — spot it and save the arithmetic.

ANSWER: OQ = 10 m.

CASE 2 (c)

How far is the rose bed from the tap at O?

1 Distance = $\sqrt{(4^2 + 3^2)} = \sqrt{25} =$ **5 m** — exactly half the diagonal, as a centre should be.

ANSWER: The rose bed is 5 m from the tap.

CASE 2 (d)

A second tap is fitted at Q(8, 6). Show that the rose bed is equidistant from both taps.

1 Distance to Q = $\sqrt{((8-4)^2 + (6-3)^2)} = \sqrt{(16 + 9)} =$ **5 m**, equal to the distance from O found in (c).

ANSWER: Both distances are 5 m — the centre is equidistant from opposite corners.

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Case Study 3 — The Cricket Throw

On a coaching app, a fielder stands at F(0, 0). The batsman is at B(12, 5), and the stumps are at S(24, 0) (units in metres).

CASE 3 (a)

How far must the fielder throw to reach the batsman?

1 FB = $\sqrt{(12^2 + 5^2)} = \sqrt{(144 + 25)} = \sqrt{169} =$ **13 m**.

ANSWER: FB = 13 m.

CASE 3 (b)

Find the midpoint of the pitch segment FS.

1 Midpoint = $((0 + 24)/2, (0 + 0)/2) =$ **(12, 0)**.

ANSWER: The midpoint is (12, 0).

CASE 3 (c)

How far is the batsman from this midpoint?

1 Distance from (12, 0) to B(12, 5) = $\sqrt{(0 + 25)} =$ **5 m** — a purely vertical gap.

ANSWER: The batsman is 5 m from the midpoint.

CASE 3 (d)

The coach says: 'The batsman stands exactly above the midpoint of FS.' Is he right?

1 B = (12, 5) and the midpoint is (12, 0): same x-coordinate 12, B is 5 m straight above. Yes — exactly above.

ANSWER: Yes — B(12, 5) is vertically above the midpoint (12, 0).

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Case Study 4 — The Delivery Hub

A warehouse W(1, 2) supplies two shops, $S_1(4, 6)$ and $S_2(7, 2)$, on a city grid (units in km).

CASE 4 (a)

Find the distance from the warehouse to shop S_1.

1. $WS_1 = \sqrt{((4-1)^2 + (6-2)^2)} = \sqrt{(9 + 16)} =$ **5 km**.

ANSWER: WS_1 = 5 km.

CASE 4 (b)

Find the distance from the warehouse to shop S_2.

1. $WS_2 = \sqrt{((7-1)^2 + (2-2)^2)} = \sqrt{36} =$ **6 km**.

ANSWER: WS_2 = 6 km.

CASE 4 (c)

Find the distance between the two shops.

1. $S_1S_2 = \sqrt{((7-4)^2 + (2-6)^2)} = \sqrt{(9 + 16)} =$ **5 km**.

ANSWER: S_1S_2 = 5 km.

CASE 4 (d)

A pickup hub is placed midway between the warehouse and S_2. Find its coordinates, and name the triangle W S_1 S_2's type.

1. Hub = midpoint of W(1, 2) and $S_2(7, 2) = ((1 + 7)/2, (2 + 2)/2) =$ **(4, 2)**.
2. Sides: $WS_1 = 5$, $S_1S_2 = 5$, $WS_2 = 6$ — two equal sides, so the triangle is **isosceles**.

ANSWER: Hub at (4, 2); $\triangle WS_1S_2$ is isosceles.

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Case Study 5 — The Ground Stations

Two satellite ground stations stand at $G_1(-4, 3)$ and $G_2(4, -3)$ (units in 100 km). A control centre sits at the origin O(0, 0).

CASE 5 (a)

Find the distance between the two ground stations.

1. $G_1G_2 = \sqrt{((4+4)^2 + (-3-3)^2)} = \sqrt{(64 + 36)} = \sqrt{100} =$ **10 units** = 1000 km.

ANSWER: The stations are 10 units (1000 km) apart.

CASE 5 (b)

Show that the control centre is the midpoint of G_1G_2.

1. Midpoint = $((-4 + 4)/2, (3 + (-3))/2) = (0, 0) = O$. Shown.

ANSWER: O(0, 0) is indeed the midpoint.

CASE 5 (c)

How far is each station from the control centre?

1. $OG_1 = \sqrt{((-4)^2 + 3^2)} = \sqrt{25} =$ **5 units**; $OG_2 = \sqrt{(4^2 + (-3)^2)} =$ **5 units**. Equal — as a midpoint demands.

ANSWER: Each station is 5 units (500 km) from the centre.

CASE 5 (d)

A third station G_3 is planned at (4, 3). Find the distance G_1G_3 and G_3G_2.

1. $G_1G_3 = \sqrt{((4+4)^2 + (3-3)^2)} =$ **8 units** (purely horizontal).
2. $G_3G_2 = \sqrt{((4-4)^2 + (3+3)^2)} =$ **6 units** (purely vertical).

ANSWER: G_1G_3 = 8 units and G_3G_2 = 6 units.

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Assertion–Reason 1–4

For each question, choose: **(a)** Both A and R are true and R is the correct explanation of A. **(b)** Both true, but R is NOT the correct explanation of A. **(c)** A is true, R is false. **(d)** A is false, R is true.

ASSERTION-REASON 1

Assertion (A): The distance between (2, 3) and (5, 7) is 5 units. Reason (R): The distance between (x_1, y_1) and (x_2, y_2) is $\sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2]}$.

1. A: $\sqrt{((5-2)^2 + (7-3)^2)} = \sqrt{(9 + 16)} = 5$. A is TRUE.
2. R states the distance formula correctly. R is TRUE, and it is exactly what proves A.

ANSWER: Option (a).

ASSERTION-REASON 2

Assertion (A): The midpoint of (2, 6) and (8, 4) is (5, 5). Reason (R): The midpoint of a segment is the average of the endpoints' coordinates.

1. A: midpoint = ((2+8)/2, (6+4)/2) = (5, 5). TRUE.
2. R is the correct definition and the direct explanation of A. TRUE.

ANSWER: Option (a).

ASSERTION-REASON 3

Assertion (A): The point (3, 0) divides the join of (1, 0) and (6, 0) in the ratio 2 : 3. Reason (R): The section formula is $((mx_2 + nx_1)/(m + n), (my_2 + ny_1)/(m + n))$.

1. A: dividing (1, 0)–(6, 0) in 2 : 3 gives $x = (2\times6 + 3\times1)/5 = 15/5 = 3$. Point (3, 0). TRUE.
2. R states the section formula correctly and explains A. TRUE.

ANSWER: Option (a).

ASSERTION-REASON 4

Assertion (A): The distance of the point (−5, 12) from the origin is 13 units. Reason (R): The distance of (x, y) from the origin is x + y.

1. A: $\sqrt{(25 + 144)} = \sqrt{169} = 13$. TRUE.

Assertion–Reason 5–7

ASSERTION-REASON 5

Assertion (A): The points (0, 0), (2, 3) and (4, 6) are collinear. Reason (R): Three points are collinear if the area of the triangle formed by them is zero.

1. A: area = ½|0(3−6) + 2(6−0) + 4(0−3)| = ½|0 + 12 − 12| = 0. TRUE.
2. R is the standard area test — TRUE, and it is precisely how A was verified.

ANSWER: Option (a).

ASSERTION-REASON 6

Assertion (A): The x-axis divides the join of (2, −3) and (4, 5) in the ratio 3 : 5. Reason (R): The x-axis divides a segment in the ratio $(-y_1) : y_2$.

1. A: ratio = (−(−3)) : 5 = 3 : 5. TRUE.
2. R states the correct rule and explains A. TRUE.

ANSWER: Option (a).

ASSERTION-REASON 7

Assertion (A): The midpoint of the segment joining (1, 2) and (6, 6) is the same point as the midpoint of the segment joining (4, 3) and (3, 5). Reason (R): In a parallelogram, the diagonals bisect each other.

1. Midpoint of (1, 2)–(6, 6) = (7/2, 4). Midpoint of (4, 3)–(3, 5) = (7/2, 4). Identical — A is TRUE.
2. The four points, taken in order, form a parallelogram (opposite sides: √10, √13, √10, √13), and the two segments are its diagonals. R is TRUE and is exactly why the midpoints coincide.

ANSWER: Option (a).

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Assertion–Reason 8–10

ASSERTION-REASON 8

Assertion (A): The point (−1, 6) divides the join of (−3, 10) and (6, −8) in the ratio 2 : 7.
Reason (R): If a point P(x, y) divides AB in the ratio m : n, then $x = (mx_2 + nx_1)/(m + n)$.

1. From x: the fraction along = (−1 − (−3))/(6 − (−3)) = 2/9, so the ratio is 2 : (9 − 2) = 2 : 7.
2. From y: (6 − 10)/(−8 − 10) = (−4)/(−18) = 2/9. Same fraction — A is TRUE.
   Why: A claimed dividing point must give the SAME fraction from x and from y; checking both is the full verification.
3. R states the section formula's x-part correctly and explains A. TRUE.

ANSWER: Option (a). (This pair also appears in the Previous Year Questions — it is a genuine board favourite.)

ASSERTION-REASON 9

Assertion (A): The distance between (a, b) and (−a, −b) is $2\sqrt{(a^2 + b^2)}$. Reason (R): The distance between two points is always positive.

1. A: $\sqrt{((2a)^2 + (2b)^2)} = 2\sqrt{(a^2 + b^2)}$. TRUE (Ex 7.1 Q1(iii)).
2. R is TRUE as a fact — but it does not EXPLAIN the computation in A. The explanation is the distance formula, not positivity.

ANSWER: Option (b).

ASSERTION-REASON 10

Assertion (A): The trisection points of (0, 0)–(6, 9) are (2, 3) and (4, 6). Reason (R): Points of trisection divide a segment in the ratios 1 : 2 and 2 : 1.

1. A: ratio 1 : 2 gives ((1×6)/3, (1×9)/3) = (2, 3); ratio 2 : 1 gives ((2×6)/3, (2×9)/3) = (4, 6). TRUE.
2. R is TRUE and is exactly the fact used.

ANSWER: Option (a).

TOPPER TIP: *Assertion–Reason marking is ruthless: judge A alone, then R alone, then — only if both are true — ask whether R EXPLAINS A. Three separate verdicts, in that order, every time.*

15 Olympiad & Foundation Challenges

8D • CHALLENGE — FOR STUDENTS WHO FINISH EARLY AND THINK DEEP

OLYMPIAD 1

Find the point on the y-axis which is equidistant from (3, 5) and (−4, 2).

THINK FIRST: Fractions are welcome in answers. A coordinate need not be a whole number to be exactly right.

1. Let the point be P(0, y). $PA^2 = (0-3)^2 + (y-5)^2 = 9 + (y-5)^2$.
   Why: On the y-axis, x = 0 — the whole point of the (0, y) form.
2. $PB^2 = (0+4)^2 + (y-2)^2 = 16 + (y-2)^2$.
3. Equate: $9 + y^2 - 10y + 25 = 16 + y^2 - 4y + 4 \Rightarrow 34 - 10y = 20 - 4y \Rightarrow 14 = 6y \Rightarrow y = 7/3$.

ANSWER: The point is (0, 7/3).

OLYMPIAD 2

A point P(x, 3) is equidistant from A(5, 2) and B(7, 6). Find x.

1. $PA^2 = (x-5)^2 + (3-2)^2 = (x-5)^2 + 1$.
2. $PB^2 = (x-7)^2 + (3-6)^2 = (x-7)^2 + 9$.
3. Equate and expand: $x^2 - 10x + 26 = x^2 - 14x + 58 \Rightarrow 4x = 32 \Rightarrow x = 8$.

ANSWER: P = (8, 3).

OLYMPIAD 3 (EXTENSION)

Using the area formula, find the area of the triangle with vertices (0, 0), (4, 3) and (2, 5) — and confirm it by the rectangle method of Derivation D.

1. Formula: $\frac{1}{2}|0(3-5) + 4(5-0) + 2(0-3)| = \frac{1}{2}|0 + 20 - 6| = \frac{1}{2}(14) = \mathbf{7}$.
2. Rectangle check (Derivation D): bounding box 4 × 5 = 20; corner triangles 6, 2 and 5; 20 − 13 = 7. Same.
   Why: Two independent methods agreeing is the strongest verification mathematics offers.

ANSWER: Area = 7 square units, confirmed twice.

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Olympiad Challenges 4–6

OLYMPIAD 4

The points A(1, 4), B(5, 7) and C(x, 10) are collinear. Find x.

1. Collinear ⇒ area of △ABC = 0 (extension test).
   Why: With one unknown coordinate, the area test gives ONE clean linear equation — much faster than three distances.
2. ½|1(7−10) + 5(10−4) + x(4−7)| = 0 ⇒ |−3 + 30 − 3x| = 0.
3. 27 − 3x = 0 ⇒ x = **9**.
4. Check by slopes of thought: from A to B, x grows 4 while y grows 3; from B to C(9, 10), x grows 4 while y grows 3. Same rhythm — collinear indeed.

ANSWER: x = 9.

OLYMPIAD 5 (EXTENSION)

Find the centroid of the triangle with vertices (1, 2), (3, 4) and (5, 0) — first by the median-and-section method, then by the averaging shortcut.

1. Take the median from C(5, 0) to the midpoint of AB: midpoint of AB = ((1+3)/2, (2+4)/2) = (2, 3).
   Why: The centroid lies on every median, 2/3 of the way from the vertex.
2. The centroid divides the median in the ratio 2 : 1 (from the vertex): point = ((2×2 + 1×5)/3, (2×3 + 1×0)/3) = (9/3, 6/3) = **(3, 2)**.
3. Shortcut check: average of vertices = ((1 + 3 + 5)/3, (2 + 4 + 0)/3) = (3, 2). Identical.
   Why: The centroid is just the 'centre of mass' — the average address of the three corners.

ANSWER: The centroid is (3, 2).

OLYMPIAD 6

Two opposite vertices of a square are (3, 4) and (1, −1). Find the other two vertices.

1. The segment joining opposite vertices is a DIAGONAL. Its midpoint — the square's centre — is ((3+1)/2, (4−1)/2) = (2, 3/2).
2. Half-diagonal vector (from centre to (3, 4)) = (3−2, 4−3/2) = (1, 5/2).

Olympiad Challenges 7–8

OLYMPIAD 7

Find the point which lies one-quarter of the way from A(−4, 5) to B(8, −1).

1. One-quarter of the way means AP : PB = 1 : 3.
   Why: 'Fraction of the way' always converts to a ratio first: 1/4 taken, 3/4 remaining.
2. $x = (1\times 8 + 3\times(-4))/4 = (8 - 12)/4 = \mathbf{-1}$.
3. $y = (1\times(-1) + 3\times 5)/4 = (-1 + 15)/4 = \mathbf{7/2}$.

ANSWER: The point is (−1, 7/2).

OLYMPIAD 8

Three vertices of a parallelogram are (1, 4), (2, 7) and (5, 6) — but nobody recorded which vertex is missing, or the order. Find ALL possible positions of the fourth vertex.

1. Any two of the three points could be opposite corners. The fourth vertex D satisfies: midpoint of one pair = midpoint of the other pair, i.e. D = (sum of one pair) − (the third point).
   Why: Three choices for which point is 'left out' – three parallelograms.
2. Leave out (1, 4): D = (2 + 5, 7 + 6) − (1, 4) = **(6, 9)**.
3. Leave out (2, 7): D = (1 + 5, 4 + 6) − (2, 7) = **(4, 3)**.
4. Leave out (5, 6): D = (1 + 2, 4 + 7) − (5, 6) = **(−2, 5)**.

ANSWER: The fourth vertex can be (6, 9), (4, 3) or (−2, 5) — three different parallelograms from the same three points.

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Olympiad Challenges 9–10

OLYMPIAD 9

The points (0, 0), (6, 0) and (0, 8) form a triangle. Find its circumcentre and circumradius (the centre and radius of the circle through all three vertices).

1. The triangle is right-angled at (0, 0) (its sides lie on the axes), with hypotenuse from (6, 0) to (0, 8).
2. By Reasoning 2, the midpoint of the hypotenuse is equidistant from all three vertices — that IS the circumcentre: ((6+0)/2, (0+8)/2) = **(3, 4)**.
3. Radius = distance from (3, 4) to (0, 0) = √(9 + 16) = **5**. (Check with (6, 0): √(9 + 16) = 5; with (0, 8): √(9 + 16) = 5.)

ANSWER: Circumcentre (3, 4), circumradius 5.

OLYMPIAD 10

In △ABC with A(1, 2), B(5, 4) and C(3, 6), find the length of the median drawn from B to the side AC.

1. The median from B lands on the midpoint of AC: ((1 + 3)/2, (2 + 6)/2) = (2, 4).
2. Median length = distance from B(5, 4) to (2, 4) = $\sqrt{((5-2)^2 + 0)}$ = **3 units**.

   Why: A horizontal median — the y-coordinates match, so the answer falls out in one line.

ANSWER: The median from B has length 3 units.

TOPPER TIP: *Olympiad habit: every challenge above was solved by NCERT tools plus ONE extra idea (rotate a vector, average three points, leave-one-out). There is no separate 'Olympiad mathematics' — there is only basic mathematics, aimed better.*

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16 Chapter Mind Map

THE WHOLE CHAPTER ON ONE PAGE — REVISE FROM HERE

COORDINATE GEOMETRY

Heart of the chapter: **position becomes numbers** — then Pythagoras and the average answer everything.

The Plane

Axes, origin, quadrants. Abscissa = x, ordinate = y. On x-axis: (x, 0). On y-axis: (0, y). Distance from x-axis = $|y|$; from y-axis = $|x|$.

→

Distance Formula

$\sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2]}$ — Pythagoras in a grid. From origin: $\sqrt{(x^2 + y^2)}$. Never negative.

Section Formula

Divide in m : n: $((mx_2 + nx_1)/(m+n), (my_2 + ny_1)/(m+n))$. Weights CROSS. Trisection = 1 : 2 and 2 : 1.

Midpoint

Section formula at m = n: the plain average. Unlocks: parallelogram diagonals, centre of a circle, 'halfway' problems.

Division by the Axes

x-axis cuts in $(-y_1) : y_2$. y-axis cuts in $(-x_1) : x_2$. Because the dividing point has a zero coordinate.

Equidistant Points

Set the two SQUARED distances equal; x^2 and y^2 cancel; a linear equation (perpendicular bisector) remains.

Detective Work with Distances

Collinear: largest distance = sum of other two. Square: sides equal + diagonals equal. Rhombus: sides equal only. Parallelogram: opposite sides equal (or diagonals share a midpoint).

→

Area of Triangle (Extension)

$\frac{1}{2}|x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)|$. Zero ⇔ collinear. Centroid = average of the three vertices.

Key Results to Quote

Towns A–B: 39 • Classroom ABCD is a square (Champa was right!) • Sports-day flags: $\sqrt{61}$ m apart, blue flag (5, 22.5) • Rhombus area: 24 • Equidistant line for (3, 6), (−3, 4): $3x + y = 5$

TOPPER TIP: *Revise actively: cover a box, speak its contents aloud from memory, then uncover and check. A mind map you can REPRODUCE from memory is worth ten readings.*

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17 Pin-Point Flash Cards

THE WHOLE CHAPTER IN YOUR POCKET — CUT THEM OUT, CARRY THEM, MASTER THEM

How to use these cards

Cut along the dashed borders and keep the 16 cards in your pocket or a small box. On Smart Memory **Day 3, Day 7 and Day 21**, shuffle the cards and test yourself: read the title, say the pin-point aloud from memory, then turn the card over (flip it in your hand) to check. Any card you miss goes back into the box for tomorrow. Two minutes a day — that is all it takes to keep this chapter alive all year.

CARD 1 — Distance formula

PQ = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$. It is Pythagoras wearing a grid — derive it, don't cram it.

CARD 2 — From the origin & the axes

Distance of (x, y) from origin = $\sqrt{(x^2 + y^2)}$. From the x-axis = $|y|$. From the y-axis = $|x|$.

CARD 3 — Section formula

Dividing in m : n: $x = (m x_2 + n x_1)/(m + n)$. The weights CROSS — m holds the far point.

CARD 4 — Midpoint

Midpoint = $((x_1 + x_2)/2, (y_1 + y_2)/2)$. Just the average, twice. It is the section formula at 1 : 1.

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Flash Cards 5–10

CARD 5 — Trisection

Two points cut a segment into three equal parts: they divide it in the ratios 1 : 2 and 2 : 1. Check: coordinates step evenly.

CARD 6 — Cut by the x-axis

The x-axis divides a segment in the ratio $(-y_1) : y_2$. The zero of the axis writes the equation for you.

CARD 7 — Cut by the y-axis

The y-axis divides a segment in the ratio $(-x_1) : x_2$. Same trick, other coordinate.

CARD 8 — Collinearity tests

Distances: largest = sum of other two. OR area of the triangle = 0. 'Nearly equal' is NOT collinear.

CARD 9 — Parallelogram secret

Diagonals of a parallelogram bisect each other: midpoint of one diagonal = midpoint of the other. Solves every 'find x, y' problem.

CARD 10 — Equidistant points

Set the SQUARED distances equal. The squares cancel and a straight line (perpendicular bisector) falls out.

TOPPER TIP: *Say each card aloud once before you test yourself on it — hearing your own voice say the fact makes the memory stick twice as fast.*

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Flash Cards 11–16

CARD 11 — Area of a triangle (extension)

$\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$. Cyclic: 1→2→3→1. Zero means collinear.

CARD 12 — Centroid (extension)

Centroid = average of the three vertices: $((x_1 + x_2 + x_3)/3, (y_1 + y_2 + y_3)/3)$. It divides every median 2 : 1.

CARD 13 — Fraction of a segment

AP = (3/7) AB means P divides AB in the ratio 3 : 4 — taken : remaining. Always convert the fraction before using the formula.

CARD 14 — Shape tests

Square: sides equal + diagonals equal. Rhombus: sides equal, diagonals unequal. Rectangle: diagonals equal, sides not all equal.

CARD 15 — Points on the axes

On the x-axis every point is (x, 0); on the y-axis, (0, y). Half of all 'find the point' problems start with this line.

CARD 16 — Distance is never negative

Distances come out of a square root — always take the POSITIVE value. But a squared-distance equation can give TWO positions: report both.

MAKE YOUR OWN CARD

Write one pin-point YOU keep forgetting, in your own words. A card you write yourself is the card you remember best.

_____

_____

MAKE YOUR OWN CARD

One more: the mistake you made most often in this chapter, and the one-line rule that fixes it.

_____

18 Active Recall

8D • RECALL — CLOSE THE BOOK, THEN ANSWER

How this works: close every other page. Answer all ten questions on rough paper, from memory alone. Only then open the Answer Key. Anything you could not recall is exactly what your Day-3 revision must target — write it in the Error Tracker (Section 19).

Ten rapid questions

R1. Find the distance between (0, 0) and (6, 8).

R2. Find the midpoint of (1, 5) and (3, 9).

R3. State the distance formula.

R4. Find the point dividing (2, 2)–(8, 5) in the ratio 1 : 2.

R5. Find the distance of the point (5, 12) from the origin.

R6. In what ratio does the x-axis divide the join of (1, 2) and (3, −4)?

R7. Find the points of trisection of (0, 0)–(3, 6).

R8. Are (1, 1), (2, 3) and (3, 5) collinear? Justify in one line.

R9. (0, 0), (3, 1) and (4, 4) are three vertices of a parallelogram, taken in order. Find the fourth vertex.

R10. (Extension) Find the area of the triangle (0, 0), (4, 0), (0, 3).

Five recall prompts — say each answer aloud, in a full sentence

P1. Define abscissa and ordinate.

P2. State the section formula for internal division in the ratio m : n.

P3. State the midpoint formula, and say which formula it comes from.

P4. Give TWO different tests for the collinearity of three points.

P5. A point divides a segment in the ratio 1 : 1. What is that point called, and what else does it do to the diagonals of a parallelogram?

> **TOPPER TIP:** *Scoring: 13–15 correct — chapter mastered, move on. 9–12 — revise the missed topics, retake in 3 days. Below 9 — re-study Sections 05–07, then retake. Record your score in the Mastery Tracker.*

19 Self-Assessment & Mastery Tracker

KALVIYOGI SMART MEMORY SYSTEM — LEARN IT, THEN KEEP IT

Memory fades on a predictable curve — unless you revisit at the right moments. This schedule is your revisit plan for THIS chapter. Tick each day off when done.

Day	Your activity for Chapter 7	Done
Day 0	Pre-learning assessment: attempt the Quick Check (Section 02) and Rapid Questions R1–R10 (Section 18) BEFORE studying. Note your score.	☐
Day 1	Learn the chapter fully (Sections 01–10) and complete Level 1–3 practice.	☐
Day 3	Without-book recall: reproduce the Mind Map (Section 16) on a blank page; redo Rapid Questions; finish Level 4.	☐
Day 7	Independent problem-solving: Level 5 + Olympiad Challenges, timed, book closed. Then take the **Day 7 Mini-Test** (Practice Plus) and run the Flash Cards (Section 17).	☐
Day 14	Mixed practice: re-attempt every question in your Error Tracker; solve Ex 7.1 Q6 and Ex 7.2 Q3 fresh.	☐
Day 21	Long-term recall test: write the section-formula derivation and the distance derivation from memory, then verify line by line. Then take the **Day 21 Mini-Test** (Practice Plus).	☐
Day 30	Chapter mastery assessment: take the **Day 30 Mastery Test** (Practice Plus) under exam conditions; update your Concept Mastery Score below.	☐

Formula Recall Tracker — tick when you can say it with the book closed

Formula / Statement	Day 1	Day 3	Day 7	Day 21
Distance formula (and its Pythagoras picture)	☐	☐	☐	☐
Section formula (weights cross over)	☐	☐	☐	☐
Midpoint formula	☐	☐	☐	☐
Ratios in which the axes divide a segment	☐	☐	☐	☐
Collinearity tests (distance sum / zero area)	☐	☐	☐	☐
Area of a triangle (extension) and centroid	☐	☐	☐	☐

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Mastery Tracker — Errors and Concept Scores

Error Tracker — every mistake becomes a lesson

Date	Question	What went wrong	The correct idea

Concept Mastery Score — rate yourself honestly, out of 5

Concept	Score / 5	Status
Distance formula and its uses	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Section formula and trisection	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Midpoint — parallelograms and circles	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Equidistant-point problems	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Shape identification by coordinates	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Area test and centroid (extension)	___ / 5	☐ Weak ☐ Okay ☐ Mastered

Learning Time Tracker

Day / Date	Minutes studied	What I studied

Mastery rule: a concept counts as MASTERED only when you score 4 or 5 AND can reproduce it on Day 21 without the book. Anything less — revisit, don't just re-read.

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Practice Plus — 25 Concept MCQs (1–13)

One mark each. Cover the Answer Key and choose the single best option — then check the key, where the tricky ones carry a one-line explanation.

Q1. The distance between (0, 0) and (3, 4) is
(a) 7 (b) 1 (c) 5 (d) 25

Q2. The distance of the point (−3, 4) from the x-axis is
(a) 3 (b) −3 (c) 4 (d) 5

Q3. The distance of the point (5, −2) from the y-axis is
(a) 5 (b) 2 (c) −2 (d) $\sqrt{29}$

Q4. The midpoint of the segment joining (2, −5) and (−2, 5) is
(a) (0, 0) (b) (2, −5) (c) (0, 5) (d) (2, 0)

Q5. The distance between (a, b) and (−a, −b) is
(a) $2\sqrt{(a^2 + b^2)}$ (b) $\sqrt{(a^2 + b^2)}$ (c) 2(a + b) (d) 0

Q6. The point dividing (1, −2)–(4, 7) in the ratio 1 : 2 is
(a) (2, 1) (b) (3, 4) (c) (1, 0) (d) (2, 3)

Q7. The distance between (2, −2) and (−1, x) is 5. One of the values of x is
(a) −6 (b) 2 (c) −2 (d) 1

Q8. If the midpoint of (k, 3) and (2, 5) is (3, 4), then k =
(a) 2 (b) 4 (c) 6 (d) 8

Q9. The distance between (5, 0) and (−3, 0) is
(a) 2 (b) 5 (c) 3 (d) 8

Q10. The point on the x-axis equidistant from (2, −5) and (−2, 9) is
(a) (−7, 0) (b) (7, 0) (c) (−4, 0) (d) (4, 0)

Q11. The point dividing (−1, 7)–(4, −3) in the ratio 2 : 3 is
(a) (3, 1) (b) (0, 3) (c) (1, 3) (d) (3, 0)

Q12. The distance of the point (−12, 5) from the origin is
(a) 17 (b) 13 (c) 7 (d) 169

Q13. Which of the following sets of points is collinear?
(a) (0, 0), (1, 2), (2, 5) (b) (1, 1), (3, 3), (5, 5) (c) (2, 3), (4, 5), (6, 8) (d) (0, 1), (1, 3), (2, 4)

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25 Concept MCQs (14–25)

Q14. The x-axis divides the join of (1, −5) and (−4, 5) in the ratio
(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 5 : 4

Q15. The point dividing (2, 3)–(4, 7) in the ratio 3 : 1 is
(a) (3, 5) (b) (5/2, 5) (c) (7/2, 6) (d) (3, 6)

Q16. The distance between (2, 3) and (2, 7) is
(a) 2 (b) 4 (c) 5 (d) 10

Q17. The centre of the circle whose diameter has endpoints (3, −2) and (−5, 4) is
(a) (−1, 1) (b) (1, −1) (c) (−4, 3) (d) (4, −3)

Q18. If the distance between (x, 2) and (3, 4) is $2\sqrt{2}$, then one value of x is
(a) 0 (b) 2 (c) 3 (d) 5

Q19. (Extension) The area of the triangle with vertices (0, 0), (3, 0) and (0, 4) is
(a) 12 (b) 6 (c) 24 (d) 7

Q20. The point dividing (7, −6)–(3, 4) in the ratio 1 : 2 is
(a) (17/3, −8/3) (b) (5, −1) (c) (13/3, −2/3) (d) (17/3, 8/3)

Q21. If P(2, 4) is the midpoint of A(1, 2) and B(x, y), then B is
(a) (1, 2) (b) (2, 4) (c) (3, 6) (d) (4, 8)

Q22. The distance of the point (6, −8) from the origin is
(a) 14 (b) 10 (c) 2 (d) 48

Q23. The first point of trisection of the segment joining (0, 0) and (6, 9) is
(a) (2, 3) (b) (3, 4) (c) (4, 6) (d) (1, 2)

Q24. The point P(3, 5) divides the join of A(0, 2) and B(4, 6) in the ratio
(a) 1 : 3 (b) 1 : 1 (c) 3 : 1 (d) 2 : 1

Q25. The quadrilateral with vertices (0, 0), (4, 0), (4, 3) and (0, 3) is a
(a) square (b) rectangle (c) rhombus (d) kite

TOPPER TIP: Scored below 20? Do not just re-read the key — find WHICH pin-point card covers each missed question and revise that card first.

Find the Error — Questions 1–5

ERROR 1

"Distance between (1, 2) and (4, 6) = √[(4 − 1) + (6 − 2)] = √7."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 2

"Distance between (−3, 4) and (3, −4): Δy = −4 − 4 = −8, and $(-8)^2 = -64$, so distance = √(36 − 64)."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 3

"Midpoint of (2, 5) and (4, 9) = ((2 + 4)/2, (5 − 9)/2) = (3, −2)."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 4

"The point dividing (0, 0)–(10, 0) in the ratio 2 : 3 is x = (2×0 + 3×10)/5 = 6."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 5

"Distance between (2, 3) and (5, 7) = $(5-2)^2 + (7-3)^2 = 9 + 16 = 25$ units."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

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Find the Error — Questions 6–10

ERROR 6

"For (1, 2), (2, 4), (3, 5): AB = √5, BC = √2, AC = √13. Since √5 + √2 = √13, the points are collinear."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 7

"The x-axis divides the join of (2, −4) and (6, 8) in the ratio 8 : 4 = 2 : 1."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 8

"Point (x, 0) equidistant from (2, −5) and (−2, 9): $(x - 2)^2 + 25 = (x + 2)^2 + 9$, so x = −5."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 9

"Area of the triangle (0, 0), (2, 5), (5, 2) = ½[0(5−2) + 2(2−0) + 5(0−5)] = ½(−21) = −21/2 square units."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 10

"Trisection of (4, −1)–(−2, −3): both points divide in the ratio 1 : 2, so both are (2, −5/3)."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

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Previous Year Questions — Real CBSE Board Questions (1)

These are genuine CBSE board questions. Where the year and paper code were verified against board-question records, they are printed; the rest are labelled simply "CBSE Board Exam Question". Solve them exactly as you would in the exam hall.

CBSE 2023 (30/6/1) — 3 MARKS

Find the ratio in which the line segment joining the points A(6, 3) and B(−2, −5) is divided by the x-axis. Also find the coordinates of the point of division.

1. Let the x-axis divide AB in the ratio k : 1 at P. P's y-coordinate = (−5k + 3)/(k + 1).
2. P is on the x-axis, so y = 0: −5k + 3 = 0 ⇒ k = 3/5. The ratio is **3 : 5**.
3. Point: x = (3×(−2) + 5×6)/(3 + 5) = (−6 + 30)/8 = 24/8 = **3**; y = 0. Point of division: **(3, 0)**.

ANSWER: Ratio 3 : 5; point of division (3, 0).

CBSE 2024 (30/3/1) — 3 MARKS

Find the ratio in which the line segment joining the points (5, 3) and (−1, 6) is divided by the y-axis. Also find the point of division.

1. Let the y-axis divide the segment in the ratio k : 1 at P. P's x-coordinate = (−k + 5)/(k + 1).
2. P is on the y-axis, so x = 0: −k + 5 = 0 ⇒ k = 5. The ratio is **5 : 1**.
3. Point: y = (5×6 + 1×3)/(5 + 1) = 33/6 = **11/2**; x = 0. Point of division: **(0, 11/2)**.

ANSWER: Ratio 5 : 1; point of division (0, 11/2).

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Previous Year Questions (2)

CBSE 2024 (30/1/1) — MCQ

If the distance between the points (3, −5) and (x, −5) is 15 units, then the values of x are: (a) 12, −18 (b) −12, 18 (c) 18, 5 (d) −9, −12

1. Both points have y = −5, so the segment is horizontal: distance = $|x - 3|$.
   Why: Equal y-coordinates — the distance formula collapses to one subtraction.
2. $|x - 3| = 15 \Rightarrow x - 3 = 15$ or $x - 3 = -15 \Rightarrow x = 18$ or $x = -12$.
3. Option **(b)**.

ANSWER: Option (b): x = −12 or 18.

CBSE 2025 (30/1/1) — 2 MARKS

The coordinates of the centre of a circle are (2a, a − 7). Find the value(s) of a if the circle passes through the point (11, −9) and has diameter 10√2 units.

1. Diameter $10\sqrt{2} \Rightarrow$ radius = $5\sqrt{2}$. The distance from the centre to any point on the circle equals the radius.
2. Distance from (2a, a − 7) to (11, −9) = $5\sqrt{2}$. Square both sides: $(2a - 11)^2 + (a - 7 + 9)^2 = 50$.
3. Expand: $4a^2 - 44a + 121 + a^2 + 4a + 4 = 50 \Rightarrow 5a^2 - 40a + 75 = 0$.
4. Divide by 5: $a^2 - 8a + 15 = 0 \Rightarrow (a - 3)(a - 5) = 0 \Rightarrow$ **a = 3 or a = 5**.
   Why: Two valid centres — two different circles satisfy the conditions. Report both.

ANSWER: a = 3 or a = 5.

CBSE 2025 (30/2/1) — MCQ

The equation of a line parallel to the x-axis and at a distance of 3 units below the x-axis is: (a) x = 3 (b) x = −3 (c) y = −3 (d) y = 3

1. A line parallel to the x-axis has every point at the SAME y. Below the axis by 3 means y = −3 everywhere.
2. Option **(c)**.

ANSWER: Option (c): y = −3.

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Previous Year Questions (3)

CBSE BOARD EXAM QUESTION

Using the distance formula, prove that the points A(2, 3), B(−7, 0) and C(−1, 2) are collinear.

1. AC = $\sqrt{((-1-2)^2 + (2-3)^2)} = \sqrt{(9 + 1)} = \mathbf{\sqrt{10}}$.
2. CB = $\sqrt{((-7+1)^2 + (0-2)^2)} = \sqrt{(36 + 4)} = \sqrt{40} = \mathbf{2\sqrt{10}}$.
3. AB = $\sqrt{((-7-2)^2 + (0-3)^2)} = \sqrt{(81 + 9)} = \sqrt{90} = \mathbf{3\sqrt{10}}$.
4. AC + CB = $\sqrt{10} + 2\sqrt{10} = 3\sqrt{10}$ = AB. Hence C lies on segment AB — the three points are collinear. ∎

ANSWER: A, B and C are collinear (with C between A and B).

CBSE BOARD EXAM QUESTION — MCQ

If the vertices of a parallelogram ABCD are A(6, 1), B(8, 2), C(9, 4) and D(p, 3), then the value of p is: (a) 5 (b) 6 (c) 7 (d) 8

1. Diagonals of a parallelogram share their midpoint: midpoint of AC = midpoint of BD.
2. $(6 + 9)/2 = (8 + p)/2 \Rightarrow 15 = 8 + p \Rightarrow p = \mathbf{7}$. Option **(c)**.

ANSWER: Option (c): p = 7.

TOPPER TIP: *Board pattern watch: in the last several papers, Coordinate Geometry's favourite children are (1) ratio in which an AXIS divides a segment, (2) equidistant points, (3) the parallelogram midpoint trick. All three are drilled in this chapter — and all three appear on this very page set.*

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Chapter Model Test A — Basics (20 Marks)

Time: 40 minutes. Attempt all questions. Show every step — the marking key in the Answer Key shows where each mark lives.

Q1. Find the distance between (0, 0) and (6, 8). **[1 marks]**

Q2. Find the midpoint of (1, 3) and (5, 7). **[1 marks]**

Q3. Find the distance of the point (7, 24) from the origin. **[1 marks]**

Q4. Find the point dividing (0, 0)–(4, 2) in the ratio 1 : 1. **[1 marks]**

Q5. Find the distance between (−3, 4) and (0, 0). **[2 marks]**

Q6. Find the midpoint of (−4, 6) and (2, −2). **[2 marks]**

Q7. Find the point dividing (1, 2)–(7, 8) in the ratio 1 : 2. **[2 marks]**

Q8. Find the point on the y-axis equidistant from (2, 3) and (−1, 4). **[3 marks]**

Q9. Find the distance between (2, 5) and (−4, 13). **[3 marks]**

Q10. Find the coordinates of the points of trisection of the segment joining (1, 1) and (7, 7). **[4 marks]**

Chapter Model Test B — Standard (20 Marks)

Time: 40 minutes. Board-pattern questions at full Class 10 level.

Q1. If the distance between (3, −5) and (x, −5) is 15 units, write one value of x. **[1 marks]**

Q2. In what ratio does the x-axis divide the join of (3, −4) and (9, 2)? **[1 marks]**

Q3. Find the point dividing (5, 5)–(−5, −5) in the ratio 2 : 3. **[1 marks]**

Q4. Find the distance between (a, 0) and (−a, 0), where a > 0. **[1 marks]**

Q5. Find the values of y for which the distance between (2, −3) and (10, y) is 10 units. **[2 marks]**

Q6. A circle has centre (1, 2) and one end of a diameter is (−3, 5). Find the other end. **[2 marks]**

Q7. Find the midpoint of the segment joining (−2, 2) and (2, 8). **[2 marks]**

Q8. If (1, 2), (4, y), (x, 6) and (3, 5) are vertices of a parallelogram taken in order, find x and y. **[3 marks]**

Q9. Determine if the points (1, 5), (2, 3) and (−2, −11) are collinear. Justify. **[3 marks]**

Q10. Find the area of the rhombus with vertices (3, 0), (4, 5), (−1, 4) and (−2, −1) taken in order. **[4 marks]**

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Chapter Model Test C — Advanced (20 Marks)

Time: 45 minutes. Proof-heavy, HOTS-level — this is the paper that separates 95 from 100.

Q1. (Extension) Find the centroid of the triangle with vertices (0, 0), (6, 0) and (0, 6). **[1 marks]**

Q2. (Extension) Find the area of the triangle with vertices (1, 1), (4, 1) and (4, 5). **[1 marks]**

Q3. Find the point dividing (2, 3)–(6, 11) in the ratio 3 : 1. **[1 marks]**

Q4. Find the distance between (1, 2) and the midpoint of (3, 4)–(7, 8). **[1 marks]**

Q5. Find the circumcentre of the triangle with vertices (0, 0), (6, 0) and (0, 8). **[2 marks]**

Q6. Find the points on the x-axis which are at a distance of 5 units from (3, 4). **[2 marks]**

Q7. In what ratio does the y-axis divide the join of (5, 3) and (−1, 6)? **[2 marks]**

Q8. Two opposite vertices of a square are (3, 4) and (1, −1). Find the other two vertices. **[3 marks]**

Q9. In △ABC, A(1, 2), B(5, 4), C(3, 6). Find the length of the median from B to side AC. **[3 marks]**

Q10. Prove that the midpoint of the hypotenuse of a right-angled triangle is equidistant from all three vertices. **[4 marks]**

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Smart Memory Mini-Tests — Day 7 and Day 21

Day 7 Mini-Test — independent solving (10 questions)

Q1. Find the distance between (1, 1) and (4, 5).

Q2. Find the midpoint of (3, −2) and (7, 6).

Q3. Find the point dividing (0, 6)–(6, 0) in the ratio 1 : 2.

Q4. Find the distance of (−8, 6) from the origin.

Q5. In what ratio does the x-axis divide the join of (2, −3) and (4, 5)?

Q6. Three vertices of a parallelogram, in order, are (0, 0), (2, 3) and (5, 3). Find the fourth vertex.

Q7. Find the distance between (7, 1) and (−2, 1).

Q8. Find the first point of trisection of (3, 3)–(9, 15).

Q9. Find the point on the x-axis equidistant from (1, 4) and (5, 4).

Q10. (Extension) Find the area of the triangle (0, 0), (5, 0), (0, 12).

Day 21 Mini-Test — long-term recall (10 questions)

Q1. Find the distance between (−1, 2) and (2, 6).

Q2. Find the midpoint of (−3, −5) and (5, 1).

Q3. Find the point dividing (4, 4)–(−2, −2) in the ratio 2 : 1.

Q4. The distance between (1, 2) and (1, y) is 3 units. Find the values of y.

Q5. In what ratio does the y-axis divide the join of (−6, 2) and (4, 8)?

Q6. Find the centre of the circle whose diameter has endpoints (2, 4) and (8, 10).

Q7. Are (2, 2), (4, 4) and (7, 7) collinear? Answer yes or no, with a one-line reason.

Q8. Find the point dividing (10, 5)–(0, 5) in the ratio 3 : 2.

Q9. Find the distance of the point (12, −5) from the origin.

Q10. (Extension) Find the centroid of the triangle (3, 3), (6, 9), (9, 0).

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Smart Memory Mini-Test — Day 30 Mastery Test

Exam conditions: 30 minutes, book closed. This is the test your Day 0 self could not have passed. Show every step.

Q1. Find the distance between (2, −1) and (−3, 11).

Q2. Find the midpoint of (−7, 3) and (3, −9).

Q3. Find the point dividing (−4, 6)–(8, −2) in the ratio 1 : 3.

Q4. Find the point on the y-axis equidistant from (4, 1) and (−2, 5).

Q5. In what ratio does the point (2, 3) divide the join of (1, 2) and (5, 6)?

Q6. (Extension) Find the area of the triangle (2, 3), (6, 3), (4, 7).

Q7. Find the point which lies three-quarters of the way from (0, 0) to (8, 12).

Q8. Find the distance between the midpoints of (0, 0)–(4, 0) and (0, 6)–(4, 6).

Q9. If (k, 4) is the midpoint of (2, 2) and (6, 6), find k.

Q10. Write, in one line, the distance of the point (t, −t) from the origin, in terms of t.

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Answer Key — Graded Practice and Active Recall

Graded Practice — Level 1

L1.1 5 units. **L1.2** 10 units. **L1.3** (4, 6). **L1.4** 5 units. **L1.5** (2, 1). **L1.6** 6 units (same x — just subtract the y's).

Graded Practice — Level 2

L2.1 13 units. **L2.2** (−1, 3). **L2.3** (6, 7). **L2.4** AB = 2√2, BC = 2√2, AC = 4√2; AB + BC = AC, so collinear. **L2.5** y = 3 or y = −5. **L2.6** (0, −1).

Graded Practice — Level 3

L3.1 10 km. **L3.2** Fountain (5, 3); distance √34 ≈ 5.83 units. **L3.3** 13 units; halfway at (6, 5/2). **L3.4** (4, 5).

Graded Practice — Levels 4 and 5

L4.1 k = 7. **L4.2** D = (2, 4). **L4.3** AB = 3, BC = 3, AC = 3√2: $AB^2 + BC^2 = 9 + 9 = 18 = AC^2$, so right-angled at B, and AB = BC makes it isosceles; midpoint of hypotenuse AC = (7/2, 5/2). **L5.1** Centre (2, 1); half-diagonal vector (3, 3), rotated (−3, 3): other vertices (−1, 4) and (5, −2); every side = 6. **L5.2** The equidistant points satisfy 3x + y = 5 (Ex 7.1 Q10) and $x^2 + y^2 = 25$; solving gives (0, 5) and (3, −4).

Active Recall

R1 10. **R2** (2, 7). **R3** $\sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2]}$. **R4** (4, 3). **R5** 13. **R6** 1 : 2. **R7** (1, 2) and (2, 4). **R8** Yes: AB = BC = √5 and AC = 2√5 = AB + BC. **R9** (1, 3). **R10** 6 square units. **P1** Abscissa = x-coordinate (distance from y-axis); ordinate = y-coordinate (distance from x-axis). **P2** $((mx_2 + nx_1)/(m + n), (my_2 + ny_1)/(m + n))$. **P3** Average of the coordinates; it is the section formula with m = n = 1. **P4** Largest distance = sum of the other two; or area of the triangle = 0. **P5** The midpoint; the two diagonals of a parallelogram share it (bisect each other).

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Answer Key — MCQs and Find the Error

25 Concept MCQs

1 (c) **2** (c) **3** (a) **4** (a) **5** (a) **6** (a) **7** (b) **8** (b) **9** (d) **10** (a) **11** (c) **12** (b) **13** (b) **14** (a) **15** (c) **16** (b) **17** (a) **18** (d) **19** (b) **20** (a) **21** (c) **22** (b) **23** (a) **24** (c) **25** (b)

Q7: $(x + 2)^2 = 16$ gives $x = 2$ or -6 — the question asks for ONE value, and 2 is listed. **Q13:** only (b) has all three points on one line ($y = x$). **Q18:** $(x - 3)^2 = 8 - 4 = 4$, so $x = 5$ or 1; 5 is the listed value. **Q24:** from x: $3 = 4k/(k + 1)$ gives $k = 3$, and y confirms it.

Find the Error — corrections

E1. The differences must be SQUARED: $\sqrt{[(4-1)^2 + (6-2)^2]} = \sqrt{25} = 5$.

E2. $(-8)^2 = +64$, never -64: distance $= \sqrt{(36 + 64)} = 10$.

E3. Midpoint ADDS both coordinates: $((2+4)/2, (5+9)/2) = (3, 7)$.

E4. Weights swapped: m multiplies the FAR point: $x = (2\times10 + 3\times0)/5 = 4$.

E5. 25 is the SQUARED distance; the distance is $\sqrt{25} = 5$ units.

E6. $\sqrt{5} + \sqrt{2} \neq \sqrt{13}$ (square both: $7 + 2\sqrt{10} \neq 13$). AB + BC > AC, so the points are NOT collinear.

E7. The x-axis ratio is $(-y_1) : y_2 = 4 : 8 = 1 : 2$, not 2 : 1.

E8. $(0 - 9)^2 = 81$, not 9: $(x-2)^2 + 25 = (x+2)^2 + 81$ gives $x = -7$, point $(-7, 0)$.

E9. Area is never negative — take the absolute value: 21/2 square units.

E10. Trisection uses TWO ratios: 1 : 2 gives $(2, -5/3)$; 2 : 1 gives $(0, -7/3)$. Two different points.

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Answer Key — Model Tests and Day Tests

Model Test A (Basics)

Q1 10. **Q2** (3, 5). **Q3** 25. **Q4** (2, 1). **Q5** 5. **Q6** (−1, 2). **Q7** (3, 4). **Q8** (0, 2). **Q9** 10. **Q10** (3, 3) and (5, 5).

Model Test B (Standard)

Q1 18 (or −12). **Q2** 2 : 1. **Q3** (1, 1). **Q4** 2a. **Q5** y = 3 or −9. **Q6** (5, −1). **Q7** (0, 5). **Q8** x = 6, y = 3. **Q9** Not collinear: AB + BC ≈ 16.80 ≠ AC ≈ 16.28 (or area = 11 ≠ 0). **Q10** 24 square units.

Model Test C (Advanced)

Q1 (2, 2). **Q2** 6 square units. **Q3** (5, 9). **Q4** $4\sqrt{2}$ (midpoint is (5, 6)). **Q5** (3, 4). **Q6** (0, 0) and (6, 0). **Q7** 5 : 1. **Q8** (−1/2, 5/2) and (9/2, 1/2). **Q9** 3 units. **Q10** Full proof in Section 12, Reasoning 2.

Day 7 Mini-Test

1 5. **2** (5, 2). **3** (2, 4). **4** 10. **5** 3 : 5. **6** (3, 0). **7** 9. **8** (5, 7). **9** (3, 0). **10** 30 square units.

Day 21 Mini-Test

1 5. **2** (1, −2). **3** (0, 0). **4** y = 5 or −1. **5** 3 : 2. **6** (5, 7). **7** Yes — all three lie on y = x. **8** (4, 5). **9** 13. **10** (6, 4).

Day 30 Mastery Test

1 13. **2** (−2, −3). **3** (−1, 4). **4** (0, 3/2). **5** 1 : 3. **6** 8 square units. **7** (6, 9). **8** 6. **9** k = 4. **10** $\sqrt{2}\,|t|$.

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Revision Card — Chapter 7 on One Page

The three formulas (derive, don't cram)

Distance: PQ = $\sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2]}$ — Pythagoras in a grid. From origin: $\sqrt{(x^2 + y^2)}$.
Section: divide in m : n → $((mx_2 + nx_1)/(m + n), (my_2 + ny_1)/(m + n))$ — weights cross.
Midpoint: $((x_1 + x_2)/2, (y_1 + y_2)/2)$ — section formula at 1 : 1.

The five reflexes

1. **Axis division:** x-axis cuts in $(-y_1) : y_2$; y-axis cuts in $(-x_1) : x_2$.
2. **Equidistant:** equate the SQUARED distances; expect a linear equation.
3. **Parallelogram:** the diagonals share their midpoint.
4. **Collinear:** largest distance = sum of other two, or area = 0.
5. **Fraction of a segment:** AP = (p/q) AB → ratio p : (q − p).

Extensions in your pocket

Area: $\frac{1}{2}|x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)|$ — cyclic 1→2→3→1.
Centroid: the average of the three vertices; divides each median 2 : 1.

Results worth quoting

Towns A–B distance 39 • Champa's classroom IS a square • Sports-day flags $\sqrt{61}$ m apart, blue flag at (5, 22.5) • Rhombus area 24 • Equidistant line for (3, 6) and (−3, 4): 3x + y = 5 • Board favourites: axis-division ratios (2023, 2024), circle-centre equation (2025).

TOPPER TIP: *Final word: this chapter rewards the student who PLOTS. Thirty seconds spent sketching the points will save you from almost every mistake in Section 10. Draw first, compute second — that is the topper's order of work.*

— End of Chapter 7 —

Next: Chapter 8 — Introduction to Trigonometry. Keep your Flash Cards in your pocket; the Day 3 bell is already ringing.

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CBSE · CLASS 10 · MATHEMATICS

MATHEMATICS BEYOND THE FORMULA

CHAPTER 8 · INTRODUCTION TO TRIGONOMETRY

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01 Chapter Introduction

8D • DISCOVER — WHY SHOULD I LEARN THIS?

Stand outside at noon and look at a tall coconut tree. You cannot climb it with a tape — yet, using only its **shadow** and one angle, you can find its height without leaving the ground. A pilot descending to land, an engineer designing a ramp, a surveyor measuring a river she cannot cross — all of them use the same magical idea: **in a right-angled triangle, the ratios of the sides depend only on the angle, never on the size of the triangle.**

That idea is called **trigonometry** — from the Greek words *trigonon* (triangle) and *metron* (measure): **the measurement of triangles**. In this chapter you will meet six ratios — sin, cos, tan, cosec, sec and cot — you will **derive** their values for the standard angles 0°, 30°, 45°, 60° and 90° from pure geometry (no memorised table!), and you will master the three great identities that power hundreds of board-exam proofs.

What you will master in this chapter

- The six trigonometric ratios of an acute angle — and which side is opposite, adjacent, hypotenuse.
- Why the ratios depend **only on the angle** (similar triangles), never on the triangle's size.
- The standard table — every value **derived** from two special triangles.
- The master identity $\sin^2 A + \cos^2 A = 1$, derived from Pythagoras, and its two children: $1 + \tan^2 A = \sec^2 A$ and $1 + \cot^2 A = \operatorname{cosec}^2 A$.
- Complementary-angle relations: $\sin(90° - A) = \cos A$, and friends.
- Proving identities the board way: LHS → RHS, one step at a time, each identity named.
- Finding heights and distances you can never measure directly (Chapter 9 will take this further).

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Your Chapter Map — 19 Stops, One Journey

This chapter is a journey with 19 stops. Tick each stop as you finish it.

Discover & Visualise	Understand & Derive	Solve, Apply, Recall
☐ 01 Chapter Introduction ☐ 02 Prerequisite Knowledge ☐ 03 Real-Life Story ☐ 04 Visual Concepts (CPA)	☐ 05 Mathematical Foundations ☐ 06 Formula Derivations ☐ 08 Multiple Solution Methods ☐ 10 Common Mistakes	☐ 07 Detailed Solved Examples ☐ 09 Hands-on Activities ☐ 11 Graded Practice (5 Levels) ☐ 12 Mathematical Reasoning
☐ 13 Real-Life Applications ☐ 14 Competency-Based Questions	☐ 15 Olympiad Challenges ☐ Previous Year Questions	☐ 16 Chapter Mind Map ☐ 17 Pin-Point Flash Cards ☐ 18 Active Recall ☐ 19 Mastery Tracker

How to use this chapter (read this first!)

1. **Never read a solution passively.** Cover the steps, try yourself, then uncover one step at a time.
2. Every solution shows **Step 1, Step 2, Step 3…** with a small *Why* note in purple — the reason behind the step. Examiners award marks for these steps.
3. In identity proofs, watch the **name of the identity** used at each step — that naming habit is what separates a 3-mark proof from a 1-mark attempt.
4. Finish with the Mind Map (16), Flash Cards (17), closed-book Recall (18) and your Mastery Tracker (19). Return on Day 3, 7, 14, 21 and 30.

KALVIYOGI DIGITAL COMPANION — Chapter 8: Video lessons for every derivation • an interactive right-triangle simulator (drag the angle, watch the ratios) • online chapter tests • audio revision shorts. **QR code and link activate with the KalviYogi app/website at print.** (Placeholder — no link is printed in this edition.)

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02 Prerequisite Knowledge

CLASS 9 RECAP — THE TOOLS YOU ALREADY OWN

Trigonometry stands on three Class 9 pillars. Refresh them now — five minutes here will save you fifty minutes later.

1. Pythagoras' theorem — the engine of this chapter

In a right-angled triangle, **(hypotenuse)² = (base)² + (perpendicular)²**. The hypotenuse is the longest side, always opposite the 90° angle. The famous triples — (3, 4, 5), (5, 12, 13), (7, 24, 25), (8, 15, 17), (9, 40, 41), (20, 21, 29) — appear again and again in this chapter. Spot them and you save a full step.

2. Similar triangles — why ratios survive resizing

If two triangles have the same angles, their corresponding sides are in the **same ratio** (AA similarity). A tiny paper triangle and a giant one on a hill, if both have a 30° angle, have identical side-*ratios*. This single fact is the soul of trigonometry.

3. Angles, degrees and complements

An **acute** angle is between 0° and 90°. In a right triangle, the two acute angles add to 90° — they are **complementary**: if one is A, the other is 90° − A. Section 06 will turn this into six free formulas.

4. Square roots you must know by heart

√1 = 1, √2 ≈ 1.414, √3 ≈ 1.732, √4 = 2. And the rationalising move: 1/√2 = √2/2, 1/√3 = √3/3 — multiply top and bottom by the root. You will use this in almost every solution.

PIN POINT: In every right triangle of this chapter: name the hypotenuse FIRST. Every ratio is counted from the angle you are standing at — change the angle, and opposite/adjacent swap their roles.

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Quick Check — Are You Ready? (5 Questions)

Try all five without looking back. Cover the answers with your hand first!

Q1. A right triangle has legs 6 cm and 8 cm. Find the hypotenuse.

Q2. A right triangle has hypotenuse 25 cm and one leg 7 cm. Find the other leg.

Q3. In $\triangle ABC$, $\angle B = 90°$ and $\angle A = 35°$. Find $\angle C$.

Q4. Rationalise: $1/\sqrt{3}$ and $2/\sqrt{2}$.

Q5. Two right triangles both have an acute angle of 40°. The smaller has sides 3, 4, 5 (in cm); the larger is exactly double. Write the ratio (side opposite 40°) ÷ (hypotenuse) for both triangles.

Answers — check only after trying

Q1. $\sqrt{(6^2 + 8^2)} = \sqrt{100} =$ **10 cm** (the 3-4-5 family, doubled).

Q2. $\sqrt{(25^2 - 7^2)} = \sqrt{(625 - 49)} = \sqrt{576} =$ **24 cm** (the 7-24-25 triple).

Q3. $\angle C = 180° - 90° - 35° =$ **55°** — and note $35° + 55° = 90°$: A and C are complementary.

Q4. $1/\sqrt{3} =$ **$\sqrt{3}/3$**; $2/\sqrt{2} =$ **$\sqrt{2}$**.

Q5. Smaller: 4/5 (if the side opposite 40° is 4)... precisely: the ratio is the **same number for both triangles** — doubling every side doubles top and bottom of the ratio, so it cancels. *That* is why one ratio table can serve every triangle in the world.

TOPPER TIP: *Got 4 or 5 right? You are ready. Q5 is the whole chapter in one question — if it clicked, trigonometry will feel easy.*

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03 Real-Life Mathematical Story

DISCOVER • VISUALISE — THE KITE AND THE FLAGPOLE

Before any formula, live the problem. Read this story slowly — the ratios will walk out of it by themselves.

*It is the school Sports Day. **Meena** is flying her bright red kite from the school ground. The string is fully out — **50 metres** of it — and it makes an angle of **60°** with the ground. Her little brother asks the question nobody can answer by looking: "Akka, how HIGH is the kite right now?"*

*Meena cannot climb into the sky with a tape. But she knows the string, the ground and the vertical line below the kite form a **right-angled triangle** — the string is its hypotenuse. If only she knew how the **height** is related to the **string length** when the angle is 60°...*

*Meanwhile, her friend **Arun** has the opposite problem. The school flagpole casts a shadow. At the moment when the Sun's rays make a **45°** angle with the ground, he measures the shadow: exactly **12 metres**. "So the flagpole must be..." he begins — and stops. He needs a rule that connects **height** and **shadow** when the angle is 45°.*

*Two problems, one missing tool: a dictionary between **angles** and **side-ratios**. That dictionary is trigonometry — and in this chapter you will not memorise it. You will **build** it.*

Hold both questions in your mind. By the end of Section 06 you will own every tool needed; in Section 13 you will solve Meena's and Arun's problems completely, step by step.

PIN POINT: Every height-and-distance problem is a right triangle in a costume. Your first job is always the same: find the right angle, name the hypotenuse, and stand at the given angle.

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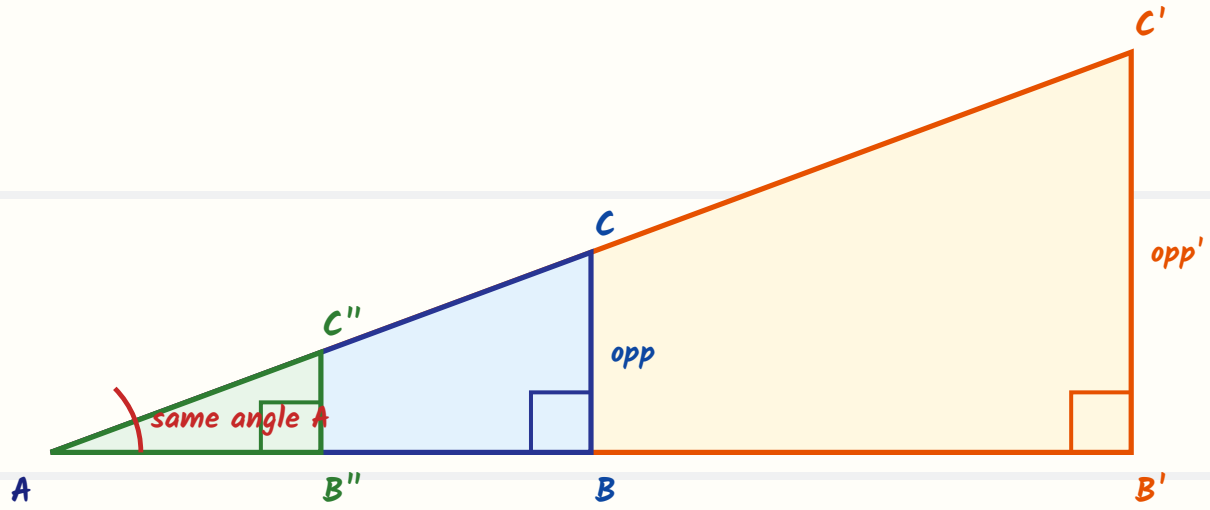
04 Visual Concept Explanation

SINGAPORE CPA — CONCRETE → PICTORIAL → ABSTRACT

Concrete first. Take a set square from your geometry box. It is a right triangle. Measure its sides, then find a bigger right triangle with the same angles — a sun-shadow triangle, a ramp. The sizes differ wildly; something hidden stays the same. Let us see it in a picture.

Now the picture. Three right triangles share the **same angle A**. They are nested like Russian dolls:



Because all three triangles have the same angles, they are **similar** (AA). So the ratio **opposite ÷ hypotenuse** is identical in all three: BC/AC = B′C′/AC′ = B″C″/AC″. The triangle can grow or shrink — **the ratio only remembers the angle**.

Then the abstract. Since this ratio depends only on A, it deserves a name: it is called **sin A**. Five more ratios are waiting in Section 05. Symbols are just the picture, compressed — never memorise the symbol before you can see the picture.

> **TOPPER TIP:** *This is the most important page of the chapter. Whenever a ratio confuses you, come back to these three nested triangles.*

Watch the Ratio Change — Only the Angle Matters

Now keep the triangle the same size but **open the angle wider**. The side opposite the angle stretches, the adjacent side shrinks, and the hypotenuse stays put:

Angle A	Opposite side	Adjacent side	opp ÷ hyp (sin A)	What you see
very small, near 0°	almost 0	almost = hyp	almost 0	The triangle is nearly flat.
30°	exactly half of hyp	√3/2 of hyp	1/2	Section 06 will prove this!
45°	= adjacent side	= opposite side	1/√2	The triangle is isosceles.
60°	√3/2 of hyp	half of hyp	√3/2	The 30° picture, mirrored.
near 90°	almost = hyp	almost 0	almost 1	The triangle is nearly a vertical line.

Read the sin column top to bottom: **0 → 1/2 → 1/√2 → √3/2 → 1**. As the angle grows from 0° to 90°, sin A grows from 0 to 1. You have just seen the skeleton of the standard table — Section 06 will put exact flesh on these bones, with proofs.

PIN POINT: sin A lives between 0 and 1 for acute A — always. If your calculation gives sin A = 4/3 or cos A = 5/4, you have made an error: a part can never exceed the hypotenuse.

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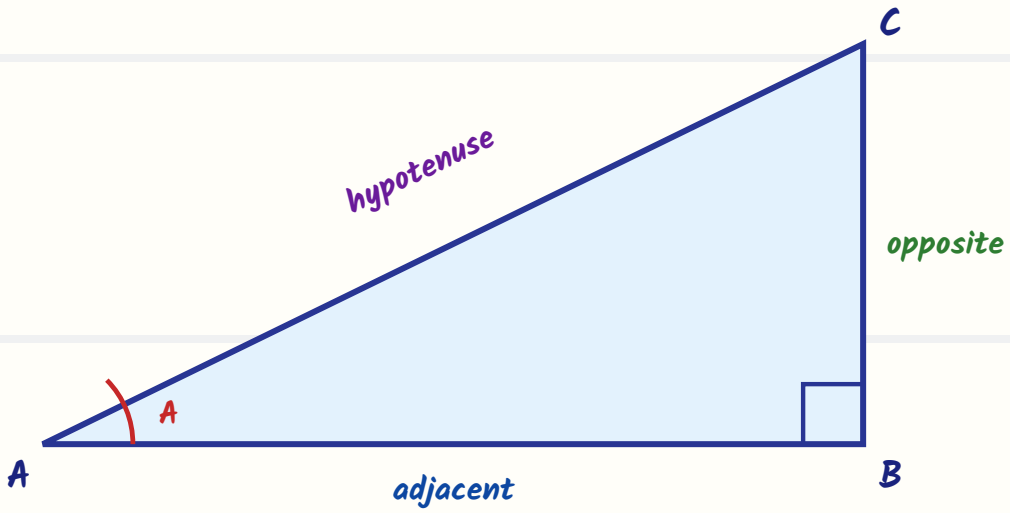
05 Mathematical Foundations

UNDERSTAND — THE SIX RATIOS, DEFINED PRECISELY

Stand at angle A in the right triangle ABC, right-angled at B. Facing you across the triangle is the **opposite side** BC; under your feet runs the **adjacent side** AB; the slanted **hypotenuse** AC is opposite the right angle:



Ratio	Definition (at angle A)	Short memory form
sin A (sine)	opposite ÷ hypotenuse = BC/AC	opp/hyp
cos A (cosine)	adjacent ÷ hypotenuse = AB/AC	adj/hyp
tan A (tangent)	opposite ÷ adjacent = BC/AB	opp/adj
cosec A (cosecant)	hypotenuse ÷ opposite = AC/BC = 1/sin A	hyp/opp
sec A (secant)	hypotenuse ÷ adjacent = AC/AB = 1/cos A	hyp/adj
cot A (cotangent)	adjacent ÷ opposite = AB/BC = 1/tan A	adj/opp

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The Same Triangle from Angle C — Roles Swap!

Now walk across the triangle and stand at angle C instead. The hypotenuse is still AC — it never changes. But the side **opposite** C is AB, and the side **adjacent** to C is BC. So:

At angle A	At angle C	The pattern
sin A = BC/AC	sin C = AB/AC	sin A = cos C, because cos C = BC/AC
cos A = AB/AC	cos C = BC/AC	cos A = sin C
tan A = BC/AB	tan C = AB/BC	tan A = cot C, tan A × tan C = 1

Since A + C = 90° (the two acute angles of a right triangle), C = 90° − A. So the pattern above says: **sin A = cos(90° − A)** — you have just discovered the complementary-angle relations, two sections before we "officially" derive them. Mathematics rewards those who look at pictures carefully.

Three facts to fix forever

1. sin A is **one symbol** — it is NOT sin × A. Similarly cos A, tan A are single quantities. Writing "sin" alone is meaningless.
2. tan A = sin A / cos A and cot A = cos A / sin A — divide the definitions: (opp/hyp) ÷ (adj/hyp) = opp/adj. The hyp cancels!
3. Each ratio is a pure **number** — cm ÷ cm cancels. Ratios have no units, and 0 < sin A, cos A < 1 for acute A, while tan A can be any positive number.

PIN POINT: sin A means 'the sine OF A' — one single number. And tan A = sin A / cos A is not a new formula to memorise: it falls out of the definitions in one line.

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06 Formula Derivations

DERIVE — EVERY VALUE, BUILT FROM GEOMETRY

Most guides print the standard table and say "learn it". We will do the opposite: **build** the table from two triangles you can draw yourself. Once you have built it, you can rebuild it in the exam hall if you ever forget a value.

Derivation 1 — The 45° values (the isosceles right triangle)

1. Take a right triangle with the acute angles 45° and 45° (they must be equal, since they add to 90°).
   Why: A right triangle whose acute angles are equal is isosceles.
2. Equal angles face equal sides, so let the two legs be 1 unit each: AB = BC = 1.
   Why: Choosing 1 unit is allowed — ratios do not depend on size (Section 04).
3. Find the hypotenuse by Pythagoras: AC = $\sqrt{(1^2 + 1^2)} = \sqrt{2}$.
4. Now read off the ratios at 45°: sin 45° = opp/hyp = $1/\sqrt{2}$; cos 45° = adj/hyp = $1/\sqrt{2}$; tan 45° = opp/adj = 1/1 = 1.
   Why: At 45°, opposite = adjacent, so sin = cos and tan = 1 — the picture forces it.
5. The reciprocals follow: cosec 45° = $\sqrt{2}$, sec 45° = $\sqrt{2}$, cot 45° = 1.
   Why: Reciprocal ratios are just the fractions flipped.

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Derivation 2 — The 30° and 60° Values

1. Start with an **equilateral** triangle of side 2 units. Every angle is 60°.

   Why: The equilateral triangle is the most symmetric triangle — a perfect place to hunt for exact values.

2. Drop a perpendicular from the top vertex to the base. It cuts the base into two halves of 1 unit each, and cuts the 60° top angle into two 30° angles.

   Why: In an equilateral triangle, the altitude is also a median and an angle bisector.

3. Look at one half: a right triangle with hypotenuse 2, base 1, and angles 30° and 60°.

   Why: This small triangle is the famous 30-60-90 triangle.

4. Find its height by Pythagoras: $\sqrt{(2^2 - 1^2)} = \sqrt{3}$.

5. Stand at the 30° angle (at the base). Opposite side = 1, adjacent = $\sqrt{3}$, hypotenuse = 2. So $\sin 30° = 1/2$, $\cos 30° = \sqrt{3}/2$, $\tan 30° = 1/\sqrt{3}$.

   Why: Read straight from the picture — nothing memorised.

6. Now stand at the 60° angle (at the top). Opposite and adjacent swap: $\sin 60° = \sqrt{3}/2$, $\cos 60° = 1/2$, $\tan 60° = \sqrt{3}$.

   Why: The 60° row is the 30° row read in the mirror — complementary angles at work.

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Derivation 3 — The Edge Values: 0° and 90°

There is no real triangle with a 0° or 90° acute angle — but we can watch what happens as the angle **approaches** these edges (remember the growing table in Section 04):

1. Let angle A shrink towards 0°. The opposite side shrinks towards 0, while the adjacent side grows to meet the hypotenuse.
   Why: The triangle flattens onto the base.

2. So sin 0° = 0/hyp = **0**, cos 0° = hyp/hyp = **1**, tan 0° = 0/adj = **0**.
   Why: These are limiting values — the natural end of the pattern.

3. Now let A grow towards 90°. The adjacent side shrinks towards 0 and the opposite side grows to meet the hypotenuse.
   Why: The triangle stands up into a vertical line.

4. So sin 90° = **1**, cos 90° = **0**, and tan 90° = opp/0 — division by zero — **not defined**.
   Why: This is why tan 90° has no value: its denominator (adjacent) has vanished. Similarly cosec 0° and cot 0°... cot 0 = adj/opp = hyp/0 — also not defined.

The Standard Table — Now Fully Earned

Angle A	0°	30°	45°	60°	90°
sin A	0	1/2	1/√2	√3/2	1
cos A	1	√3/2	1/√2	1/2	0
tan A	0	1/√3	1	√3	not defined
cosec A	not defined	2	√2	2/√3	1
sec A	1	2/√3	√2	2	not defined
cot A	not defined	√3	1	1/√3	0

TOPPER TIP: *Read the sin row left to right: 0, 1/2, 1/√2, √3/2, 1 — that is √0/2, √1/2, √2/2, √3/2, √4/2. The cos row is the same pattern backwards. A pattern you can regenerate beats a table you must memorise.*

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Derivation 4 — The Master Identity, from Pythagoras

Every identity proof in this chapter runs on one engine. Let us build the engine.

1. In our triangle, sin A = BC/AC and cos A = AB/AC. Compute $\sin^2A + \cos^2A = (BC/AC)^2 + (AB/AC)^2 = (BC^2 + AB^2) / AC^2$.
   Why: Square each ratio and add with the common denominator AC^2.
2. By Pythagoras' theorem, $AB^2 + BC^2 = AC^2$ (the triangle is right-angled at B).
   Why: This is the only fact we borrow — and it is exactly what a right triangle guarantees.
3. Substitute: $\sin^2A + \cos^2A = AC^2 / AC^2 =$ **1**.
   Why: Numerator and denominator are the same quantity.

THE MASTER IDENTITY: $\sin^2A + \cos^2A = 1$ — true for EVERY angle A.

Derivation 5 — The Two Child Identities (divide and conquer)

1. Divide every term of $\sin^2A + \cos^2A = 1$ by $\cos^2A$: $\tan^2A + 1 = \sec^2A$, i.e. **$1 + \tan^2A = \sec^2A$**.
   Why: sin/cos = tan and 1/cos = sec — the definitions do all the work.
2. Start again and divide every term by $\sin^2A$: $1 + \cot^2A = \text{cosec}^2A$.
   Why: cos/sin = cot and 1/sin = cosec.
3. Memorise the family as one line each: $\sin^2 + \cos^2 = 1$ • $\sec^2 - \tan^2 = 1$ • $\text{cosec}^2 - \cot^2 = 1$.
   Why: The rearranged forms ($\sec^2 - \tan^2 = 1$) are the ones proofs actually use — watch for them in Section 07.

Derivation 6 — Complementary-Angle Relations

From Section 05's role-swap table, with C = 90° − A:

$\sin(90° - A) = \cos A$	$\cos(90° - A) = \sin A$	$\tan(90° - A) = \cot A$
$\cot(90° - A) = \tan A$	$\sec(90° - A) = \text{cosec } A$	$\text{cosec}(90° - A) = \sec A$

One line to remember them by: **"co" pairs swap** — sine ↔ cosine, tangent ↔ cotangent, secant ↔ cosecant, whenever the angle is replaced by its complement.

PIN POINT: All three identities are Pythagoras wearing different clothes. If you forget one in the exam, re-derive it: write $\sin^2 + \cos^2 = 1$ and divide.

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07 Detailed Solved Examples

SOLVE — NCERT EXERCISE 8.1, EVERY QUESTION, EVERY STEP

These are the complete NCERT Exercise 8.1 solutions (rationalised edition). Notice the rhythm of every solution: **find the missing side first (Pythagoras), then read the ratios off the picture.**

EX 8.1 Q1

In △ABC, right-angled at B, AB = 24 cm and BC = 7 cm. Determine: (i) sin A, cos A (ii) sin C, cos C.

THINK FIRST: Two sides are given and every ratio needs all three sides. Pythagoras first, ratios second.

1. The hypotenuse is AC (opposite the right angle at B). By Pythagoras: AC = $\sqrt{(AB^2 + BC^2)}$ = $\sqrt{(24^2 + 7^2)}$ = $\sqrt{(576 + 49)}$ = $\sqrt{625}$ = **25 cm**.

   Why: 7-24-25 is a Pythagorean triple — spotting it saves the whole calculation.

2. (i) Stand at A. Opposite side = BC = 7, adjacent = AB = 24, hypotenuse = 25. So sin A = 7/25 and cos A = 24/25.

   Why: Ratios are always read from the angle you stand at.

3. (ii) Stand at C. Now the opposite side is AB = 24 and the adjacent side is BC = 7. So sin C = 24/25 and cos C = 7/25.

   Why: Same triangle, different viewpoint — the roles of the legs swap.

4. Notice: sin A = cos C and cos A = sin C.

   Why: A + C = 90° — the complementary relations of Derivation 6, alive in numbers.

ANSWER: (i) sin A = 7/25, cos A = 24/25 (ii) sin C = 24/25, cos C = 7/25

CBSE-STYLE MARKING SCHEME: Correct hypotenuse AC = 25 [1] • sin A and cos A [1] • sin C and cos C [1]

COMMON MISTAKE: Writing sin A = 24/25 by copying the first side you see. **The sides have no fixed names** — opposite and adjacent are decided by the ANGLE, not by the letter.

Exercise 8.1 — Questions 2 and 3

EX 8.1 Q2

In the figure, △PQR is right-angled at Q, with PQ = 5 cm, QR = 12 cm and PR = 13 cm. Find tan P − cot R.

THINK FIRST: tan P is read at P; cot R is read at R. Opposite/adjacent swap between the two angles — watch carefully.

1. Check the triple: $5^2 + 12^2 = 25 + 144 = 169 = 13^2$. So PR = 13 is indeed the hypotenuse, and the right angle is at Q.

   Why: Always confirm which side is the hypotenuse before reading any ratio.

2. At angle P: opposite side = QR = 12, adjacent side = PQ = 5. So tan P = 12/5.

3. At angle R: opposite side = PQ = 5, adjacent side = QR = 12. So cot R = adjacent/opposite = 12/5.

   Why: cot is adjacent over opposite — the flip of tan at the same angle, but here we are at R, not P.

4. Subtract: tan P − cot R = 12/5 − 12/5 = **0**.

   Why: Not a coincidence: tan P = cot(90° − P) = cot R, always.

ANSWER: tan P − cot R = 0

CBSE-STYLE MARKING SCHEME: tan P = 12/5 [1] • cot R = 12/5 [1] • difference 0 [½]

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Exercise 8.1 — Question 3

EX 8.1 Q3

If sin A = 3/4, calculate cos A and tan A.

THINK FIRST: One ratio is given as a fraction. Turn it into a triangle: opposite = 3, hypotenuse = 4, then find the third side.

1. sin A = opposite/hypotenuse = 3/4. Build the triangle with opposite side 3 and hypotenuse 4 (any scale works — ratios ignore size).
   Why: A ratio IS a triangle, compressed. Unpack it.
2. Find the adjacent side by Pythagoras: $\sqrt{(4^2 - 3^2)} = \sqrt{(16 - 9)} = \sqrt{7}$.
   Why: Hypotenuse² MINUS the known leg² — subtraction, because we solve for a leg.
3. cos A = adjacent/hypotenuse = **√7/4**.
4. tan A = opposite/adjacent = **3/√7** (= 3√7/7 rationalised).
5. Sanity check with the master identity: $\sin^2 A + \cos^2 A = 9/16 + 7/16 = 1$.
   Why: Thirty seconds of checking catches most silly errors in this chapter.

ANSWER: cos A = √7/4 and tan A = 3/√7

CBSE-STYLE MARKING SCHEME: Adjacent side √7 by Pythagoras [1] • cos A [½] • tan A [½]

Exercise 8.1 — Questions 4, 5 and 6

EX 8.1 Q4

Given 15 cot A = 8, find sin A and sec A.

THINK FIRST: First free cot A from the equation, then build its triangle: cot = adjacent/opposite.

1. 15 cot A = 8 gives cot A = 8/15.
2. cot A = adjacent/opposite = 8/15, so take adjacent = 8 and opposite = 15.
3. Hypotenuse = $\sqrt{(8^2 + 15^2)} = \sqrt{(64 + 225)} = \sqrt{289} = \mathbf{17}$.
   Why: 8-15-17 — another triple worth recognising on sight.
4. sin A = opposite/hypotenuse = **15/17**.
5. sec A = hypotenuse/adjacent = **17/8**.
   Why: sec is the reciprocal of cos: cos A = 8/17, flip it.

ANSWER: sin A = 15/17 and sec A = 17/8

CBSE-STYLE MARKING SCHEME: cot A = 8/15 [½] • hypotenuse 17 [1] • sin A [½] • sec A [½]

EX 8.1 Q5

Given sec θ = 13/12, calculate all other trigonometric ratios.

THINK FIRST: sec gives cos at once (flip it). Then one Pythagoras step unlocks the whole family.

1. sec θ = hypotenuse/adjacent = 13/12. Take hypotenuse = 13, adjacent = 12.
2. Opposite side = $\sqrt{(13^2 - 12^2)} = \sqrt{(169 - 144)} = \sqrt{25} = \mathbf{5}$.
   Why: 5-12-13, the hardest-working triple in this chapter.
3. Now read all six: sin θ = 5/13, cos θ = 12/13, tan θ = 5/12, cosec θ = 13/5, sec θ = 13/12 (given), cot θ = 12/5.
   Why: With all three sides known, every ratio is just a choice of two sides.

ANSWER: sin θ = 5/13, cos θ = 12/13, tan θ = 5/12, cosec θ = 13/5, cot θ = 12/5

CBSE-STYLE MARKING SCHEME: Opposite side 5 [1] • any three correct ratios [1] • all five correct [½]

Exercise 8.1 — Question 6

EX 8.1 Q6

If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.

THINK FIRST: Equal cosines, two different triangles. Build both triangles, let the equal ratio force the triangles to be similar — similar triangles have equal angles.

1. In △ABC (right-angled at C, say), cos A = AC/AB. In a second right triangle PQR (right-angled at R), cos B = PR/PQ.

   Why: Write the definition of cos in each triangle separately.

2. Given cos A = cos B, so AC/AB = PR/PQ = k (call the common value k). Then AC = k·AB and PR = k·PQ.

   Why: Naming the common ratio k turns an equation into two usable facts.

3. By Pythagoras in each triangle, the third sides also come out in the same ratio: BC = AB$\sqrt{(1 - k^2)}$ and QR = PQ$\sqrt{(1 - k^2)}$. So all three pairs of sides are proportional.

   Why: The hypotenuse-leg pair forces the remaining legs into the same proportion.

4. Hence the two triangles are similar (SSS), so their corresponding angles are equal: ∠A = ∠B. ∎

   Why: Similar triangles are the deep reason: a ratio determines the angle completely.

ANSWER: ∠A = ∠B, proved via similarity of the two triangles

CBSE-STYLE MARKING SCHEME: cos written in both triangles [1] • proportional sides via Pythagoras [1] • similarity ⇒ equal angles [½]

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Exercise 8.1 — Questions 7 and 8

EX 8.1 Q7

If cot θ = 7/8, evaluate: (i) [(1 + sin θ)(1 − sin θ)] / [(1 + cos θ)(1 − cos θ)] (ii) $\cot^2 \theta$.

THINK FIRST: The brackets are begging to be multiplied: $(1 + x)(1 - x) = 1 - x^2$. The master identity then converts everything to cot.

1. (i) Numerator: $(1 + \sin\theta)(1 - \sin\theta) = 1 - \sin^2\theta = \cos^2\theta$.
   Why: Algebra identity first, then $\sin^2 + \cos^2 = 1$ in the form $1 - \sin^2 = \cos^2$.
2. Denominator: $(1 + \cos\theta)(1 - \cos\theta) = 1 - \cos^2\theta = \sin^2\theta$.
   Why: Same two moves, mirrored.
3. So the fraction = $\cos^2\theta / \sin^2\theta = \cot^2\theta$.
   Why: cos/sin = cot, squared throughout.
4. From cot θ = 7/8: $\cot^2\theta = 49/64$. So (i) = **49/64**.
5. (ii) $\cot^2\theta = (7/8)^2$ = **49/64** directly.
   Why: Part (ii) was hidden inside part (i) all along.

ANSWER: (i) 49/64 (ii) 49/64

CBSE-STYLE MARKING SCHEME: (i) expansion + identity [1] • reduction to $\cot^2\theta$ [½] • value 49/64 [½] • (ii) [½]

TOPPER TIP: *When you see (1 + sin)(1 − sin) in ANY question, your pen should write cos² before your brain finishes reading. Pattern reflexes win marks.*

Exercise 8.1 — Question 8

EX 8.1 Q8

If 3 cot A = 4, check whether $(1 - \tan^2 A)/(1 + \tan^2 A) = \cos^2 A - \sin^2 A$ or not.

THINK FIRST: A 'check whether' question: compute the LHS and the RHS separately and compare. Never assume — verify.

1. 3 cot A = 4 gives cot A = 4/3, so tan A = 3/4 (flip it).
   Why: tan and cot are reciprocals.
2. LHS: $(1 - \tan^2 A)/(1 + \tan^2 A) = (1 - 9/16)/(1 + 9/16) = (7/16)/(25/16) =$ **7/25**.
3. For the RHS we need sin A and cos A. cot A = 4/3 gives adjacent 4, opposite 3, hypotenuse 5 (3-4-5!). So cos A = 4/5 and sin A = 3/5.
4. RHS: $\cos^2 A - \sin^2 A = 16/25 - 9/25 =$ **7/25**.
5. LHS = RHS = 7/25, so YES — the equality holds.
   Why: Both roads reached the same number; the check is complete.

ANSWER: Yes — both sides equal 7/25, so the equality is true

CBSE-STYLE MARKING SCHEME: tan A = 3/4 [½] • LHS = 7/25 [1] • sin, cos from the 3-4-5 triangle [½] • RHS = 7/25 and conclusion [½]

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Exercise 8.1 — Questions 9, 10 and 11

EX 8.1 Q9

In △ABC, right-angled at B, if tan A = 1/√3, find: (i) sin A cos C + cos A sin C (ii) cos A cos C − sin A sin C.

THINK FIRST: tan A = 1/√3 is the table knocking at the door: it says A = 30°. Then C = 60°, and every ratio is a table value.

1. tan A = 1/√3 = tan 30°, so A = 30° and C = 90° − 30° = 60°.
   Why: Recognising standard values inside problems is a core exam skill.

2. Collect the four values: sin A = 1/2, cos A = √3/2, sin C = √3/2, cos C = 1/2.
   Why: From the derived table — sin 60° = cos 30°, exactly as Derivation 6 promised.

3. (i) sin A cos C + cos A sin C = (1/2)(1/2) + (√3/2)(√3/2) = 1/4 + 3/4 = **1**.

4. (ii) cos A cos C − sin A sin C = (√3/2)(1/2) − (1/2)(√3/2) = √3/4 − √3/4 = **0**.
   Why: The two products are identical, so they cancel exactly.

ANSWER: (i) 1 (ii) 0

CBSE-STYLE MARKING SCHEME: A = 30°, C = 60° [1] • four ratio values [½] • (i) = 1 [½] • (ii) = 0 [½]

Exercise 8.1 — Question 10

EX 8.1 Q10

In △PQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine sin P, cos P and tan P.

THINK FIRST: Neither PR nor QR is given alone — only their sum. Let QR = x, write PR in terms of x, and let Pythagoras solve for x.

1. Let QR = x cm. Then PR = (25 − x) cm, since PR + QR = 25.

   Why: One unknown, one equation — Pythagoras will supply it.

2. PR is the hypotenuse (opposite the right angle at Q). Pythagoras: $PR^2 = PQ^2 + QR^2$, so $(25 - x)^2 = 5^2 + x^2$.

3. Expand: $625 - 50x + x^2 = 25 + x^2$. The x^2 terms cancel: $625 - 50x = 25$, so $50x = 600$ and $x = \mathbf{12}$.

   Why: The quadratic collapses to a linear equation — a gift; take it.

4. So QR = 12 cm and PR = 25 − 12 = 13 cm. The triangle is 5-12-13.

5. At P: opposite = QR = 12, adjacent = PQ = 5, hypotenuse = PR = 13. sin P = 12/13, cos P = 5/13, tan P = 12/5.

ANSWER: sin P = 12/13, cos P = 5/13, tan P = 12/5

CBSE-STYLE MARKING SCHEME: Setting up $(25 - x)^2 = 25 + x^2$ [1] • QR = 12, PR = 13 [1] • three ratios [½]

Exercise 8.1 — Question 11

EX 8.1 Q11

State whether the following are true or false. Justify your answer. (i) The value of tan A is always less than 1. (ii) sec A = 12/5 for some value of angle A. (iii) cos A is the abbreviation used for the cosecant of angle A. (iv) cot A is the product of cot and A. (v) sin θ = 4/3 for some angle θ.

THINK FIRST: Each part tests one fact from Section 05. Answer with a reason, not just T/F — the justification carries the marks.

1. (i) **False.** tan A = opposite/adjacent can exceed 1: tan 60° = √3 > 1, whenever the opposite side is the longer leg.

   Why: A single counterexample destroys an 'always' claim.

2. (ii) **True.** sec A = hypotenuse/adjacent ≥ 1 always, and 12/5 = 2.4 ≥ 1, so such an angle exists (cos A = 5/12).

   Why: sec can be any number from 1 upwards.

3. (iii) **False.** cos A is the abbreviation for the **cosine** of angle A. The cosecant is written cosec A.

   Why: Names matter: cos ≠ cosec.

4. (iv) **False.** cot A is a single symbol — the cotangent of A — not a product of 'cot' and 'A'.

   Why: Just like sin A in Section 05, fact 1.

5. (v) **False.** sin θ = opposite/hypotenuse, and the opposite side can never exceed the hypotenuse, so sin θ ≤ 1 < 4/3 for every angle.

   Why: The PIN POINT from Section 04, doing real work.

ANSWER: (i) False (ii) True (iii) False (iv) False (v) False

CBSE-STYLE MARKING SCHEME: Each correct verdict with a valid reason [½ × 5]

PIN POINT: The five verdicts of Q11 are five flash-card facts in disguise: tan can exceed 1 • sec ≥ 1 • cos = cosine • cot A is one symbol • sin and cos never exceed 1.

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Exercise 8.2 — The Standard Table at Work (Q1)

Every value below comes from the table you derived in Section 06. The method never changes: **substitute → simplify → rationalise if needed.**

EX 8.2 Q1

Evaluate: (i) $\sin 60° \cos 30° + \sin 30° \cos 60°$ (ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$ (iii) $\cos 45° / (\sec 30° + \operatorname{cosec} 30°)$ (iv) $(\sin 30° + \tan 45° - \operatorname{cosec} 60°) / (\sec 30° + \cos 60° + \cot 45°)$ (v) $(5\cos^2 60° + 4\sec^2 30° - \tan^2 45°) / (\sin^2 30° + \cos^2 30°)$

THINK FIRST: Write every value from the table first, in one line, before any arithmetic. Most errors in this exercise are substitution errors, not idea errors.

1. (i) Substitute: $(\sqrt{3}/2)(\sqrt{3}/2) + (1/2)(1/2) = 3/4 + 1/4 =$ **1**.

2. (ii) Substitute: $2(1)^2 + (\sqrt{3}/2)^2 - (\sqrt{3}/2)^2 = 2 + 3/4 - 3/4 =$ **2**.

   Why: The two squared terms are identical — they cancel before you compute anything.

3. (iii) Substitute: $(1/\sqrt{2}) / (2/\sqrt{3} + 2)$. Write the denominator as $(2 + 2\sqrt{3})/\sqrt{3}$, so the value = $(1/\sqrt{2}) \times (\sqrt{3}/(2 + 2\sqrt{3})) = \sqrt{3} / (2\sqrt{2}(1 + \sqrt{3}))$.

   Why: Division by a sum: rewrite the sum as a single fraction first.

4. Rationalise with $(\sqrt{3} - 1)$: numerator $\sqrt{3}(\sqrt{3} - 1) = 3 - \sqrt{3}$; denominator $2\sqrt{2}(3 - 1) = 4\sqrt{2}$. Value = $(3 - \sqrt{3})/(4\sqrt{2}) =$ **$(3\sqrt{2} - \sqrt{6})/8$**.

   Why: Multiply top and bottom by $\sqrt{2}$ at the end to clear the last root from the denominator.

ANSWER: (i) 1 (ii) 2 (iii) $(3\sqrt{2} - \sqrt{6})/8$ — parts (iv) and (v) continue on the next page

CBSE-STYLE MARKING SCHEME: Correct substitution in each part [½ each] • simplifications [1]

Exercise 8.2 Q1 — Parts (iv) and (v)

EX 8.2 Q1 (CONTINUED)

Evaluate: (iv) $(\sin 30° + \tan 45° - \operatorname{cosec} 60°) / (\sec 30° + \cos 60° + \cot 45°)$ (v) $(5\cos^2 60° + 4\sec^2 30° - \tan^2 45°) / (\sin^2 30° + \cos^2 30°)$

1. (iv) Numerator: $1/2 + 1 - 2/\sqrt{3} = (3\sqrt{3} - 4)/(2\sqrt{3})$. Denominator: $2/\sqrt{3} + 1/2 + 1 = (4 + 3\sqrt{3})/(2\sqrt{3})$. The $(2\sqrt{3})$ cancels in the division, leaving $(3\sqrt{3} - 4)/(3\sqrt{3} + 4)$.

   Why: Put each part over the common denominator $2\sqrt{3}$ and watch the structure appear.

2. Rationalise with $(3\sqrt{3} - 4)$: numerator $(3\sqrt{3} - 4)^2 = 27 + 16 - 24\sqrt{3} = 43 - 24\sqrt{3}$; denominator $27 - 16 = 11$. Value = **$(43 - 24\sqrt{3})/11$**.

   Why: $(a - b)(a + b) = a^2 - b^2$ clears the root in the denominator.

3. (v) Denominator first: $\sin^2 30° + \cos^2 30° = 1$ — the master identity, no arithmetic needed!

   Why: Always scan for $\sin^2 + \cos^2$ hiding in a problem.

4. Numerator: $5(1/2)^2 + 4(2/\sqrt{3})^2 - (1)^2 = 5/4 + 16/3 - 1 = (15 + 64 - 12)/12 = 67/12$. Value = **67/12**.

   Why: $\sec 30° = 2/\sqrt{3}$, and its square is $4/3$ — the root vanishes on squaring.

ANSWER: (i) 1 (ii) 2 (iii) $(3\sqrt{2} - \sqrt{6})/8$ (iv) $(43 - 24\sqrt{3})/11$ (v) 67/12

CBSE-STYLE MARKING SCHEME: Correct substitution in each part [½ each] • simplifications [1] • rationalised final forms [1]

COMMON MISTAKE: Writing $\cos^2 30° = (\cos 30°)^2$ correctly but then computing $(\sqrt{3}/2)^2$ as $\sqrt{3}/4$. **Square the numerator AND the denominator: 3/4.**

Exercise 8.2 — Questions 2, 3 and 4

EX 8.2 Q2

Choose the correct option and justify: (i) $2\tan 30°/(1+\tan^2 30°)$ equals (A) sin 60° (B) cos 60° (C) tan 60° (D) sin 30°. (ii) $(1-\tan^2 45°)/(1+\tan^2 45°)$ equals (A) tan 90° (B) 1 (C) sin 45° (D) 0. (iii) sin 2A = 2 sin A is true when A = (A) 0° (B) 30° (C) 45° (D) 60°. (iv) $2\tan 30°/(1-\tan^2 30°)$ equals (A) cos 60° (B) sin 60° (C) tan 60° (D) sin 30°.

THINK FIRST: Do not guess from memory — substitute tan 30° = 1/√3 and tan 45° = 1 and let the arithmetic choose the option.

1. (i) Numerator $2/\sqrt{3}$; denominator $1 + 1/3 = 4/3$. Value = $(2/\sqrt{3}) \times (3/4) = \sqrt{3}/2 = \sin 60°$. **Option (A).**

2. (ii) tan 45° = 1, so numerator = 1 − 1 = 0 and the value is **0. Option (D).**

   Why: When the numerator is 0, the denominator hardly matters (it is 2, not 0).

3. (iii) Test A = 0°: LHS sin 0° = 0, RHS 2 sin 0° = 0. Equal! **Option (A).** (For 30°: sin 60° = √3/2 but 2 sin 30° = 1 — not equal.)

   Why: For 'true when' MCQs, test the options one by one – start with the easiest, 0°.

4. (iv) Numerator $2/\sqrt{3}$; denominator $1 - 1/3 = 2/3$. Value = $(2/\sqrt{3}) \times (3/2) = \sqrt{3} = \tan 60°$. **Option (C).**

ANSWER: (i) A (ii) D (iii) A (iv) C

CBSE-STYLE MARKING SCHEME: Each option with a substitution-based justification [½ × 4]

Exercise 8.2 — Questions 3 and 4

EX 8.2 Q3

If tan(A + B) = √3 and tan(A − B) = 1/√3, with 0° < A + B ≤ 90° and A > B, find A and B.

THINK FIRST: tan of something equals √3 — the table says that 'something' is 60°. Two equations, two unknowns.

1. tan(A + B) = √3 = tan 60°, so A + B = 60°. ...(1)

   Why: The condition 0° < A + B ≤ 90° guarantees this is the only possibility in range.

2. tan(A − B) = 1/√3 = tan 30°, so A − B = 30°. ...(2)

3. Add (1) and (2): 2A = 90°, so A = **45°**.

   Why: Adding eliminates B — the elimination method from Chapter 3, back again.

4. Subtract (2) from (1): 2B = 30°, so B = **15°**. Check: A > B and A + B = 60° ≤ 90°.

   Why: Always verify the given conditions at the end.

ANSWER: A = 45° and B = 15°

CBSE-STYLE MARKING SCHEME: A + B = 60° and A − B = 30° [1] • solving the pair [1] • verification [½]

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Exercise 8.2 — Question 4

EX 8.2 Q4

State whether the following are true or false. Justify. (i) sin(A + B) = sin A + sin B. (ii) The value of sin θ increases as θ increases (0° to 90°). (iii) The value of cos θ increases as θ increases (0° to 90°). (iv) sin θ = cos θ for all values of θ. (v) cot A is not defined for A = 0°.

THINK FIRST: One numerical test kills most false claims. Use A = B = 30° or θ = 0°, 45°, 90° from the table.

1. (i) **False.** Take A = 30°, B = 60°: LHS = sin 90° = 1, but RHS = 1/2 + √3/2 > 1. Not equal.

   Why: Ratios do NOT distribute over sums of angles — a classic trap.

2. (ii) **True.** From the table: sin 0° = 0, sin 30° = 1/2, sin 45° = 1/√2, sin 60° = √3/2, sin 90° = 1 — steadily increasing.

   Why: The Section 04 picture: the opposite side grows as the angle opens.

3. (iii) **False.** cos falls: 1, √3/2, 1/√2, 1/2, 0. It *decreases* as θ increases.

   Why: cos is the sin row read backwards.

4. (iv) **False.** At θ = 30°: sin = 1/2 but cos = √3/2. They are equal only at θ = 45°.

   Why: 'For all' claims die by one counterexample.

5. (v) **True.** cot 0° = cos 0°/sin 0° = 1/0 — division by zero, so cot 0° is not defined.

   Why: Same family as tan 90°: a zero denominator.

ANSWER: (i) False (ii) True (iii) False (iv) False (v) True

CBSE-STYLE MARKING SCHEME: Each verdict with a table-based justification [½ × 5]

Exercise 8.3 — Expressing Ratios and Complementary Angles

The final NCERT exercise (8.3 in the rationalised edition) has one theme: **convert everything into one chosen ratio**, and let the identities do the rest.

EX 8.3 Q1

Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

THINK FIRST: cot A is the anchor. Every other ratio must be built from it using only the three identities.

1. tan A = 1/cot A.
   Why: tan and cot are reciprocals — definition.
2. Use $1 + \cot^2 A = \text{cosec}^2 A$: cosec A = $\sqrt{(1 + \cot^2 A)}$, so sin A = 1/cosec A = $\mathbf{1/\sqrt{(1 + \cot^2 A)}}$.
   Why: The identity $\text{cosec}^2 - \cot^2 = 1$ (Derivation 5) is the bridge.
3. Use the master identity for cos: $\cos^2 A = 1 - \sin^2 A = 1 - 1/(1 + \cot^2 A) = \cot^2 A/(1 + \cot^2 A)$, so cos A = cot A/$\sqrt{(1 + \cot^2 A)}$.
   Why: A is acute, so we take the positive root.
4. sec A = 1/cos A = $\mathbf{\sqrt{(1 + \cot^2 A)}/\cot A}$.
5. Collect: sin A = 1/$\sqrt{(1 + \cot^2 A)}$, sec A = $\sqrt{(1 + \cot^2 A)}$/cot A, tan A = 1/cot A.
   Why: Check with A = 45° (cot = 1): sin = 1/√2, sec = √2, tan = 1.

ANSWER: sin A = $1/\sqrt{(1 + \cot^2 A)}$, sec A = $\sqrt{(1 + \cot^2 A)}/\cot A$, tan A = 1/cot A

CBSE MARKING SCHEME: tan A [½] • sin A via cosec identity [1] • sec A via cos [1]

Exercise 8.3 — Question 2

EX 8.3 Q2

Write all the other trigonometric ratios of ∠A in terms of sec A.

THINK FIRST: Same game, new anchor: sec A. cos falls out immediately; the rest follow through tan.

1. cos A = 1/sec A.
   Why: Reciprocal pair.
2. Use $1 + \tan^2 A = \sec^2 A$: $\tan^2 A = \sec^2 A - 1$, so $\tan A = \sqrt{(\sec^2 A - 1)}$.
3. $\cot A = 1/\tan A = 1/\sqrt{(\sec^2 A - 1)}$.
4. $\sin^2 A = 1 - \cos^2 A = 1 - 1/\sec^2 A = (\sec^2 A - 1)/\sec^2 A$, so $\sin A = \sqrt{(\sec^2 A - 1)}/\sec A$.
   Why: Master identity, then the positive root (A acute).
5. $\text{cosec } A = 1/\sin A = \sec A/\sqrt{(\sec^2 A - 1)}$.
6. Check with A = 60° (sec = 2): tan = √3, sin = √3/2, cosec = 2/√3.
   Why: A 10-second table check validates the whole chain.

ANSWER: cos A = 1/sec A, $\tan A = \sqrt{(\sec^2 A - 1)}$, $\cot A = 1/\sqrt{(\sec^2 A - 1)}$, $\sin A = \sqrt{(\sec^2 A - 1)}/\sec A$, $\text{cosec } A = \sec A/\sqrt{(\sec^2 A - 1)}$

CBSE MARKING SCHEME: cos A and tan A [1] • sin A [1] • cot A and cosec A [½]

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Exercise 8.3 — Questions 3 and 4

EX 8.3 Q3

Evaluate: (i) $(\sin^2 63° + \sin^2 27°) / (\cos^2 17° + \cos^2 73°)$ (ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$.

THINK FIRST: Every angle here has its complement hiding nearby: 27 = 90 − 63, 73 = 90 − 17, 65 = 90 − 25. Convert, then $\sin^2 + \cos^2 = 1$ finishes.

1. (i) $\sin 27° = \sin(90° - 63°) = \cos 63°$, so the numerator = $\sin^2 63° + \cos^2 63° = 1$.
   Why: Complementary relation (Derivation 6) + master identity.
2. $\cos 73° = \cos(90° - 17°) = \sin 17°$, so the denominator = $\cos^2 17° + \sin^2 17° = 1$.
   Why: Same two moves.
3. Value = 1/1 = **1**.
4. (ii) $\cos 65° = \cos(90° - 25°) = \sin 25°$ and $\sin 65° = \cos 25°$. So the expression = $\sin^2 25° + \cos^2 25° =$ **1**.
   Why: Both terms convert, and the master identity appears again.

ANSWER: (i) 1 (ii) 1

CBSE MARKING SCHEME: (i) conversions [1] • value 1 [½] • (ii) conversions [½] • value 1 [½]

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Exercise 8.3 — Question 4

EX 8.3 Q4

Choose the correct option and justify: (i) $9\sec^2 A - 9\tan^2 A$ = (A) 1 (B) 9 (C) 8 (D) 0. (ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$ = (A) 0 (B) 1 (C) 2 (D) −1. (iii) $(\sec A + \tan A)(1 - \sin A)$ = (A) sec A (B) sin A (C) cosec A (D) cos A. (iv) $(1 + \tan^2 A)/(1 + \cot^2 A)$ = (A) $\sec^2 A$ (B) −1 (C) $\cot^2 A$ (D) $\tan^2 A$.

THINK FIRST: Factor before you expand. In (ii) and (iii), converting to sin and cos turns a monster into two lines.

1. (i) $9(\sec^2 A - \tan^2 A) = 9 \times 1 =$ **9. Option (B).**

   Why: $\sec^2 - \tan^2 = 1$ — the rearranged identity.

2. (ii) Convert: $1 + \tan\theta + \sec\theta = (\cos\theta + \sin\theta + 1)/\cos\theta$, and $1 + \cot\theta - \operatorname{cosec}\theta = (\sin\theta + \cos\theta - 1)/\sin\theta$.

   Why: Express everything in sin and cos over a common denominator.

3. Multiply: $[(\sin\theta + \cos\theta)^2 - 1] / (\sin\theta\cos\theta) = [1 + 2\sin\theta\cos\theta - 1] / (\sin\theta\cos\theta) =$ **2. Option (C).**

   Why: $(a + 1)(a - 1) = a^2 - 1$ with $a = \sin + \cos$, and $\sin^2 + \cos^2 = 1$.

4. (iii) $(\sec A + \tan A)(1 - \sin A) = ((1 + \sin A)/\cos A)(1 - \sin A) = (1 - \sin^2 A)/\cos A = \cos^2 A/\cos A =$ **cos A. Option (D).**

   Why: sec + tan over a common denominator is $(1 + \sin)/\cos$ — remember this shape.

5. (iv) $(1 + \tan^2 A)/(1 + \cot^2 A) = \sec^2 A/\operatorname{cosec}^2 A = (1/\cos^2 A) \times \sin^2 A = \tan^2 A$. **Option (D).**

   Why: Numerator is $\sec^2$, denominator is cosec^2 — by the two child identities.

ANSWER: (i) B (ii) C (iii) D (iv) D

CBSE MARKING SCHEME: Each option with justification [½ × 4]

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Exercise 8.3 Q5 — Proving Identities (i) to (v)

The board's favourite question type. The method is always the same: **start from the harder side (usually LHS), name the identity used at every step, and walk to the other side.** Never move terms across the '=' sign.

PROOF (i)

Prove that $(\text{cosec}\,\theta - \cot\theta)^2 = (1 - \cos\theta)/(1 + \cos\theta)$.

THINK FIRST: The RHS has $(1 - \cos\theta)$ on top — so aim to manufacture exactly that factor on the LHS.

1. LHS: write $\text{cosec}\,\theta = 1/\sin\theta$ and $\cot\theta = \cos\theta/\sin\theta$. Then $\text{cosec}\,\theta - \cot\theta = (1 - \cos\theta)/\sin\theta$.
   Why: Convert to sin and cos — the universal first move.
2. Square it: LHS = $(1 - \cos\theta)^2/\sin^2\theta$.
3. Replace $\sin^2\theta = 1 - \cos^2\theta = (1 - \cos\theta)(1 + \cos\theta)$ [master identity + difference of squares].
   Why: Identity used: $\sin^2\theta + \cos^2\theta = 1$.
4. Cancel the common factor $(1 - \cos\theta)$: LHS = $(1 - \cos\theta)/(1 + \cos\theta)$ = RHS. ∎

ANSWER: Hence proved.

PROOF (ii)

Prove that $\cos A/(1 + \sin A) + (1 + \sin A)/\cos A = 2\sec A$.

1. Take the common denominator $\cos A(1 + \sin A)$: LHS = $[\cos^2 A + (1 + \sin A)^2] / [\cos A(1 + \sin A)]$.
   Why: Add the fractions honestly — no shortcuts.
2. Expand the numerator: $\cos^2 A + 1 + 2\sin A + \sin^2 A = (\sin^2 A + \cos^2 A) + 1 + 2\sin A = 2 + 2\sin A = 2(1 + \sin A)$.
   Why: Identity used: $\sin^2 A + \cos^2 A = 1$.
3. Cancel $(1 + \sin A)$: LHS = $2/\cos A = 2\sec A$ = RHS. ∎
   Why: $1/\cos = \sec$, by definition.

ANSWER: Hence proved.

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Exercise 8.3 Q5 — Proof (iii)

PROOF (iii)

Prove that $\tan\theta/(1-\cot\theta) + \cot\theta/(1-\tan\theta) = 1 + \sec\theta\,\mathrm{cosec}\,\theta$.

THINK FIRST: The hardest of the ten. The whole proof is one plan: common denominator, then the $a^3 - b^3$ factorisation.

1. Convert to sin and cos: $\tan\theta = \sin\theta/\cos\theta$, $\cot\theta = \cos\theta/\sin\theta$. First term: $(\sin\theta/\cos\theta) / ((\sin\theta - \cos\theta)/\sin\theta) = \sin^2\theta / [\cos\theta(\sin\theta - \cos\theta)]$.

   Why: Divide the fractions carefully, one at a time.

2. Second term: $(\cos\theta/\sin\theta) / ((\cos\theta - \sin\theta)/\cos\theta) = \cos^2\theta / [\sin\theta(\cos\theta - \sin\theta)] = -\cos^2\theta / [\sin\theta(\sin\theta - \cos\theta)]$.

   Why: Flipping (cos − sin) to (sin − cos) costs a minus sign — the step everyone drops.

3. Put both over the common denominator $\sin\theta\cos\theta(\sin\theta - \cos\theta)$: numerator $= \sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)(\sin^2\theta + \sin\theta\cos\theta + \cos^2\theta)$.

   Why: Identity used: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

4. Cancel $(\sin\theta - \cos\theta)$ and use $\sin^2 + \cos^2 = 1$: LHS $= (1 + \sin\theta\cos\theta)/(\sin\theta\cos\theta) = 1/(\sin\theta\cos\theta) + 1 = \sec\theta\,\mathrm{cosec}\,\theta + 1 =$ RHS. ∎

   Why: Identity used: $\sin^2\theta + \cos^2\theta = 1$.

ANSWER: Hence proved.

Exercise 8.3 Q5 — Proving Identities (iv) to (vii)

PROOF (iv)

Prove that (1 + sec A)/sec A = $\sin^2 A/(1 - \cos A)$.

1. Work on the RHS this time (it looks harder). Multiply top and bottom by (1 + cos A): RHS = $\sin^2 A(1 + \cos A) / [(1 - \cos A)(1 + \cos A)] = \sin^2 A(1 + \cos A)/\sin^2 A$.
   Why: Identity used: $(1 - \cos)(1 + \cos) = 1 - \cos^2 = \sin^2$ — rationalising the denominator.
2. Cancel $\sin^2 A$: RHS = 1 + cos A.
3. LHS: (1 + sec A)/sec A = 1/sec A + sec A/sec A = cos A + 1.
   Why: Split the fraction; 1/sec = cos.
4. LHS = 1 + cos A = RHS. ∎
   Why: Meeting in the middle is a legal proof strategy — simplify BOTH sides to the same thing.

ANSWER: Hence proved.

Exercise 8.3 Q5 — Proof (v)

PROOF (v)

Prove that (cos A − sin A + 1)/(cos A + sin A − 1) = cosec A + cot A.

THINK FIRST: The hardest-working trick in this chapter: replacing the innocent-looking number 1 by (cosec − cot)(cosec + cot). Watch for it whenever a denominator contains a stray 1.

1. Divide numerator and denominator by sin A: LHS = (cot A − 1 + cosec A)/(cot A + 1 − cosec A) = (cosec A + cot A − 1)/((cot A − cosec A) + 1).

   Why: NCERT's own hint – dividing by sin A manufactures cot and cosec, which is where the RHS lives.

2. In the denominator, replace the number 1 by (cosec A − cot A)(cosec A + cot A).

   Why: Identity used: $\text{cosec}^2A - \cot^2A = 1$, written as a product by the difference of squares.

3. Denominator = (cot A − cosec A) + (cosec A − cot A)(cosec A + cot A) = (cosec A − cot A)(cosec A + cot A − 1), taking (cosec A − cot A) common — note (cot A − cosec A) = −(cosec A − cot A).

   Why: One sign flip, then a clean factorisation.

4. Cancel the common factor (cosec A + cot A − 1) between numerator and denominator: LHS = 1/(cosec A − cot A).

5. Multiply top and bottom by (cosec A + cot A): LHS = (cosec A + cot A) / [(cosec A − cot A)(cosec A + cot A)] = (cosec A + cot A)/1 = cosec A + cot A = RHS. ∎

   Why: The denominator is again $\text{cosec}^2A - \cot^2A = 1$.

ANSWER: Hence proved.

Exercise 8.3 Q5 — Proofs (vi) and (vii)

PROOF (vi)

Prove that $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$.

1. Multiply inside the root, top and bottom, by $(1 + \sin A)$: $(1 + \sin A)^2 / [(1 - \sin A)(1 + \sin A)] = (1 + \sin A)^2/(1 - \sin^2 A) = (1 + \sin A)^2/\cos^2 A$.
   Why: Identity used: $1 - \sin^2 A = \cos^2 A$.
2. Take the square root (A acute, all quantities positive): LHS = $(1 + \sin A)/\cos A$.
   Why: $\sqrt{(x^2)} = x$ for positive x.
3. Split: $1/\cos A + \sin A/\cos A = \sec A + \tan A$ = RHS. ∎
   Why: Definitions of sec and tan.

ANSWER: Hence proved.

PROOF (vii)

Prove that $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$.

1. Factorise top and bottom: numerator = $\sin\theta(1 - 2\sin^2\theta)$; denominator = $\cos\theta(2\cos^2\theta - 1)$.
   Why: Take out the common factor first — always.
2. In the numerator, replace $\sin^2\theta = 1 - \cos^2\theta$: $1 - 2\sin^2\theta = 1 - 2(1 - \cos^2\theta) = 2\cos^2\theta - 1$.
   Why: Identity used: $\sin^2\theta + \cos^2\theta = 1$.
3. Now numerator = $\sin\theta(2\cos^2\theta - 1)$ and denominator = $\cos\theta(2\cos^2\theta - 1)$. Cancel the common bracket: LHS = $\sin\theta/\cos\theta = \tan\theta$ = RHS. ∎
   Why: The ugly bracket was designed to cancel — trust the plan.

ANSWER: Hence proved.

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Exercise 8.3 Q5 — Proving Identities (viii) to (x)

PROOF (viii)

Prove that $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$.

1. Expand both squares: $\sin^2 A + 2 + \operatorname{cosec}^2 A + \cos^2 A + 2 + \sec^2 A$ (the cross terms: $2 \sin A \operatorname{cosec} A = 2$ and $2 \cos A \sec A = 2$).
   Why: sin × cosec = 1 and cos × sec = 1 — reciprocal pairs collapse the cross terms to constants.
2. Group: $(\sin^2 A + \cos^2 A) + 4 + (\operatorname{cosec}^2 A + \sec^2 A) = 1 + 4 + \operatorname{cosec}^2 A + \sec^2 A$.
   Why: Identity used: $\sin^2 + \cos^2 = 1$.
3. Replace $\operatorname{cosec}^2 A = 1 + \cot^2 A$ and $\sec^2 A = 1 + \tan^2 A$: total $= 5 + (1 + \cot^2 A) + (1 + \tan^2 A) = 7 + \tan^2 A + \cot^2 A$ = RHS. ∎
   Why: Identities used: both child identities of Derivation 5.

ANSWER: Hence proved.

PROOF (ix)

Prove that $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = 1/(\tan A + \cot A)$.

1. Simplify each bracket: $\operatorname{cosec} A - \sin A = (1 - \sin^2 A)/\sin A = \cos^2 A/\sin A$.
   Why: Identity used: $1 - \sin^2 A = \cos^2 A$.
2. $\sec A - \cos A = (1 - \cos^2 A)/\cos A = \sin^2 A/\cos A$.
   Why: Identity used: $1 - \cos^2 A = \sin^2 A$.
3. Multiply: LHS $= (\cos^2 A \sin^2 A)/(\sin A \cos A) = \sin A \cos A$.
   Why: Cancel one sin and one cos.
4. RHS check: $\tan A + \cot A = \sin A/\cos A + \cos A/\sin A = (\sin^2 A + \cos^2 A)/(\sin A \cos A) = 1/(\sin A \cos A)$, so $1/(\tan A + \cot A) = \sin A \cos A$.
   Why: Identity used: $\sin^2 A + \cos^2 A = 1$.
5. LHS $= \sin A \cos A$ = RHS. ∎
   Why: Both sides were simplified to the same meeting point.

ANSWER: Hence proved.

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Exercise 8.3 Q5 — Proof (x)

PROOF (x)

Prove that $(1 + \tan^2 A)/(1 + \cot^2 A) = [(1 - \tan A)/(1 - \cot A)]^2 = \tan^2 A$.

THINK FIRST: Two separate mini-proofs glued by $\tan^2 A$. Prove each side equals $\tan^2 A$ and the chain is complete.

1. First equality: $1 + \tan^2 A = \sec^2 A$ and $1 + \cot^2 A = \operatorname{cosec}^2 A$, so LHS $= \sec^2 A/\operatorname{cosec}^2 A = (1/\cos^2 A) \times \sin^2 A = \tan^2 A$.

   Why: Identities used: both child identities.

2. Second: $(1 - \tan A)/(1 - \cot A)$. Write $\cot A = 1/\tan A$: denominator $= (\tan A - 1)/\tan A$, so the fraction $= (1 - \tan A) \times \tan A/(\tan A - 1) = -\tan A$.

   Why: Flipping (tan − 1) to (1 − tan) costs one minus sign.

3. Square it: $(-\tan A)^2 = \tan^2 A$.

   Why: The square swallows the minus sign — that is why the identity needs the square.

4. All three expressions equal $\tan^2 A$. ∎

ANSWER: Hence proved.

TOPPER TIP: *Identity-proof toolkit, in order of use: (1) convert to sin/cos, (2) $\sin^2 + \cos^2 = 1$, (3) $\sec^2 - \tan^2 = 1$ and $\operatorname{cosec}^2 - \cot^2 = 1$, (4) $(1 \pm \sin)(1 \mp \sin)$ patterns, (5) factorise — $a^3 - b^3$, difference of squares. Ninety percent of board proofs use only these five moves.*

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08 Multiple Solution Methods

CHALLENGE — SAME ANSWER, DIFFERENT ROADS

A master is not the student who knows one method, but the one who can choose. Here are the chapter's two most common tasks, each solved two ways.

Task 1 — Given tan A = 4/3, find sin A and cos A

Method 1 — The Triangle (visual)

tan A = opp/adj = 4/3. Take opp = 4, adj = 3.
Hypotenuse = √(16 + 9) = 5 (Pythagoras).
sin A = 4/5, cos A = 3/5.
Best when: the ratio is a friendly fraction.

Method 2 — The Identity (algebraic)

$\sec^2 A = 1 + \tan^2 A = 1 + 16/9 = 25/9$, so sec A = 5/3 and cos A = 3/5.
sin A = tan A × cos A = (4/3)(3/5) = 4/5.
Best when: no triangle is drawn, or the ratio is awkward like 5/7.

Task 2 — Evaluate $\sin^2 45° + \cos^2 45°$ — and a harder cousin

1. Direct: $\sin^2 45° + \cos^2 45° = 1/2 + 1/2 = 1$.
   Why: Substitute the table values.
2. By identity: it is $\sin^2 A + \cos^2 A$ with A = 45° — the answer is 1 for ANY angle, no substitution needed.
   Why: The identity is the method; the table is only a check.
3. Cousin: $\sin^2 17° + \sin^2 73°$. The angles are not in the table — but 73° = 90° − 17°, so sin 73° = cos 17°, and the sum is $\sin^2 17° + \cos^2 17° = 1$.
   Why: Complementary conversion (Derivation 6) + master identity. This is exactly NCERT Ex 8.3 Q3 thinking.

PIN POINT: Method choice rule: friendly numbers → draw the triangle. Strange angles or proofs → reach for the identities. Marks are awarded for the answer, not the road — but the right road is faster and safer.

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09 Interactive Simulations & Hands-on Activities

VISUALISE • APPLY — DO IT WITH YOUR HANDS

Activity 1 — The Ratio Never Changes (similarity lab)

Draw three right triangles on squared paper, all with a 30° angle (use a protractor), of sizes small, medium, large. Measure the side opposite 30° and the hypotenuse of each, and complete:

Triangle	Opposite 30° (cm)	Hypotenuse (cm)	opp ÷ hyp
Small			
Medium			
Large			

Every ratio should come out ≈ 0.5 — sin 30°, measured by your own hand. Small errors are measurement errors; the *idea* is exact.

Activity 2 — Build a Clinometer (measure the unmeasurable)

Fix a straw along the top of a protractor, hang a weighted thread from the centre. Sight the top of your school building through the straw; your partner reads the thread's angle — that is the angle of elevation θ. Measure your distance d from the building. Height ≈ d × tan θ + (your eye height). You have just done Chapter 9's job with a 20-rupee instrument.

Activity 3 — The Shadow Clock

Plant a 1-metre stick vertically. Measure its shadow at 9 a.m., noon and 3 p.m. Each time, tan(Sun's angle) = stick height ÷ shadow length. Watch tan grow and shrink through the day — and notice the shadow is shortest when the Sun's angle is greatest.

DIGITAL SIMULATION (KalviYogi Digital Companion): drag-the-angle right-triangle explorer • shadow-and-sun simulator • identity visualiser. **QR code and link activate with the KalviYogi app/website at print.** (Placeholder — no link is printed in this edition.)

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10 Common Mistakes

UNDERSTAND — THE SIX TRAPS, DISARMED

Every mistake below has cost real students real marks. Read the wrong way, feel why it tempts you, then fix the right way into memory.

COMMON MISTAKE: Trap 1 — Ratios glued to letters. 'BC is always the opposite side.' Opposite/adjacent are decided by the angle you stand at: in Q1 of Ex 8.1, BC is opposite A but adjacent to C. **Right way: stand at the angle, then name the sides.**

COMMON MISTAKE: Trap 2 — sin A = sin × A. Cancelling 'sin' from both sides of an equation, or writing sin 2A = 2 × sin A as a rule. sin A is one indivisible number (Ex 8.2 Q2(iii) shows sin 2A = 2 sin A only at A = 0°). **Right way: treat sin A, cos A, tan A as single quantities, like √2.**

COMMON MISTAKE: Trap 3 — Squaring only the top. $(\sqrt{3}/2)^2 = \sqrt{3}/4$. Right way: square numerator and denominator separately: 3/4.

COMMON MISTAKE: Trap 4 — Moving terms across '=' in proofs. Starting a proof by assuming LHS = RHS and rearranging both sides. That assumes the very thing to be proved. **Right way: start from one side only (or simplify both sides separately to a common meeting point).**

COMMON MISTAKE: Trap 5 — Accepting sin A > 1. Reporting sin θ = 4/3 or cos A = 5/4 as final answers. A leg can never exceed the hypotenuse. **Right way: if sin or cos exceeds 1, hunt the error — usually a swapped side.**

COMMON MISTAKE: Trap 6 — The vanishing minus sign. Writing (cos θ − sin θ) = (sin θ − cos θ) when forming a common denominator (Ex 8.3 Q5(iii)). Right way: (a − b) = −(b − a) — flip costs a minus; write it, every time.

TOPPER TIP: Before submitting any trigonometry answer, run the 5-second audit: sin and cos within [0, 1]? tan positive? hypotenuse the biggest side? proof started from one side only?

11 Graded Practice Problems

SOLVE — FIVE LEVELS, FOUNDATION TO GENIUS

Work level by level. Do not peek at the Answer Key until a level is finished — and if a level defeats you, drop back one level, repair, and climb again.

LEVEL 1 — FOUNDATION

L1.1. In a right triangle, the hypotenuse is 10 cm and the side opposite angle A is 6 cm. Find sin A and cos A.

L1.2. If tan A = 5/12, find sin A and cos A. (Build the triangle.)

L1.3. Evaluate: sin 45° + cos 45°. Is the answer more than 1? Is that allowed? Say why in one line.

L1.4. Find the value of tan 60° ÷ cot 30°. What do you notice?

L1.5. If cos A = 12/13, find tan A.

L1.6. Write cosec 30°, sec 60° and cot 45° from the table.

LEVEL 2 — SKILL BUILDER

L2.1. If sin A = 7/25, find all the other trigonometric ratios of A.

L2.2. Evaluate: $2\sin^2 30° + 3\cos^2 60° - \tan^2 45°$.

L2.3. If 5 tan θ = 4, find the value of $(5\sin\theta - 3\cos\theta)/(5\sin\theta + 3\cos\theta)$. (Hint: divide top and bottom by cos θ.)

L2.4. Prove that $(1 - \sin A)(1 + \sin A) = \cos^2 A$, and state the identity you used.

L2.5. Given sin A = cos B, where A and B are acute angles, prove that A + B = 90°. (Hint: write cos B = sin(90° − B) and use Reasoning-2 logic: equal sines of acute angles force equal angles.)

L2.6. Evaluate: $(\tan^2 60° + \sin^2 45°)/(1 + \cos^2 60°)$.

Graded Practice — Levels 3, 4 and 5

LEVEL 3 — APPLICATION

L3.1. A ladder 10 m long leans against a vertical wall, making 60° with the ground. How high up the wall does it reach?

L3.2. A tree casts a shadow 30 m long when the Sun's elevation is 30°. Find the height of the tree.

L3.3. A kite string is 50 m long and makes 60° with the horizontal. Find the height of the kite (ignore the height of the flyer's hand).

L3.4. From a point 20 m from the foot of a tower, the top of the tower is seen at an angle of elevation of 45°. Find the height of the tower.

LEVEL 4 — ADVANCED REASONING

L4.1. Prove that $(\tan A + \sec A - 1)/(\tan A - \sec A + 1) = (1 + \sin A)/\cos A$. (Hint: replace 1 by $\sec^2 A - \tan^2 A$ in the numerator.)

L4.2. If $x = a \cos \theta$ and $y = b \sin \theta$, prove that $x^2/a^2 + y^2/b^2 = 1$. Why does the angle disappear completely?

L4.3. If $\sin \theta + \cos \theta = \sqrt{2}$, prove that $\sin \theta - \cos \theta = 0$, and hence find θ. (Hint: square the given equation to find $\sin \theta \cos \theta$; then compute $(\sin \theta - \cos \theta)^2 = (\sin^2\theta + \cos^2\theta) - 2 \sin \theta \cos \theta$.)

LEVEL 5 — MATHEMATICAL GENIUS

L5.1. Prove that $\tan^2 A/(1 + \tan^2 A) + \cot^2 A/(1 + \cot^2 A) = 1$ for every acute angle A — without using a specific value of A.

L5.2. If $A + B = 90°$, prove that $(\sin^2 A + \sin^2 B) + (\tan A - \cot B)^2 = 1$. (Each bracket is a friend from Derivation 6 in disguise.)

> ***TOPPER TIP:*** *Level 5 answers are in the Key — but the real prize is the method. Write your proof, THEN compare line by line.*

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12 Mathematical Reasoning

CHALLENGE — THINK LIKE A MATHEMATICIAN

REASONING 1

Prove that tan A + cot A = sec A cosec A. Then explain why this means tan A + cot A ≥ 2 for every acute A... or find the flaw in that claim.

THINK FIRST: Reasoning questions reward the second sentence as much as the first — always ask 'what else does this tell me?'

1. Convert to sin and cos: LHS = sin A/cos A + cos A/sin A = $(\sin^2 A + \cos^2 A)/(\sin A \cos A) = 1/(\sin A \cos A)$.
   Why: Common denominator + master identity.
2. RHS: sec A cosec A = (1/cos A)(1/sin A) = 1/(sin A cos A). Equal. ∎
   Why: Definitions of sec and cosec.
3. For the claim: sin A cos A ≤ 1/2 for acute A (its maximum, at 45°, is 1/2), so 1/(sin A cos A) ≥ 2 — the claim is TRUE, with equality only at A = 45°.
   Why: A bonus: a proved identity often hides an inequality inside it.

ANSWER: tan A + cot A = sec A cosec A; and indeed tan A + cot A ≥ 2, equality at 45°

REASONING 2

A student claims: 'If sin A = sin B for acute angles A and B, then A = B must hold.' Prove or disprove the claim.

THINK FIRST: This is why the standard table can be read backwards: tan A = √3 forces A = 60°, no other acute choice exists.

1. Suppose sin A = sin B = k. Build right triangles for A and B with hypotenuse 1 and opposite side k (scale is free).
   Why: Use the unpack-the-ratio move from Ex 8.1 Q3.
2. In each triangle the adjacent side is $\sqrt{(1 - k^2)}$ by Pythagoras — the same for both.
   Why: The hypotenuse and one leg fix the other leg uniquely.
3. So the two triangles are congruent (SSS), hence A = B. The claim is TRUE. ∎
   Why: Same spirit as Ex 8.1 Q6: a ratio determines the angle.

ANSWER: The claim is true — on acute angles, each sine value belongs to exactly one angle.

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Mathematical Reasoning — Question 3

REASONING 3

Show that $\sin^2\theta + \cos^2\theta = 1$ can be used to prove: if $\sin\theta = (x^2 - y^2)/(x^2 + y^2)$ for positive $x > y$, then $\cos\theta = 2xy/(x^2 + y^2)$ (taking $\cos\theta$ positive).

1. $\cos^2\theta = 1 - \sin^2\theta = [(x^2 + y^2)^2 - (x^2 - y^2)^2] / (x^2 + y^2)^2$.
   Why: Common denominator $(x^2 + y^2)^2$.

2. Numerator: expand — $(x^4 + 2x^2y^2 + y^4) - (x^4 - 2x^2y^2 + y^4) = 4x^2y^2$.
   Why: The x^4 and y^4 terms cancel; the middle terms add.

3. So $\cos^2\theta = 4x^2y^2/(x^2 + y^2)^2$, and taking the positive root: $\cos\theta = 2xy/(x^2 + y^2)$. ∎
   Why: θ acute ⇒ cos positive.

4. Recognise the pattern? With $x = 2$, $y = 1$: $\sin\theta = 3/5$, $\cos\theta = 4/5$ — this machine manufactures Pythagorean triples!
   Why: Every (x, y) pair prints a fresh triple: $(x^2 - y^2, 2xy, x^2 + y^2)$.

ANSWER: $\cos\theta = 2xy/(x^2 + y^2)$, proved from the master identity alone.

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13 Real-Life Applications

APPLY — THE STORY PROBLEMS, SOLVED AT LAST

Return to Section 03. Meena's kite and Arun's flagpole have waited long enough.

APPLICATION 1

Arun's flagpole: the shadow is 12 m long when the Sun's rays make 45° with the ground. Find the height of the flagpole.

THINK FIRST: At 45° the answer is free. Examiners love 45° for exactly this reason.

1. Draw the triangle: the flagpole is the opposite side, the shadow is the adjacent side, the Sun's ray is the hypotenuse. The angle at the ground is 45°.

   Why: Every application begins as a labelled picture — never skip the picture.

2. The ratio linking opposite and adjacent is tan: tan 45° = height/12.

   Why: Choose the ratio that contains BOTH the known side and the wanted side.

3. tan 45° = 1, so height = 12 × 1 = **12 m**.

   Why: At 45°, height and shadow are always equal — remember Arun's noon discovery.

ANSWER: The flagpole is 12 m tall.

APPLICATION 2

Meena's kite: the string is 50 m long and makes 60° with the ground. Find the height of the kite above the ground.

1. Draw the triangle: string = hypotenuse (50 m), height = side opposite the 60° angle.

2. The ratio linking opposite and hypotenuse is sin: sin 60° = height/50.

3. height = 50 × √3/2 = 25√3 ≈ **43.3 m**.

   Why: 25 × 1.732 = 43.3 — a very high kite indeed.

ANSWER: The kite is 25√3 m (about 43.3 m) above the ground.

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Real-Life Applications — 3 and 4

APPLICATION 3

A guy wire is attached to the top of a 20 m tower and fixed to the ground at 60° to the horizontal. Find the length of the wire.

1. The tower is the opposite side (20 m); the wire is the hypotenuse; the angle is 60°.
2. sin 60° = 20/wire, so wire = 20/sin 60° = 20 × 2/√3 = 40/√3 = 40√3/3 ≈ **23.1 m**.
   Why: This time the unknown is in the denominator — rearrange before substituting.

ANSWER: The wire is 40√3/3 m (about 23.1 m) long.

APPLICATION 4

Standing 30√3 m from the foot of a minaret, the angle of elevation of its top is 30°. Find the height of the minaret.

1. Adjacent side = 30√3 m (given), opposite = height wanted, angle 30° — the ratio is tan.
   Why: Angle of elevation: the angle between the horizontal and the line of sight, measured upwards.
2. tan 30° = height/(30√3), so height = 30√3 × 1/√3 = **30 m**.
   Why: The √3 cancels exactly — problem-setters choose such numbers on purpose; take the hint and expect clean answers.

ANSWER: The minaret is 30 m tall.

PIN POINT: The 3-step application ritual: (1) draw and label the triangle, (2) pick the ratio containing the known and the wanted side, (3) substitute and solve. Chapter 9 is this ritual, scaled up.

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14 Competency-Based Questions

APPLY • CHALLENGE — FIVE CASE STUDIES, TWENTY QUESTIONS

Read each case, then answer its four questions. Fully worked answers follow each case — try before you look.

CASE 1 — The School Ramp

A wheelchair ramp leads up to the school stage. The ramp surface is 6 m long and the stage is 3 m above the ground. The ramp makes an angle θ with the ground.

(i) Find sin θ.

(j) Find the angle θ.

(k) Find the horizontal distance covered by the ramp.

(l) Find tan θ, and say whether the ramp is 'gentle' (angle less than 45°) or steep.

Case 1 — Worked Answers

(i) sin θ = opposite/hypotenuse = 3/6 = **1/2**. **(ii)** sin θ = 1/2 = sin 30°, so **θ = 30°**. **(iii)** Horizontal distance = $\sqrt{(6^2 - 3^2)} = \sqrt{27}$ = **$3\sqrt{3}$ m** (Pythagoras). **(iv)** tan θ = $3/(3\sqrt{3})$ = **$1/\sqrt{3}$**; since 30° < 45°, the ramp is **gentle**.

CASE 2 — The Staircase

In a house, each step of a staircase rises 20 cm and goes forward (tread) 25 cm. The staircase makes an angle θ with the floor.

(i) Find tan θ.

(j) Is θ more or less than 45°? Give a reason without finding θ.

(k) Find sin θ and cos θ. (First find the slanted length of one step.)

(l) If there are 12 steps, what is the total height reached?

Case 2 — Worked Answers

(i) tan θ = rise/tread = 20/25 = **4/5**. **(ii)** tan θ = 4/5 < 1 = tan 45°, and tan increases with the angle, so **θ < 45°**. **(iii)** Slanted length = $\sqrt{(20^2 + 25^2)} = \sqrt{1025} = 5\sqrt{41}$ cm. sin θ = $20/(5\sqrt{41})$ = **$4/\sqrt{41}$**, cos θ = **$5/\sqrt{41}$**. **(iv)** 12 × 20 cm = 240 cm = **2.4 m**.

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Competency Cases 3, 4 and 5

CASE 3 — The Solar Panel

A solar panel of length 2 m is fixed on a flat roof, tilted at 30° to the roof so that it faces the Sun. One end of the panel touches the roof; the other end is raised.

(i) How high is the raised end above the roof?

(j) How much roof length does the panel cover horizontally?

(k) Write tan 30° using the panel's rise and horizontal cover, and verify it matches the table.

(l) If the panel is tilted at 60° instead, how high is the raised end?

Case 3 — Worked Answers

(i) Rise = 2 sin 30° = 2 × 1/2 = **1 m**. **(ii)** Cover = 2 cos 30° = 2 × √3/2 = **√3 m** ≈ 1.73 m. **(iii)** tan 30° = rise/cover = 1/√3 — **matches the table**. **(iv)** Rise = 2 sin 60° = **√3 m** ≈ 1.73 m — the 30° and 60° answers have swapped, exactly as complementary angles promise.

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Competency Cases 4 and 5

CASE 4 — The Moving Shadow

A vertical pole is 10 m tall. A student measures its shadow at three times of the day and finds the Sun's angle of elevation to be 30°, 45° and 60° at different times.

(i) Find the shadow length when the angle is 45°.

(j) Find the shadow length when the angle is 30°.

(k) Find the shadow length when the angle is 60°.

(l) As the Sun climbs higher, what happens to the shadow? Connect your answer to the behaviour of tan.

Case 4 — Worked Answers

Shadow = height/tan(angle) throughout. **(i)** 10/tan 45° = 10/1 = **10 m**. **(ii)** 10/tan 30° = 10√3 ≈ **17.32 m**. **(iii)** 10/tan 60° = 10/√3 ≈ **5.77 m**. **(iv)** The shadow **shrinks** as the Sun climbs: the angle grows, tan(angle) grows, and height ÷ tan therefore falls.

CASE 5 — The Park Slide

A slide in a children's park is 5 m long (along the slope) and its top platform is 4 m above the ground. The slide makes an angle θ with the ground.

(i) Find sin θ, cos θ and tan θ.

(j) Is θ more than 45°? Justify using one ratio.

(k) Find the horizontal distance from the foot of the ladder to the end of the slide.

(l) A second slide has the same length 5 m but top height 3 m. Without calculating, say which slide is steeper, and why.

Case 5 — Worked Answers

(i) The ground distance = √(25 − 16) = 3 m (3-4-5). sin θ = **4/5**, cos θ = **3/5**, tan θ = **4/3**. **(ii)** tan θ = 4/3 > 1 = tan 45°, so **θ > 45°** — steep! **(iii) 3 m**. **(iv)** The first slide: with the same hypotenuse, a greater height means a greater sin θ, hence a greater angle.

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Assertion – Reason Questions (10)

For each question, choose: **(a)** Both A and R are true and R is the correct explanation of A. **(b)** Both true, but R does not explain A. **(c)** A is true, R is false. **(d)** A is false, R is true. — Answers with reasons are in the Answer Key.

AR1. A: $\sin 60° = 2 \sin 30° \cos 30°$ numerically holds (both sides equal $\sqrt{3}/2$... check!). **R:** $\sin^2 A + \cos^2 A = 1$.

AR2. A: If $\sin A = 1/2$, then $A = 30°$ (A acute). **R:** The sine of an acute angle determines the angle uniquely.

AR3. A: $\tan 90°$ is not defined. **R:** $\cos 90° = 0$, and $\tan A = \sin A/\cos A$.

AR4. A: $\sec^2 A - \tan^2 A = 1$. **R:** $\sec^2 A - \tan^2 A$ is obtained by dividing $\sin^2 A + \cos^2 A = 1$ by $\cos^2 A$.

AR5. A: $\sin(90° - A) = \cos A$. **R:** In a right triangle, the side opposite $(90° - A)$ is the side adjacent to A.

AR6. A: $\cos A$ can be $5/4$ for some acute angle A. **R:** The hypotenuse is the longest side of a right-angled triangle.

AR7. A: If $\tan A = \cot B$ for acute A, B, then $A + B = 90°$. **R:** $\cot B = \tan(90° - B)$, and equal tangents of acute angles force equal angles.

AR8. A: $(1 - \sin A)(1 + \sin A) = \cos^2 A$. **R:** $(x - y)(x + y) = x^2 - y^2$ and $\sin^2 A + \cos^2 A = 1$.

AR9. A: The value of $\tan A$ is always greater than 1. **R:** The opposite side is always longer than the adjacent side.

AR10. A: $\operatorname{cosec} 0°$ is not defined. **R:** $\sin 0° = 0$ and $\operatorname{cosec} A = 1/\sin A$.

> **TOPPER TIP:** *Assertion-Reason is a reading test disguised as maths: even when both statements are true, ask 'does R CAUSE A?' before choosing (a).*

15 Olympiad & Foundation Challenges

CHALLENGE — TEN PROBLEMS FOR THE BRAVE

CHALLENGE 1

Prove that $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$.

1. Write $\sin^6 + \cos^6$ as $(\sin^2)^3 + (\cos^2)^3$ and use $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$ with $a = \sin^2\theta$, $b = \cos^2\theta$.

   Why: The cube identity does the heavy lifting.

2. $a + b = \sin^2\theta + \cos^2\theta = 1$ and $ab = \sin^2\theta\cos^2\theta$. So the sum $= 1 - 3\sin^2\theta\cos^2\theta(1) = 1 - 3\sin^2\theta\cos^2\theta$. ∎

   Why: Identity used: the master identity, twice.

ANSWER: Hence proved.

CHALLENGE 2

If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $(m^2 - n^2)^2 = 16mn$.

1. $m + n = 2\tan\theta$ and $m - n = 2\sin\theta$, so $m^2 - n^2 = (m + n)(m - n) = 4\tan\theta\sin\theta$.

   Why: Add and subtract the two given equations.

2. $mn = \tan^2\theta - \sin^2\theta = \sin^2\theta(\sec^2\theta - 1) = \sin^2\theta\tan^2\theta$, so $16mn = (4\sin\theta\tan\theta)^2$.

   Why: Identity used: $\tan^2 = \sin^2/\cos^2$ and $\sec^2 - 1 = \tan^2$.

3. $(m^2 - n^2)^2 = (4\tan\theta\sin\theta)^2 = 16mn$. ∎

   Why: Both sides are the same quantity in two costumes.

ANSWER: Hence proved.

Olympiad Challenges 3 and 4

CHALLENGE 3

If $\sin\theta + \sin^2\theta = 1$, prove that $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta - 1 = 0$.

1. From $\sin\theta = 1 - \sin^2\theta = \cos^2\theta$: the given says $\cos^2\theta = \sin\theta$.
   Why: Master identity, rearranged.

2. Factor $\cos^6\theta$ out of the target sum: $\cos^{12} + 3\cos^{10} + 3\cos^8 + \cos^6 = \cos^6\theta\,(\cos^6\theta + 3\cos^4\theta + 3\cos^2\theta + 1) = \cos^6\theta\,(1 + \cos^2\theta)^3$.
   Why: The bracket is the binomial cube $(1 + a)^3 = 1 + 3a + 3a^2 + a^3$ with $a = \cos^2\theta$ — the 1, 3, 3, 1 pattern in the coefficients is the fingerprint.

3. Now $1 + \cos^2\theta = 1 + \sin\theta$, and $\cos^6\theta = (\cos^2\theta)^3 = \sin^3\theta$. So the sum $= \sin^3\theta(1 + \sin\theta)^3 = [\sin\theta(1 + \sin\theta)]^3 = [\sin\theta + \sin^2\theta]^3 = 1^3 = 1$.
   Why: Use $\cos^2\theta = \sin\theta$ from Step 1, then the given condition itself.

4. Therefore the expression $= 1 - 1 = 0$. ∎

ANSWER: Hence proved.

CHALLENGE 4

Given $\operatorname{cosec} A + \cot A = 3$, find the value of $\cos A$. (You may first prove $(\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A) = 1$.)

1. $(\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A) = \operatorname{cosec}^2 A - \cot^2 A = 1$.
   Why: Identity used: $\operatorname{cosec}^2 - \cot^2 = 1$.

2. So $\operatorname{cosec} A - \cot A = 1/3$ (divide 1 by the given value 3).
   Why: The conjugate pair multiplies to 1 — knowing one gives the other free.

3. Add the two equations: $2\operatorname{cosec} A = 3 + 1/3 = 10/3$, so $\operatorname{cosec} A = 5/3$ and $\sin A = 3/5$.

4. $\cos A = \sqrt{(1 - 9/25)} = \sqrt{(16/25)} = \mathbf{4/5}$ (A acute).
   Why: Master identity.

ANSWER: $\cos A = 4/5$

Olympiad Challenges 5 to 10

CHALLENGE 5

If $x \sin^3\theta + y \cos^3\theta = \sin\theta \cos\theta$ and $x \sin\theta = y \cos\theta$, prove that $x^2 + y^2 = 1$.

1. From the second equation, $y = x \sin\theta/\cos\theta$. Substitute in the first: $x \sin^3\theta + x \sin\theta \cos^2\theta = \sin\theta \cos\theta$, i.e. $x \sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta \cos\theta$.

   Why: Factor $x \sin\theta$ out of the left side.

2. $\sin^2 + \cos^2 = 1$ gives $x \sin\theta = \sin\theta \cos\theta$, so $x = \cos\theta$. Then $y = \cos\theta \cdot \sin\theta/\cos\theta = \sin\theta$.

3. $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$. ∎

   Why: The angle was a mask: x and y were cos and sin all along.

ANSWER: Hence proved.

CHALLENGE 6

Find the minimum value of $\sin^2\theta + \cos^2\theta + \tan^2\theta + \cot^2\theta + \sec^2\theta + \text{cosec}^2\theta$ for acute θ.

1. Regroup: $(\sin^2 + \cos^2) + (\tan^2 + \sec^2) + (\cot^2 + \text{cosec}^2) = 1 + (2\sec^2\theta - 1) + (2\,\text{cosec}^2\theta - 1)$, using $\tan^2 = \sec^2 - 1$ and $\cot^2 = \text{cosec}^2 - 1$.

   Why: Replace every ratio by its identity form.

2. Total $= 2(\sec^2\theta + \text{cosec}^2\theta) - 1$. And $\sec^2 + \text{cosec}^2 = (\sin^2 + \cos^2)/(\sin^2\theta \cos^2\theta) = 1/(\sin^2\theta \cos^2\theta)$.

   Why: Common denominator $\sin^2\cos^2$.

3. $\sin^2\theta \cos^2\theta \le 1/4$ (maximum at 45°), so $1/(\sin^2\cos^2) \ge 4$, and the total $\ge 2(4) - 1 = \mathbf{7}$, attained at $\theta = 45°$.

   Why: Check at 45°: 1 + 1 + 1 + 2 + 2 = 7.

ANSWER: Minimum value = 7, at $\theta = 45°$

Olympiad Challenge 7

CHALLENGE 7

Prove that $\tan\theta/(1+\tan^2\theta) + \cot\theta/(1+\cot^2\theta) = 2\sin\theta\cos\theta$.

1. $1 + \tan^2\theta = \sec^2\theta$, so the first term = $\tan\theta/\sec^2\theta = (\sin\theta/\cos\theta) \times \cos^2\theta = \sin\theta\cos\theta$.

   Why: Identity used: $1 + \tan^2 = \sec^2$.

2. $1 + \cot^2\theta = \operatorname{cosec}^2\theta$, so the second term = $\cot\theta/\operatorname{cosec}^2\theta = (\cos\theta/\sin\theta) \times \sin^2\theta = \sin\theta\cos\theta$.

   Why: Identity used: $1 + \cot^2 = \operatorname{cosec}^2$.

3. Sum = $2\sin\theta\cos\theta$. ∎

   Why: Two different-looking terms, one identical heart.

ANSWER: Hence proved.

Olympiad Challenges 8 to 10

CHALLENGE 8

If sec θ + tan θ = p, prove that sin θ = (p² − 1)/(p² + 1).

1. The conjugate: (sec θ − tan θ)(sec θ + tan θ) = sec²θ − tan²θ = 1, so sec θ − tan θ = 1/p.
   Why: Identity used: sec² − tan² = 1.
2. Subtract: 2 tan θ = p − 1/p = (p² − 1)/p. Add: 2 sec θ = p + 1/p = (p² + 1)/p.
3. sin θ = tan θ/sec θ = [(p² − 1)/p] / [(p² + 1)/p] = (p² − 1)/(p² + 1). ∎
   Why: sin = tan/sec — from tan = sin/cos and sec = 1/cos.

ANSWER: Hence proved.

CHALLENGE 9

Prove that √((1 + cos A)/(1 − cos A)) = cosec A + cot A.

1. Multiply inside the root by (1 + cos A) top and bottom: (1 + cos A)²/[(1 − cos A)(1 + cos A)] = (1 + cos A)²/sin²A.
   Why: Identity used: 1 − cos²A = sin²A.
2. Square root (A acute): (1 + cos A)/sin A = 1/sin A + cos A/sin A = cosec A + cot A. ∎
   Why: The mirror image of NCERT Proof (vi) — same move, cos instead of sin.

ANSWER: Hence proved.

CHALLENGE 10

If sin θ + cos θ = p and sec θ + cosec θ = q, show that q(p² − 1) = 2p.

1. p² = sin²θ + cos²θ + 2 sin θ cos θ = 1 + 2 sin θ cos θ, so p² − 1 = 2 sin θ cos θ.
   Why: Square p and use the master identity.
2. q = 1/cos θ + 1/sin θ = (sin θ + cos θ)/(sin θ cos θ) = p/(sin θ cos θ).
   Why: Common denominator.
3. q(p² − 1) = [p/(sin θ cos θ)] × 2 sin θ cos θ = 2p. ∎
   Why: The sinθcosθ cancels exactly.

ANSWER: Hence proved.

Previous Year Questions — From Real CBSE Papers

These questions appeared in actual CBSE board examinations (years verified). Solve them exactly as you solved Section 07 — they are the truest mirror of the exam.

CBSE 2023 (BASIC)

If $\sin\theta = 3/4$, then the value of $[(\sec^2\theta - 1)\cos^2\theta] / \sin\theta$ is: (A) 3/5 (B) 3/4 (C) 4/3 (D) 9/16.

THINK FIRST: Do not build the triangle — the identities collapse the expression before any number is needed.

1. $\sec^2\theta - 1 = \tan^2\theta$ (identity: $\sec^2 - \tan^2 = 1$).
2. $\tan^2\theta \cos^2\theta = (\sin^2\theta/\cos^2\theta) \times \cos^2\theta = \sin^2\theta$.
   Why: tan = sin/cos; the $\cos^2$ cancels.
3. Divide by $\sin\theta$: expression = $\sin\theta$ = **3/4. Option (B)**.
   Why: The given value was the answer in disguise.

ANSWER: Option (B) — 3/4

CBSE MARKING SCHEME: Identity $\sec^2 - 1 = \tan^2$ [1] • simplification to $\sin\theta$ [1]

CBSE 2023 (BASIC)

For A = 30° and B = 60°, verify that $\sin(A + B) = \sin A \cos B + \cos A \sin B$.

THINK FIRST: Compute both sides separately from the table and compare — a verification, not a proof for all angles.

1. LHS: $\sin(A + B) = \sin 90° = 1$.
2. RHS: $\sin 30° \cos 60° + \cos 30° \sin 60° = (1/2)(1/2) + (\sqrt{3}/2)(\sqrt{3}/2) = 1/4 + 3/4 = 1$.
3. LHS = RHS = 1. Verified.

ANSWER: Both sides equal 1 — verified

CBSE MARKING SCHEME: LHS [1] • RHS with correct table values [1]

Previous Year Questions — One More

CBSE 2023 (STANDARD)

If $\cos A + \cos^2 A = 1$, find the value of $\sin^2 A + \sin^4 A$.

THINK FIRST: Rearrange the given equation until sin appears; the target is built from the same blocks.

1. $\cos A = 1 - \cos^2 A = \sin^2 A$ (master identity). So $\sin^2 A = \cos A$.
   Why: The given equation IS a conversion machine: it turns cos into sin².

2. Square both sides of $\sin^2 A = \cos A$: $\sin^4 A = (\sin^2 A)^2 = \cos^2 A$.
   Why: Squaring a true equation keeps it true.

3. $\sin^2 A + \sin^4 A = \cos A + \cos^2 A = 1$ (the given).
   Why: We have come full circle — the target equals the given.

ANSWER: $\sin^2 A + \sin^4 A = 1$

CBSE MARKING SCHEME: $\sin^2 A = \cos A$ [1] • $\sin^4 A = \cos^2 A$ [½] • sum = 1 [½]

Previous Year Questions — The Proof Papers

CBSE 2023 (STANDARD)

Prove that $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$.

THINK FIRST: Two building blocks first: $\sin^6 + \cos^6$ and $\sin^4 + \cos^4$, each in terms of $\sin^2\theta\cos^2\theta$. (Challenge 1 built the first already.)

1. $\sin^4\theta + \cos^4\theta = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta = 1 - 2\sin^2\theta\cos^2\theta$.
   Why: $a^2 + b^2 = (a + b)^2 - 2ab$ with $a = \sin^2$, $b = \cos^2$.

2. $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$ (Challenge 1).
   Why: $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$.

3. Substitute, writing $s = \sin^2\theta\cos^2\theta$: LHS $= 2(1 - 3s) - 3(1 - 2s) + 1 = 2 - 6s - 3 + 6s + 1 = 0$. ∎
   Why: The s terms cancel — the identity was engineered that way.

ANSWER: Hence proved

CBSE MARKING SCHEME: $\sin^4 + \cos^4$ block [1] • $\sin^6 + \cos^6$ block [1] • substitution and cancellation [1]

Previous Year Questions — The Proof Papers

CBSE SAMPLE PAPER 2019

Prove that sin θ(1 + tan θ) + cos θ(1 + cot θ) = sec θ + cosec θ.

THINK FIRST: Convert tan and cot to sin/cos, then hunt for $\sin^2 + \cos^2$ inside the brackets.

1. LHS = sin θ + $\sin^2\theta$/cos θ + cos θ + $\cos^2\theta$/sin θ.
   Why: Expand both brackets after writing tan = sin/cos, cot = cos/sin.
2. Regroup: (sin θ + $\cos^2\theta$/sin θ) + (cos θ + $\sin^2\theta$/cos θ) = ($\sin^2\theta$ + $\cos^2\theta$)/sin θ + ($\cos^2\theta$ + $\sin^2\theta$)/cos θ.
   Why: Pair each term with the fraction whose denominator it can join.
3. = 1/sin θ + 1/cos θ = cosec θ + sec θ = RHS. ∎
   Why: Identity used: $\sin^2 + \cos^2 = 1$, twice.

ANSWER: Hence proved

CBSE MARKING SCHEME: Conversion to sin/cos [1] • regrouping with $\sin^2 + \cos^2 = 1$ [1] • final form [1]

CBSE BOARD EXAM QUESTION

Prove that (sin θ − cos θ + 1)/(sin θ + cos θ − 1) = 1/(sec θ − tan θ).

THINK FIRST: Divide top and bottom by cos θ, then use the conjugate pair (sec + tan)(sec − tan) = 1.

1. Divide numerator and denominator by cos θ: LHS = (tan θ − 1 + sec θ)/(tan θ + 1 − sec θ) = ((sec θ + tan θ) − 1)/(1 − (sec θ − tan θ)).
   Why: Group the denominator as 1 − (sec − tan).
2. Let s = sec θ − tan θ. Then sec θ + tan θ = 1/s, because (sec − tan)(sec + tan) = $\sec^2$ − $\tan^2$ = 1.
   Why: Identity used: $\sec^2 - \tan^2 = 1$.
3. LHS = (1/s − 1)/(1 − s) = [(1 − s)/s] / (1 − s) = 1/s = 1/(sec θ − tan θ) = RHS. ∎
   Why: The (1 − s) cancels — the same conjugate trick as Challenge 8.

ANSWER: Hence proved

CBSE MARKING SCHEME: Division by cos θ [1] • conjugate substitution [1] • cancellation and finish [1]

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16

Chapter Mind Map

RECALL — THE WHOLE CHAPTER ON ONE PAGE

INTRODUCTION TO TRIGONOMETRY

Six ratios of a right-angled triangle — depending only on the angle

THE SIX RATIOS (at A)

sin = opp/hyp • cos = adj/hyp • tan = opp/adj
cosec = 1/sin • sec = 1/cos • cot = 1/tan
tan = sin/cos • cot = cos/sin

WHY RATIOS WORK

Similar triangles (AA): resize the triangle, the ratios do not change.
A ratio determines its angle uniquely (acute angles).

THE STANDARD TABLE

Built from the 45-45-90 and 30-60-90 triangles.
sin row: 0, 1/2, 1/√2, √3/2, 1
cos row: the same, backwards.
tan 90°: not defined.

THE THREE IDENTITIES

$\sin^2 A + \cos^2 A = 1$ (from Pythagoras)
$1 + \tan^2 A = \sec^2 A$ (divide by $\cos^2$)
$1 + \cot^2 A = \text{cosec}^2 A$ (divide by $\sin^2$)

COMPLEMENTS (90° – A)

sin ↔ cos • tan ↔ cot • sec ↔ cosec
sin(90° − A) = cos A, and five friends.

PROOF TOOLKIT

1. Convert to sin/cos
2. $\sin^2 + \cos^2 = 1$
3. $(1 \pm \sin)(1 \mp \sin)$ patterns
4. Conjugate pairs = 1
5. Factorise: $a^3 \pm b^3$, $a^2 - b^2$

Applications: height = distance × tan(angle) • height = string × sin(angle) — the 3-step ritual: draw, choose the ratio, solve. → Chapter 9 takes this to towers, rivers and the sky.

TOPPER TIP: Cover this page and redraw the map from memory on a blank sheet. Whatever you cannot redraw is exactly what you have not yet learnt — the map grades you honestly.

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17 Pin-Point Flash Cards

RECALL — CUT THEM OUT, CARRY THEM, MASTER THEM

Sixteen yellow cards carry the whole chapter's pin-points. Cut along the dashed borders, keep them in your pocket or pencil box, and revise on Smart Memory days (Day 3, 7, 21): read the title, say the point aloud from memory, then flip to check.

sin A

sin A = opposite ÷ hypotenuse. The FIRST ratio — 'SOH' of SOH-CAH-TOA.

cos A

cos A = adjacent ÷ hypotenuse. 'CAH' — the side hugging the angle, over the longest side.

tan A

tan A = opposite ÷ adjacent = sin A ÷ cos A. 'TOA' — the only ratio with no hypotenuse.

The Reciprocals

cosec A = 1/sin A • sec A = 1/cos A • cot A = 1/tan A. Flip the fraction, flip the name.

Master Identity

$\sin^2 A + \cos^2 A = 1$ — Pythagoras in disguise. The engine of every proof.

sec – tan Pair

$\sec^2 A - \tan^2 A = 1$, i.e. $1 + \tan^2 A = \sec^2 A$. Divide the master identity by $\cos^2 A$.

cosec – cot Pair

$\text{cosec}^2 A - \cot^2 A = 1$, i.e. $1 + \cot^2 A = \text{cosec}^2 A$. Divide the master identity by $\sin^2 A$.

The sin Row

0°, 30°, 45°, 60°, 90° → 0, 1/2, 1/√2, √3/2, 1 — that is √0, √1, √2, √3, √4, all over 2.

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Flash Cards 9 to 16

The cos Row

cos is the sin row read BACKWARDS: 1, √3/2, 1/√2, 1/2, 0. Learn one row, own both.

The tan Row

0, 1/√3, 1, √3, NOT DEFINED. tan = sin ÷ cos, computed fresh each time.

Complement Swap

sin(90° − A) = cos A • tan(90° − A) = cot A • sec(90° − A) = cosec A. 'Co' pairs swap at 90° − A.

Safe Ranges

For acute A: 0 < sin A, cos A < 1, but tan A can be ANY positive number. sin = 4/3 is impossible — always.

Not Defined Club

tan 90°, sec 90°, cosec 0°, cot 0° — each hides a division by zero. cos 90° = 0 and sin 0° = 0 are the culprits.

The (1 ± sin) Trick

(1 + sin A)(1 − sin A) = $\cos^2 A$ • (1 + cos A)(1 − cos A) = $\sin^2 A$. See this shape, strike instantly.

Conjugate Pairs = 1

(sec A − tan A)(sec A + tan A) = 1 and (cosec A − cot A)(cosec A + cot A) = 1. Know one twin, get the other free.

Only the Angle Matters

Ratios depend ONLY on the angle (similar triangles), never on the triangle's size. One table serves every triangle in the universe.

TOPPER TIP: *Shuffle the cards and test a friend. Teaching a card to someone else is the fastest way to move it into long-term memory — that is the Feynman trick, and it is built into Day 21 of your schedule.*

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18 Active Recall

RECALL — CLOSED BOOK, NO PEEKING

Close the chapter. Answer all ten rapid questions and five prompts aloud or on paper — answers are in the Answer Key. Struggling to recall is not failure; the struggle IS the learning.

R1. In the 7-24-25 triangle, write sin, cos and tan of the angle opposite the side 24.

R2. Find $\cos 60° + \sin 30°$.

R3. Write $1 + \tan^2 A$ as a single ratio squared.

R4. $\sin^2 40° + \cos^2 40° = ?$

R5. cos 70° is equal to sin of which angle?

R6. If $\sin A = 1/\sqrt{2}$, find A.

R7. $1 - \cos^2 A = ?$

R8. $\tan 45° - \cot 45° = ?$

R9. If sec A = 5/4, find cos A.

R10. $\sin 0° + \cos 90° = ?$

Five Prompts — Say the Full Sentence

P1. "sin A is defined as..."

P2. "The master identity states... and it comes from..."

P3. "tan A can be written using sin and cos as..."

P4. "tan 90° is not defined because..."

P5. "sin 30°, sin 45° and sin 60° are... (in order)"

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19 Self-Assessment & Mastery Tracker

RECALL — THE SMART MEMORY SYSTEM

Learning once is not learning. The KalviYogi Smart Memory System revisits this chapter on a spaced schedule — each visit is short, and each visit makes the memory harder to lose.

Day	Learning Activity	Your record (date & score)
Day 0	Pre-learning assessment: attempt the 5 Quick Check questions (Section 02) BEFORE studying.	
Day 1	Learn Sections 01–07 fully; complete Level 1 and Level 2 practice.	
Day 3	Without-book recall: Flash Cards (Section 17) + Active Recall (Section 18), book closed.	
Day 7	Independent problem-solving: Day 7 Mini-Test + Model Test A.	
Day 14	Mixed concept practice: Levels 3–4 + Competency Cases 1–3.	
Day 21	Long-term recall test: Day 21 Mini-Test + Model Test B + teach 3 flash cards to a friend.	
Day 30	Chapter mastery assessment: Day 30 Mastery Test + Model Test C.	

Error Tracker — Every Mistake, Logged and Killed

Question	My mistake	Trap number (Section 10)	Fixed? (date)

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Formula Recall Tracker & Concept Mastery Score

Formula Recall Tracker

Cover the right column. Can you produce each item from memory? Tick only honest successes.

Prompt	Recall it	Day 3 / 7 / 21
The six ratios at angle A	opp/hyp, adj/hyp, opp/adj + reciprocals	☐ ☐ ☐
sin row for 0°, 30°, 45°, 60°, 90°	0, 1/2, 1/√2, √3/2, 1	☐ ☐ ☐
The master identity + its source	$\sin^2 + \cos^2 = 1$, from Pythagoras	☐ ☐ ☐
The two child identities	$1 + \tan^2 = \sec^2$ • $1 + \cot^2 = \text{cosec}^2$	☐ ☐ ☐
Complementary swaps	$\sin(90° - A) = \cos A$, etc.	☐ ☐ ☐
Conjugate pairs	$(\sec - \tan)(\sec + \tan) = 1$, etc.	☐ ☐ ☐

Concept Mastery Score

Concept	Learning (1)	Confident (2)	Mastered (3)
Naming sides & writing the six ratios	☐	☐	☐
Given one ratio, finding all others	☐	☐	☐
Standard values & evaluations	☐	☐	☐
Complementary-angle questions	☐	☐	☐
Proving identities (all ten patterns)	☐	☐	☐
Height-and-distance applications	☐	☐	☐

Scoring: add your six rows (maximum 18). 15–18: chapter mastered — move on, keep the Day 30 date. 10–14: revisit your two weakest rows in Sections 07/10. Below 10: re-learn Sections 05–06 slowly, then re-test. Your Learning Time Tracker: Day 0 ___ min • Day 1 ___ min • Day 3 ___ min • Day 7 ___ min • Day 14 ___ min • Day 21 ___ min • Day 30 ___ min.

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Practice Plus — 25 Concept MCQs

One mark each, board style. Answers and explanations for the tricky ones are in the Answer Key.

1. If sin A = 5/13 (A acute), then cos A =
(a) 12/13 (b) 5/12 (c) 13/12 (d) 8/13

2. tan 45° + cot 45° =
(a) 1 (b) 0 (c) 2 (d) √2

3. $\sin^2 60° + \cos^2 60°$ =
(a) √3 (b) 3/2 (c) 1/2 (d) 1

4. If cos A = 1/2, then A =
(a) 30° (b) 45° (c) 60° (d) 90°

5. $1 - \sin^2 A$ equals
(a) $\tan^2 A$ (b) $\cos^2 A$ (c) $\sec^2 A$ (d) $\cot^2 A$

6. tan A × cot A =
(a) 0 (b) 2 (c) 1 (d) −1

7. $\sec^2 A - \tan^2 A$ =
(a) 0 (b) −1 (c) 2 (d) 1

8. If tan A = 1, then sin A =
(a) 1 (b) 1/√2 (c) √3/2 (d) 1/2

9. sin(90° − A) =
(a) sin A (b) tan A (c) cos A (d) sec A

10. sin 90° + cos 0° =
(a) 1 (b) 0 (c) 1/2 (d) 2

11. If tan A = √3, then A =
(a) 30° (b) 60° (c) 45° (d) 90°

12. $\sin^2 27° + \sin^2 63°$ =
(a) 0 (b) 2 (c) 1 (d) 1/2

13. If sin A = cos A (A acute), then tan A =
(a) 0 (b) 1 (c) √3 (d) 1/√3

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Practice Plus — MCQs 14 to 25

14. cosec 60° =

(a) 2 (b) √3/2 (c) 2/√3 (d) 1/2

15. In △ABC, right-angled at B, sin C equals

(a) BC/AC (b) AB/BC (c) BC/AB (d) AB/AC

16. If cosec A = 5/3, then sin A =

(a) 5/3 (b) 4/5 (c) 3/5 (d) 5/4

17. $2 \sin^2 45° =$

(a) 2 (b) 1/2 (c) √2 (d) 1

18. tan 30° × tan 60° =

(a) √3 (b) 3 (c) 1 (d) 1/3

19. If sec A = 2 (A acute), then sin A + cos A =

(a) (1 + √3)/2 (b) 3/2 (c) √3 (d) 1

20. $1 + \tan^2 A =$

(a) $\text{cosec}^2 A$ (b) $\cot^2 A$ (c) $\sin^2 A$ (d) $\sec^2 A$

21. If sin A = 1/2 and cos B = 1/2 (A, B acute), then A + B =

(a) 60° (b) 90° (c) 120° (d) 45°

22. As an acute angle A grows towards 90°, sin A approaches

(a) 0 (b) 1/2 (c) √3/2 (d) 1

23. cot(90° − A) =

(a) cot A (b) tan A (c) sin A (d) cos A

24. If tan A = 4/3, then sin A − cos A =

(a) 1/5 (b) 7/5 (c) −1/5 (d) 0

25. (sec A + tan A)(sec A − tan A) =

(a) 0 (b) 2 (c) $\sec^2 A$ (d) 1

Find the Error — 10 Broken Solutions

Each solution below contains exactly one planted error. Find it, name it, and write the corrected step. Full corrections are in the Answer Key.

E1. "sin A = 3/5, so the hypotenuse is 5 and opposite is 3. Therefore cos A = hypotenuse/adjacent = 5/4."

E2. "tan 60° = 1/√3, because 60° is the smaller standard angle."

E3. "$\sin^2 30° + \cos^2 30° = 1$, therefore sin 30° + cos 30° = 1 also."

E4. "In △ABC, right-angled at B, sin A = AB/AC, since AB comes first in the alphabet."

E5. "To prove (1 + sin A)/cos A = sec A + tan A, I cross-multiplied first and then simplified both sides until they matched."

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Find the Error — E6 to E10

E6. "$\cos^2 30° = (\sqrt{3}/2)^2 = \sqrt{3}/4$."

E7. "cot A = 1, so A = 0°, because cot starts from zero."

E8. "sec A = 13/12, so sin A = 12/13, since 12 is the bigger number."

E9. "sin(A + B) = sin A + sin B, so sin 75° = sin 45° + sin 30° = $1/\sqrt{2} + 1/2$."

E10. "$1 + \cot^2 A = \text{cosec}^2 A$, therefore $\cot^2 A = \text{cosec}^2 A + 1$."

TOPPER TIP: *Error-spotting is the highest form of understanding: you cannot catch a mistake you do not truly understand. These ten errors are the ten most common real exam errors in this chapter.*

Chapter Model Test A — Basics (20 Marks)

Time: 45 minutes. Direct ratios, table values and one friendly proof.

Q1. If sin A = 3/5 (A acute), find cos A and tan A. **[2]**

Q2. Evaluate: $\tan^2 45° + \sin^2 60° - \cos^2 30°$. **[2]**

Q3. In △ABC, right-angled at B, AB = 8 cm and BC = 15 cm. Find sin A, cos A and tan A. **[3]**

Q4. If cos A = √3/2, find sin A and tan A. **[3]**

Q5. In △PQR, right-angled at Q, PQ = 7 cm and PR = 25 cm. Find sin P, cos P, tan P and sin R. **[5]**

Q6. Prove that cos A/(1 + sin A) + (1 + sin A)/cos A = 2 sec A. **[5]**

Chapter Model Test B — Standard (20 Marks)

Time: 45 minutes. Board-paper level — this is the paper to benchmark yourself with.

Q1. Evaluate: $2\tan^2 45° + \cos^2 30° - \sin^2 60°$. **[3]**

Q2. If $4\cot A = 3$, check whether $(1 - \tan^2 A)/(1 + \tan^2 A) = \cos^2 A - \sin^2 A$ or not. **[3]**

Q3. Prove that $(\operatorname{cosec}\theta - \cot\theta)^2 = (1 - \cos\theta)/(1 + \cos\theta)$. **[4]**

Q4. A ladder 15 m long leans against a wall, making 60° with the ground. Find the height it reaches and the distance of its foot from the wall. **[4]**

Q5. Express $\sin A$ and $\tan A$ in terms of $\sec A$. **[3]**

Q6. State, with justification, whether each is possible: (i) $\sin\theta = 5/3$ for some angle θ; (ii) $\tan\theta = 5/3$ for some angle θ. **[3]**

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Chapter Model Test C — Advanced (20 Marks)

Time: 45 minutes. Proof-heavy, HOTS-level — the paper that separates 95 from 100.

Q1. Prove that $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$. **[5]**

Q2. If $\sec\theta + \tan\theta = p$, prove that $\sin\theta = (p^2 - 1)/(p^2 + 1)$. **[4]**

Q3. If $\text{cosec } A + \cot A = 5$, find $\cos A$. **[4]**

Q4. Prove that $\tan\theta/(1 - \cot\theta) + \cot\theta/(1 - \tan\theta) = 1 + \sec\theta \text{ cosec } \theta$. **[4]**

Q5. If $\sin\theta + \cos\theta = \sqrt{2}$, prove that $\theta = 45°$. **[3]**

> ***TOPPER TIP:*** *Take these three tests on three different days — Test A after Section 11, Test B after Section 14, Test C in the final revision week. Record every score in your Mastery Tracker.*

My Test Record — Chapter 8

Test	Date	Score	Weakest topic — revise this first
Model Test A — Basics		/ 20	
Model Test B — Standard		/ 20	
Model Test C — Advanced		/ 20	
Day 7 Mini-Test		/ 10	
Day 21 Mini-Test		/ 10	
Day 30 Mastery Test		/ 10	

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Smart Memory Mini-Tests — Day 7 and Day 21

Ten quick questions each, closed book, five minutes. Answers are in the Answer Key.

DAY 7 TEST

1. Write the value of sin 60°.

2. If tan A = 3/4, find sin A.

3. Complete: $\sin^2 A + \cos^2 A =$ ___

4. tan(90° − A) = which ratio of A?

5. Write the value of sec 30°.

6. If cos A = 4/5, find tan A.

7. Simplify: $1 - \sin^2 A$.

8. sin 25° is equal to which cos value?

9. Write the value of cot 45°.

10. In a right triangle, hypotenuse 13 and side opposite A is 5. Find sin A.

DAY 21 TEST

1. Complete: $\operatorname{cosec}^2 A - \cot^2 A =$ ___

2. Evaluate: $\sin^2 45° + \cos^2 45°$.

3. If sin A = 7/25, find cosec A.

4. tan A × cot A = ?

5. Write tan A using sin A and cos A.

6. Simplify: (sec A − tan A)(sec A + tan A).

7. Evaluate: sin 90° − cos 90°.

8. If tan A = √3, find A.

9. Complete: $1 + \tan^2 A =$ ___

10. Evaluate: $\sin^2 63° + \sin^2 27°$.

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Smart Memory Mini-Test — Day 30 Mastery Test

The final checkpoint. Ten questions, closed book — 9 or 10 means this chapter is yours for the whole year.

DAY 30 MASTERY TEST

1. In the 5-12-13 triangle, write sin A and tan A for the angle opposite the side 5.

2. Prove in two lines: $(1 + \sin A)(1 - \sin A) = \cos^2 A$.

3. Evaluate: $2\tan^2 45° + \cos^2 30° - \sin^2 60°$.

4. If $\sec \theta = 25/7$, find $\tan \theta$.

5. State, in one line, why $\tan 90°$ is not defined.

6. True or false: $\sin(A + B) = \sin A + \sin B$ for all A, B. Justify in five words or fewer.

7. If $\operatorname{cosec} A + \cot A = 3$, find $\operatorname{cosec} A - \cot A$.

8. Evaluate: $9\sec^2 A - 9\tan^2 A$.

9. $\cos 80°$ equals $\sin$ of which angle?

10. If $\sin \theta + \cos \theta = \sqrt{2}$, find θ.

> **TOPPER TIP:** *Missed any Day 30 question? That question's Flash Card goes back into your pocket box for one more week.*

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Answer Key — Graded Practice, Levels 1 and 2

Level 1 — Foundation

L1.1 Adjacent = $\sqrt{(100 - 36)} = 8$. $\sin A = 3/5$, $\cos A = 4/5$. **L1.2** Hypotenuse = 13: $\sin A = 5/13$, $\cos A = 12/13$. **L1.3** $1/\sqrt{2} + 1/\sqrt{2} = \sqrt{2} \approx 1.41 > 1$ — allowed: the [0,1] limit applies to each ratio alone, not to their sum. **L1.4** $\sqrt{3} \div \sqrt{3} = 1$; tan 60° and cot 30° are complementary twins, always equal. **L1.5** Opposite = $\sqrt{(169 - 144)} = 5$, $\tan A = 5/12$. **L1.6** cosec 30° = 2, sec 60° = 2, cot 45° = 1.

Level 2 — Skill Builder

L2.1 Adjacent = 24: $\cos A = 24/25$, $\tan A = 7/24$, $\text{cosec } A = 25/7$, $\sec A = 25/24$, $\cot A = 24/7$.

L2.2 $2(1/4) + 3(1/4) - 1 = 1/2 + 3/4 - 1 =$ **1/4**.

L2.3 $\tan \theta = 4/5$. Divide top and bottom by $\cos \theta$: $(5 \tan \theta - 3)/(5 \tan \theta + 3) = (4 - 3)/(4 + 3) =$ **1/7**.

L2.4 $(1 - \sin A)(1 + \sin A) = 1 - \sin^2 A = \cos^2 A$, using $\sin^2 A + \cos^2 A = 1$.

L2.5 $\cos B = \sin(90° - B)$, so $\sin A = \sin(90° - B)$; equal sines of acute angles force $A = 90° - B$, i.e. $A + B = 90°$.

L2.6 $(3 + 1/2)/(1 + 1/4) = (7/2)/(5/4) =$ **14/5**.

TOPPER TIP: *Marked yourself wrong at Level 2? Compare your working with the matching Section 07 solution step by step — find the FIRST step where you differ. That step is the lesson.*

Answer Key — Levels 3, 4, 5 and Active Recall

Level 3 — Application

L3.1 Height = 10 sin 60° = **5√3 m** ≈ 8.66 m. **L3.2** Height = 30 tan 30° = 30/√3 = **10√3 m** ≈ 17.32 m. **L3.3** Height = 50 sin 60° = **25√3 m** ≈ 43.3 m. **L3.4** Height = 20 tan 45° = **20 m**.

Level 4 — Advanced Reasoning

L4.1 Replace 1 in the numerator by $\sec^2 A - \tan^2 A$: numerator = (tan A + sec A) − (sec A − tan A)(sec A + tan A) = (sec A + tan A)(1 − sec A + tan A) = (sec A + tan A)(tan A − sec A + 1). Cancel the denominator (tan A − sec A + 1): result = sec A + tan A = (1 + sin A)/cos A. ∎

L4.2 $x^2/a^2 = \cos^2\theta$ and $y^2/b^2 = \sin^2\theta$; their sum is 1 by the master identity. The angle disappears because $\sin^2 + \cos^2$ erases it — the identity is angle-blind.

L4.3 Squaring: 1 + 2 sin θ cos θ = 2, so sin θ cos θ = 1/2. Then $(\sin\theta - \cos\theta)^2 = 1 - 2(1/2) = 0$, so sin θ = cos θ, tan θ = 1, **θ = 45°**.

Level 5 — Mathematical Genius

L5.1 $\tan^2 A/(1 + \tan^2 A) = \tan^2 A/\sec^2 A = \sin^2 A$. Similarly $\cot^2 A/(1 + \cot^2 A) = \cot^2 A/\operatorname{cosec}^2 A = \cos^2 A$. Sum = $\sin^2 A + \cos^2 A = 1$. ∎

L5.2 B = 90° − A, so sin B = cos A and cot B = tan A. Then $\sin^2 A + \sin^2 B = \sin^2 A + \cos^2 A = 1$, and $(\tan A - \cot B)^2 = 0$. Total = 1. ∎

Active Recall (Section 18)

R1 sin = 24/25, cos = 7/25, tan = 24/7. **R2** 1/2 + 1/2 = 1. **R3** $\sec^2 A$. **R4** 1. **R5** sin 20°. **R6** 45°. **R7** $\sin^2 A$. **R8** 0. **R9** 4/5. **R10** 0 + 0 = 0.

P1 sin A = opposite/hypotenuse in a right triangle. **P2** $\sin^2 A + \cos^2 A = 1$; from Pythagoras. **P3** tan A = sin A/cos A. **P4** Because tan 90° = sin 90°/cos 90° = 1/0, and division by zero is undefined. **P5** 1/2, 1/√2, √3/2.

Assertion – Reason (Section 14)

AR1 (c) — A is true numerically (√3/2 = 2 · 1/2 · √3/2), but R is a different fact and does not explain it. **AR2** (a). **AR3** (a). **AR4** (a). **AR5** (a). **AR6** (d) — A is false because cos A = adjacent/hypotenuse ≤ 1 (the hypotenuse is the longest side), while R is true. **AR7** (a). **AR8** (a). **AR9** (d) — A false (tan 30° < 1), R false as stated (either leg can be longer). **AR10** (a).

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Answer Key — Practice Plus

25 Concept MCQs

1 (a) **2** (c) **3** (d) **4** (c) **5** (b) **6** (c) **7** (d) **8** (b) **9** (c) **10** (d) **11** (b) **12** (c) **13** (b)
14 (c) **15** (d) **16** (c) **17** (d) **18** (c) **19** (a) **20** (d) **21** (b) **22** (d) **23** (b) **24** (a) **25** (d)

Explanations for the tricky ones. Q3: $\sin^2 + \cos^2$ of the SAME angle is always 1, whatever the angle. Q12: sin 63° = cos 27°, so the sum is $\sin^2 27° + \cos^2 27° = 1$. Q15: at angle C, the opposite side is AB, so sin C = AB/AC — Trap 1 in reverse. Q19: sec A = 2 gives cos A = 1/2, A = 60°, sin A = √3/2; sum = (1 + √3)/2. Q21: A = 30° and B = 60° (cos B = 1/2), total 90°. Q24: tan A = 4/3 gives the 3-4-5 triangle: sin A − cos A = 4/5 − 3/5 = 1/5.

Find the Error — Corrections

E1 cos A is adjacent/hypotenuse, never the reverse — and a ratio of a part to the whole can never exceed 1. Adjacent = 4, so cos A = **4/5**.

E2 60° is not 'smaller'; tan grows with the angle: tan 60° = **√3** (1/√3 is tan 30°).

E3 $(\sin + \cos)^2 = \sin^2 + \cos^2 + 2 \sin \cos \neq \sin^2 + \cos^2$. In fact sin 30° + cos 30° = (1 + √3)/2 ≈ 1.37.

E4 Sides are named by the ANGLE, not the alphabet: at A, the opposite side is BC, so sin A = **BC/AC**.

E5 Cross-multiplying assumes the equality is already true — circular reasoning. Start from ONE side only.

E6 Square numerator and denominator: $(\sqrt{3}/2)^2$ = **3/4**.

E7 cot 45° = 1, so A = **45°**; cot 0° is not even defined.

E8 12/13 is cos A (flip of sec). Opposite = √(169 − 144) = 5, so sin A = **5/13**.

E9 There is NO rule sin(A + B) = sin A + sin B (Ex 8.2 Q4(i) disproved it). The 'answer' is doubly wrong.

E10 Sign slip on rearrangement: $\cot^2 A = \text{cosec}^2 A$ **− 1**.

Answer Key — Model Tests and Day Tests

Model Test A — Basics (20)

Q1 cos A = 4/5, tan A = 3/4. **Q2** 1 + 3/4 − 3/4 = 1. **Q3** AC = 17: sin A = 15/17, cos A = 8/17, tan A = 15/8. **Q4** sin A = 1/2, tan A = 1/√3. **Q5** QR = 24: sin P = 24/25, cos P = 7/25, tan P = 24/7, sin R = 7/25. **Q6** Proof as in Ex 8.3 Q5(ii): common denominator, $\sin^2 + \cos^2 = 1$, cancel (1 + sin A), get 2/cos A = 2 sec A.

Model Test B — Standard (20)

Q1 2. **Q2** cot A = 3/4, tan A = 4/3. LHS = (1 − 16/9)/(1 + 16/9) = −7/25. From the 3-4-5 triangle, cos A = 3/5, sin A = 4/5: RHS = 9/25 − 16/25 = −7/25. Equal — YES, it holds (both sides negative; a value below zero is fine for a difference of squares). **Q3** Proof as in Ex 8.3 Q5(i). **Q4** Height = 15 sin 60° = 15√3/2 ≈ 12.99 m; foot distance = 15 cos 60° = 7.5 m. **Q5** sin A = √(sec²A − 1)/sec A; tan A = √(sec²A − 1) (Ex 8.3 Q2). **Q6** (i) Impossible — sin ≤ 1. (ii) Possible — tan can be any positive number (e.g. opposite 5, adjacent 3).

Model Test C — Advanced (20)

Q1 As in the CBSE 2023 (Standard) solution: build the $\sin^4 + \cos^4$ and $\sin^6 + \cos^6$ blocks, substitute, cancel. **Q2** As in Challenge 8. **Q3** cosec A − cot A = 1/5; adding: 2 cosec A = 26/5, cosec A = 13/5, sin A = 5/13, cos A = **12/13**. **Q4** As in Ex 8.3 Q5(iii). **Q5** Square: sin θ cos θ = 1/2; then $(\sin\theta - \cos\theta)^2 = 0$, so tan θ = 1 and θ = 45°.

Day 7 Test

1 √3/2 **2** 3/5 **3** 1 **4** cot A **5** 2/√3 **6** 3/4 **7** $\cos^2 A$ **8** cos 65° **9** 1 **10** 5/13

Day 21 Test

1 1 **2** 1 **3** 25/7 **4** 1 **5** sin A/cos A **6** 1 **7** 1 **8** 60° **9** $\sec^2 A$ **10** 1 (sin 27° = cos 63°)

Day 30 Mastery Test

1 sin A = 5/13, tan A = 5/12 **2** 1 − $\sin^2 A$ = $\cos^2 A$ by the master identity **3** 2 **4** 24/7 (opposite = √(625 − 49) = 24) **5** cos 90° = 0 makes the denominator zero **6** False — e.g. A = B = 30° fails **7** 1/3 (conjugate pair) **8** 9 **9** sin 10° **10** 45°

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Revision Card — Chapter 8 on One Page

The six ratios (at angle A, right triangle)

sin A = opp/hyp • cos A = adj/hyp • tan A = opp/adj • cosec A = hyp/opp • sec A = hyp/adj • cot A = adj/opp. tan A = sin A/cos A, cot A = cos A/sin A.

The table — regenerate, don't memorise

sin: 0, 1/2, 1/√2, √3/2, 1 for 0°, 30°, 45°, 60°, 90°. cos = the same row backwards. tan = sin/cos: 0, 1/√3, 1, √3, not defined. Sources: the 45-45-90 triangle (legs 1, 1, √2) and half an equilateral (1, √3, 2).

The three identities

$\sin^2 A + \cos^2 A = 1$ • $1 + \tan^2 A = \sec^2 A$ • $1 + \cot^2 A = \text{cosec}^2 A$. All three are Pythagoras; the last two come from dividing the first by $\cos^2 A$ and $\sin^2 A$.

Complements and conjugates

$\sin(90° - A) = \cos A$ • $\tan(90° - A) = \cot A$ • $\sec(90° - A) = \text{cosec } A$ (and reverses). $(\sec A - \tan A)(\sec A + \tan A) = 1$ • $(\text{cosec } A - \cot A)(\text{cosec } A + \cot A) = 1$ • $(1 \pm \sin A)(1 \mp \sin A) = \cos^2 A$.

Exam-eve checklist

☐ Hypotenuse named first in every figure ☐ Ratios read from the CORRECT angle ☐ sin, cos ≤ 1 sanity check ☐ Proofs start from one side, identity named at each step ☐ (a − b) = −(b − a) minus sign written ☐ tan 90° / cot 0°: not defined

Chapter 8 complete. You did not memorise trigonometry — you discovered, visualised, derived and mastered it.

Next: Chapter 9 — Some Applications of Trigonometry, in the same 8D format. • KalviYogi • Mathematics – Beyond the Formula • 369 Tesla × KalviYogi • © Edunivra Pvt Ltd

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CHAPTER 9 · SOME APPLICATIONS OF TRIGONOMETRY

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01 Chapter Introduction

8D • DISCOVER — WHY SHOULD I LEARN THIS?

Here is a bold claim: after this chapter, you will be able to find the **height of a temple tower**, the **width of a river you cannot cross**, and the **height of a flying kite** — using nothing but a measuring tape on the ground and one angle. No climbing. No guessing. Just one right triangle and the trigonometry you learnt in Chapter 8.

Surveyors, architects, pilots and sailors have used this exact idea for centuries. When you look **up** at the top of a tower, your line of sight makes an **angle of elevation** with the horizontal. When a lighthouse keeper looks **down** at a ship, the line of sight makes an **angle of depression**. Those two angles, plus one distance you can measure, are enough to unlock heights and distances that no tape can reach.

What you will master in this chapter

- The line of sight, the horizontal line, and the two angles — elevation and depression.
- Drawing the correct diagram for ANY heights-and-distances problem — the skill that decides half the marks.
- Why **tan** is the workhorse ratio of this chapter — derived, not memorised.
- One-triangle problems: towers, kites, ladders, broken trees, slides and ropes.
- Two-triangle problems: walking towards a building, towers on buildings, statues on pedestals, ships from a lighthouse, a moving balloon and a moving car.
- The two-observation-point method — derived once, then used everywhere.
- The complementary-angles theorem: if the angles from two points are complementary, height = √(product of the distances).

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Your Chapter Map — 19 Stops, One Journey

This chapter is a journey with 19 stops. Tick each stop as you finish it.

Discover & Visualise	Understand & Derive	Solve, Apply, Recall
☐ 01 Chapter Introduction ☐ 02 Prerequisite Knowledge ☐ 03 Real-Life Story ☐ 04 Visual Concepts (CPA)	☐ 05 Mathematical Foundations ☐ 06 Formula Derivations ☐ 08 Multiple Solution Methods ☐ 10 Common Mistakes	☐ 07 Detailed Solved Examples ☐ 09 Hands-on Activities ☐ 11 Graded Practice (5 Levels) ☐ 12 Mathematical Reasoning
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How to use this chapter (read this first!)

1. **Never read a solution passively.** Cover the steps, draw the diagram yourself, then uncover one step at a time.
2. In this chapter, **Step 1 of every solution is always the same: DRAW THE DIAGRAM.** A correct diagram is worth marks by itself in the board exam — and a wrong diagram makes every later step wrong.
3. Every solution shows **Step 1, Step 2, Step 3...** with a small *Why* note in purple — the reason behind the step.
4. After Section 07, test yourself in Sections 11–15 **before** looking at the Answer Key.
5. Finish with the Mind Map (16), the Flash Cards (17), the closed-book Recall (18) and your Mastery Tracker (19). Come back on Day 3, 7, 14, 21 and 30.

KALVIYOGI DIGITAL COMPANION: This chapter also lives online — video lessons, interactive simulations (move the observer, watch the angle change), online tests and the AI Math Tutor for Chapter 9. **QR code and link activate with the KalviYogi app/website at print.** Point your phone camera at the chapter QR in the printed book to open the Chapter 9 digital pack.

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02 Prerequisite Knowledge

CHAPTER 8 RECAP — THE TOOLS YOU ALREADY OWN

This whole chapter runs on four tools from Chapter 8. Refresh them here — five minutes now will save you fifty minutes later.

1. The three ratios in a right triangle

For an acute angle θ in a right triangle: **sin θ** = opposite / hypotenuse, **cos θ** = adjacent / hypotenuse, **tan θ** = opposite / adjacent. The side opposite the right angle is the hypotenuse; the side opposite θ is its perpendicular; the remaining side is the base (adjacent). If θ moves to the other acute corner, opposite and adjacent swap — always name the sides **from the angle you are using**.

2. The standard values — you must know these by heart

θ	0°	30°	45°	60°	90°
sin θ	0	1/2	1/√2	√3/2	1
cos θ	1	√3/2	1/√2	1/2	0
tan θ	0	1/√3	1	√3	not defined

Only 30°, 45° and 60° appear in this chapter's problems — but they appear in **every single one**.

3. Complementary angles

sin(90° − θ) = cos θ and tan(90° − θ) = cot θ = 1/tan θ. Two angles that add to 90° are **complementary**. This one idea powers the last NCERT question of the chapter (Q16) and Reasoning 1 in Section 12.

4. Parallel lines and alternate angles

When a transversal cuts two **parallel** lines, the alternate interior angles are equal. In this chapter the two horizontal lines — one through the observer's eye, one through the object's base level — are parallel, and the line of sight is the transversal. That is why an angle of depression can always be re-drawn as an angle of elevation (proved fully in Section 06).

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Quick Check — Are You Ready? (5 Questions)

Try all five without looking back. Answers are given right below — cover them with your hand first!

Q1. Write the values of tan 30°, tan 45° and tan 60°.

Q2. In a right triangle, the side opposite 30° is 5 cm. What is the hypotenuse?

Q3. If tan θ = 1/√3, what is θ? If tan φ = √3, what is φ?

Q4. A vertical pole and its shadow: which side of the triangle is the shadow — opposite, adjacent or hypotenuse (for the sun's angle at the tip of the shadow)?

Q5. Two horizontal lines are cut by a slanted line. Are the alternate interior angles equal or supplementary?

Answers — check only after trying

Q1. tan 30° = 1/√3, tan 45° = 1, tan 60° = √3.

Q2. sin 30° = 5/hypotenuse = 1/2, so the hypotenuse is 10 cm.

Q3. θ = 30° and φ = 60°. Notice the pair: 1/√3 and √3 belong to complementary angles.

Q4. The shadow is the **adjacent** (base) — the pole is the opposite side, and the sun's ray is the hypotenuse. So tan(sun's angle) = pole height / shadow length.

Q5. Equal — alternate interior angles between parallel lines are always equal.

TOPPER TIP: *Got 4 or 5 right? You are fully ready. Got fewer? Revisit the standard-values table once — in this chapter, a forgotten tan value costs more marks than any hard idea.*

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03 Real-Life Mathematical Story

THE TANK TOWER THAT NOBODY COULD CLIMB

*It was the last week of the school year, and KalviYogi School was getting ready for the annual inspection. Behind the main building stood the school's **water-tank tower** — a tall concrete tower holding the big black tank that supplied water to the whole school. The painter, Mr. Velu, had come to give a quotation.*

"My rate is per metre of height," he said, looking up. "So first, tell me — how tall is the tower?"

There was a problem. The tower's ladder had rusted and broken years ago. Nobody could climb it. The headmaster shook his head. "We don't know the height. It was built before any of us came here."

Meena, who had just finished Chapter 8 in mathematics, smiled. "Sir, we don't need to climb it. Give me a measuring tape and ten minutes."

Arun was puzzled. "The tape is only 30 metres long, and we can't even reach the top!"

*"We won't measure the tower," said Meena. "We will measure the **ground** — and one **angle**. The triangle will tell us the rest."*

Meena walked out from the foot of the tower, unrolling the tape along the ground. She stopped at exactly **15 metres** from the tower. Kavin held the school's clinometer — a protractor with a straw and a hanging thread — and sighted the top of the tank through the straw.

"The angle is **60 degrees**," he announced. "My eye is looking up at 60° from the horizontal."

Meena drew a triangle in her notebook: a horizontal line for the ground, a vertical line for the tower, and a slanted line for Kavin's line of sight. "The tower, the ground and your line of sight make a **right triangle**," she said. "In that triangle, the angle at your eye is 60°, the side next to it is 15 m, and the tower is the side opposite it. And opposite divided by adjacent is..."

"**tan!**" said Arun. "tan 60° = height / 15. And tan 60° is √3!"

The Story Becomes Mathematics

On the back page of her notebook, Meena wrote the whole solution in four lines:

$$\tan 60° = (\text{height above eye level}) / 15$$

$$\sqrt{3} = (\text{height above eye level}) / 15$$

$$\text{height above eye level} = 15\sqrt{3} \approx 15 \times 1.732 = \mathbf{25.98\ m}$$

*"Wait," said Kavin. "My eye is not on the ground. I am standing — my eye is **1.5 metres** above the ground. The triangle started at my eye, not at the floor!"*

*"Exactly," laughed Meena. "That is the trap in every such problem. The triangle gives only the part **above your eye**. The full height of the tower = 25.98 + 1.5 = **27.48 metres**, about 27.5 m."*

Mr. Velu checked the figure twice, then nodded slowly. "Twenty-seven and a half metres. In thirty years of painting, no one ever measured a tower for me from the ground."

That evening, standing on the school terrace, Arun looked down at the school gate far below. "Meena — from up here, my line of sight tilts down. Is that also an angle?"

*"That," she said, "is the **angle of depression** — and it is the second hero of this chapter. With it, we can even find how far the gate is from the tower... without walking there."*

The two questions this story leaves us with

Question 1 (looking UP): If I know my distance from an object and the angle at which I look up at its top, can I always find its height? — **Yes: height = distance × tan(angle), plus eye height.**

Question 2 (looking DOWN): If I stand at a known height and look down at an object at some angle, can I find how far away it is? — **Yes — and this chapter will show that the downward angle secretly equals an upward angle.**

Every one of the 16 NCERT problems in Section 07 is one of these two questions in disguise. Learn to see the disguise, and the chapter is yours.

PIN POINT: The triangle never starts at the ground when a person is observing — it starts at the EYE. Always ask: 'Is an eye height (or observer height) hiding in this problem?'

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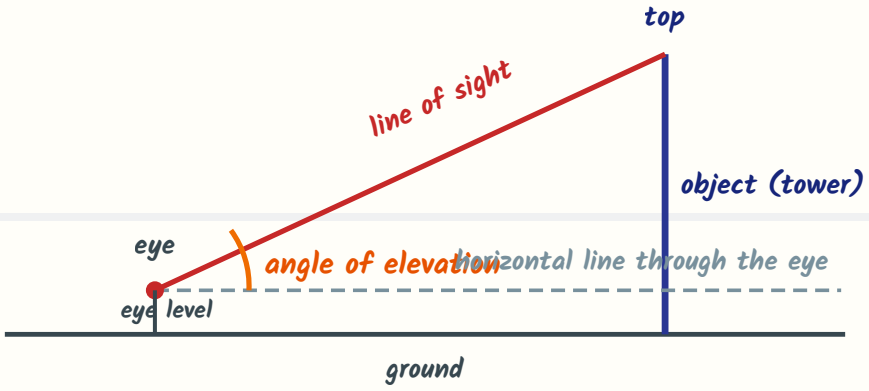
04 Visual Concept Explanation

SINGAPORE CPA — CONCRETE → PICTORIAL → ABSTRACT

Concrete. Stand in a room. Look straight ahead at the wall — your line of sight is horizontal. Now look up at the ceiling fan: your line of sight tilts **up**. Now look down at your shoes: it tilts **down**. Those two tilts are the only new ideas in this chapter. Everything else is Chapter 8.

Pictorial. Here is looking UP, drawn carefully. Three lines matter: the **horizontal line** through the eye, the **line of sight** from the eye to the top, and the angle between them.

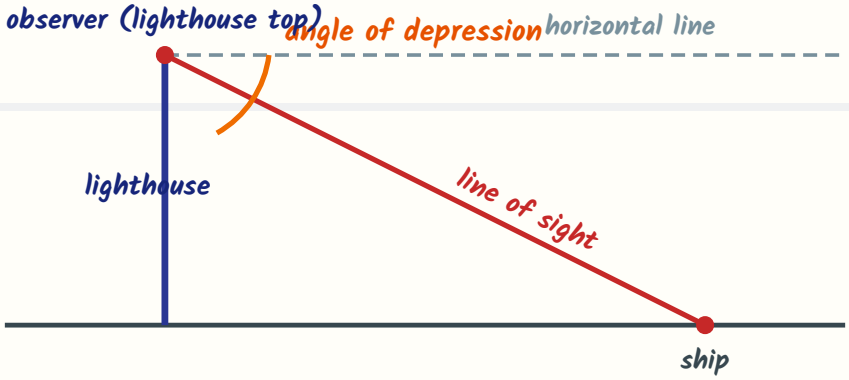


Looking UP: the angle of ELEVATION is measured from the horizontal, at the eye.

The **angle of elevation** of a point above an observer is the angle between the horizontal line through the observer's eye and the line of sight to that point. Notice two things: the angle lives **at the eye**, and it is measured from the **horizontal** — never from the vertical, never from the ground line.

Looking Down — and the Only Triangle You Will Ever Need

Pictorial, continued. Now the observer is high up — on a lighthouse — looking DOWN at a ship.



Looking DOWN: the angle of DEPRESSION is also measured from the horizontal, at the eye.

The **angle of depression** of a point below an observer is the angle between the horizontal line through the observer's eye and the line of sight down to that point. Same rules: at the eye, from the horizontal.

Abstract. Strip away the tower, the ship and the story, and every problem in this chapter becomes the same skeleton:

The master skeleton — one right triangle

Vertical side = the height (of the object, or the height difference). **Horizontal side** = the distance along the ground. **Slanted side** = the line of sight (or a rope, string, ladder — anything slanted). **The angle** sits at the observer's eye, between the horizontal side and the slanted side.

Your whole job in every problem: **(1)** find this triangle in the story, **(2)** label what is given and what is wanted, **(3)** pick the ratio that connects them. Section 06 shows why step (3) is almost always **tan**.

PIN POINT: Elevation = eye looks UP. Depression = eye looks DOWN. Both angles are measured from the HORIZONTAL — if your diagram measures an angle from the vertical, the diagram is wrong.

05 Mathematical Foundations

8D • UNDERSTAND — THE DEFINITIONS, MADE EXACT

1. Line of sight

The **line of sight** is the straight line drawn from the eye of the observer to the point in the object that is being viewed. It is the hypotenuse of our master triangle (or a rope/string when the problem gives one).

2. Angle of elevation

When the viewed point is **above** the horizontal line through the eye, the angle between that horizontal line and the line of sight is the **angle of elevation** of the point, as seen from the eye.

3. Angle of depression

When the viewed point is **below** the horizontal line through the eye, the angle between that horizontal line and the line of sight is the **angle of depression** of the point, as seen from the eye.

4. The three working equations of this chapter

Let θ be the angle at the eye, **h** the vertical height (difference) and **d** the horizontal distance. Then:

tan θ = h / d | **sin θ = h / (line of sight)** | **cos θ = d / (line of sight)**

Read them as a menu: if the problem speaks of **height and distance**, use tan. If it gives or asks a **slanted length** (string, rope, ladder, wire), sin or cos will join in.

5. The eye-height rule

When a **person** observes, the triangle's vertical side is the height **above the person's eye**, not the full height of the object. Full height = (height above eye) + (observer's height). NCERT Q6 and Q14 are built entirely on this rule — and it is the single most-lost mark in the chapter.

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Foundations — Reading a Problem Like an Examiner

Before any formula, a topper translates the story into the skeleton. Practise on these four translations:

The story says	The skeleton says
"The angle of elevation of the top of the tower from a point 30 m away is 30°."	d = 30, θ = 30°, find h. One triangle, tan.
"The string of the kite makes 60° with the ground; the kite is 60 m high."	h = 60, θ = 60°, find the slanted side (string). One triangle, sin.
"As he walks towards the building, the angle grows from 30° to 60°."	The SAME height seen from TWO distances. Two triangles sharing one vertical side.
"From the top of the lighthouse, the angles of depression of two ships are 30° and 45°."	Convert each depression into an elevation from the ship (Section 06). Two triangles sharing the lighthouse.

The 4-line ritual for every problem in this chapter

Line 1: Draw the diagram and name the points (A, B, C...). **Line 2:** Write what is given and what is to be found, in symbols. **Line 3:** Choose the ratio and write the equation. **Line 4:** Solve, then state the answer in the story's words with units.

Sections 07, 12, 13 and 15 follow this ritual in every single solution, step by numbered step. Imitate the ritual until it becomes habit — examiners reward it with step marks even when a final arithmetic slip occurs.

TOPPER TIP: *In the board exam, the diagram + the ratio equation usually carry 1 of the 3 marks by themselves. Even if you cannot finish a problem, draw and set up — never leave a heights-and-distances question blank.*

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06 Formula Derivations

8D • DERIVE — WHERE THE WORKING RULES COME FROM

Derivation A — Why tan is the workhorse: height = distance × tan θ

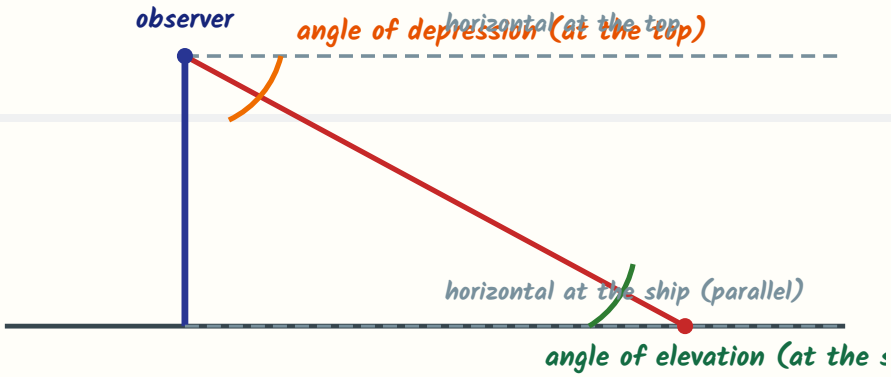
Start from the definition, nothing else. In our master right triangle, the angle θ sits at the observer's eye. The side **opposite** θ is the height h; the side **adjacent** to θ is the ground distance d.

1. By the definition of the tangent ratio (Chapter 8): tan θ = opposite / adjacent = **h / d**.
   Why: This is not a new formula — it is the definition of tan, read in this triangle.
2. Multiply both sides by d: **h = d × tan θ**.
   Why: One algebraic step — and the chapter's main engine stands derived.
3. Divide the same definition differently: **d = h / tan θ**.
   Why: The same equation, rearranged, finds a distance from a known height.
4. Now compare with sin and cos: sin θ = h / (line of sight) and cos θ = d / (line of sight). Both involve the **hypotenuse** — the one side we usually cannot measure and are not asked for.
5. Conclusion: heights-and-distances problems almost always give and ask the two **legs** of the triangle (height and ground distance), never the slanted side. tan is the only ratio built from exactly those two legs — that is why tan does nearly all the work in this chapter.
   Why: Reach for sin or cos only when a slanted length — a string, rope, ladder or wire — enters the story.

PIN POINT: h = d tan θ is not a formula to memorise — it is the DEFINITION of tan with the sides renamed. If you ever forget it, re-derive it in ten seconds: tan = opposite/adjacent.

Derivation B — Angle of Depression = Angle of Elevation

This is the chapter's master key: it lets us throw away every 'looking down' triangle and replace it with a 'looking up' one, which our tan engine already knows how to solve.



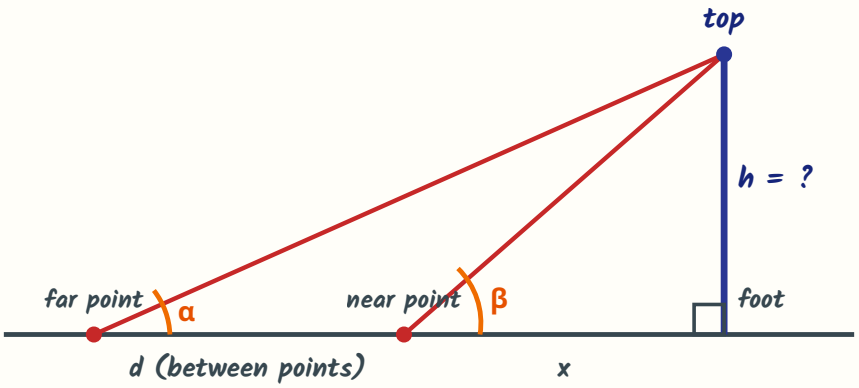
The two horizontals are parallel; the line of sight is the transversal.

1. Through the observer (top of the lighthouse) draw the horizontal line. Through the ship draw its horizontal line (the sea-level line).
   Why: Both lines are horizontal, and all horizontal lines are parallel to each other.
2. The angle of depression is the angle between the TOP horizontal and the line of sight.
   Why: By the definition in Section 05.
3. The angle of elevation of the lighthouse top, as seen from the ship, is the angle between the BOTTOM horizontal and the same line of sight.
4. The line of sight cuts the two parallel horizontals, so these two angles are **alternate interior angles**.
   Why: Chapter 9 geometry (and Prerequisite 4): a transversal cutting parallel lines.
5. Alternate interior angles are equal. Therefore: **angle of depression (from the top) = angle of elevation (from the object).** ∎
   Why: From now on, every depression problem is solved by re-drawing the angle at the bottom and using tan as usual.

TOPPER TIP: *In the exam, write this reason in one line inside your solution: 'The angle of depression equals the angle of elevation (alternate interior angles, horizontals are parallel).' It is a scoring step in 5-mark questions.*

Derivation C — The Two-Observation-Point Method

Many problems watch the **same height** from **two points** on the same line — a walker approaching a building, a balloon drifting away, a car approaching a tower. Let us derive the machine once, completely.



Far point: angle α (smaller). Near point: angle β (bigger). The points are d apart; the near point is x from the foot.

1. Let the height be h, the near point be x from the foot, and the gap between the two points be d. Then the far point is (x + d) from the foot.

   Why: Naming the unknowns is Step 1 of every two-triangle problem.

2. From the NEAR point: tan β = h / x, so **h = x tan β** ... (i)

   Why: The bigger angle belongs to the nearer point — the closer you stand, the steeper you look.

3. From the FAR point: tan α = h / (x + d), so **h = (x + d) tan α** ... (ii)

4. Equate (i) and (ii): x tan β = (x + d) tan α. Expand: x tan β = x tan α + d tan α.

   Why: Both expressions equal the same height h — that is the whole trick.

5. Collect x: x (tan β − tan α) = d tan α, so **x = d tan α / (tan β − tan α)**, and then h = x tan β from (i).

   Why: Derived once — now it works for every two-point problem in the chapter.

6. **The favourite special case:** α = 30°, β = 60°. Then x = d × (1/√3) / (√3 − 1/√3) = d × (1/√3) / (2/√3) = **d/2**, and h = (d/2) × √3 = **d√3/2**.

   Why: When the angles are 30° and 60°, the near point is exactly HALFWAY — a gift in timed tests.

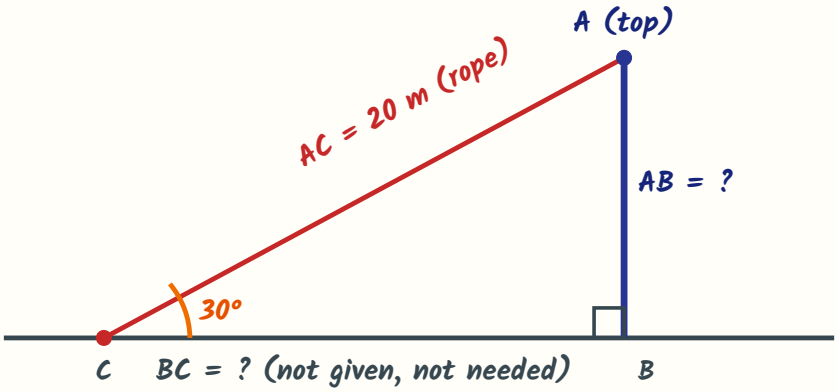
7. **Worked check:** points 20 m apart, angles 30° and 60°. Then x = 10 m and h = 10√3 ≈ 17.32 m. Verify from the far point: (10 + 20) × tan 30° = 30/√3 = 10√3. Same h — the machine is consistent.

   Why: Always run this far-point check; it catches algebra slips instantly.

PIN POINT: Two points, one height → write h in TWO ways and equate. That single move solves Q6, Q10, Q11, Q13, Q14 and Q15 of the NCERT exercise.

07 Detailed Solved Examples

NCERT EXERCISE 9.1 — EVERY QUESTION, EVERY STEP • STEP 1 IS ALWAYS: DRAW THE DIAGRAM



Ex 9.1 Q1 — the rope AC is the HYPOTENUSE of the triangle.

EXERCISE 9.1 — QUESTION 1

A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30°.

THINK FIRST: Height and a SLANTED length (the rope) are involved — so this is a sin problem, not a tan problem. The rope is the hypotenuse.

1. **Draw the diagram.** Let AB be the vertical pole, C the point where the rope is tied to the ground. AC is the rope (the line of sight direction), making 30° with the ground at C. Triangle ABC is right-angled at B.

   Why: Step 1 is always the diagram — and here it reveals that the 20 m is the hypotenuse, not the ground distance.

2. In triangle ABC: the side opposite the 30° angle is AB (the pole), and the hypotenuse is AC = 20 m.

   Why: Name the sides from the angle you are using — the angle sits at C.

3. Choose the ratio connecting opposite and hypotenuse: **sin 30° = AB / AC**.

   Why: Opposite + hypotenuse → sin. (tan would wrongly pull in the ground distance BC, which we neither know nor need.)

4. Substitute: 1/2 = AB / 20.

   Why: sin 30° = 1/2 — from the standard table.

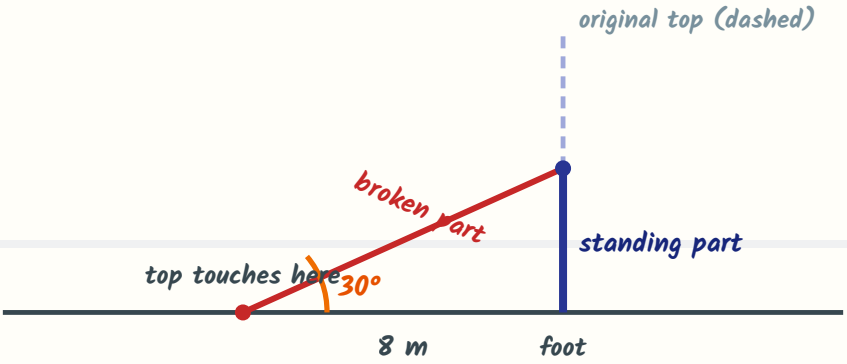
5. AB = 20 × 1/2 = **10 m**.

ANSWER: The height of the pole is 10 m.

CBSE-STYLE MARKING SCHEME: Diagram with the rope as hypotenuse [1] • sin 30° = AB/AC [1] • AB = 10 m [1]

TOPPER TIP: *Whenever a problem hands you a slanted object — rope, string, ladder, wire — that object is the hypotenuse. Your ratio will be sin or cos, decided by which leg you want.*

Exercise 9.1 — Question 2: The Broken Tree



Ex 9.1 Q2 — the broken part becomes the hypotenuse; the tree's height = standing part + broken part.

EXERCISE 9.1 — QUESTION 2

A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground, making an angle of 30° with it. The distance between the foot of the tree and the point where the top touches the ground is 8 m. Find the height of the tree.

THINK FIRST: The tree is in TWO pieces now: a standing stump (opposite side) and a bent-over piece (hypotenuse). We need both pieces, then add.

1. **Draw the diagram.** Let the foot be B, the break point be A on the trunk, and C the point where the top touches the ground. BC = 8 m, angle at C is 30°, and the triangle ABC is right-angled at B.

2. The required height of the original tree = AB (standing part) + AC (broken part).

   Why: The tree did not lose any length when it broke — the two pieces together are the whole tree.

3. Find the standing part AB: it is opposite the 30° angle, with adjacent BC = 8. Use tan: tan 30° = AB / 8.

   Why: Opposite + adjacent → tan.

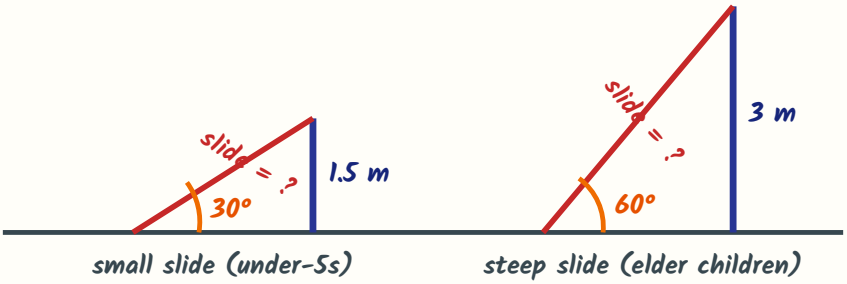
4. AB = 8 × tan 30° = 8/√3 m.

5. Find the broken part AC: it is the hypotenuse, with adjacent BC = 8. Use cos: cos 30° = BC / AC = 8 / AC.

   Why: Adjacent + hypotenuse → cos.

6. AC = 8 / cos 30° = 8 / (√3/2) = 16/√3 m.

Exercise 9.1 — Question 3: The Two Slides



Ex 9.1 Q3 — two independent triangles; in each, the slide itself is the hypotenuse.

EXERCISE 9.1 — QUESTION 3

A contractor plans to install two slides for the children to play in a park. For children below the age of 5 years, she prefers a slide whose top is at a height of 1.5 m and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

THINK FIRST: Two separate mini-problems. In each, height (opposite) and the slanted slide (hypotenuse) appear — sin both times.

1. **Draw the diagram.** Each slide forms a right triangle: the vertical drop is the height of the top, the slide surface is the hypotenuse, and the given angle is at the ground.

2. **Small slide:** let its length be l_1. sin 30° = opposite / hypotenuse = 1.5 / l_1.

   Why: Opposite + hypotenuse → sin.

3. 1/2 = 1.5 / l_1, so l_1 = 1.5 × 2 = **3 m**.

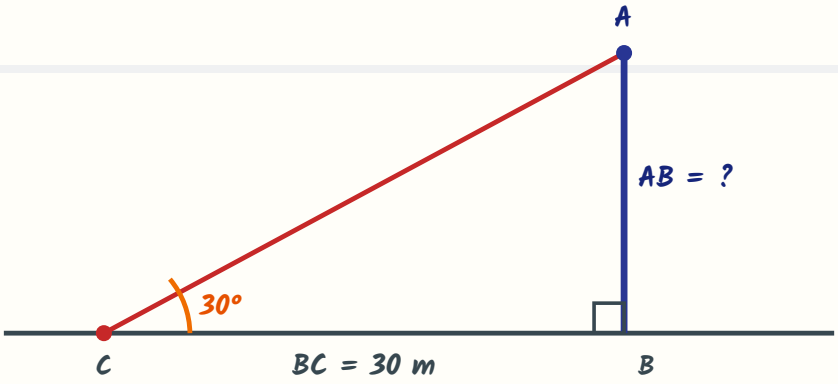
4. **Steep slide:** let its length be l_2. sin 60° = 3 / l_2.

5. √3/2 = 3 / l_2, so l_2 = 6/√3 = 2√3 ≈ **3.46 m**.

   Why: Rationalise 6/√3 = 6√3/3 = 2√3.

ANSWER: The small slide should be 3 m long and the steep slide 2√3 m (about 3.46 m) long.

CBSE-STYLE MARKING SCHEME: Two diagrams/triangles identified [1] • sin 30° giving 3 m [1] • sin 60° giving 2√3 m [1]



Ex 9.1 Q4 — one tower, one point, one angle: the workhorse triangle.

EXERCISE 9.1 — QUESTION 4

The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.

THINK FIRST: The purest problem of the chapter: height and distance, nothing slanted. This is Derivation A in action.

1. **Draw the diagram.** Let AB be the tower and C the point on the ground, with BC = 30 m and the angle of elevation at C equal to 30°.

2. In triangle ABC, tan 30° = AB / BC.

   Why: Height (opposite) over distance (adjacent) — the workhorse ratio.

3. $1/\sqrt{3} = AB / 30$.

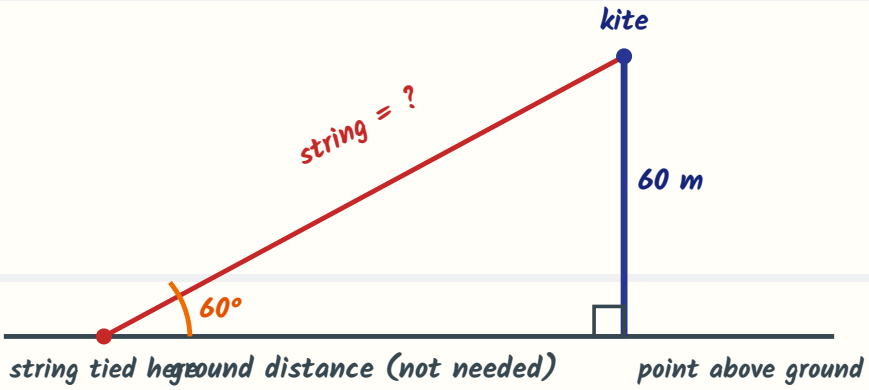
4. $AB = 30/\sqrt{3} = 30\sqrt{3}/3 = \mathbf{10\sqrt{3}\ m} \approx 17.32$ m.

   Why: Rationalise the denominator — board answers are expected in this form.

ANSWER: The height of the tower is $10\sqrt{3}$ m (about 17.32 m).

CBSE-STYLE MARKING SCHEME: Diagram [1] • tan 30° = AB/30 [1] • AB = $10\sqrt{3}$ m [1]

Exercise 9.1 — Question 5: The Kite



Ex 9.1 Q5 — the string is the hypotenuse; the height is the opposite side.

EXERCISE 9.1 — QUESTION 5

A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

THINK FIRST: Height (opposite) + string (hypotenuse) — sin again, exactly like the circus rope of Q1.

1. **Draw the diagram.** Let A be the kite, B the point on the ground directly below it, and C the point where the string is tied. AB = 60 m, angle at C is 60°, AC is the string.

2. In triangle ABC, sin 60° = AB / AC.

   Why: Opposite over hypotenuse — the two quantities this problem connects.

3. $\sqrt{3}/2 = 60 / AC$.

4. $AC = 120/\sqrt{3} = 120\sqrt{3}/3 = \mathbf{40\sqrt{3}\ m} \approx 69.28$ m.

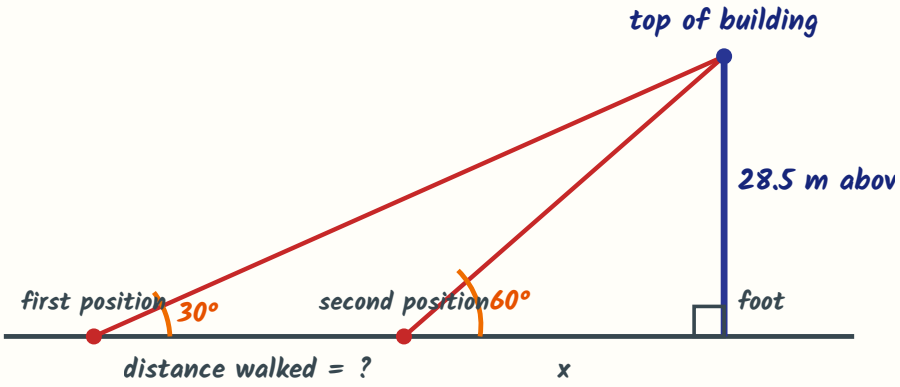
   Why: Multiply out and rationalise.

ANSWER: The length of the string is $40\sqrt{3}$ m (about 69.28 m).

CBSE-STYLE MARKING SCHEME: Diagram with string as hypotenuse [1] • sin 60° = 60/AC [1] • AC = $40\sqrt{3}$ m [1]

TOPPER TIP: *Q1, Q3 and Q5 are one family: a slanted object (rope, slide, string) is the hypotenuse. Spot the family and the ratio chooses itself.*

Exercise 9.1 — Question 6: Walking Towards the Building



Ex 9.1 Q6 — one building, two eye positions. The triangle's height is 30 − 1.5 = 28.5 m.

EXERCISE 9.1 — QUESTION 6

A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

THINK FIRST: Two positions, one building — Derivation C. But first, the eye-height trap: the triangle sits on the boy's eye, 1.5 m up.

1. **Draw the diagram.** The boy's eye is 1.5 m above the ground, so the part of the building inside the triangle is 30 − 1.5 = **28.5 m**. Let x be his final distance from the building and d the distance he walked.

   Why: The eye-height rule of Section 05 — forgetting this subtraction is the classic error here.

2. From the SECOND (near) position: tan 60° = 28.5 / x, so x = 28.5/√3.

   Why: Near point, bigger angle.

3. From the FIRST (far) position: tan 30° = 28.5 / (x + d), so x + d = 28.5 × √3 = 28.5√3.

   Why: Far point distance = final distance + distance walked.

4. Subtract: d = 28.5√3 − 28.5/√3 = 28.5 × (√3 − 1/√3) = 28.5 × (2/√3).

   Why: √3 − 1/√3 = (3 − 1)/√3 = 2/√3.

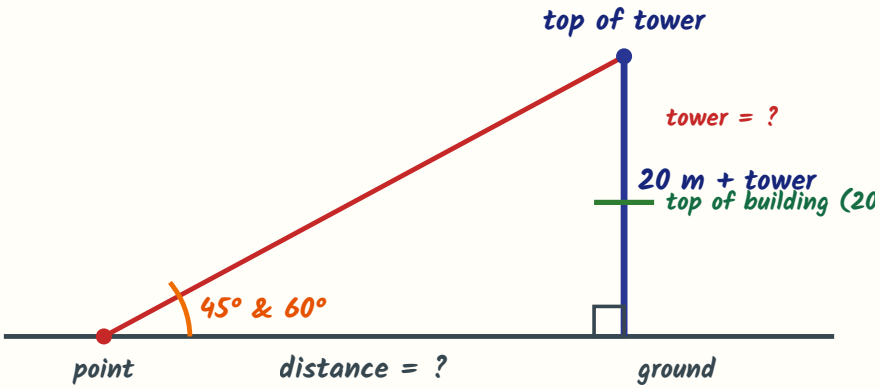
5. d = 57/√3 = 57√3/3 = **19√3 m** ≈ 32.91 m.

ANSWER: The boy walked 19√3 m (about 32.91 m) towards the building.

CBSE-STYLE MARKING SCHEME: Diagram with eye-level correction 28.5 m [1] • Two tan equations [1] • Subtraction giving 19√3 m [1]

COMMON MISTAKE: Wrong way: using 30 m as the triangle's height. **Right way:** the boy's eyes are 1.5 m up, so the triangle only contains 28.5 m of the building.

Exercise 9.1 — Question 7: Tower on a Building



Ex 9.1 Q7 — ONE point, TWO angles: 45° to the bottom of the tower, 60° to its top.

EXERCISE 9.1 — QUESTION 7

From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

THINK FIRST: Two angles from one point give two equations in the same triangle family. The 45° angle is the friendly one — it hands us the distance for free.

1. **Draw the diagram.** Let the building be BC (20 m), the tower CD on top of it, and A the point on the ground. Angle to C (bottom of tower) is 45°; angle to D (top of tower) is 60°. Let AB = x and CD = h.

2. Use the 45° angle first: in triangle ABC, tan 45° = BC / AB = 20 / x.

   Why: The bottom-of-tower angle looks at the known 20 m building.

3. 1 = 20 / x, so **x = 20 m**.

   Why: tan 45° = 1 — height equals distance. A gift.

4. Now the 60° angle: in triangle ABD, tan 60° = BD / AB = (20 + h) / 20.

   Why: The top-of-tower angle sees building + tower together.

5. $\sqrt{3} = (20 + h) / 20$, so $20 + h = 20\sqrt{3}$.

6. $h = 20\sqrt{3} - 20 =$ **$20(\sqrt{3} - 1)$ m** ≈ 14.64 m.

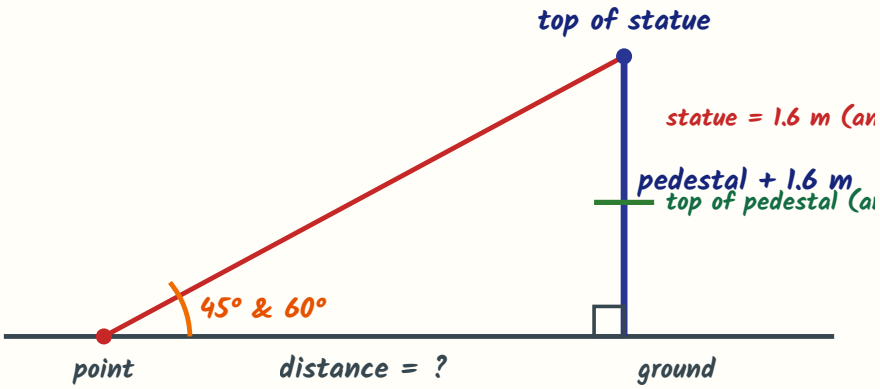
   Why: Subtract the building — the question asks only for the tower.

ANSWER: The height of the transmission tower is $20(\sqrt{3} - 1)$ m (about 14.64 m).

CBSE-STYLE MARKING SCHEME: Diagram with both angles [1] • Distance 20 m from tan 45° [1] • Total $20\sqrt{3}$ from tan 60° [1] • Tower = $20(\sqrt{3} - 1)$ m [1]

COMMON MISTAKE: Wrong way: answering $20\sqrt{3}$ m. **Right way:** $20\sqrt{3}$ m is the height of building + tower; the tower alone needs the building subtracted.

Exercise 9.1 — Question 8: The Statue and the Pedestal



Ex 9.1 Q8 — same skeleton as Q7, but now the SMALL piece (the statue) is known and the BIG piece is wanted.

EXERCISE 9.1 — QUESTION 8

A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

THINK FIRST: Let the pedestal be h and the distance be x. Two angles, two equations — and the 45° equation will say x = h, the great simplifier.

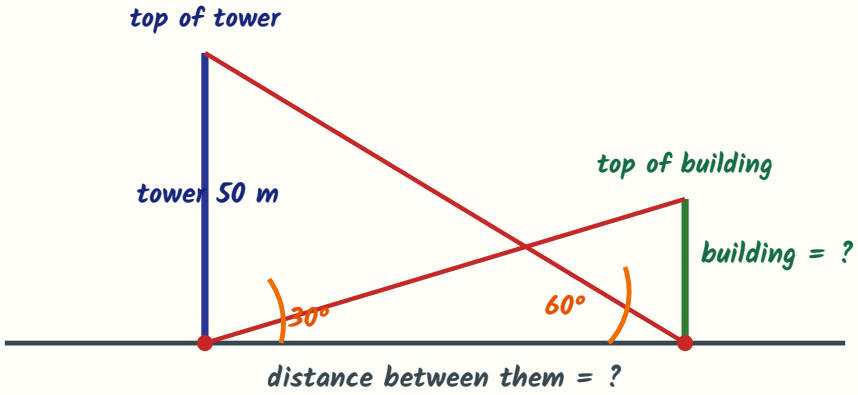
1. **Draw the diagram.** Let the pedestal be AB = h, the statue BC = 1.6 m on top of it, and D the point on the ground at distance x from A. Angle to B (top of pedestal) is 45°; angle to C (top of statue) is 60°.
2. From the 45° angle: tan 45° = AB / AD = h / x.
3. 1 = h / x, so **x = h**.
   Why: Distance and pedestal height are the same number — now there is only one unknown.
4. From the 60° angle: tan 60° = AC / AD = (h + 1.6) / x = (h + 1.6) / h.
   Why: The statue angle sees pedestal + statue.
5. √3 = (h + 1.6) / h, so √3 h = h + 1.6.
6. h(√3 − 1) = 1.6, so h = 1.6 / (√3 − 1).
   Why: Collect the h terms on one side.
7. Rationalise: h = 1.6(√3 + 1) / ((√3 − 1)(√3 + 1)) = 1.6(√3 + 1) / 2 = **0.8(√3 + 1) m** ≈ 2.19 m.
   Why: Multiply top and bottom by (√3 + 1); the denominator becomes 3 − 1 = 2.

ANSWER: The height of the pedestal is 0.8(√3 + 1) m (about 2.19 m).

CBSE-STYLE MARKING SCHEME: Diagram [1] • x = h from tan 45° [1] • Equation √3 h = h + 1.6 [1] • h = 0.8(√3 + 1) m after rationalising [1]

TOPPER TIP: *Q7 and Q8 are mirror twins: in Q7 the lower piece is known, in Q8 the upper piece is known. In both, fire the 45° angle first — it always converts one unknown into the other.*

Exercise 9.1 — Question 9: The Tower and the Building Face Each Other



Ex 9.1 Q9 — two verticals, two angles, ONE shared distance between them.

EXERCISE 9.1 — QUESTION 9

The angle of elevation of the top of a building from the foot of a tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

THINK FIRST: Each foot is the other's observation point. The 60° angle involves the known 50 m — use it first to capture the shared distance.

1. **Draw the diagram.** Let the tower be AB = 50 m and the building CD = h, standing d metres apart (BD = d). From D (foot of tower), the top of the building C is seen at 30°. From B (foot of building), the top of the tower A is seen at 60°.

   Why: Careful reading: each angle is measured at the foot of the OTHER building.

2. From the foot of the building: tan 60° = AB / BD = 50 / d.

   Why: This triangle contains the known height 50 m.

3. √3 = 50 / d, so d = 50/√3 m.

   Why: The shared distance is captured.

4. From the foot of the tower: tan 30° = CD / BD = h / d.

   Why: Now the same d serves the second triangle.

5. 1/√3 = h / (50/√3), so h = 50/3 m ≈ **16.67 m**.

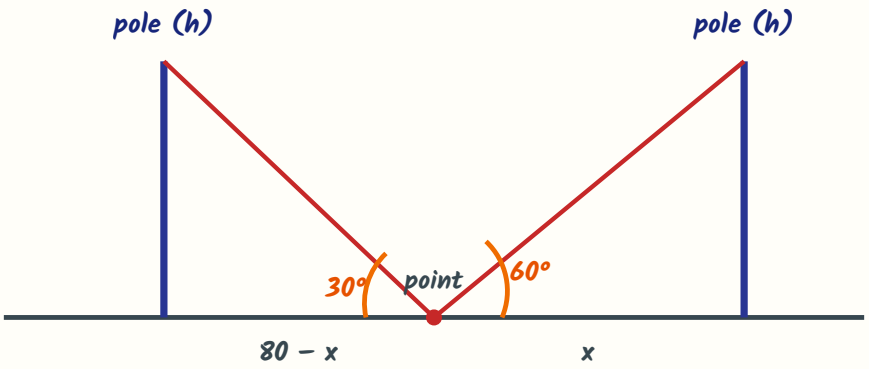
   Why: The √3 cancels beautifully — a sign the method is right.

ANSWER: The height of the building is 50/3 m (about 16.67 m).

CBSE-STYLE MARKING SCHEME: Diagram with both verticals [1] • d = 50/√3 from tan 60° [1] • h = 50/3 m from tan 30° [1]

TOPPER TIP: *When two objects face each other, solve the triangle that contains the KNOWN height first. The unknown distance it gives you is the key that opens the second triangle.*

Exercise 9.1 — Question 10: Two Poles Across a Road



Ex 9.1 Q10 — equal poles, one point between them; the point splits the 80 m road into x and 80 − x.

EXERCISE 9.1 — QUESTION 10

Two poles of equal heights are standing opposite each other on either side of a road, which is 80 m wide. From a point between them on the road, the angles of elevation of the tops of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.

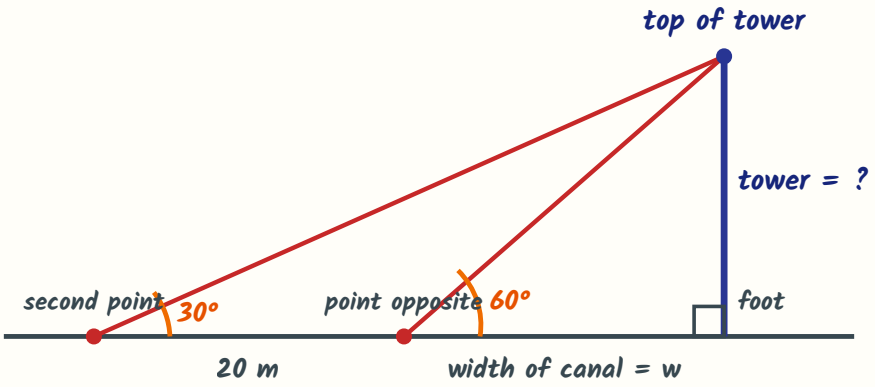
THINK FIRST: Equal heights is the gift: the same h is written in two ways — Derivation C again, with the two points being the two pole-feet.

1. **Draw the diagram.** Let each pole be h metres tall. Let the point be x metres from the pole whose top is seen at 60°; then it is (80 − x) metres from the other pole, seen at 30°.
   Why: The bigger angle (60°) must belong to the NEARER pole — so x is the smaller distance. This sanity check fixes the labelling.
2. From the 60° triangle: tan 60° = h / x, so **h = x√3** … (i)
3. From the 30° triangle: tan 30° = h / (80 − x), so **h = (80 − x)/√3** … (ii)
4. Equate (i) and (ii): x√3 = (80 − x)/√3. Multiply by √3: 3x = 80 − x.
5. 4x = 80, so **x = 20 m**. The other distance is 80 − 20 = **60 m**.
6. From (i): h = 20√3 ≈ **34.64 m**.
   Why: Check with (ii): (80 − 20)/√3 = 60/√3 = 20√3. Same — verified.

ANSWER: Each pole is 20√3 m (about 34.64 m) high; the point is 20 m from one pole and 60 m from the other.

CBSE-STYLE MARKING SCHEME: Diagram splitting 80 m into x and 80−x [1] • Two expressions for h [1] • x = 20 m [1] • h = 20√3 m with both distances stated [1]

Exercise 9.1 — Question 11: The TV Tower and the Canal



Ex 9.1 Q11 — both points are on the far bank, on the line through the tower's foot.

EXERCISE 9.1 — QUESTION 11

A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.

THINK FIRST: The canal's width IS the near distance — call it w. The far point is then (w + 20) from the foot. One height, two expressions: Derivation C.

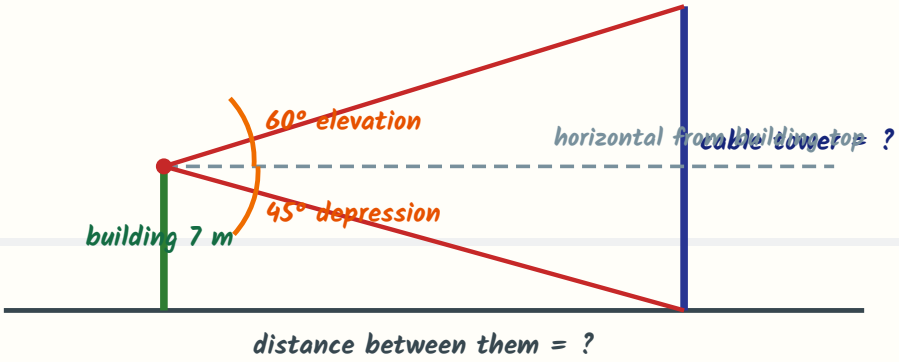
1. **Draw the diagram.** Let the tower be AB = h, its foot B on the near bank. Let C be the point on the other bank directly opposite, so BC = w = width of the canal. Let D be the second point, 20 m farther along the same line, so BD = w + 20.
2. From C (near, 60°): tan 60° = h / w, so **h = w√3** ... (i)
3. From D (far, 30°): tan 30° = h / (w + 20), so **h = (w + 20)/√3** ... (ii)
4. Equate: w√3 = (w + 20)/√3. Multiply by √3: 3w = w + 20.
5. 2w = 20, so **w = 10 m** — the canal is 10 m wide.
6. From (i): h = 10√3 ≈ **17.32 m**.
   Why: Check with (ii): (10 + 20)/√3 = 30/√3 = 10√3. Same — verified.

ANSWER: The tower is 10√3 m (about 17.32 m) high and the canal is 10 m wide.

CBSE-STYLE MARKING SCHEME: Diagram with width w and far point w + 20 [1] • Two expressions for h [1] • w = 10 m [1] • h = 10√3 m [1]

TOPPER TIP: Notice the pattern of Derivation C's special case hiding here: with angles 30° and 60°, the near distance came out exactly half the 20 m gap ($w = 10$). Recognising the pattern lets you sanity-check your algebra in seconds.

Exercise 9.1 — Question 12: The Cable Tower



Ex 9.1 Q12 — the observer stands ON the 7 m building: one angle looks UP (60°), one looks DOWN (45°).

EXERCISE 9.1 — QUESTION 12

From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

THINK FIRST: The depression angle is worked FIRST — converted to an elevation (Derivation B), it quietly hands us the distance between the buildings.

1. **Draw the diagram.** Let AB be the 7 m building (A at top), CE the cable tower, and E its foot. From A, draw the horizontal line meeting the tower at D. Then DE is the part of the tower above A's level, and CD = AB = 7 m.

   Why: Splitting the tower at the observer's eye level is the key drawing move — the tower becomes two stacked pieces.

2. Convert the depression: the angle of depression of E from A (45°) equals the angle of elevation of A from E (alternate interior angles). In triangle ABE: tan 45° = AB / BE = 7 / BE.

   Why: Derivation B at work — the distance BE is the same as AD, the horizontal gap.

3. 1 = 7 / BE, so **BE = 7 m**, and hence AD = 7 m.

4. Now look UP from A: in triangle ADE, tan 60° = DE / AD = DE / 7.

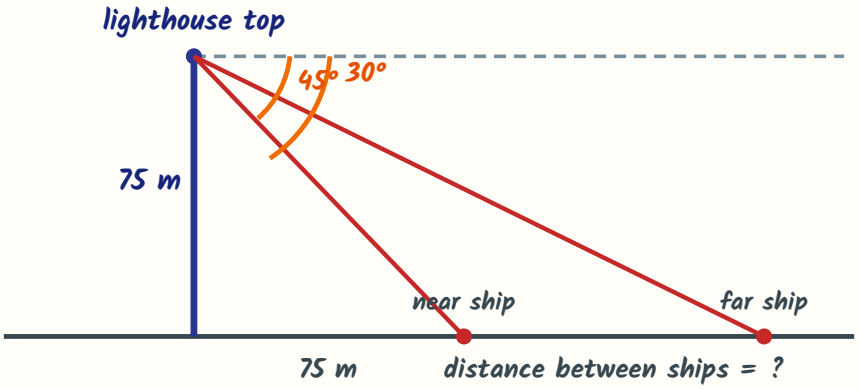
   Why: The elevation triangle sits on the horizontal through A.

5. DE = 7 × √3 = 7√3 m.

6. Total height of the tower = CD + DE = 7 + 7√3 = **7(√3 + 1) m** ≈ 19.12 m.

   Why: Add back the piece below eye level — exactly like adding the boy's eye height in Q6, in reverse.

Exercise 9.1 — Question 13: Two Ships from a Lighthouse



Ex 9.1 Q13 — both ships on the SAME side, one exactly behind the other: the answer is a DIFFERENCE of distances.

EXERCISE 9.1 — QUESTION 13

As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

THINK FIRST: First convert both depressions into elevations at the ships (Derivation B). The nearer ship has the BIGGER angle — check your diagram against that fact.

1. **Draw the diagram.** Let AB be the lighthouse (75 m), C the nearer ship and D the farther ship, on the same side. The angles of depression from A are 45° (to C) and 30° (to D).

2. Convert: angle of elevation of A from C is 45°, and from D is 30° (alternate interior angles).
   Why: Now both angles sit at the bottom, where tan lives.

3. Near ship: tan 45° = AB / BC = 75 / BC, so **BC = 75 m**.

4. Far ship: tan 30° = AB / BD = 75 / BD, so BD = 75√3 m.

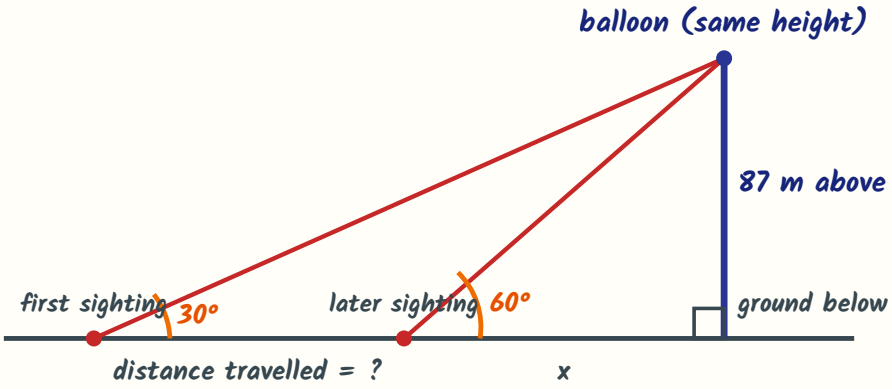
5. Distance between the ships = BD − BC = 75√3 − 75 = **75(√3 − 1) m** ≈ 54.90 m.
   Why: 'One behind the other on the same side' means SUBTRACT. (Opposite sides would mean add — read the words every time.)

ANSWER: The distance between the two ships is 75(√3 − 1) m (about 54.90 m).

CBSE-STYLE MARKING SCHEME: Diagram with both ships on one side [1] • Depressions converted to elevations [1] • BC = 75 m and BD = 75√3 m [1] • Difference 75(√3 − 1) m [1]

COMMON MISTAKE: Wrong way: adding the two distances ($75\sqrt{3} + 75$). **Right way:** same side → subtract; opposite sides of the lighthouse → add. The story's geometry decides, not habit.

Exercise 9.1 — Question 14: The Drifting Balloon



Ex 9.1 Q14 — the balloon flies LEVEL: only its distance changes. Height above the girl's eyes = 88.2 − 1.2 = 87 m.

EXERCISE 9.1 — QUESTION 14

A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.

THINK FIRST: A falling angle means a departing object. Eye-height rule first: the triangle's height is 88.2 − 1.2 = 87 m, and it never changes because the balloon flies in a horizontal line.

1. **Draw the diagram.** Let the girl's eye be E. The balloon is first at P and later at Q, both 87 m above eye level. Let x be the horizontal distance of P from E's vertical, and d the distance travelled PQ.

2. First position (60°): tan 60° = 87 / x, so x = 87/√3 = 29√3 m.
   Why: Near position, bigger angle.

3. Later position (30°): tan 30° = 87 / (x + d), so x + d = 87√3 m.
   Why: The balloon's height is unchanged — only the distance grew.

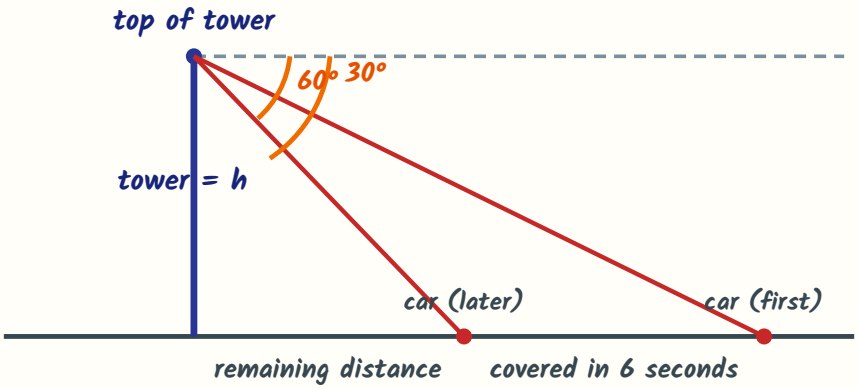
4. Subtract: d = 87√3 − 29√3 = **58√3 m** ≈ 100.46 m.
   Why: 87√3 = 3 × 29√3, so the subtraction is clean: (3 − 1) × 29√3.

ANSWER: The balloon travelled 58√3 m (about 100.46 m) during the interval.

CBSE-STYLE MARKING SCHEME: Diagram with constant height 87 m [1] • Eye-height correction 88.2 − 1.2 [1] • Two distances 29√3 and 87√3 [1] • Travelled 58√3 m [1]

TOPPER TIP: The angle FALLING from 60° to 30° is the story's way of saying 'moving away'. If an angle grows, the object approaches; if it shrinks, it departs. Say this sentence in every moving-object problem before you compute.

Exercise 9.1 — Question 15: The Approaching Car



Ex 9.1 Q15 — notice: the tower's height is never given. Watch it cancel out.

EXERCISE 9.1 — QUESTION 15

A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

THINK FIRST: No height is given — so the answer must not depend on it. Call the height h, work symbolically, and watch h disappear.

1. **Draw the diagram.** Let the tower be AB = h. The car is first at D (depression 30°) and 6 seconds later at C (depression 60°), moving towards B.

2. Convert the depressions to elevations at the car (alternate interior angles): at D the elevation of A is 30°; at C it is 60°.

3. Distance of the first position: tan 30° = h / BD, so **BD = h√3**.

4. Distance of the second position: tan 60° = h / BC, so **BC = h/√3**.

5. Distance covered in 6 s: CD = BD − BC = h√3 − h/√3 = h(3 − 1)/√3 = **2h/√3**.
   Why: √3 = 3/√3 — put both terms over √3.

6. Speed of the car = distance / time = (2h/√3) / 6 = **h/(3√3)** per second.
   Why: Uniform speed — the same speed covers the remaining distance.

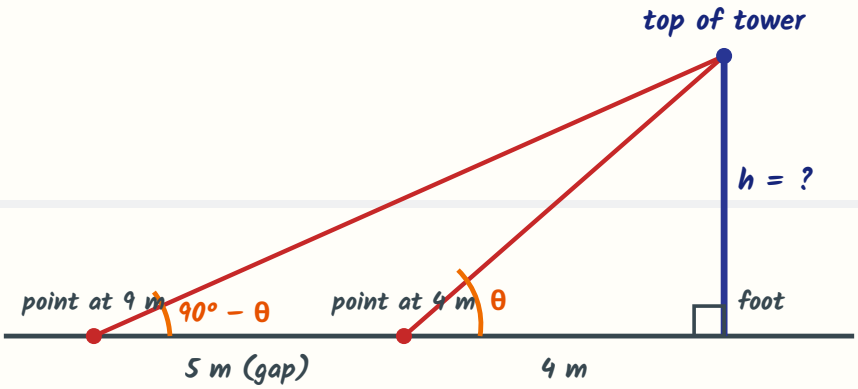
7. Remaining distance is BC = h/√3. Time to cover it = (h/√3) ÷ (h/(3√3)) = (h/√3) × (3√3/h) = **3 seconds**.
   Why: h cancels completely — that is why the problem never gave it. The answer is pure ratio.

ANSWER: The car will take 3 more seconds to reach the foot of the tower.

CBSE-STYLE MARKING SCHEME: Diagram with both car positions [1] • BD = h√3 and BC = h/√3 [1] • Distance covered 2h/√3 and speed [1] • Remaining time 3 s with h cancelling [1]

PIN POINT: If a problem gives NO height, the height must cancel. Keep it as h, never invent a number — and expect a clean answer like 3 seconds.

Exercise 9.1 — Question 16: The Complementary-Angles Theorem



Ex 9.1 Q16 — the two angles are complementary: they add to 90°. The nearer point (4 m) takes the bigger angle θ.

EXERCISE 9.1 — QUESTION 16

The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

THINK FIRST: 'Complementary' is the trigger word: if one angle is θ, the other is (90° − θ), and tan(90° − θ) = cot θ = 1/tan θ. The two tan equations will multiply into h^2.

1. **Draw the diagram.** Let AB be the tower of height h. Let C be the point 4 m from B and D the point 9 m from B, on the same line. Let the angle at C be θ; then the angle at D is (90° − θ).

   Why: The nearer point (4 m) must have the BIGGER angle — call that one θ.

2. From C: tan θ = AB / BC = **h / 4** ... (i)

3. From D: tan(90° − θ) = AB / BD = **h / 9** ... (ii)

4. But tan(90° − θ) = cot θ = 1 / tan θ. So (ii) says 1/tan θ = h/9, i.e. tan θ = **9/h** ... (iii)

   Why: The complementary identity from Prerequisite 3 — this is the step the whole proof stands on.

5. Equate (i) and (iii): h/4 = 9/h.

   Why: Two expressions for the same tan θ.

6. Cross-multiply: $h^2 = 36$.

7. h = 6 (taking the positive value, since height is positive).

   Why: h = ±6 algebraically; a tower's height is the positive root.

08 Multiple Solution Methods

8D • CHALLENGE (PART 1) — ONE PROBLEM, MANY ROADS

A topper owns more than one road to every answer. Here is Exercise 9.1 Q4 — tower, 30 m away, angle 30° — solved **three** ways. All three must agree; when they do, you know you are right.

Method 1 — The tan formula (the standard road)

tan 30° = h/30 → 1/√3 = h/30 → h = 30/√3 = **10√3 m**. Two lines. This is the road to write in the exam.

Method 2 — The special-triangle ratio (no formula at all)

In a 30°–60°–90° triangle the sides stand in the ratio 1 : √3 : 2 (shortest : middle : hypotenuse). Here the angles are 30° at the ground and 60° at the top, so the side opposite 60° is the ground distance 30 m (the 'middle' side, √3 parts), and the height is the 'shortest' side (1 part). One part = 30/√3, so height = **10√3 m**. Same answer — reached by pure ratio.

Method 3 — The cot road (stand the triangle on its head)

cot θ = adjacent / opposite = distance / height. cot 30° = 30/h → √3 = 30/h → h = 30/√3 = **10√3 m**. Cot is rarely needed, but it is the natural language of DEPRESSION problems worked from the top (see below).

Q13 (the lighthouse) by a second road

Road used in Section 07: convert each depression to an elevation at the ship, then tan: distances 75 m and 75√3 m; difference 75(√3 − 1) m.

Second road — stay at the top: from the top, distance of a ship = h × cot(angle of depression). Near ship: 75 × cot 45° = 75 × 1 = 75 m. Far ship: 75 × cot 30° = 75√3 m. Difference = 75(√3 − 1) m. Identical — because cot(depression) is just tan(elevation) wearing a coat.

Situation	Fastest road
Height & distance only	tan (Method 1) — always the exam road
Angles 30/60 and a quick mental check	Special-triangle ratio 1 : √3 : 2
Working from the top (depression given, distance wanted)	distance = h × cot(depression)

09 Interactive Simulations & Hands-on Activities

8D • VISUALISE IN REAL LIFE — DO THESE, DO NOT JUST READ THEM

Activity 1 — Build a clinometer and measure your school

You need: a protractor, a drinking straw, thread, a small weight (a key works), tape, and a 30 m measuring tape.
Build: tape the straw along the straight edge of the protractor; tie the thread at the protractor's centre so the weight hangs freely.
Measure: stand at a measured distance d from your school building. Sight the roof through the straw. Your partner reads the angle between the thread and the 90° mark — that is your angle of elevation θ. Height of building = d × tan θ + (your eye height).
Record: distance d = _____ m; angle θ = _____°; tan θ = _____; height above eye = _____ m; eye height = _____ m; **building height = _____ m**. Compare with a friend who stood at a different distance — your answers should agree within half a metre.

Activity 2 — The shadow method (no angle instrument at all)

At the same moment, the sun's rays make the SAME angle with every object. Plant a 1 m stick vertically and measure its shadow. Then measure the shadow of a tree (or pole, or building). Because the sun's angle is shared, height / shadow is the same ratio for both: **object height = its shadow × (1 / stick's shadow)**. Try it: stick shadow = _____ m; tree shadow = _____ m; tree height = _____ m. Then check your answer with the clinometer from Activity 1.

Activity 3 — Table-top simulation: the moving observer

Draw a ‘tower’ 20 cm tall on chart paper. Tie a string at its top. Hold the string's free end at different distances along the desk and measure the angle with a protractor. Fill the table — two rows are done for you. In every row, check that distance × tan(angle) ≈ 20 cm.

Distance (cm)	Angle at the desk	tan(angle)	distance × tan(angle)
20	45°	1	20 cm — correct
34.6	30°	0.577	≈ 20 cm — correct

You have just simulated, on a desk, exactly what the KalviYogi digital simulation does on screen: move the observer, and the height stays recoverable from any distance.

KALVIYOGI DIGITAL COMPANION: Chapter 9 simulations — ‘Drag the Observer’ (heights and distances), ‘Elevation or Depression?’ angle trainer, video lessons and online tests. **QR code and link activate with the KalviYogi app/ website at print.**

10 Common Mistakes

THE SEVEN TRAPS OF THIS CHAPTER — AND HOW TOPPERS DODGE THEM

COMMON MISTAKE: Trap 1 — Measuring the angle from the vertical. Wrong way: drawing the angle between the line of sight and the tower itself. **Right way:** elevation and depression are always measured from the **horizontal** line through the eye. If your angle touches the vertical, every ratio that follows is the complementary one — and wrong.

COMMON MISTAKE: Trap 2 — Forgetting the observer's eye height. Wrong way: in Q6, using the full 30 m of the building inside the triangle. **Right way:** the triangle starts at the eye: 30 − 1.5 = 28.5 m. Same in Q14: 88.2 − 1.2 = 87 m. Whenever a PERSON observes, subtract first.

COMMON MISTAKE: Trap 3 — Using tan when a slanted length is involved. Wrong way: in Q1 (the rope) or Q5 (the kite string), writing tan and hunting for a ground distance nobody gave. **Right way:** a rope/string/ladder is the hypotenuse — sin or cos. tan connects only the two legs.

COMMON MISTAKE: Trap 4 — Answering the wrong height. Wrong way: in Q7, answering $20\sqrt{3}$ m (building + tower) when only the tower was asked; in Q12, forgetting to add the building's 7 m back. **Right way:** after solving, re-read the question's last line and check: is my answer the piece, or the whole? Add or subtract the eye-level piece as needed.

COMMON MISTAKE: Trap 5 — Adding when the story says subtract. Wrong way: in Q13, adding the ships' distances although both ships are on the SAME side. **Right way:** same side → the wanted distance is the DIFFERENCE of the two distances; opposite sides → the SUM. Let the diagram, not habit, decide.

COMMON MISTAKE: Trap 6 — The complementary slip. Wrong way: in Q16, writing $\tan(90° - \theta) = \tan\theta$, or putting the bigger angle at the farther point. **Right way:** $\tan(90° - \theta) = \cot\theta = 1/\tan\theta$, and the nearer point always owns the bigger angle.

COMMON MISTAKE: Trap 7 — Rounding √3 too early. Wrong way: replacing √3 by 1.73 in the first line and dragging rounding errors through five steps. **Right way:** keep √3 symbolic until the final line; only then, if a decimal is wanted, use $\sqrt{3} \approx 1.732$. Board answers are happiest in exact form: $10\sqrt{3}$ m beats 17.32 m.

TOPPER TIP: *Before submitting any answer in this chapter, run the four-point check: (1) Is my angle measured from the horizontal? (2) Did I subtract the eye height? (3) Does my ratio match the sides I actually know? (4) Is my answer the piece or the whole?*

PIN POINT: Six of these seven traps are DIAGRAM traps, not algebra traps. Ninety seconds spent drawing carefully is the highest-value time in this chapter.

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11 Graded Practice Problems

FIVE LEVELS — CLIMB ONE LEVEL AT A TIME • ANSWERS IN THE ANSWER KEY

Rules of the climb: finish a level with at most one error before moving up. **Draw the diagram first in every question** — answers without diagrams earn nothing here.

LEVEL 1 — FOUNDATION (one triangle, direct)

L1.1 The angle of elevation of the top of a tower from a point 25 m from its foot is 45°. Find the height of the tower.

L1.2 The shadow of a vertical pole is 18 m long when the angle of elevation of the sun is 30°. Find the height of the pole.

L1.3 A kite's string is 40 m long and makes an angle of 30° with the ground. At what height is the kite flying? (Assume no slack.)

L1.4 A ladder leans against a wall. Its foot is 4 m from the wall and it makes an angle of 60° with the ground. Find the length of the ladder.

L1.5 From the top of a 60 m high cliff, the angle of depression of a boat is 30°. How far is the boat from the foot of the cliff?

L1.6 A tower is 45 m high. From a point on the ground, the angle of elevation of its top is 60°. How far is the point from the foot of the tower?

LEVEL 2 — SKILL BUILDER (textbook standard)

L2.1 From a point 15 m from the foot of a tower, the angle of elevation of the top is 60°. Find the height of the tower.

L2.2 A tree breaks in a storm and its top touches the ground 12 m from the foot, making an angle of 30° with the ground. Find the original height of the tree.

L2.3 A 1.5 m tall boy stands at some distance from a 21.5 m tall building. The angle of elevation from his eyes to the top of the building is 45°. How far is he from the building?

L2.4 A pole $6\sqrt{3}$ m high casts a shadow 6 m long. Find the angle of elevation of the sun.

L2.5 The angles of elevation of the top of a tower from two points on the same side, 10 m apart, are 60° (nearer point) and 30° (farther point). Find the height of the tower.

L2.6 A statue 1 m tall stands on a pedestal. From a point on the ground, the angles of elevation of the top of the pedestal and the top of the statue are 45° and 60° respectively. Find the height of the pedestal.

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Graded Practice — Levels 3, 4 and 5

LEVEL 3 — APPLICATION (real-life problems)

L3.1 A surveyor wants the width of a river. She stands at a point A directly opposite a tree on the far bank, walks 30 m along her bank to a point B, and finds that the line from B to the tree makes an angle of 60° with the bank. Find the width of the river.

L3.2 An aeroplane flies in a straight horizontal line. Two observers on the ground, 1000 m apart on the same side, see it at angles of elevation 60° and 30° at the same instant. Find the height at which the aeroplane is flying.

L3.3 From the top of a bridge 30 m above a river, the angles of depression of two boats on the same side are 60° and 30°. Find the distance between the boats.

L3.4 From a point 200 m above a lake, the angle of elevation of a cloud is 30° and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud above the lake.

LEVEL 4 — ADVANCED REASONING (multi-concept)

L4.1 From a point on the ground, the angle of elevation of the top of a tower is 30°. After walking 30 m towards the tower, the angle becomes 60°. Find the height of the tower.

L4.2 Two ships are on opposite sides of a 100 m high lighthouse. Their angles of depression from the top of the lighthouse are 45° and 30°. Find the distance between the ships.

L4.3 A building is 20 m high. From the top of the building, the angle of elevation of the top of a tower is 45°; from the foot of the building, the angle of elevation of the top of the tower is 60°. Find the height of the tower and the distance between the building and the tower.

LEVEL 5 — MATHEMATICAL GENIUS (olympiad flavour)

L5.1 A man in a boat rows away from the foot of a 60 m high lighthouse. The angle of depression of the boat from the top changes from 60° to 30° in 2 minutes. Find the distance the boat travelled and its speed (in m/s and in km/h).

L5.2 The shadow of a tower standing on level ground is found to be 40 m longer when the sun's altitude is 30° than when it is 60°. Find the height of the tower.

> **TOPPER TIP:** *Stuck at a level? Section 07 contains a twin of almost every question here. Find the twin, study its steps, close the book, and try again.*

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12 Mathematical Reasoning

JUSTIFY AND PROVE — FULL SOLUTIONS

REASONING 1 — THE COMPLEMENTARY-ANGLES THEOREM

The angles of elevation of the top of a tower from two points at distances a and b from its base, in the same straight line with it, are complementary. Prove that the height of the tower is √(ab).

THINK FIRST: Whenever a board question says 'complementary', your hand should write tan(90° − θ) = cot θ before your brain even finishes reading.

1. Let the tower be AB = h, and the two points be C (at distance a) and D (at distance b) from B, on the same line.
   Why: This is NCERT Q16 with the numbers replaced by letters — a particular proof becomes a theorem.
2. Let the angle of elevation at C be θ. Since the angles are complementary, the angle at D is (90° − θ).
3. From C: tan θ = h / a ... (i)
4. From D: tan(90° − θ) = h / b ... (ii)
5. Use the complementary identity tan(90° − θ) = cot θ = 1 / tan θ. Equation (ii) becomes 1/ tan θ = h/b, so tan θ = b/h ... (iii)
   Why: The identity is the engine — exactly as in Q16.
6. Equate (i) and (iii): h/a = b/h.
7. Cross-multiply: h^2 = ab, so **h = √(ab)** (positive root, as height is positive). ∎
   Why: Check with Q16: a = 4, b = 9 gives h = √ 36 = 6 m. The theorem and the exercise agree.

ANSWER: For complementary angles of elevation, the height is the geometric mean of the two distances: h = √(ab).

Reasoning 2 and 3 — Two More Proofs You Own

REASONING 2 — WHY DEPRESSION CAN ALWAYS BE RE-DRAWN AS ELEVATION

Prove that the angle of depression of a point from an observer equals the angle of elevation of the observer from that point. Hence find the distance of a car from the foot of a 25 m tower, if its angle of depression from the top of the tower is 45°.

THINK FIRST: Examiners award a reasoning mark for the words 'alternate interior angles, horizontals being parallel'. Never use a depression angle in a bottom triangle without writing that licence line.

1. Draw the horizontal line through the observer O (top of the tower) and the horizontal line through the point P (the car). Both are horizontal, so the two lines are parallel.
   Why: This is Derivation B, now written as a formal proof.
2. The line of sight OP is a transversal cutting these two parallel lines.
3. The angle of depression at O and the angle of elevation at P are alternate interior angles formed by this transversal.
   Why: One sits above-left of the transversal, the other below-right — the classic alternate pair.
4. Alternate interior angles between parallel lines are equal. Hence angle of depression = angle of elevation. ∎
5. Apply it: the car's angle of elevation of the tower top is 45°. So tan 45° = 25 / distance, giving distance = 25 / 1 = **25 m**.
   Why: The proof is not decoration — it is the licence that lets you solve every depression problem with tan.

ANSWER: Angle of depression = angle of elevation (proved); the car is 25 m from the foot of the tower.

REASONING 3 — THE LENGTHENING SHADOW

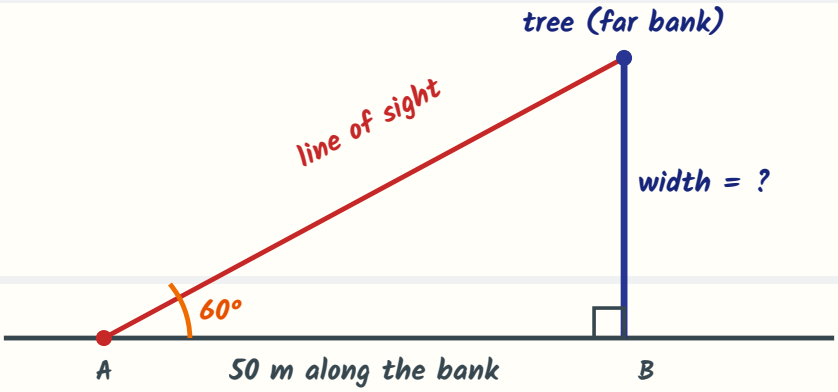
A vertical pole has height h. Prove that when the sun's angle of elevation falls from 45° to 30°, the pole's shadow lengthens by exactly h(√3 − 1).

THINK FIRST: Proofs with a letter (h) instead of a number are stronger: one proof settles every pole, tree and tower at once.

1. Let the shadow lengths at 45° and 30° be s_1 and s_2. The pole, its shadow and the sun's ray form a right triangle with the pole as the opposite side.
2. At 45°: tan 45° = h / s_1, so s_1 = h / 1 = h.

13 Real-Life Applications

8D • APPLY — THE CHAPTER AT WORK IN THE WORLD



Application 1 — the surveyor never gets her feet wet: the river's width is the opposite side.

APPLICATION 1 — MEASURING A RIVER WITHOUT CROSSING IT

A surveyor stands at a point A on one bank of a river, directly opposite a tree T on the other bank. She walks 50 m along her bank to a point B, from where the line to the tree makes an angle of 60° with the bank. Find the width of the river.

THINK FIRST: Any 'cannot reach it' distance — a river, a gorge, a compound wall — is an adjacent side waiting for an angle.

1. **Draw the diagram.** TA is the river's width w (perpendicular to the banks). AB = 50 m along the bank. The angle at B, between the bank line BA and the sight line BT, is 60°. Triangle TAB is right-angled at A.

   Why: Rivers are measured exactly like this in real surveys — the perpendicular crossing is replaced by an angled sighting.

2. In triangle TAB: tan 60° = TA / AB = w / 50.

   Why: Width is opposite the 60° angle; the walked distance is adjacent.

3. $\sqrt{3} = w / 50$, so $w = 50\sqrt{3} \approx$ **86.60 m**.

ANSWER: The river is $50\sqrt{3}$ m (about 86.60 m) wide.

APPLICATION 2 — THE HEIGHT OF A HILL

Two observation posts stand on a straight road leading to a hill, 200 m apart. From the nearer post, the angle of elevation of the top of the hill is 45°; from the farther post, it is 30°. Find the height of the hill.

THINK FIRST: Two posts + two angles is Derivation C wearing a uniform. The 45° angle makes it even shorter: near distance = height.

Real-Life Applications 3 and 4

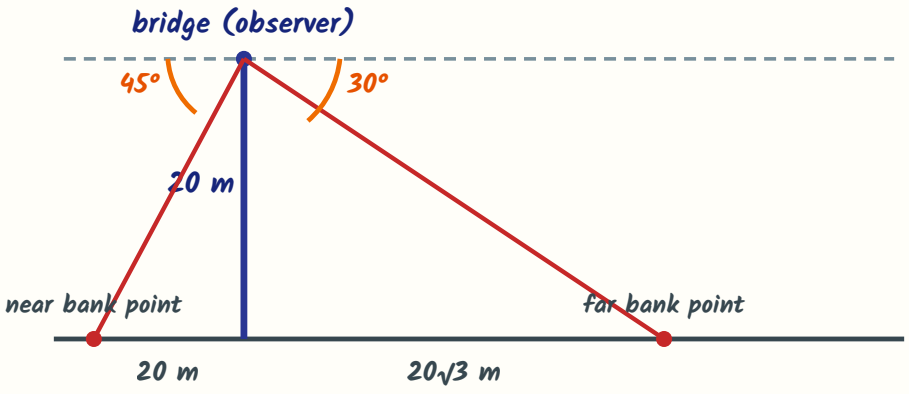
APPLICATION 3 — HOW HIGH IS THAT AEROPLANE?

Two children standing on the ground 3 km apart, on the same side of an aeroplane's path, observe it at the same instant at angles of elevation 60° and 30°. Find the height at which the aeroplane is flying.

THINK FIRST: Pilots really are tracked this way — two ground stations, two angles, one height. It is called triangulation.

1. **Draw the diagram.** Let the height be h and the nearer child be x from the point directly below the plane; the farther child is (x + 3000) from it (working in metres).
   Why: Same-side observers, one instant — the plane is a single fixed point for both triangles.
2. From the nearer child (60°): tan 60° = h / x, so h = x√3 … (i)
3. From the farther child (30°): tan 30° = h / (x + 3000), so h = (x + 3000)/√3 … (ii)
4. Equate: x√3 = (x + 3000)/√3 → 3x = x + 3000 → x = 1500 m.
5. From (i): h = 1500√3 ≈ **2598 m**, about 2.6 km.
   Why: This is Derivation C's special case again: with 30°/60°, the near distance (1500) is half the gap (3000).

ANSWER: The aeroplane is flying at 1500√3 m (about 2.6 km) high.



Application 4 — from the bridge, the two banks lie on OPPOSITE sides: this time the distances ADD.

APPLICATION 4 — THE WIDTH OF A RIVER FROM A BRIDGE

A bridge stands 20 m above a river. From a point on the bridge, the angles of depression of the two banks are 45° and 30°, on opposite sides of the point's vertical. Find the width of the river.

THINK FIRST: Q13 and Application 4 are the same picture with one word changed — 'same side' versus 'opposite sides'. That one word flips subtraction into addition. Examiners love testing exactly this.

1. **Draw the diagram.** Let P be the point on the bridge, 20 m above the water. The banks are at A (depression 45°) and B (depression 30°), on opposite sides of the point directly below P.

   Why: Opposite sides — so unlike Q13, the final step will be an ADDITION.

2. Convert both depressions to elevations at the banks (alternate interior angles).

3. Bank A: $\tan 45° = 20 / (\text{distance of A}) \rightarrow$ distance of A = 20 m.

4. Bank B: $\tan 30° = 20 / (\text{distance of B}) \rightarrow$ distance of B = $20\sqrt{3}$ m.

5. Width of the river = $20 + 20\sqrt{3} =$ **$20(1 + \sqrt{3})$ m** ≈ 54.64 m.

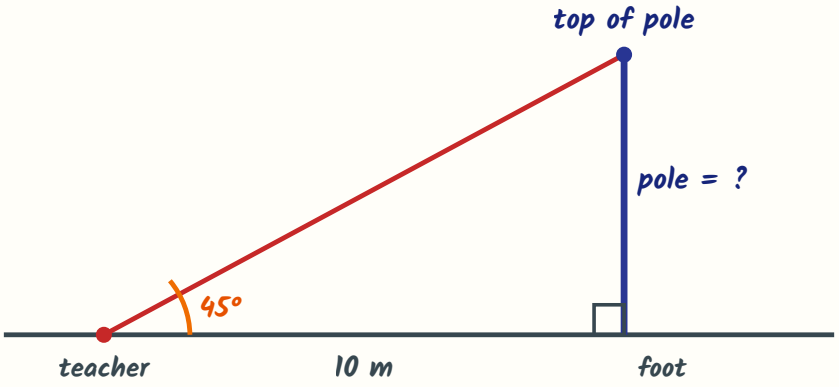
   Why: The banks lie on opposite sides of the bridge point, so the two distances add to give the full width.

ANSWER: The river is $20(1 + \sqrt{3})$ m (about 54.64 m) wide.

14 Competency-Based Questions

CASE STUDIES • ASSERTION-REASON — THE CBSE WAY: READ, MODEL, SOLVE

Case Study 1 — The School Flagpole. *For the Republic Day parade, KalviYogi School is preparing its flagpole. The parade teacher stands* ***10 m*** *from the foot of the pole and finds the angle of elevation of the top of the pole to be* ***45°****. A rope will be tied from the top of the pole to a peg in the ground for extra support. Later in the day, the pole's shadow is measured for a science activity.*



Case 1 — one pole, several questions. The diagram is drawn once and reused.

CASE 1 (i)

Find the height of the flagpole.

1. tan 45° = height / 10.
   Why: Height over distance, angle at the teacher.
2. 1 = height / 10, so the height is **10 m**.
   Why: At 45°, height and distance are always equal.

ANSWER: The flagpole is 10 m high.

CASE 1 (ii)

The support rope is tied from the top of the pole to a peg in the ground, making an angle of 30° with the ground. Find the length of the rope.

1. The rope is the hypotenuse; the pole (10 m) is the side opposite the 30° angle.
   Why: Slanted object → sin.
2. sin 30° = 10 / rope, so 1/2 = 10 / rope.
3. Rope = **20 m**.

ANSWER: The support rope is 20 m long.

Case Study 1 (continued)

CASE 1 (iii)

At a certain time, the angle of elevation of the sun is 60°. Find the length of the pole's shadow.

1. Now the pole is the opposite side and the shadow is the adjacent side, with the sun's angle at the tip of the shadow.
   Why: Same triangle, viewed from the sun's side.
2. tan 60° = 10 / shadow, so shadow = 10/√3 = 10√3/3 ≈ **5.77 m**.

ANSWER: The shadow is 10/√3 m (about 5.77 m) long.

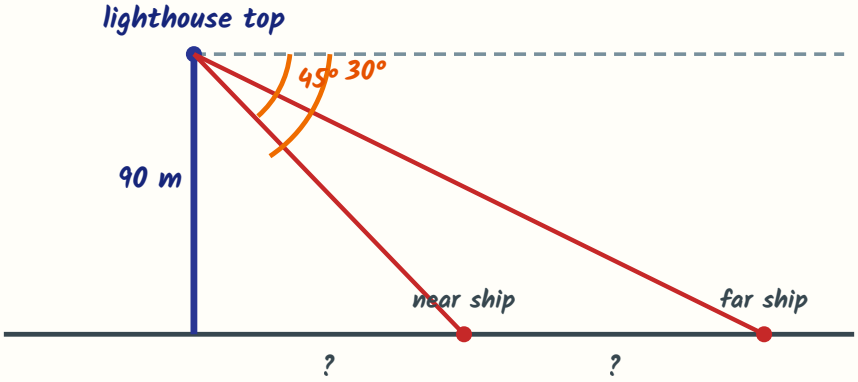
CASE 1 (iv)

Later, the shadow of the pole is found to be 10√3 m long. Find the angle of elevation of the sun at that time.

1. tan(angle) = height / shadow = 10 / (10√3) = 1/√3.
   Why: This time the RATIO is known and the ANGLE is the unknown – read the tan table backwards.
2. 1/√3 is tan 30°, so the sun's angle of elevation is **30°**.
   Why: A longer shadow means a lower sun – 30° < 60°, consistent with (iii).

ANSWER: The sun's angle of elevation is 30°.

Case Study 2 — The Lighthouse Keeper. *A lighthouse stands **90 m** above the sea. The keeper watches two ships on the **same side** of the lighthouse. The nearer ship is seen at an angle of depression of **45°** and the farther ship at **30°**. A third ship approaches the lighthouse during the day.*



Case 2 — depressions from the top; convert them to elevations at the ships.

Case Study 2 — Questions

CASE 2 (i)

How far is the nearer ship from the foot of the lighthouse?

1. Convert the depression: the elevation of the lighthouse top from the near ship is also 45° (alternate interior angles).
2. tan 45° = 90 / distance, so distance = **90 m**.

ANSWER: The nearer ship is 90 m from the foot of the lighthouse.

CASE 2 (ii)

How far is the farther ship from the foot of the lighthouse?

1. Elevation from the far ship is 30°: tan 30° = 90 / distance.
2. Distance = 90 × √3 = 90√3 ≈ **155.88 m**.

ANSWER: The farther ship is 90√3 m (about 155.88 m) away.

CASE 2 (iii)

Find the distance between the two ships.

1. Both ships are on the same side, so subtract: 90√3 − 90 = **90(√3 − 1) m** ≈ 65.88 m.
   Why: The Q13 rule — same side, subtract.

ANSWER: The ships are 90(√3 − 1) m (about 65.88 m) apart.

CASE 2 (iv)

The nearer ship sails towards the lighthouse until its angle of depression from the top is 60°. How far did it sail?

1. At depression 60°, its distance is 90 / tan 60° = 90/√3 = 30√3 ≈ 51.96 m.
   Why: Bigger angle → nearer ship — the distance must shrink from 90 m, and 51.96 < 90. Consistent.
2. Distance sailed = 90 − 30√3 ≈ **38.04 m**.

ANSWER: The ship sailed about 38.04 m towards the lighthouse.

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Case Study 3 — The Kite Festival

Case Study 3 — The Kite Festival. *At the school kite festival, many kites are flying at the same height of* ***75 m*** *above the ground. Students hold the strings at ground level (take the string as tied at the ground point). Three kites — K1, K2 and K3 — are studied by the mathematics club.*

CASE 3 (i)

Kite K1's string makes an angle of 60° with the ground. Find the length of its string.

1. sin 60° = 75 / string → √3/2 = 75 / string.
   Why: Height (opposite) and string (hypotenuse) → sin.
2. String = 150/√3 = **50√3 m** ≈ 86.60 m.

ANSWER: K1's string is 50√3 m (about 86.60 m) long.

CASE 3 (ii)

Kite K2 flies at the same height, but its string makes an angle of 30° with the ground. Find the length of its string.

1. sin 30° = 75 / string → 1/2 = 75 / string.
2. String = **150 m**.
   Why: Same height, shallower angle, much longer string — a result worth pausing on.

ANSWER: K2's string is 150 m long.

CASE 3 (iii)

Find the horizontal distance of K1 from the student holding it.

1. tan 60° = 75 / distance → distance = 75/√3 = **25√3 m** ≈ 43.30 m.
   Why: Or by cos on the string: 50√3 × cos 60° = 25√3 — two roads, one answer.

ANSWER: K1 is 25√3 m (about 43.30 m) away horizontally.

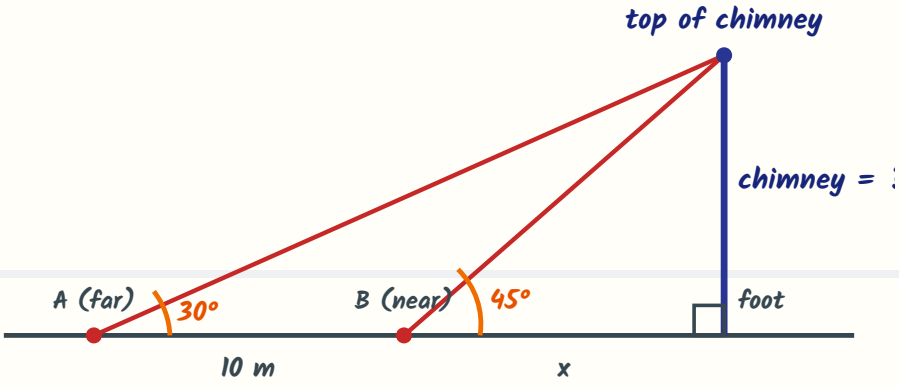
CASE 3 (iv)

Kite K3's string is 100 m long and makes an angle of 60° with the ground. At what height is K3 flying?

1. sin 60° = height / 100 → height = 100 × √3/2 = **50√3 m** ≈ 86.60 m.

Case Study 4 — The Factory Chimney

Case Study 4 — The Factory Chimney. *An engineer must find the height of a factory chimney before fitting a lightning rod. She cannot climb it. From a point A on the ground, the angle of elevation of the top of the chimney is* ***30°****. She walks* ***10 m*** *closer, to a point B, and the angle becomes* ***45°****. A cable will run from the top of the chimney down to point B.*



Case 4 — Derivation C in engineer's clothes.

CASE 4 (i)

Find the distance of point B from the foot of the chimney.

1. Let B be x from the foot and the height be h. From B (45°): tan 45° = h/x, so h = x.

2. From A (30°): tan 30° = h/(x + 10), so x + 10 = h√3 = x√3.

   Why: Substitute h = x — one unknown left.

3. 10 = x(√3 − 1), so x = 10/(√3 − 1) = **5(√3 + 1) m** ≈ 13.66 m.

   Why: Rationalise as in Q8.

ANSWER: Point B is 5(√3 + 1) m (about 13.66 m) from the foot of the chimney.

CASE 4 (ii)

Find the height of the chimney.

1. From Step 1 of part (i), h = x.

   Why: At 45° the height equals the distance — no fresh work needed.

2. Height = **5(√3 + 1) m** ≈ 13.66 m.

ANSWER: The chimney is 5(√3 + 1) m (about 13.66 m) high.

Case Study 4 (continued) and Case Study 5

CASE 4 (iii)

Find the length of the cable from the top of the chimney to point B.

1. The cable is the hypotenuse of the 45° triangle with legs h and x, where h = x = 5(√3 + 1).

2. Cable = $\sqrt{(h^2 + x^2)} = h\sqrt{2} = 5(\sqrt{3} + 1)\sqrt{2} \approx$ **19.32 m**.

   Why: In a 45°–45°–90° triangle the hypotenuse is leg × √2 — or use sin 45° on the same triangle.

ANSWER: The cable is about 19.32 m long.

CASE 4 (iv)

Verify, using your answers, that the angle at A really comes out to 30°.

1. Distance of A from the foot = x + 10 ≈ 13.66 + 10 = 23.66 m.

2. tan(angle at A) = h / 23.66 ≈ 13.66 / 23.66 ≈ 0.577 ≈ 1/√3.

   Why: And 1/√3 is exactly tan 30°.

3. So the angle at A is 30°, matching the data. The solution is consistent.

   Why: This 'plug it back' habit is the cheapest error-detector in all of trigonometry.

ANSWER: Verified: tan = 1/√3 at A, so the angle is 30°, as given.

Case Study 5 — The Balloon Show. *At a fair, a hot-air balloon hangs in the air on a line between two friends, Divya and Farhan, who stand* ***120 m*** *apart on level ground. The balloon is somewhere above the line joining them. From Divya, its angle of elevation is* ***45°****; from Farhan, it is* ***60°****.*

CASE 5 (i)

How far is the point directly below the balloon from Farhan (the 60° friend)?

1. Let that distance be y; then Divya's distance is (120 − y), and the height h satisfies h = y tan 60° = y√3 and h = (120 − y) tan 45° = 120 − y.

   Why: Two friends, one balloon — h written two ways (Derivation C, opposite-directions edition).

2. y√3 = 120 − y → y(√3 + 1) = 120 → y = 120/(√3 + 1) = **60(√3 − 1) m** ≈ 43.92 m.

   Why: Rationalise: multiply by (√3 − 1)/2.

ANSWER: The point below the balloon is 60(√3 − 1) m (about 43.92 m) from Farhan.

CASE 5 (ii)

Find the height of the balloon.

1

h = 120 − y = 120 − 60(√3 − 1) = **60(3 − √3) m** ≈ 76.08 m.

Why: Check with Farhan: y√3 = 60(√3 − 1)√3 = 60(3 − √3). Same — verified.

ANSWER: The balloon is 60(3 − √3) m (about 76.08 m) high.

Case Study 5 (continued)

CASE 5 (iii)

Find the straight-line distance from Divya to the balloon.

1. Divya's triangle has angle 45° and opposite side h, so the line of sight is the hypotenuse: distance = h / sin 45° = h√2.
2. = 60(3 − √3) × √2 ≈ 76.08 × 1.414 ≈ **107.59 m**.

ANSWER: Divya is about 107.59 m from the balloon in a straight line.

CASE 5 (iv)

Find the straight-line distance from Farhan to the balloon.

1. Farhan's triangle has angle 60°: distance = h / sin 60° = 2h/√3.
2. = 2 × 60(3 − √3) / √3 = 120(3 − √3)/√3 = **120(√3 − 1) m** ≈ 87.85 m.
   Why: Simplify (3 − √3)/√3 = √3 − 1. Farhan is nearer the balloon than Divya — matching his steeper angle.

ANSWER: Farhan is 120(√3 − 1) m (about 87.85 m) from the balloon.

Assertion–Reason (choose: (a) both true, R explains A (b) both true, R does not explain A (c) A true, R false (d) A false, R true)

AR 1. Assertion (A): The angle of depression of a car from the top of a tower is equal to the angle of elevation of the top of the tower from the car. **Reason (R):** The horizontal lines through the top of the tower and through the car are parallel, and the line of sight is a transversal, so the two angles are alternate interior angles.
Solution: A is true (Derivation B), and R states exactly the parallel-lines reason that proves it. **Answer: (a)**.

AR 2. A: If the angle of elevation of the top of a tower from a point d metres away is θ, then the height of the tower is d tan θ. **R:** In a right triangle, tan θ = opposite side / adjacent side.
Solution: With the height as opposite and d as adjacent, R gives tan θ = height/d at once — A is R applied. **Answer: (a)**.

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Assertion–Reason 3–6

Same choices: (a) both true, R explains A (b) both true, R does not explain A (c) A true, R false (d) A false, R true

AR 3. A: If the angle of elevation of a tower from a fixed point doubles from 30° to 60°, the height of the tower doubles. **R:** tan 60° = 3 × tan 30°.
Solution: R is true: tan 60° = √3 and tan 30° = 1/√3, and √3 = 3 × (1/√3). But height = d tan θ would become THREE times, not double — A is false. **Answer: (d)**.

AR 4. A: From the top of a 50 m lighthouse, a boat seen at an angle of depression of 45° is 50 m (horizontally) from the foot of the lighthouse. **R:** The angle of elevation of the top from the boat is 45°, and tan 45° = 1.
Solution: By R, distance = 50 / tan 45° = 50 m — A follows from R. **Answer: (a)**.

AR 5. A: A kite whose string is 100 m long, inclined at 30° to the ground, is flying at a height of 100 m. **R:** sin 30° = 1/2.
Solution: R is true, but it gives height = 100 × sin 30° = 50 m, not 100 m — A is false.
Answer: (d).

AR 6. A: When the sun's angle of elevation is 30°, the shadow of a pole is √3 times the height of the pole. **R:** tan 30° = height / shadow = 1/√3.
Solution: From R, shadow = √3 × height — exactly A. **Answer: (a)**.

Assertion–Reason 7–10

Assertion–Reason (continued)

AR 7. A: The angle of elevation of the top of a tower from a point 30 m away is 30°, so the height of the tower is $10\sqrt{3}$ m. **R:** $\sqrt{3} \approx 1.732$.
Solution: A is true ($30 \times \tan 30° = 30/\sqrt{3} = 10\sqrt{3}$) and R is a true numerical fact — but A follows from the tan ratio, not from the decimal value of $\sqrt{3}$. R does not explain A. **Answer: (b)**.

AR 8. A: As an observer walks towards a building, the angle of elevation of its top increases. **R:** The height of the building increases as the observer approaches it.
Solution: A is true — the distance shrinks, so $\tan\theta = h/d$ grows. But the building's height never changes; R is false. **Answer: (c)**.

AR 9. A: For a fixed tower, the angle of elevation of its top from a point farther away is smaller than from a point nearer to it. **R:** For a fixed height h, $\tan\theta = h/d$ becomes smaller as d becomes larger.
Solution: R is precisely the mechanism behind A. **Answer: (a)**.

AR 10. A: The angle of depression of an object is measured between the line of sight and the vertical line through the observer. **R:** The angle of depression is measured between the line of sight and the horizontal line through the observer.
Solution: R states the correct definition; A contradicts it. A is false, R is true. **Answer: (d)**.

TOPPER TIP: *In Assertion–Reason, judge A and R separately first. Only when both are true, ask the third question: does R actually explain A? AR 7 is the classic trap — a true fact that explains nothing.*

15 Olympiad & Foundation Challenges

8D • CHALLENGE (PART 2) — TEN PROBLEMS THAT STRETCH THE METHOD

CHALLENGE 1

A flagstaff stands on the top of a tower. From a point on the ground 20 m from the foot of the tower, the angles of elevation of the top of the tower and the top of the flagstaff are 45° and 60° respectively. Find the height of the flagstaff.

THINK FIRST: Stacked objects always surrender to the same order: lower angle first, total second, subtract.

1. **Draw the diagram.** Tower AB = T, flagstaff BC = F, point D at 20 m. Angles at D: 45° to B, 60° to C.

   Why: Q7/Q8's skeleton with both pieces unknown — but the 45° angle still unlocks it.

2. tan 45° = T/20 → T = 20 m.

3. tan 60° = (T + F)/20 → T + F = 20√3.

4. F = 20√3 − 20 = **20(√3 − 1) m** ≈ 14.64 m.

ANSWER: The flagstaff is 20(√3 − 1) m (about 14.64 m) high.

CHALLENGE 2

Two towers stand 60 m apart. From the top of the shorter tower, which is 30 m high, the angle of elevation of the top of the taller tower is 45°. Find the height of the taller tower.

THINK FIRST: From-the-top problems are eye-height problems standing on a building. Split at the eye level, solve, add back.

1. **Draw the diagram.** Draw the horizontal line from the top of the shorter tower to the taller one. The part of the taller tower ABOVE this line forms a right triangle with the 60 m gap.

   Why: The observer is on a tower now — the triangle floats at the 30 m level.

2. In that triangle: tan 45° = (part above) / 60.

3. Part above = 60 × 1 = 60 m.

4. Height of taller tower = 30 + 60 = **90 m**.

   Why: Add back the observer's own height — the eye-height rule in reverse.

ANSWER: The taller tower is 90 m high.

Olympiad Challenges 3 and 4

CHALLENGE 3

A ladder 10 m long is placed against a wall on one side of a street, making an angle of 60° with the ground. Keeping its foot at the same point, the ladder is turned over to lean against the wall on the opposite side of the street, now making an angle of 30° with the ground. Find the width of the street, and the heights the ladder reaches on the two walls.

THINK FIRST: A fixed foot with walls on both sides → the street is a SUM of two adjacent sides. The complementary-angle swap (sin 60 = cos 30) is the hidden beauty here.

1. **Draw the diagram.** The foot F is fixed. On side 1 the ladder reaches height H_1 at 60°; on side 2 it reaches H_2 at 30°. The street's width = (distance from F to wall 1) + (distance from F to wall 2).

   Why: One ladder, two triangles, sharing the hypotenuse 10 m.

2. Side 1: $H_1 = 10 \sin 60° =$ **$5\sqrt{3}$ m** ≈ 8.66 m; foot-to-wall distance $= 10 \cos 60° =$ **5 m**.

   Why: sin for the height (opposite), cos for the ground distance (adjacent) – the ladder is the hypotenuse in both.

3. Side 2: $H_2 = 10 \sin 30° =$ **5 m**; foot-to-wall distance $= 10 \cos 30° =$ **$5\sqrt{3}$ m** ≈ 8.66 m.

   Why: Notice the swap: the two heights and the two distances exchange values – complementary angles at work.

4. Width of the street $= 5 + 5\sqrt{3} =$ **$5(1 + \sqrt{3})$ m** ≈ 13.66 m.

ANSWER: The street is $5(1 + \sqrt{3})$ m (about 13.66 m) wide; the ladder reaches $5\sqrt{3}$ m (about 8.66 m) on the first wall and 5 m on the second.

CHALLENGE 4

An aeroplane is flying at a height of 3000 m. From the aeroplane, the angles of depression of two milestones on the ground, on opposite sides of the aeroplane, are 45° and 60°. Find the distance between the milestones.

THINK FIRST: 'From the aeroplane' just means the observer is at the top. The machinery does not care who is up and who is down.

1. **Draw the diagram.** The milestones are at ground points A and B on opposite sides of the point directly below the plane. Convert the depressions to elevations at the milestones.

Olympiad Challenges 5 and 6

CHALLENGE 5

From the top of a 60 m high building, the angle of elevation of the top of a chimney is 45° and the angle of depression of the foot of the chimney is 60°. Find the height of the chimney and the distance between the building and the chimney.

THINK FIRST: Compare with NCERT Q12: identical skeleton, sterner numbers. Master the skeleton once and the numbers are just weather.

1. **Draw the diagram.** Split the chimney at the building-top level (the Q12 move). Let the gap between building and chimney be d.
2. Depression of the foot (60°), converted: tan 60° = 60 / d, so d = 60/√3 = **20√3 m** ≈ 34.64 m.

   Why: The depression angle works on the building's own height – it hands us the distance.
3. Elevation of the top (45°): part of chimney above the building-top level = d × tan 45° = 20√3 m.
4. Chimney height = 60 + 20√3 ≈ **94.64 m**.

ANSWER: The chimney is 60 + 20√3 m (about 94.64 m) high, standing 20√3 m (about 34.64 m) from the building.

CHALLENGE 6

The shadow of a tower is 30 m longer when the sun's angle of elevation is 30° than when it is 45°. Find the height of the tower.

THINK FIRST: Shadow-difference problems are two-point problems where the 'two points' are the two tips of the shadow.

1. **Draw the diagram.** Let the height be h. Shadow at 45° is s_1; at 30° it is $s_2 = s_1 + 30$.

   Why: Reasoning 3 proved the pattern; now feed it a number.
2. At 45°: $s_1 = h / \tan 45° = h$.
3. At 30°: $s_2 = h / \tan 30° = h\sqrt{3}$.
4. Difference: $h\sqrt{3} - h = 30 \rightarrow h(\sqrt{3} - 1) = 30$.
5. $h = 30 / (\sqrt{3} - 1) = 30(\sqrt{3} + 1)/2 =$ **$15(\sqrt{3} + 1)$ m** ≈ 40.98 m.

ANSWER: The tower is 15(√3 + 1) m (about 40.98 m) high.

Olympiad Challenges 7 and 8

CHALLENGE 7

From the top of a 75 m tower, a car approaching the tower is observed at an angle of depression of 30°. Five seconds later, the angle of depression is 60°. Find the speed of the car in m/s and in km/h.

THINK FIRST: Always end a speed answer with the smell test: 62 km/h for a car on a highway is believable; 620 or 6.2 would expose an error.

1. **Draw the diagram.** Convert the depressions to elevations at the car. First distance: 75 / tan 30° = 75√3. Later distance: 75 / tan 60° = 75/√3.

   Why: NCERT Q15 with a height supplied — now a speed can be computed, not just a time.

2. Distance covered in 5 s = 75√3 − 75/√3 = 75 × (2/√3) = **50√3 m** ≈ 86.60 m.

3. Speed = 50√3 / 5 = **10√3 m/s** ≈ 17.32 m/s.

4. Convert: 17.32 × 18/5 ≈ **62.35 km/h**.

   Why: m/s to km/h: multiply by 18/5. A realistic highway speed — the answer smells right.

ANSWER: The car's speed is 10√3 m/s (about 17.32 m/s, i.e. about 62.35 km/h).

CHALLENGE 8

Two vertical poles, 20 m and 30 m high, stand 40 m apart on level ground. A wire is stretched from the top of the shorter pole to the top of the taller pole. Find the length of the wire and the angle of elevation of the top of the taller pole as seen from the top of the shorter pole.

THINK FIRST: When BOTH legs are known and the hypotenuse is wanted, skip trig entirely — Pythagoras is the shortest road.

1. **Draw the diagram.** Draw the horizontal from the top of the 20 m pole to the 30 m pole. It cuts the taller pole 20 m up, leaving a 10 m piece above it. The wire is the hypotenuse of a triangle with legs 40 m (horizontal) and 10 m (vertical).

   Why: The floating-triangle move of Challenge 2 — the wire's triangle lives at the 20 m level.

2. Wire length = √(40² + 10²) = √(1600 + 100) = √1700 = **10√17 m** ≈ 41.23 m.

   Why: Pythagoras — no trig ratio gives a hypotenuse from two legs faster.

3. For the angle θ at the top of the shorter pole: tan θ = 10 / 40 = 1/4, so θ = $\tan^{-1}(0.25)$ ≈ **14.04°**, about 14°.

Olympiad Challenges 9 and 10

CHALLENGE 9

A tower stands on the top of a hill. From a point on the ground 500 m from the foot of the hill, the angle of elevation of the top of the hill is 30° and the angle of elevation of the top of the tower is 45°. Find the height of the hill and the height of the tower.

THINK FIRST: Hill + tower, building + statue, pedestal + flag — one skeleton, endless costumes.

1. **Draw the diagram.** One point, two angles, stacked objects — the Q7 skeleton on a grand scale. Hill = H, tower = T.
2. Angle to the hill top (30°): tan 30° = H / 500 → H = 500/√3 ≈ **288.68 m**.
3. Angle to the tower top (45°): tan 45° = (H + T) / 500 → H + T = 500 m.
4. T = 500 − 500/√3 ≈ 500 − 288.68 = **211.32 m**.

   Why: Exact form: T = 500(1 − 1/√3) = 500(√3 − 1)/√3 m.

ANSWER: The hill is 500/√3 m (about 288.68 m) high and the tower on it is about 211.32 m high.

CHALLENGE 10

A man observes the top of a tower at an angle of elevation of 45°. He walks 20 m directly away from the tower, and the angle of elevation becomes 30°. Find the height of the tower.

THINK FIRST: Approach or retreat, the machine is identical: write h two ways, equate, solve. Only the sign of the 20 m changes.

1. **Draw the diagram.** Walking AWAY this time: the 45° point is the NEAR one. Let the height be h; the near distance is then h itself (tan 45°).

   Why: Direction matters: away → the second distance is (first + 20).
2. Near point: tan 45° = h / (near distance) → near distance = h.
3. Far point: tan 30° = h / (h + 20) → h + 20 = h√3.
4. 20 = h(√3 − 1) → h = 20/(√3 − 1) = **10(√3 + 1) m** ≈ 27.32 m.

   Why: The same rationalisation as Q8 and Case 4 — by now your hand should do it automatically.

ANSWER: The tower is 10(√3 + 1) m (about 27.32 m) high.

16 Chapter Mind Map

THE WHOLE CHAPTER ON ONE PAGE — REVISE FROM HERE

SOME APPLICATIONS OF TRIGONOMETRY

Heart of the chapter: **every story hides ONE right triangle** — vertical height, horizontal distance, slanted line of sight, and an angle at the eye measured from the horizontal.

The Two Angles

Elevation: eye looks UP. **Depression:** eye looks DOWN. Both measured from the HORIZONTAL at the eye. Depression (from top) = Elevation (from object) — alternate interior angles.

→

tan — the Workhorse

tan θ = height / distance, i.e. **h = d tan θ** (derived, Section 06). Slanted length in play (rope, string, ladder)? Switch to sin / cos.

The Eye-Height Rule

A person's triangle starts at the EYE: use (object height − observer height) inside the triangle; add the observer back at the end. Q6, Q14.

Two-Triangle Families

Moving observer/object: walker (Q6), balloon (Q14), car (Q15) — write h two ways, equate. **Stacked objects:** tower on building (Q7), statue (Q8), hill + tower — lower angle first, subtract. **Facing objects:** Q9 — solve the known-height triangle first.

Ships & Bridges

Same side → SUBTRACT the distances (Q13: 75(√3 − 1)). Opposite sides → ADD (Application 4). The story's geometry decides.

Gift Patterns

45°: height = distance. **30°/60° pair:** near distance = half the gap, h = gap × √3/2. **Complementary angles:** h = √(ab) (Q16: √(4×9) = 6). **No height given:** keep h — it cancels (Q15: 3 s).

Key Results to Quote

Q4 tower = 10√3 m • Q5 string = 40√3 m • Q10 poles = 20√3 m, point at 20 m / 60 m • Q11 tower = 10√3 m, canal = 10 m • Q14 balloon travels 58√3 m • Q16 height = 6 m (proved)

TOPPER TIP: *Revise actively: cover a box, speak its contents aloud from memory, then uncover and check. A mind map you can REPRODUCE from memory is worth ten readings.*

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17 Pin-Point Flash Cards

THE WHOLE CHAPTER IN YOUR POCKET — CUT THEM OUT, CARRY THEM, MASTER THEM

How to use these cards

Cut along the dashed borders and keep the 16 cards in your pocket or a small box. On Smart Memory **Day 3, Day 7 and Day 21**, shuffle the cards and test yourself: read the title, say the pin-point aloud from memory, then flip the card to check. Any card you miss goes back into the box for tomorrow. Two minutes a day keeps this chapter alive all year.

CARD 1 — Angle of Elevation

The angle between the HORIZONTAL line through the eye and the line of sight, when the eye looks UP at the object.

CARD 2 — Angle of Depression

The angle between the HORIZONTAL line through the eye and the line of sight, when the eye looks DOWN at the object.

CARD 3 — The Master Key

Angle of depression (from the top) = Angle of elevation (from the object). Reason: alternate interior angles — the two horizontals are parallel.

CARD 4 — The Workhorse

tan θ = height / distance, so height = distance × tan θ and distance = height / tan θ. It is the DEFINITION of tan, not a new formula.

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Flash Cards 5–10

CARD 5 — Slanted Lengths

A rope, string, ladder or wire is the HYPOTENUSE. Height ↔ sin; ground distance ↔ cos. tan can never touch the hypotenuse.

CARD 6 — The Eye-Height Rule

A person's triangle starts at the EYE. Inside the triangle use (object height − observer height); add the observer's height back at the end.

CARD 7 — The 45° Gift

At 45°, height = distance. Always fire a 45° angle FIRST — it converts one unknown into the other for free.

CARD 8 — The 30°/60° Pair

Two points, angles 30° and 60°, gap d: the near distance is d/2 and the height is d√3/2. Spot it and skip the algebra.

CARD 9 — Two Points, One Height

Write h in TWO ways (one from each point) and EQUATE. This single move solves the walker, the balloon, the car, the canal and the poles.

CARD 10 — Same Side / Opposite Sides

Objects on the SAME side of the observer: SUBTRACT the distances. On OPPOSITE sides: ADD them. Let the diagram decide.

TOPPER TIP: *Say each card aloud once before testing yourself on it — hearing your own voice say the fact makes the memory stick twice as fast.*

Flash Cards 11–16

CARD 11 — Complementary Angles

If the angles from two points at distances a and b are complementary, height = √(ab). Engine: tan(90° − θ) = cot θ.

CARD 12 — The Vanishing Height

No height given in the problem? Keep it as h and work symbolically — it cancels. (NCERT Q15's answer is 3 seconds for EVERY tower.)

CARD 13 — Standard tan Values

tan 30° = 1/√3 • tan 45° = 1 • tan 60° = √3. Know them cold — they appear in every single problem.

CARD 14 — The Horizontal Rule

Angles of elevation and depression are measured from the HORIZONTAL — NEVER from the vertical, NEVER from the object itself.

CARD 15 — Rising and Falling Angles

Angle GROWS → the object approaches (or you approach it). Angle FALLS → it moves away. Say it before you compute.

CARD 16 — Exact Till the End

Keep √3 symbolic through every step; rationalise denominators; use √3 ≈ 1.732 only in the final line, if at all.

MAKE YOUR OWN CARD

Write one pin-point YOU keep forgetting, in your own words. A card you write yourself is the card you remember best.

_____

_____

MAKE YOUR OWN CARD

One more: the mistake you made most often in this chapter, and the one-line rule that fixes it.

_____

18 Active Recall

8D • RECALL — CLOSE THE BOOK, THEN ANSWER

How this works: close every other page. Answer all ten questions on rough paper, from memory alone — diagrams included. Only then open the Answer Key. Anything you could not recall is exactly what your Day-3 revision must target — write it in the Error Tracker (Section 19).

Ten rapid questions

R1. Define the angle of elevation of an object, in one sentence.

R2. Define the angle of depression of an object, in one sentence.

R3. The angle of elevation of the top of a tower from a point 20 m from its foot is 60°. Find the height of the tower.

R4. A kite string 90 m long makes an angle of 30° with the ground. At what height is the kite?

R5. Why is the angle of depression from the top of a lighthouse equal to the angle of elevation from the ship? Give the geometric reason.

R6. A 1.5 m tall boy looks at the top of a 30 m building. What height goes INSIDE his triangle — and why?

R7. From a 75 m lighthouse, two ships on the same side are seen at depressions of 45° and 30°. Find the distance between the ships.

R8. The angles of elevation of a tower's top from points 4 m and 9 m away are complementary. Find the height of the tower.

R9. In the approaching-car problem (NCERT Q15), why does the answer not depend on the height of the tower?

R10. In the statue-and-pedestal problem, which angle should you use FIRST, and what gift does it give you?

Five recall prompts — say each answer aloud, in a full sentence

P1. State the workhorse equation of this chapter, and say where it comes from.

P2. State the eye-height rule.

P3. State the rule for adding versus subtracting distances in ship problems.

P4. State the complementary-angles theorem.

P5. State the two-observation-point method in one line.

> **TOPPER TIP:** *Scoring: 13–15 correct — chapter mastered, move on. 9–12 — revise the missed topics, retake in 3 days. Below 9 — re-study Sections 05–07, then retake. Record your score in the Mastery Tracker.*

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19 Self-Assessment & Mastery Tracker

KALVIYOGI SMART MEMORY SYSTEM — LEARN IT, THEN KEEP IT

Memory fades on a predictable curve — unless you revisit at the right moments. This schedule is your revisit plan for THIS chapter. Tick each day off when done.

Day	Your activity for Chapter 9	Done
Day 0	Pre-learning assessment: attempt the Quick Check (Section 02) and Rapid Questions R1–R10 (Section 18) BEFORE studying. Note your score.	☐
Day 1	Learn the chapter fully (Sections 01–10) and complete Level 1–3 practice — with a hand-drawn diagram for every question.	☐
Day 3	Without-book recall: reproduce the Mind Map (Section 16) on a blank page; redo Rapid Questions; finish Level 4.	☐
Day 7	Independent problem-solving: Level 5 + Olympiad Challenges, timed, book closed. Then take the **Day 7 Mini-Test** (Practice Plus) and run the Flash Cards (Section 17).	☐
Day 14	Mixed practice: re-attempt every question in your Error Tracker; solve Ex 9.1 Q6 and Q13 fresh, diagrams first.	☐
Day 21	Long-term recall test: write the two-point derivation (Derivation C) and the Q16 proof from memory, then verify line by line. Then take the **Day 21 Mini-Test** (Practice Plus).	☐
Day 30	Chapter mastery assessment: take the **Day 30 Mastery Test** (Practice Plus) under exam conditions; update your Concept Mastery Score below.	☐

Formula Recall Tracker — tick when you can say it with the book closed

Formula / Statement	Day 1	Day 3	Day 7	Day 21
tan θ = height / distance — and h = d tan θ	☐	☐	☐	☐
Depression = elevation (alternate interior angles)	☐	☐	☐	☐
The eye-height rule	☐	☐	☐	☐
Two-point method: write h two ways and equate	☐	☐	☐	☐
30°/60° special case: near distance = half the gap	☐	☐	☐	☐
Complementary angles: h = √(ab)	☐	☐	☐	☐

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Mastery Tracker — Errors and Concept Scores

Error Tracker — every mistake becomes a lesson

Date	Question	What went wrong	The correct idea

Concept Mastery Score — rate yourself honestly, out of 5

Concept	Score / 5	Status
Drawing the correct diagram (angle from the horizontal)	___ / 5	☐ Weak ☐ Okay ☐ Mastered
One-triangle problems (towers, kites, ladders)	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Eye-height problems (Q6, Q14 pattern)	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Two-point problems (walker, balloon, car, canal)	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Depression problems (ships, bridges, planes)	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Stacked objects (tower on building, statue, hill)	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Proofs (complementary angles, depression = elevation)	___ / 5	☐ Weak ☐ Okay ☐ Mastered

Learning Time Tracker

Day / Date	Minutes studied	What I studied

Mastery rule: a concept counts as MASTERED only when you score 4 or 5 AND can reproduce it on Day 21 without the book. Anything less — revisit, don't just re-read.

Practice Plus — 25 Concept MCQs (1–13)

One mark each. Cover the Answer Key and choose the single best option — then check the key, where the tricky ones carry a one-line explanation.

Q1. The angle of elevation of an object is measured from the
(a) vertical line (b) horizontal line (c) line of sight (d) ground itself

Q2. The angle of elevation of the top of a tower from a point 20 m from its foot is 45°. The height of the tower is
(a) 10 m (b) 20 m (c) 20√3 m (d) 40 m

Q3. In heights-and-distances problems, tan θ is the ratio of
(a) height / line of sight (b) height / distance (c) distance / height (d) height × distance

Q4. A kite string 60 m long makes an angle of 30° with the ground. The kite's height is
(a) 30 m (b) 30√3 m (c) 60 m (d) 120 m

Q5. The angle of depression of a boat from a cliff top equals the angle of elevation of the cliff top from the boat, because the two angles are
(a) vertically opposite (b) alternate interior angles (c) complementary (d) supplementary

Q6. The shadow of a pole is √3 times its height. The sun's angle of elevation is
(a) 60° (b) 45° (c) 30° (d) 90°

Q7. A tower is 10√3 m high. The angle of elevation of its top from a point 10 m from the foot is
(a) 30° (b) 45° (c) 60° (d) 15°

Q8. From the top of a 100 m tower, the angle of depression of an object on the ground is 45°. The object is at a distance of
(a) 50 m (b) 100 m (c) 100√3 m (d) 200 m

Q9. A ladder 10 m long leans against a wall at 60° to the ground. The height it reaches on the wall is
(a) 5 m (b) 5√3 m (c) 10√3 m (d) 20/√3 m

Q10. If the height of a tower and the distance of the observation point from its foot are equal, the angle of elevation is
(a) 30° (b) 45° (c) 60° (d) 90°

Q11. In the broken-tree problem, the standing part and the broken part are found using, respectively,
(a) tan and tan (b) sin and sin (c) tan and cos (d) cos and tan

Q12. The angles of elevation of a tower's top from points 4 m and 9 m from its base are complementary. The height of the tower is
(a) 5 m (b) 6 m (c) 13 m (d) 36 m

Q13. As an observer walks towards a building, the angle of elevation of its top
(a) decreases (b) increases (c) stays the same (d) becomes 90°

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25 Concept MCQs (14–25)

Q14. In the statue-on-pedestal problem, the distance of the observation point is first found using the angle of elevation of
(a) the top of the statue (b) the top of the pedestal (c) the foot of the pedestal (d) the sun

Q15. If the angle of elevation is 30° and the distance is d, the height of the object is
(a) d√3 (b) d/√3 (c) d (d) 3d

Q16. A 50 m tower casts a shadow of length 50/√3 m. The sun's angle of elevation is
(a) 30° (b) 60° (c) 45° (d) 90°

Q17. The angle of depression is measured at the
(a) object being viewed (b) observer's eye (c) foot of the tower (d) midpoint of the line of sight

Q18. A kite flies at a height of 30 m and its string makes 60° with the ground. The length of the string is
(a) 20√3 m (b) 15 m (c) 60 m (d) 10√3 m

Q19. The line of sight is the line drawn from
(a) the ground to the top (b) the eye of the observer to the point viewed (c) the top to the foot (d) one bank to the other

Q20. From the top of a 20 m tower, the angle of depression of a point on the ground is 60°. The point is at a distance of
(a) 20√3 m (b) 20/√3 m (c) 20 m (d) 60 m

Q21. From the midpoint of an 80 m wide road, the angles of elevation of the tops of two equal poles on either side are both 45°. The height of each pole is
(a) 20 m (b) 40 m (c) 80 m (d) 40√3 m

Q22. When the height and the angle of elevation are known and the ground distance is required, the ratio to use is
(a) sin (b) cos (c) tan (d) none of these

Q23. The angle of elevation of a balloon falls from 60° to 30° while its height stays constant. The balloon has
(a) come closer (b) moved away (c) risen higher (d) fallen lower

Q24. The angle of elevation of a tower's top from a point is 45°. If the point is moved to half its distance from the tower, tan of the new angle of elevation is
(a) 1/2 (b) 2 (c) 1 (d) √3

Q25. The angle of depression of a car from the top of a tower is 30°. The angle of elevation of the top of the tower from the car is
(a) 60° (b) 30° (c) 45° (d) 90°

TOPPER TIP: Scored below 20? Do not just re-read the key — find WHICH pin-point card covers each missed question and revise that card first.

Find the Error — Questions 1–5

Each piece of working below contains exactly one planted mistake — the kind real students make in board exams. Spot it before you check the key.

ERROR 1

"Height of the tower: tan 30° = h/30. Since tan 30° = √3, h = 30√3 m."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 2

"The kite's string is 80 m at 60° to the ground. Height of the kite = 80 × cos 60° = 80 × 1/2 = 40 m."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 3

"The angle of depression of the boat is the angle between the line of sight and the vertical line drawn at the top of the lighthouse."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 4

"A 1.5 m tall boy sees the top of a 30 m building at 45° from his eyes. Since tan 45° = 1, his distance from the building equals the height, 30 m."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 5

"For the statue problem: h(√3 − 1) = 1.6, so the pedestal h = 1.6 × (√3 − 1) ≈ 1.17 m."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

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Find the Error — Questions 6–10

ERROR 6

"The two ships are on the same side of the 75 m lighthouse, at distances 75 m and 75√3 m. So the distance between the ships is 75 + 75√3 m."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 7

"The car covered the first part in 6 seconds, so it will take another 6 seconds to reach the foot of the tower."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 8

"The broken tree: tan 30° = height/8, so the height of the tree is 8/√3 m."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 9

"For the transmission tower on the 20 m building: tan 60° gives the total height as 20√3 m. So the height of the tower is 20√3 m."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 10

"The height is 10√3 m. Taking √3 = 1.73, the height is 10 × 1.73 = 173 m."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

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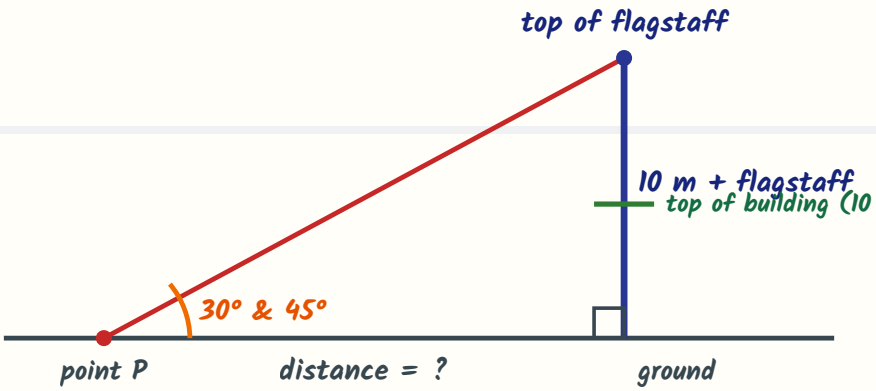
Previous Year Questions — Real CBSE Board Questions (1)

These are genuine CBSE board questions, with the year and marks verified. Solve them exactly as you would in the exam hall — full steps earn full marks.



CBSE 2012 — the flagstaff problem: two angles, one point, stacked objects.

CBSE 2012 — 2 MARKS

From a point P on the ground, the angle of elevation of the top of a 10 m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff and the distance of the building from the point P. (Take √3 = 1.73.)

1. **Draw the diagram.** Building AB = 10 m, flagstaff BC = f, P at distance d. Angles at P: 30° to B, 45° to C.
2. From the 30° angle: tan 30° = 10/d, so d = 10√3 ≈ **17.3 m**.
   Why: The distance comes first — the flagstaff needs it.
3. From the 45° angle: tan 45° = (10 + f)/d, so 10 + f = d = 10√3.
4. f = 10√3 − 10 = 10(√3 − 1) ≈ 10 × 0.73 = **7.3 m**.

ANSWER: The distance of the building from P is 10√3 m (about 17.3 m) and the flagstaff is 10(√3 − 1) m (about 7.3 m) long.

CBSE 2012 — 3 MARKS

The horizontal distance between two poles is 15 m. The angle of depression of the top of the first pole, as seen from the top of the second pole, is 30°. If the height of the second pole is 24 m, find the height of the first pole. (Use √3 = 1.732.)

1.

Previous Year Questions (2)

CBSE 2014 — 3 MARKS

Two ships are approaching a lighthouse from opposite directions. The angles of depression of the two ships from the top of the lighthouse are 30° and 45°. If the distance between the two ships is 100 m, find the height of the lighthouse. (Use √3 = 1.732.)

1. **Draw the diagram.** Let the lighthouse be h. The ships are on OPPOSITE sides, at distances h/tan 45° and h/tan 30° from the foot.
2. Ship 1 (45°): distance = h / 1 = h. Ship 2 (30°): distance = h / (1/√3) = h√3.
   Why: Distances from a known height: distance = h × cot(angle).
3. Opposite sides → ADD: h + h√3 = 100.
   Why: The Application 4 rule.
4. h(1 + √3) = 100, so h = 100/(1 + √3) = 100(√3 − 1)/2 = **50(√3 − 1) m** ≈ 36.6 m.

ANSWER: The lighthouse is 50(√3 − 1) m (about 36.6 m) high.

CBSE 2016 — 3 MARKS

A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Find the distance of the hill from the ship and the height of the hill.

1. **Draw the diagram.** The man's eye is 10 m above the water. The hill's base is at water level — 10 m BELOW his eye (depression 30°); its top is above his eye (elevation 60°). Split the hill at eye level, as in Q12.
2. From the depression of the base: tan 30° = 10 / distance, so distance = 10√3 ≈ **17.32 m**.
   Why: The below-eye triangle gives the distance.
3. From the elevation of the top: the part of the hill above eye level = distance × tan 60° = 10√3 × √3 = 30 m.
4. Height of the hill = 30 + 10 = **40 m**.
   Why: Add back the 10 m below the eye — the eye-height rule, in its grandest costume.

ANSWER: The hill is 10√3 m (about 17.32 m) from the ship and is 40 m high.

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Previous Year Questions (3)

CBSE 2018 — 4 MARKS

As observed from the top of a 100 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. (Use √3 = 1.732.)

1. **Draw the diagram.** This is NCERT Q13 with 100 m in place of 75 m — same side, so the answer will be a difference.

   Why: Board papers return to this skeleton again and again; recognising it is worth a mark of confidence.

2. Convert the depressions to elevations at the ships (alternate interior angles).

3. Near ship (45°): distance = 100 / tan 45° = 100 m.

4. Far ship (30°): distance = 100 / tan 30° = 100√3 m.

5. Distance between the ships = 100√3 − 100 = **100(√3 − 1) m** ≈ 73.2 m.

ANSWER: The distance between the two ships is 100(√3 − 1) m (about 73.2 m).

TOPPER TIP: *Five board questions, and every one of them is a Section 07 skeleton: stacked objects (2012), floating triangle (2012), opposite-side ships (2014), eye-level split (2016), same-side ships (2018). There are no new problems in the exam — only new costumes.*

Chapter Model Test A — Basics (20 Marks)

Time: 40 minutes. Attempt all questions. Draw a diagram for every question — the marking key in the Answer Key shows where each mark lives.

Q1. Define the angle of elevation and the angle of depression of an object. **[2 marks]**

Q2. The angle of elevation of the top of a tower from a point 20 m from its foot is 45°. Find the height of the tower. **[3 marks]**

Q3. A kite string is 50 m long and makes an angle of 30° with the ground. Find the height of the kite. **[3 marks]**

Q4. A ladder 12 m long leans against a wall, making an angle of 60° with the ground. How far is its foot from the wall? **[3 marks]**

Q5. From the top of a 40 m high lighthouse, the angle of depression of a boat is 30°. Find the distance of the boat from the foot of the lighthouse. **[4 marks]**

Q6. A pole casts a shadow 9 m long when the sun's angle of elevation is 60°. Find the height of the pole. **[5 marks]**

Chapter Model Test B — Standard (20 Marks)

Time: 40 minutes. Board-pattern questions at full Class 10 level.

Q1. From a point on the ground, the angle of elevation of the top of a tower is 30°. On moving 20 m nearer the tower, the angle becomes 60°. Find the height of the tower. **[4 marks]**

Q2. A tree breaks in a storm and its top touches the ground 10 m from the foot, making an angle of 30° with the ground. Find the height of the tree. **[4 marks]**

Q3. A statue 1.2 m tall stands on a pedestal. From a point on the ground, the angles of elevation of the top of the pedestal and the top of the statue are 45° and 60°. Find the height of the pedestal. **[4 marks]**

Q4. From the top of a 60 m lighthouse, the angles of depression of two ships on the same side are 60° and 30°. Find the distance between the ships. **[4 marks]**

Q5. A tower is 40 m high. The angle of elevation of the top of the tower from the foot of a building is 60°, and the angle of elevation of the top of the building from the foot of the tower is 30°. Find the distance between them and the height of the building. **[4 marks]**

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Chapter Model Test C — Advanced (20 Marks)

Time: 45 minutes. Proof-heavy, HOTS-level — this is the paper that separates 95 from 100.

Q1. The angles of elevation of the top of a tower from two points at distances a and b from its base, in the same straight line with it, are complementary. Prove that the height of the tower is √(ab). **[5 marks]**

Q2. From a point 100 m above a lake, the angle of elevation of a cloud is 30° and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud above the lake. **[5 marks]**

Q3. A car approaches a 60 m tower with uniform speed. Its angle of depression from the top of the tower changes from 30° to 60° in 4 seconds. Find the distance covered in those 4 seconds and the speed of the car. **[5 marks]**

Q4. From the top of a 25 m tower, the angle of elevation of the top of another tower is 45° and the angle of depression of its foot is 45°. Find the distance between the towers and the height of the other tower. **[5 marks]**

> *TOPPER TIP: Take these three tests on three different days — Test A after Section 11, Test B after Section 14, Test C in the final revision week. Record every score in your Mastery Tracker.*

My Test Record — Chapter 9

Test	Date	Score	Weakest topic — revise this first
Model Test A — Basics		/ 20	
Model Test B — Standard		/ 20	
Model Test C — Advanced		/ 20	
Day 7 Mini-Test		/ 10	
Day 21 Mini-Test		/ 10	
Day 30 Mastery Test		/ 10	

A rising line of scores is the real certificate of this chapter — not any single day's mark. If a score dips, the 'weakest topic' column tells you exactly which section to revisit before moving on.

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Smart Memory Mini-Tests — Day 7 and Day 21

Ten quick questions each, closed book, five minutes. Answers are in the Answer Key. These are the Day 7 and Day 21 stops of your Smart Memory schedule (Section 19).

DAY 7 TEST

1. The angle of elevation of a tower's top from a point 15 m from its foot is 45°. Find the height of the tower.

2. A kite string 30 m long makes 30° with the ground. Find the height of the kite.

3. A pole's shadow is 12 m long when the sun's angle of elevation is 60°. Find the height of the pole.

4. From the top of a 30 m tower, the angle of depression of a point on the ground is 45°. How far is the point from the foot?

5. A ladder 8 m long makes 60° with the ground. How far is its foot from the wall?

6. Using $h = d \tan \theta$, find h when d = 10 m and $\theta = 60°$.

7. If $\tan \theta = 1/\sqrt{3}$, what is θ?

8. A pole 5 m high casts a shadow 5 m long. Find the sun's angle of elevation.

9. A kite flies at a height of 40 m with its string at 60° to the ground. Find the length of the string.

10. A tree's top touches the ground 6 m from its foot at 30°. Find the height of the standing part of the tree.

DAY 21 TEST

1. The angles of elevation of a tower's top from two points 10 m apart are 45° (farther) and 60° (nearer). Find the height of the tower.

2. A statue 2 m tall stands on a pedestal; the angles to the top of the pedestal and the top of the statue are 45° and 60°. Find the height of the pedestal.

3. Two ships on opposite sides of a 50 m lighthouse are seen at depressions of 45° and 30°. Find the distance between the ships.

4. A 10 m pole's shadow grows as the sun falls from 60° to 30°. By how much does the shadow lengthen?

5. A surveyor walks 25 m along a river bank; the line to a tree on the far bank then makes 45° with the bank. Find the width of the river.

6. Find the height of a tower seen at 30° from a point 36 m from its foot.

7. A string 90 m long holds a kite at 45° to the ground. At what height is the kite?

8. State, in one line, the relation between the angle of depression of a boat from a cliff and the angle of elevation of the cliff top from the boat.

9. From the top of a 10 m house, the angle of elevation of the top of a chimney is 45° and the angle of depression of its foot is 45°. Find the height of the chimney.

10. The angles of elevation of a tower's top from points 16 m and 25 m away are complementary. Find the height of the tower.

Smart Memory Mini-Test — Day 30 Mastery Test

The final checkpoint. Ten questions, closed book — if you score 9 or 10, this chapter is yours for the whole year.

DAY 30 MASTERY TEST

1. The angles of elevation of a tower's top from points 4 m and 9 m from its base are complementary. Find the height of the tower.

2. A balloon flies 87 m above a girl's eyes; its angle of elevation falls from 60° to 30°. Find the distance it travelled.

3. A car's angle of depression from a tower top changes from 30° to 60° in 6 seconds. How much longer will it take to reach the foot of the tower?

4. Two equal poles stand on either side of an 80 m road; from a point between them the angles of elevation of their tops are 60° and 30°. Find the height of the poles and the point's distances from them.

5. A transmission tower stands on a 20 m building; the angles to its bottom and top from a point are 45° and 60°. Find the height of the tower.

6. From a point 200 m above a lake, a cloud is seen at an elevation of 30° and its reflection at a depression of 60°. Find the height of the cloud.

7. A 10 m ladder, foot fixed, leans to one wall at 60° and is then turned to the opposite wall at 30°. Find the width of the street.

8. State why the angle of depression from a tower top equals the angle of elevation from the object, naming the theorem used.

9. An aeroplane at 3000 m sees two milestones on opposite sides at depressions of 45° and 60°. Find the distance between the milestones.

10. A tower's shadow is 40 m longer when the sun is at 30° than when it is at 60°. Find the height of the tower.

> ***TOPPER TIP:*** *Missed any Day 30 question? That question's Flash Card goes back into your pocket box for one more week.*

Answer Key — Graded Practice, Levels 1 and 2

Level 1 — Foundation

L1.1 25 × tan 45° = **25 m**. **L1.2** 18 × tan 30° = 18/√3 = **6√3 m** ≈ 10.39 m. **L1.3** 40 × sin 30° = **20 m**. **L1.4** 4 / cos 60° = **8 m**. **L1.5** 60 / tan 30° = **60√3 m** ≈ 103.92 m (depression converted to elevation). **L1.6** 45 / tan 60° = **15√3 m** ≈ 25.98 m.

Level 2 — Skill Builder

L2.1 15 × tan 60° = **15√3 m** ≈ 25.98 m.

L2.2 Standing part = 12 tan 30° = 12/√3; broken part = 12 / cos 30° = 24/√3. Total = 36/√3 = **12√3 m** ≈ 20.78 m.

L2.3 Height inside the triangle = 21.5 − 1.5 = 20 m; at 45°, distance = height inside = **20 m**.

L2.4 tan θ = 6√3 / 6 = √3, so θ = **60°**.

L2.5 h = x√3 and h = (x + 10)/√3 give 3x = x + 10, x = 5; h = **5√3 m** ≈ 8.66 m.

L2.6 Distance = pedestal (45°); then p√3 = p + 1, so p = 1/(√3 − 1) = **(√3 + 1)/2 m** ≈ 1.37 m.

TOPPER TIP: *Marked yourself wrong at Level 2? Compare your diagram — not your algebra — with the twin solution in Section 07. In this chapter, almost every error is born in the diagram.*

Answer Key — Levels 3, 4, 5 and Active Recall

Level 3 — Application

L3.1 Width = 30 × tan 60° = **30√3 m** ≈ 51.96 m.

L3.2 h = x√3 = (x + 1000)/√3 gives x = 500; h = **500√3 m** ≈ 866.03 m.

L3.3 Distances: 30/tan 60° = 10√3 and 30/tan 30° = 30√3. Same side → difference = **20√3 m** ≈ 34.64 m.

L3.4 Let the cloud be h above the lake. (h − 200)/(h + 200) = tan 30°/tan 60° = 1/3, so 3h − 600 = h + 200 and h = **400 m**.

Level 4 — Advanced Reasoning

L4.1 h = x√3 = (x + 30)/√3 gives x = 15; h = **15√3 m** ≈ 25.98 m.

L4.2 Opposite sides → ADD: 100/tan 45° + 100/tan 30° = 100 + 100√3 = **100(1 + √3) m** ≈ 273.21 m.

L4.3 Let the gap be d and the tower H. From the building top: H − 20 = d. From the foot: H = d√3. So d√3 = d + 20, d = 10(√3 + 1) ≈ 27.32 m, and H = **30 + 10√3 m** ≈ 47.32 m.

Level 5 — Mathematical Genius

L5.1 Distances: 60/tan 60° = 20√3 and 60/tan 30° = 60√3. Travelled = **40√3 m** ≈ 69.28 m in 120 s. Speed = 40√3/120 = **√3/3 m/s** ≈ 0.58 m/s ≈ **2.08 km/h**.

L5.2 h√3 − h/√3 = 40 → h × (2/√3) = 40 → h = **20√3 m** ≈ 34.64 m.

Active Recall (Section 18)

R1 The angle between the horizontal line through the observer's eye and the line of sight to a point above it. **R2** The same, for a point below the eye. **R3** 20√3 m. **R4** 90 × sin 30° = 45 m. **R5** The two horizontals are parallel and the line of sight is a transversal — alternate interior angles are equal. **R6** 28.5 m — the triangle starts at his eye, 1.5 m above the ground. **R7** 75(√3 − 1) m. **R8** √(4 × 9) = 6 m. **R9** Because the height h appears in both the speed and the remaining distance and cancels — the answer is a pure ratio (3 s). **R10** The 45° angle (to the top of the pedestal) — it gives distance = pedestal height, removing one unknown.

P1 h = d tan θ — it is the definition tan θ = opposite/adjacent with the sides renamed height and distance. **P2** A person's triangle starts at the eye: subtract the observer's height inside the triangle, add it back at the end. **P3** Same side → subtract the two distances; opposite sides → add them. **P4** If the angles from distances a and b are complementary, h = √(ab). **P5** Write the same height in two ways, one from each point, and equate.

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Answer Key — Practice Plus

25 Concept MCQs

1 (b) **2** (b) **3** (b) **4** (a) **5** (b) **6** (c) **7** (c) **8** (b) **9** (b) **10** (b) **11** (c) **12** (b) **13** (b)
14 (b) **15** (b) **16** (b) **17** (b) **18** (a) **19** (b) **20** (b) **21** (b) **22** (c) **23** (b) **24** (b) **25** (b)

Explanations for the tricky ones. Q6: tan θ = height/shadow = 1/√3, so θ = 30°. Q7: tan θ = 10√3/10 = √3, so θ = 60°. Q11: the standing part is opposite the angle (tan), the broken part is the hypotenuse (cos). Q14: the 45° angle to the pedestal top gives distance = pedestal height — always open with it. Q16: tan θ = 50 ÷ (50/√3) = √3, so θ = 60°. Q21: the midpoint is 40 m from each pole, and at 45° height = distance = 40 m. Q24: halving the distance doubles tan θ (height is fixed): 2 × 1 = 2. Q25: depression and elevation are the same angle seen from the two ends (alternate interior angles).

Find the Error — Corrections

E1 tan 30° = 1/√3, not √3 (that is tan 60°). h = 30/√3 = **10√3 m**.

E2 Height is the side OPPOSITE the angle — use sin, not cos. Height = 80 × sin 60° = **40√3 m** ≈ 69.28 m. (40 m is the ground distance, which cos correctly gives.)

E3 The angle of depression is measured from the **horizontal** line through the observer, never the vertical.

E4 The triangle's height is 30 − 1.5 = 28.5 m (eye-height rule), so the distance is **28.5 m**, not 30 m.

E5 h(√3 − 1) = 1.6 means h = 1.6 / (√3 − 1) — DIVIDE. h = 0.8(√3 + 1) ≈ **2.19 m**.

E6 Same side → SUBTRACT: 75√3 − 75 = **75(√3 − 1) m**.

E7 The remaining distance (h/√3) is HALF the distance covered in the first 6 s (2h/√3), so at uniform speed it takes **3 s**, not 6 s.

E8 8/√3 is only the standing part. Add the broken part 16/√3: total = **8√3 m**.

E9 20√3 m is building + tower. The tower alone is 20√3 − 20 = **20(√3 − 1) m**.

E10 Decimal slip: 10 × 1.73 = **17.3 m**, not 173 m. (A 173 m answer for a 10√3 quantity should fail your smell test instantly.)

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Answer Key — Model Tests and Day Tests

Model Test A — Basics (20)

Q1 Elevation: angle from the horizontal at the eye to a point above [1]; depression: to a point below [1]. **Q2** Diagram [1]; tan 45° = h/20 [1]; h = **20 m** [1]. **Q3** sin 30° = h/50 [2]; h = **25 m** [1]. **Q4** cos 60° = d/12 [2]; d = **6 m** [1]. **Q5** Depression converted [1]; tan 30° = 40/d [2]; d = **40√3 m** ≈ 69.28 m [1]. **Q6** Diagram [1]; tan 60° = h/9 [2]; h = **9√3 m** ≈ 15.59 m [2].

Model Test B — Standard (20)

Q1 h = x√3 and h = (x + 20)/√3 [2]; x = 10 [1]; h = **10√3 m** ≈ 17.32 m [1]. **Q2** Standing 10/√3 [1]; broken 20/√3 [2]; total **10√3 m** [1]. **Q3** Distance = pedestal p [1]; p(√3 − 1) = 1.2 [2]; p = **0.6(√3 + 1) m** ≈ 1.64 m [1]. **Q4** Distances 60/√3 = 20√3 and 60√3 [2]; same side, subtract [1]; **40√3 m** ≈ 69.28 m [1]. **Q5** d = 40/√3 = **40√3/3 m** ≈ 23.09 m [2]; building = d/√3 = **40/3 m** ≈ 13.33 m [2].

Model Test C — Advanced (20)

Q1 tan θ = h/a [1]; tan(90° − θ) = h/b [1]; cot identity [1]; h^2 = ab [1]; h = √(ab) ∎ [1]. **Q2** Reflection is as far below the lake as the cloud is above: (h − 100)/(h + 100) = tan 30°/tan 60° = 1/3 [3]; h = **200 m** [2]. **Q3** Distances 60√3 and 20√3 [2]; covered = **40√3 m** ≈ 69.28 m [1]; speed = 40√3/4 = **10√3 m/s** ≈ 17.32 m/s [2]. **Q4** From the depression: distance = 25 m [2]; upper part = 25 × tan 45° = 25 [1]; other tower = 25 + 25 = **50 m** [2].

Day 7 Test

1 15 m **2** 15 m **3** 12√3 m ≈ 20.78 m **4** 30 m **5** 4 m **6** 10√3 m **7** 30° **8** 45° **9** 40/ sin 60° = 80√3/3 m ≈ 46.19 m **10** 6/√3 = 2√3 m ≈ 3.46 m

Day 21 Test

1 h = 15 + 5√3 m ≈ 23.66 m (near distance 5(√3 + 1)) **2** (√3 + 1) m ≈ 2.73 m **3** 50 + 50√3 m ≈ 136.60 m **4** 10√3 − 10/√3 = 20√3/3 m ≈ 11.55 m **5** 25 m **6** 12√3 m ≈ 20.78 m **7** 45√2 m ≈ 63.64 m **8** They are equal (alternate interior angles) **9** 20 m (distance 10 m from the 45° depression; upper part 10 m; total 10 + 10) **10** √(16 × 25) = 20 m

Day 30 Mastery Test

1 6 m **2** 58√3 m ≈ 100.46 m **3** 3 seconds **4** Height 20√3 m ≈ 34.64 m; distances 20 m and 60 m **5** 20(√3 − 1) m ≈ 14.64 m **6** 400 m **7** 5 + 5√3 m ≈ 13.66 m **8** The horizontals are parallel and the line of sight is a transversal — alternate interior angles are equal **9** 3000 + 1000√3 m ≈ 4732 m **10** 20√3 m ≈ 34.64 m

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Revision Card — Chapter 9 on One Page

The two definitions

Angle of elevation: from the horizontal at the eye, looking UP. **Angle of depression:** from the horizontal at the eye, looking DOWN. **Master key:** depression (from top) = elevation (from object) — alternate interior angles, horizontals parallel.

The one engine

tan θ = height / distance — so h = d tan θ and d = h / tan θ. Slanted length (rope, string, ladder) in the story? The hypotenuse has entered: use sin (height) or cos (distance). Values: tan 30° = 1/√3, tan 45° = 1, tan 60° = √3.

The two-triangle machines

Two points, one height: write h two ways and equate. **Stacked objects:** lower angle first, total second, subtract (or add the eye-level piece back). **Observer on a height:** split the other object at eye level — depression finds the distance, elevation finds the part above.

Gift patterns

45°: height = distance. **30°/60° pair, gap d:** near distance d/2, height d√3/2. **Complementary angles:** h = √(ab). **Same side:** subtract distances. **Opposite sides:** add. **No height given:** keep h — it cancels.

Exam-eve checklist

☐ Diagram drawn for EVERY question, angle at the eye from the horizontal ☐ Eye height subtracted (Q6/Q14 pattern) ☐ Depression converted with the alternate-angles line written ☐ Answer is the piece asked for — not the whole ☐ √3 kept symbolic, denominator rationalised

Chapter 9 complete. You did not memorise heights and distances — you discovered, visualised, derived and mastered them.

Next: Chapter 10 — Circles, in the same 8D format. • KalviYogi • Mathematics – Beyond the Formula • 369 Tesla × KalviYogi • © Edunivra Pvt Ltd

CBSE · CLASS 10 · MATHEMATICS

MATHEMATICS BEYOND THE FORMULA

CHAPTER 10 · CIRCLES

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01 Chapter Introduction

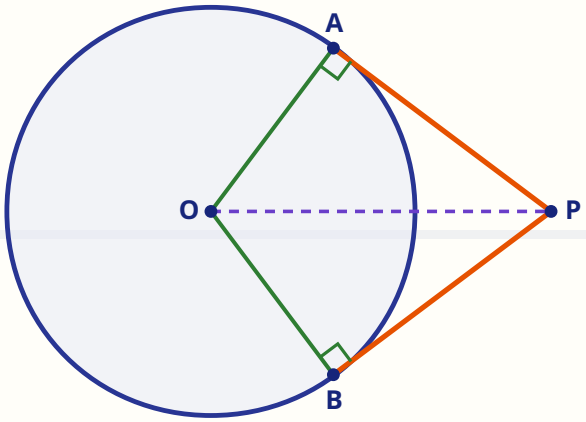
8D • DISCOVER — WHY SHOULD I LEARN THIS?

Run your finger along the edge of a coin. Now lay a ruler so that it **just touches** the coin at one point and slides past. That ruler is doing the most important job in this chapter — it is a **tangent**. A bicycle chain gripping a sprocket, a rope grazing a pulley, a road just touching a roundabout: the world is full of lines that touch a circle at **exactly one point** and then move on.

This is the shortest chapter in your book — and one of the most generous in the exam. It stands on just **two theorems**. The first says a tangent always meets the radius at a perfect right angle. The second says the two tangents drawn from one outside point are always equal. Master those two ideas — with their full proofs — and every question in this chapter, from the NCERT exercises to the board paper, becomes a variation you have already seen.

What you will master in this chapter

- What a **tangent** is, what a **secant** is, and the **point of contact**.
- How many tangents can be drawn from a point — inside, on, and outside the circle.
- **Theorem 10.1:** the tangent at any point of a circle is perpendicular to the radius through the point of contact — proved in full.
- **Theorem 10.2:** the lengths of tangents drawn from an external point to a circle are equal — proved in full.
- The golden right triangle hiding in every tangent problem: $\text{tangent}^2 = (\text{distance})^2 - (\text{radius})^2$.
- Circles inside quadrilaterals: why AB + CD = AD + BC for a circumscribing quadrilateral.
- Board-style proofs — parallel tangents, the 90° theorem, the rhombus theorem.



The whole chapter in one picture: from the outside point P, two tangents PA and PB touch the circle at A and B. By the end of Section 06 you will be able to prove PA = PB and read off every angle in this figure.

Your Chapter Map — 19 Stops, One Journey

This chapter is not a list of formulas. It is a journey with 19 stops. Tick each stop as you finish it.

Discover & Visualise	Understand & Derive	Solve, Apply, Recall
☐ 01 Chapter Introduction ☐ 02 Prerequisite Knowledge ☐ 03 Real-Life Story ☐ 04 Visual Concepts (CPA)	☐ 05 Mathematical Foundations ☐ 06 Formula Derivations ☐ 08 Multiple Solution Methods ☐ 10 Common Mistakes	☐ 07 Detailed Solved Examples ☐ 09 Hands-on Activities ☐ 11 Graded Practice (5 Levels) ☐ 12 Mathematical Reasoning
☐ 13 Real-Life Applications ☐ 14 Competency-Based Questions	☐ 15 Olympiad Challenges ☐ 16 Chapter Mind Map	☐ 17 Pin-Point Flash Cards ☐ 18 Active Recall ☐ 19 Mastery Tracker

How to use this chapter (read this first!)

1. **Never read a solution passively.** Cover the steps, try yourself, then uncover one step at a time.
2. Every solution shows **Step 1, Step 2, Step 3...** with a small *Why* note in purple — the reason behind the step. The board examiner gives marks for these steps, not just the final answer.
3. After Section 07, test yourself in Sections 11–15 **before** looking at the Answer Key.
4. Finish with the Mind Map (16), the Flash Cards (17), the closed-book Recall (18) and your Mastery Tracker (19). Come back on Day 3, 7, 14, 21 and 30 — the Smart Memory schedule on the tracker page tells you exactly what to do each day.

TOPPER TIP: *This chapter rewards figure-first thinking. In EVERY problem, draw the circle, mark the centre O, join O to each point of contact, and mark the right angles before you write a single number. Half the solution is the figure.*

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02 Prerequisite Knowledge

CLASS 9 RECAP — THE TOOLS YOU ALREADY OWN

Circles in Class 10 use no new machinery — only five tools you already own from Classes 9 and earlier. Refresh them here.

1. Circle words from Class 9

The **centre** O is the fixed middle point. A **radius** joins the centre to any point on the circle; all radii of one circle are equal. A **chord** joins two points on the circle; the longest chord, passing through the centre, is the **diameter** = 2 × radius. The perpendicular from the centre to a chord **bisects** the chord — you will use this in Exercise 10.2 Q7.

2. Pythagoras theorem

In a right-angled triangle, $(\text{hypotenuse})^2 = (\text{side 1})^2 + (\text{side 2})^2$. The hypotenuse is the side opposite the right angle — always the **longest** side. Remember the friendly triples: 3-4-5, 5-12-13, 6-8-10, 7-24-25, 8-15-17, 9-12-15. This chapter is built on them.

3. RHS congruence

Two right-angled triangles are congruent if the **hypotenuse** and **one side** of one equal the hypotenuse and one side of the other (RHS rule). Matching parts of congruent triangles are equal (CPCT). This is the engine of Theorem 10.2.

4. Angles of a quadrilateral

The four interior angles of any quadrilateral add up to **360°**. If two of them are right angles, the other two must add up to 180° — they are **supplementary**. Watch this fact prove Exercise 10.2 Q10 almost by itself.

5. Parallel lines

Two lines perpendicular to the **same** line are parallel to each other. Also, when a transversal cuts parallel lines, alternate interior angles are equal — and conversely, equal alternate angles prove lines parallel. Both facts appear in this chapter's proofs.

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Quick Check — Are You Ready? (5 Questions)

Answer these in your notebook before you move on. Answers are at the bottom of this page — try first!

Q1. A right-angled triangle has legs 9 cm and 12 cm. Find its hypotenuse.

Q2. In quadrilateral ABCD, $\angle A = 90°$, $\angle B = 90°$ and $\angle C = 110°$. Find $\angle D$.

Q3. State the RHS congruence rule in one sentence.

Q4. The radius of a circle is 7 cm. A chord is drawn and the perpendicular from the centre meets the chord at its midpoint. If half the chord is 24 cm... wait — can that happen? Answer yes or no, with a reason.

Q5. Lines l and m are both perpendicular to line n. What is the relation between l and m?

Quick Check — Answers

Q1. Hypotenuse = $\sqrt{(9^2 + 12^2)} = \sqrt{(81 + 144)} = \sqrt{225} =$ **15 cm**.
Q2. $\angle D = 360° - 90° - 90° - 110° =$ **70°**.
Q3. Two right-angled triangles are congruent if their hypotenuses and one pair of corresponding sides are equal.
Q4. No. Half the chord cannot exceed the radius: the half-chord, the perpendicular distance and the radius form a right triangle in which the radius (7 cm) is the hypotenuse, so the half-chord must be *less* than 7 cm. (Keep this picture — it returns as the concentric-circles problem.)
Q5. l and m are **parallel** to each other.

TOPPER TIP: *If Q1 or Q2 slowed you down, spend ten minutes on Pythagoras and quadrilateral angles first. Every proof in Section 06 stands on these two legs.*

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03 Real-Life Mathematical Story

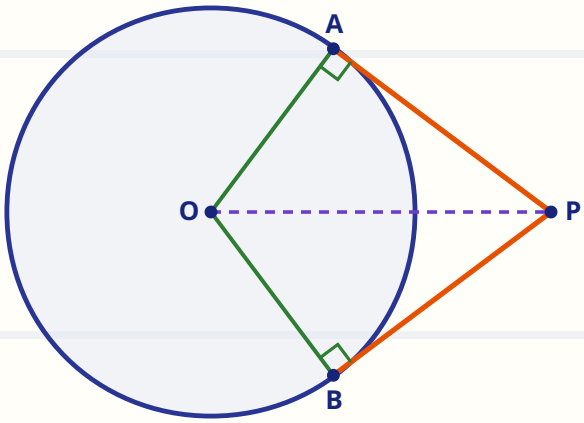
DISCOVER • VISUALISE — THE WATER TANK AND THE TWO WIRES

*The municipal water tank in Kavitha's street is a perfect circle when seen from above — a huge cylinder with a round top rim. The tank's radius is **6 metres**. One windy evening, the tank's solar lamp went dark, and electrician **Imran** was called to run a cable from the lamp post to the tank.*

*The lamp post stands on the ground at point P, exactly **10 metres** from the centre O of the tank. Imran cannot run the cable across the tank — it must run straight from P and **just graze the rim** of the tank, touching it at one point, and stop there. "If the cable merely touches the tank," he says, "how much cable do I need from P to the touching point?"*

*Kavitha, who loves puzzles, asks a second question. "Uncle, you could graze the tank on the **left side** or the **right side**. Would the two cables be different lengths? Should we measure both?" Imran smiles: "Measure one. Mathematics will give you the other for free."*

By the end of Section 06 you will know why he is right — and in Section 07 you will meet this exact story again, wearing exam clothes.



The tank (circle, radius 6 m), the lamp post P (10 m from centre O), and the two grazing cables PA and PB. Is PA really equal to PB? And how long is each?

Solving the Story — Problem 1: How Much Cable?

STORY PROBLEM 1

The cable from P grazes the tank at A. The tank's radius is 6 m and OP = 10 m. Find the cable length PA.

THINK FIRST: A grazing cable, a radius, and a right angle — wherever a tangent appears, a right triangle is hiding. Find it, and Pythagoras finishes the job.

1. Join O to A. The radius OA goes to the exact point where the cable touches the tank.

   Why: Always join the centre to the point of contact — this is the master move of the whole chapter.

2. OA is perpendicular to the cable PA, so ∠OAP = 90°.

   Why: This is Theorem 10.1 — you will prove it in Section 06. For now, feel how natural it is: the shortest line from O to the cable is the perpendicular, and that shortest distance is exactly the radius.

3. So triangle OAP is right-angled at A, with hypotenuse OP = 10 m and side OA = 6 m.

   Why: The hypotenuse is opposite the right angle — it must be the longest side, and 10 > 6. Good.

4. Apply Pythagoras: $OP^2 = OA^2 + PA^2$, that is, $100 = 36 + PA^2$.

5. $PA^2 = 100 - 36 = 64$, so $PA = \sqrt{64} =$ **8 m**.

   Why: Length is positive, so we take the positive root.

ANSWER: PA = 8 m of cable.

PIN POINT: Every tangent problem hides a right triangle: radius ⊥ tangent at the point of contact. Join O to the contact point FIRST — then use Pythagoras: $(\text{tangent})^2 = (\text{distance from centre})^2 - (\text{radius})^2$.

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Solving the Story — Problem 2: The Two Cables

STORY PROBLEM 2

Imran runs a second cable from the same post P, grazing the tank on the other side at B. Without measuring, find PB — and say why your answer must be correct.

THINK FIRST: Imran's free gift is Theorem 10.2: tangents from one external point are equal. Notice we did not measure PB at all — congruence proved it.

1. Look at the two triangles OAP and OBP in the figure. Both are right-angled: ∠OAP = ∠OBP = 90°.

   Why: Theorem 10.1 again — each radius is perpendicular to its tangent.

2. OA = OB = 6 m.

   Why: Radii of the same circle are equal.

3. OP is the hypotenuse of both triangles, and it is the same segment.

   Why: A shared hypotenuse is half of the RHS rule already.

4. So △OAP ≅ △OBP by the RHS congruence rule.

   Why: Right angle + Hypotenuse + Side — exactly the tool from Section 02.

5. Therefore PB = PA (CPCT — corresponding parts of congruent triangles).

   Why: Congruent triangles are identical in every measurement.

6. From Problem 1, PA = 8 m, so PB = **8 m**.

ANSWER: PB = 8 m — the two grazing cables are exactly equal.

The story in one breath

A grazing line is a **tangent**. The radius to the touching point is **perpendicular** to it (Theorem 10.1). Two tangents from the same outside point are **equal** (Theorem 10.2). Everything else in this chapter — all 17 NCERT questions, every board proof — is these two sentences at work. Section 05 defines the words precisely; Section 06 proves the two theorems completely.

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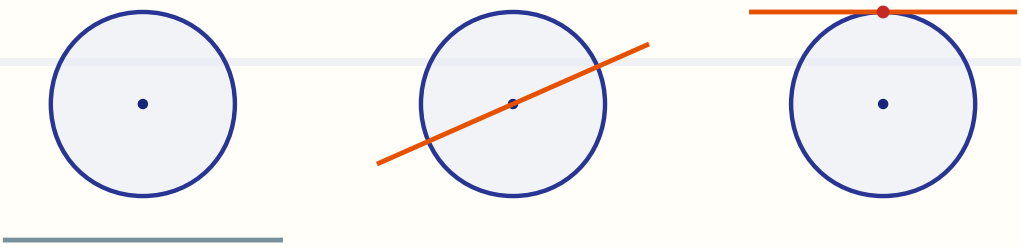
04 Visual Concept Explanation

SINGAPORE CPA — CONCRETE → PICTORIAL → ABSTRACT

Concrete. Take a bangle or a coin and a ruler. Hold the ruler so it does not touch the circle at all. Now slide it closer until it cuts across the circle. Slide it a little more, until it *just kisses* the circle and slips past. You have physically acted out the three possible meetings of a line and a circle:



No common point | Secant — 2 points | Tangent — 1 point

Pictorial — read the three pictures

No common point: the line misses the circle; its distance from the centre is *more* than the radius.
Secant: the line cuts the circle at **two** points; its distance from the centre is *less* than the radius.
Tangent: the line touches at **exactly one** point; its distance from the centre is *exactly equal* to the radius — and that distance is measured along the perpendicular, which is the radius itself.

Abstract — the same story in symbols

Let a line be at perpendicular distance d from the centre of a circle of radius r. Then:
$d > r \Rightarrow$ no meeting • $d < r \Rightarrow$ secant (2 points) • $d = r \Rightarrow$ tangent (1 point).
This single comparison — distance versus radius — is the seed from which Theorem 10.1 grows in Section 06.

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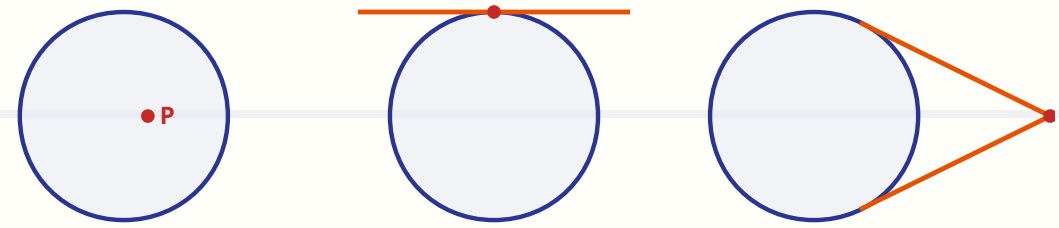
Visual Concepts — Tangents from a Point

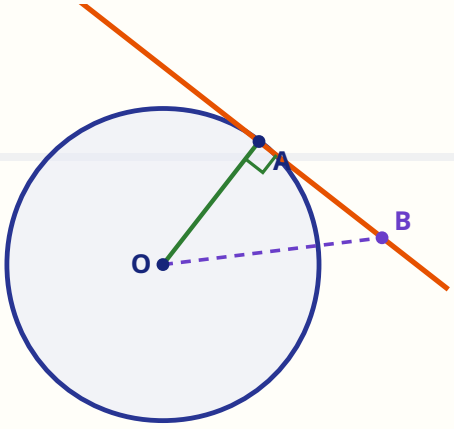
Now fix the line's job and move the **point** instead. From where can tangents be drawn, and how many?



Inside — 0 tangents | On the circle — 1 tangent | Outside — 2 tangents

The three positions of a point

Point inside the circle: every line through it must cross the circle twice — it is always a secant. **No tangent** can be drawn. (0 tangents)
Point on the circle: exactly **one** tangent — the line perpendicular to the radius at that point. (1 tangent)
Point outside the circle: exactly **two** tangents, one grazing each side — and Section 06 will prove they are always **equal in length**. (2 tangents)



Look closely: OA is the radius to the point of contact A, and the tangent stands at a perfect right angle to it. The dashed line OB reaches another point B of the tangent — and OB is visibly LONGER than OA. That one observation is the whole proof of Theorem 10.1.

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05 Mathematical Foundations

UNDERSTAND — THE PRECISE IDEAS, WORD BY WORD

Definition 1 — Tangent

A **tangent** to a circle is a line that intersects the circle at **exactly one point**. The word comes from the Latin *tangere* — "to touch". A tangent does not enter the circle at all; it only touches and passes by.

Definition 2 — Point of contact

The single common point of the tangent and the circle is called the **point of contact**. In every figure of this chapter, the point of contact is where the magic happens: the radius to this point is perpendicular to the tangent.

Definition 3 — Secant

A **secant** is a line that intersects the circle at **two distinct points**. The segment of a secant lying inside the circle is a chord. Every secant can be slid away from the centre until it becomes a tangent — the moment its distance from the centre equals the radius.

Definition 4 — Length of a tangent

When a tangent is drawn from an external point P to touch the circle at A, the segment PA is called the **length of the tangent from P**. It is a segment, not the whole infinite line — so it can be measured, compared and proved equal to its twin PB.

Foundation facts (proved in Section 06)

1. A circle can have **infinitely many** tangents — one at each of its infinitely many points.
2. At most **two** tangents can be parallel — they stand at the two ends of a diameter.
3. The tangent at a point is the **unique** line perpendicular to the radius at that point.

PIN POINT: Tangent = 1 common point. Secant = 2 common points. If a problem says a line "touches" the circle, it is a tangent — and the radius to the touching point is already perpendicular to it, whether the figure shows it or not.

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Foundations — The Position of a Point, Made Precise

Let P be any point and let OP = d, where O is the centre and r is the radius. The whole tangent story of P is decided by comparing d with r:

Position of P	Comparison	Tangents from P	Why
Inside the circle	d < r	**None (0)**	Every line through P crosses the circle at two points — always a secant.
On the circle	d = r	**Exactly 1**	Only the line perpendicular to the radius at P touches without entering.
Outside the circle	d > r	**Exactly 2**	One tangent grazes each side; the two are mirror images in the line OP.

The length formula (preview — derived in Section 06)

For an external point P, the radius to the contact point A is perpendicular to the tangent, so triangle OAP is right-angled at A and Pythagoras gives:
PA = √(OP² − OA²) = √(d² − r²).
Notice the minus sign: the distance OP is the hypotenuse, so it is the radius that gets subtracted. Section 06 derives this properly; Section 10 shows what happens to students who write a plus instead.

TOPPER TIP: *Exam questions love the comparison table in disguise: 'the point (distance) … can tangents be drawn?' Always answer in two beats — compare d with r, then state the count.*

06 Formula Derivations

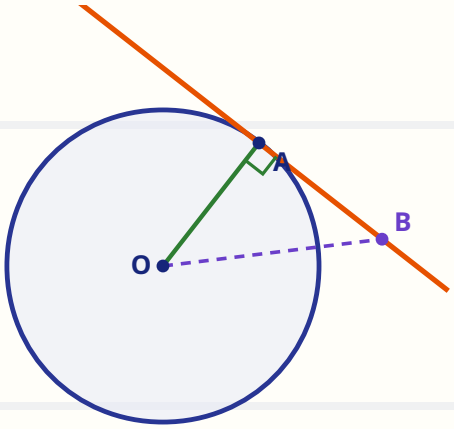
8D • DERIVE — THE TWO THEOREMS, PROVED COMPLETELY

This chapter has only two theorems, and both are proved here in full — every step justified. Learn the **logic**, not the lines: each proof is a small story with a beginning (what is given), a middle (one clever idea) and an end (what was to be proved).

Theorem 10.1 — The tangent is perpendicular to the radius

THEOREM 10.1

The tangent at any point of a circle is perpendicular to the radius through the point of contact.



Given: a circle with centre O; a tangent line touching it at point A. **To prove:** OA ⊥ tangent at A.

The clever idea: the perpendicular from a point to a line is the *shortest* distance from that point to the line. If we show OA is the shortest of all segments from O to the tangent line, then OA must be that perpendicular.

Theorem 10.1 — The Proof, Step by Step

PROOF OF THEOREM 10.1

The tangent at any point of a circle is perpendicular to the radius through the point of contact.

THINK FIRST: Notice the strategy: to prove a line is perpendicular, prove it is the SHORTEST. Whenever a board question asks for this theorem, this shortest-distance argument is what the examiner wants.

1. Take any point B on the tangent line, different from A. B lies **outside** the circle.
   Why: The tangent meets the circle only at A — every other point of the line is outside. That is the definition of a tangent doing real work.

2. Therefore OB > OA.
   Why: OA is a radius; a point outside the circle is, by definition, farther from the centre than the radius.

3. This holds for EVERY point B of the tangent except A itself. So OA is **shorter than every other segment** drawn from O to the line.
   Why: We compared OA with all rivals at once — B was any point, not a chosen one.

4. But the shortest segment from a point to a line is the **perpendicular** segment.
   Why: A Class 9 fact: of all segments from a point to a line, the perpendicular one is the shortest.

5. Hence OA is the perpendicular from O to the tangent line — that is, OA ⊥ tangent at A. **Proved.**
   Why: Shortest ⇒ perpendicular. The proof is complete: no other property of circles was needed.

ANSWER: OA ⊥ tangent at the point of contact — for EVERY point of EVERY circle.

PIN POINT: Theorem 10.1 is a licence to write ∠ = 90°. The moment you join the centre to a point of contact, you may write "radius ⊥ tangent" and claim a right angle — and with it, Pythagoras.

TOPPER TIP: *Two corollaries worth their own line in the exam: (1) at any point of the circle there is ONE and only one tangent; (2) the perpendicular to a tangent at its point of contact passes through the centre (Exercise 10.2 Q5 makes you prove this).*

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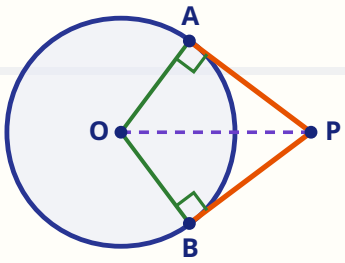
Theorem 10.2 — Tangents from an External Point are Equal

THEOREM 10.2

The lengths of tangents drawn from an external point to a circle are equal.



Given: centre O, external point P, tangents PA and PB. **To prove:** PA = PB. **Construction:** join OA, OB and OP.

PROOF OF THEOREM 10.2

The lengths of tangents drawn from an external point to a circle are equal.

THINK FIRST: One congruence settles everything — and it also gives the three bonus facts derived on the next page.

1. ∠OAP = 90° and ∠OBP = 90°.
   Why: Theorem 10.1 – each radius is perpendicular to its tangent at the point of contact. Yesterday's theorem is today's tool.
2. In right-angled triangles OAP and OBP: hypotenuse OP = OP (common), and side OA = OB (radii of the same circle).
   Why: The shared hypotenuse plus one equal side – two of the three RHS ingredients in one line.
3. So △OAP ≅ △OBP by the RHS congruence rule.
   Why: Right angle, Hypotenuse, Side – all three ingredients are on the table.
4. Therefore PA = PB (CPCT). **Proved.**
   Why: Corresponding parts of congruent triangles are equal – and PA, PB are corresponding parts.

ANSWER: PA = PB — the two tangents from any external point are always equal.

PIN POINT: Same external point ⇒ same tangent length. In any figure, the FIRST thing to mark is the pair of equal tangent segments from each outside corner — most circumscribed-figure problems collapse the moment you mark them.

Derivations — The Three Bonus Facts (Corollaries)

COROLLARY A

In the figure of Theorem 10.2, prove that OP bisects ∠APB and also bisects ∠AOB.

THINK FIRST: This is why, when the two tangents are inclined at 80°, each half-angle at the centre (like ∠POA) can be found from half of ∠AOB — Exercise 10.2 Q3 is built on exactly this.

1. From the proof of Theorem 10.2, △OAP ≅ △OBP (RHS).
   Why: We reuse the congruence — no new work needed.
2. ∠APO = ∠BPO (CPCT).
   Why: These are the two halves of ∠APB, so OP bisects ∠APB.
3. ∠AOP = ∠BOP (CPCT).
   Why: These are the two halves of ∠AOB, so OP bisects ∠AOB too.

ANSWER: OP bisects both the angle between the tangents and the angle at the centre.

COROLLARY B

Prove that ∠APB + ∠AOB = 180° (the angle between the tangents is supplementary to the angle at the centre).

THINK FIRST: This corollary IS Exercise 10.2 Q10 — you have just proved an NCERT question inside the derivation section. That is the KalviYogi way: derive once, harvest many times.

1. In quadrilateral OAPB, ∠OAP = 90° and ∠OBP = 90°.
   Why: Theorem 10.1, twice.
2. The four angles of a quadrilateral sum to 360°: ∠OAP + ∠OBP + ∠AOB + ∠APB = 360°.
   Why: Section 02's tool, used exactly as promised.
3. Substitute the two right angles: 90° + 90° + ∠AOB + ∠APB = 360°, so ∠AOB + ∠APB = 180°. **Proved.**
   Why: The two right angles eat exactly half the 360°; the remaining two angles must share the other half.

ANSWER: ∠APB + ∠AOB = 180° — always.

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Derivations — Corollary C: The Length Formula

COROLLARY C — THE LENGTH FORMULA

Derive the formula for the length of a tangent from an external point.

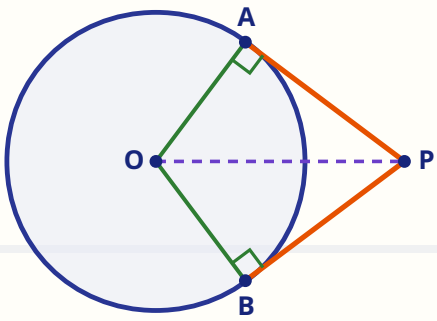
THINK FIRST: Every numerical in this chapter is this one line wearing different numbers. If you ever forget it in the exam, redraw the right triangle and re-derive it in ten seconds.

1. In right-angled △OAP (right angle at A), the hypotenuse is OP.
   Why: The hypotenuse is opposite the right angle — it is the distance d from the centre to P.

2. Pythagoras: $OP^2 = OA^2 + PA^2$.

3. Rearrange: $PA^2 = OP^2 - OA^2$, so **$PA = \sqrt{(d^2 - r^2)}$**.
   Why: The minus sign is forced: a leg is always shorter than the hypotenuse, so $d^2 - r^2$ must be the leg squared.

ANSWER: Length of tangent = $\sqrt{(d^2 - r^2)}$, where d = distance of the point from the centre, r = radius.



The one triangle behind every numerical of this chapter: radius r, distance d, tangent $\sqrt{(d^2 - r^2)}$.

07 Detailed Solved Examples

8D • SOLVE — NCERT EXERCISES 10.1 & 10.2, EVERY STEP SHOWN

All 17 NCERT questions of this chapter — 4 from Exercise 10.1 and 13 from Exercise 10.2 — solved completely, one step at a time. In Exercises 10.2 Q1–Q3, NCERT asks you to **choose the correct option and give justification**: the justification IS the solution, so write it fully.

Exercise 10.1 Q1 — How Many Tangents?

EXERCISE 10.1 Q1

How many tangents can a circle have?

THINK FIRST: One tangent lives at every point of the circle. Count the points, and you have counted the tangents.

1. At any one point of a circle, exactly one tangent can be drawn.
   Why: The tangent at a point is the unique line perpendicular to the radius there (Section 06, Corollary 1).
2. A circle has infinitely many points on it.
   Why: The circumference is an unbroken curve — between any two points there is always another.
3. One tangent at each of infinitely many points gives infinitely many tangents.

ANSWER: A circle can have infinitely many tangents.

CBSE-STYLE MARKING SCHEME: 1 mark: one tangent at each point + infinitely many points on a circle.

Exercise 10.1 Q2 — Fill in the Blanks

EXERCISE 10.1 Q2

Fill in the blanks: (i) A tangent to a circle intersects it in _____ point(s). (ii) A line intersecting a circle in two points is called a _____. (iii) A circle can have _____ parallel tangents at the most. (iv) The common point of a tangent to a circle and the circle is called _____.

THINK FIRST: These are the four definitions of Section 05, asked in reverse.

1. (i) A tangent touches, it never cuts: it intersects the circle in exactly **one** point.

   Why: By definition of a tangent.

2. (ii) A line meeting the circle in two points is a **secant**.

   Why: By definition of a secant.

3. (iii) Parallel tangents must stand at the two ends of a diameter — a circle has only two such ends facing each other, so at most **two**.

   Why: Any third line parallel to them would be at a distance different from r from the centre – a secant or a miss.

4. (iv) The common point is the **point of contact**.

   Why: By definition.

ANSWER: (i) one (ii) secant (iii) two (iv) point of contact.

CBSE-STYLE MARKING SCHEME: ½ mark for each correct blank (2 marks total).

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Exercise 10.1 Q3 — The First Right Triangle

EXERCISE 10.1 Q3

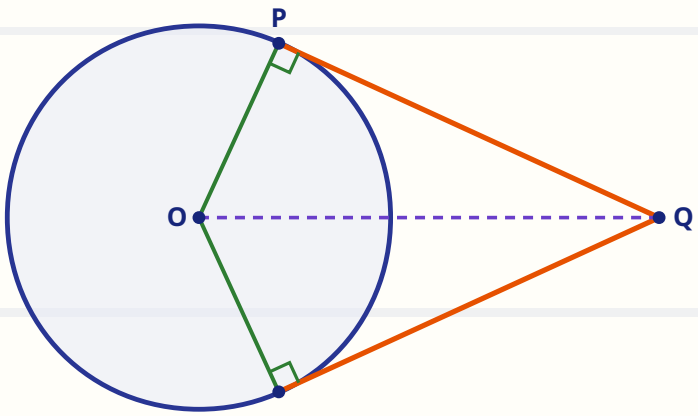
A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is: (A) 12 cm (B) 13 cm (C) 8.5 cm (D) √119 cm

THINK FIRST: Join O to P. The radius to the point of contact makes a right angle with the tangent — after that it is pure Pythagoras.

1. Join OP. OP = 5 cm (radius) and ∠OPQ = 90°.
   Why: Theorem 10.1: the radius through the point of contact is perpendicular to the tangent.
2. So △OPQ is right-angled at P, with hypotenuse OQ = 12 cm.
   Why: The hypotenuse is opposite the right angle.
3. Pythagoras: $OQ^2 = OP^2 + PQ^2$, that is, $144 = 25 + PQ^2$.
4. $PQ^2 = 144 - 25 = 119$, so PQ = √119 cm.
   Why: √119 cannot be simplified (119 = 7 × 17, no square factor) — leave it as a root.

ANSWER: Option (D) — PQ = √119 cm.

CBSE-STYLE MARKING SCHEME: 1 mark: ½ for the right angle (Theorem 10.1) + ½ for the Pythagoras computation.



Exercise 10.1 Q4 — A Drawing Question

EXERCISE 10.1 Q4

Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

THINK FIRST: Parallel lines are all at fixed distances from one another. Choose the distances cleverly: one line at distance exactly r from the centre (tangent), one at distance less than r (secant).

1. Draw the given line l, and draw a circle whose centre O is at a distance less than the radius from l — so l itself cuts the circle. (Or: draw the circle first, then the line l passing near it.)

   Why: We need room for two lines parallel to l on the same side.

2. Draw line m parallel to l, passing at distance exactly equal to the radius from O. Line m touches the circle at exactly one point.

   Why: Distance from centre = radius ⇒ tangent (Section 04's comparison).

3. Draw line n parallel to l, between l and m, at a distance less than the radius from O. Line n cuts the circle at two points.

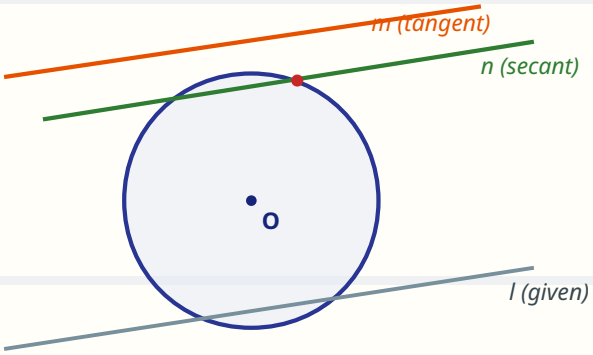
   Why: Distance from centre < radius ⇒ secant.

4. Label the point of contact of m, and the two intersection points of n.

   Why: A labelled figure is part of the answer in a drawing question.

ANSWER: Line m is the required tangent and line n is the required secant, both parallel to the given line l.

CBSE-STYLE MARKING SCHEME: 1 mark: ½ for the tangent at distance r + ½ for the secant at distance less than r, with a neat labelled figure.



One valid drawing: m just touches the circle; n cuts it at two points; both parallel to l.

Exercise 10.2 Q1 — Find the Radius

EXERCISE 10.2 Q1

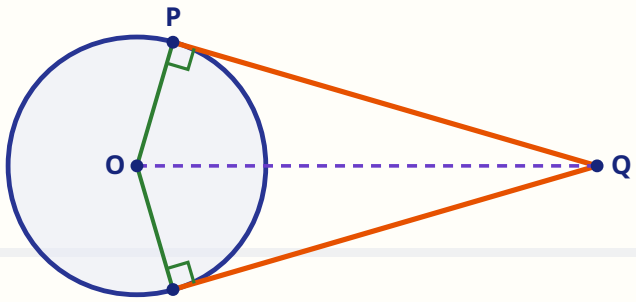
From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is: (A) 7 cm (B) 12 cm (C) 15 cm (D) 24.5 cm

THINK FIRST: This is Corollary C of Section 06 with the unknown moved: $r = \sqrt{(d^2 - \text{tangent}^2)}$.

1. Let the tangent from Q touch the circle at P, and let O be the centre. Join OP.
   Why: Name the contact point so the figure can be talked about.
2. $\angle OPQ = 90°$, so $\triangle OPQ$ is right-angled at P with hypotenuse OQ = 25 cm.
   Why: Theorem 10.1.
3. Pythagoras: $OQ^2 = OP^2 + PQ^2$, that is, $625 = OP^2 + 576$.
   Why: PQ = 24 cm is the tangent length, OP is the unknown radius.
4. $OP^2 = 625 - 576 = 49$, so OP = 7 cm.
   Why: Notice 7-24-25 — a Pythagoras triple from Section 02.

ANSWER: Option (A) — the radius is 7 cm.

CBSE-STYLE MARKING SCHEME: 1 mark: ½ for Theorem 10.1 (right angle) + ½ for the computation.



Exercise 10.2 Q2 — The Supplementary Pair

EXERCISE 10.2 Q2

In the figure, TP and TQ are the two tangents to a circle with centre O so that ∠POQ = 110°, then ∠PTQ is equal to: (A) 60° (B) 70° (C) 80° (D) 90°

THINK FIRST: Quadrilateral OPTQ has two right angles built in. The four angles must total 360° — only ∠PTQ is unknown.

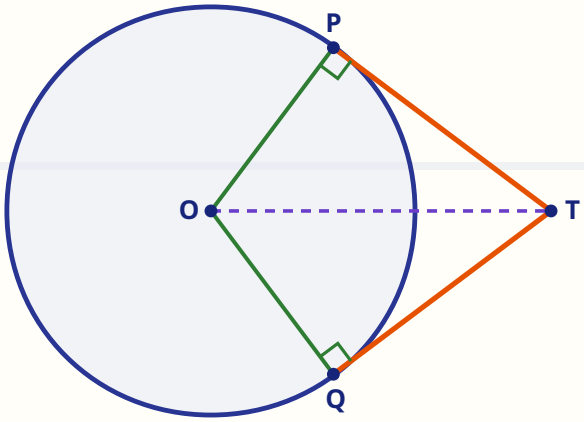
1. ∠OPT = 90° and ∠OQT = 90°.
   Why: Theorem 10.1 at both points of contact.

2. In quadrilateral OPTQ: ∠OPT + ∠OQT + ∠POQ + ∠PTQ = 360°.
   Why: Angle sum of a quadrilateral (Section 02).

3. Substitute: 90° + 90° + 110° + ∠PTQ = 360°, so ∠PTQ = 360° − 290° = 70°.
   Why: This is Corollary B of Section 06 in action: the two angles are supplementary, 180° − 110° = 70°.

ANSWER: Option (B) — ∠PTQ = 70°.

CBSE-STYLE MARKING SCHEME: 1 mark: ½ for the two right angles + ½ for the angle sum.



Exercise 10.2 Q3 — Half of the Central Angle

EXERCISE 10.2 Q3

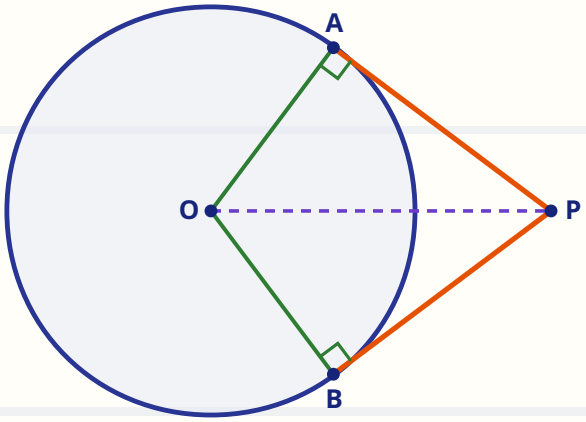
If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80°, then ∠POA is equal to: (A) 50° (B) 60° (C) 70° (D) 80°

THINK FIRST: Two facts chained: first find ∠AOB from the quadrilateral, then halve it — because OP bisects ∠AOB (Corollary A).

1. ∠OAP = ∠OBP = 90°, so in quadrilateral OAPB: ∠AOB + ∠APB = 180°.
   Why: Corollary B — or repeat the 360° angle sum.
2. ∠AOB = 180° − 80° = 100°.
3. OP bisects ∠AOB, because △OAP ≅ △OBP (RHS) gives ∠POA = ∠POB (CPCT).
   Why: This is Corollary A of Section 06.
4. Therefore ∠POA = ½ × 100° = 50°.

ANSWER: Option (A) — ∠POA = 50°.

CBSE-STYLE MARKING SCHEME: 1 mark: ½ for ∠AOB = 100° + ½ for halving via the bisector.



TOPPER TIP: *Q2 and Q3 are the same skeleton: two right angles + quadrilateral sum + (for Q3) one bisection. When you see 'tangents inclined at angle x', immediately write ∠AOB = 180° − x.*

Exercise 10.2 Q4 — Tangents at the Ends of a Diameter

EXERCISE 10.2 Q4

Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

THINK FIRST: Both tangents are perpendicular to the SAME line — the diameter. Two lines perpendicular to one line are parallel (Section 02, tool 5).

1. Let AB be a diameter of the circle with centre O, and let the tangents at A and B be the lines p and q.
   Why: Set up names for the two tangents.

2. Radius OA is part of the diameter line AB, and p is the tangent at A, so p ⊥ AB.
   Why: Theorem 10.1: tangent ⊥ radius through the point of contact — and the radius OA lies along AB.

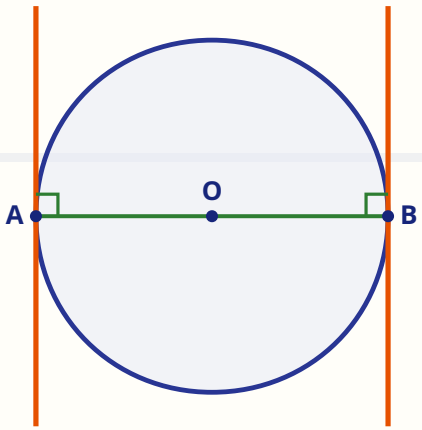
3. Similarly, q is the tangent at B and OB lies along AB, so q ⊥ AB.
   Why: Theorem 10.1 again, at the other end.

4. Lines p and q are both perpendicular to the same line AB.
   Why: This is exactly the hypothesis of the parallel-lines fact.

5. Therefore p ∥ q. **Proved.**
   Why: Two lines perpendicular to the same line are parallel to each other.

ANSWER: The tangents at the ends of any diameter are parallel.

CBSE-STYLE MARKING SCHEME: 2 marks: 1 for applying Theorem 10.1 at both ends + 1 for the parallel conclusion.



Exercise 10.2 Q5 — The Perpendicular Passes Through the Centre

EXERCISE 10.2 Q5

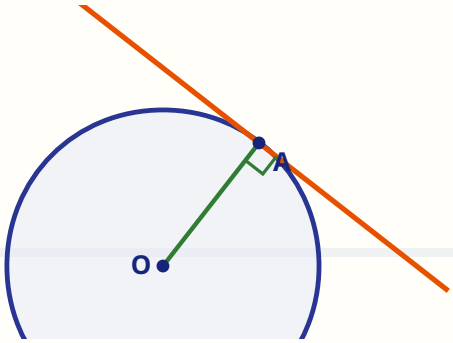
Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

THINK FIRST: There is only ONE line through the contact point perpendicular to the tangent. The radius line is such a line — so the perpendicular must be that very line, and it carries the centre on it.

1. Let the tangent touch the circle at A, and let l be the line through A perpendicular to the tangent.
   Why: This l is 'the perpendicular at the point of contact' whose journey we must track.
2. By Theorem 10.1, the radius OA is also perpendicular to the tangent, and OA passes through A.
   Why: So the line OA is a line through A perpendicular to the tangent.
3. Through a given point, exactly one line can be drawn perpendicular to a given line.
   Why: A basic Euclidean fact — two different perpendiculars through A would have to be parallel and meeting at A, impossible.
4. Therefore l is the same line as OA.
   Why: Two candidates, one slot: they coincide.
5. The line OA passes through the centre O. Hence l passes through the centre. **Proved.**

ANSWER: The perpendicular at the point of contact passes through the centre.

CBSE-STYLE MARKING SCHEME: 2 marks: 1 for Theorem 10.1 + 1 for the uniqueness argument.



TOPPER TIP: *Q4 and Q5 are converses in spirit: Q4 starts from the diameter and reaches parallel tangents; Q5 starts from the perpendicular and reaches the centre.*

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Exercise 10.2 Q6 — The 3-4-5 Circle

EXERCISE 10.2 Q6

The length of a tangent from a point A at a distance of 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

THINK FIRST: Same skeleton as Q1: $r = \sqrt{(d^2 - \text{tangent}^2)}$. Watch the 3-4-5 triple appear.

1. Let O be the centre and let the tangent from A touch the circle at P. Join OP.

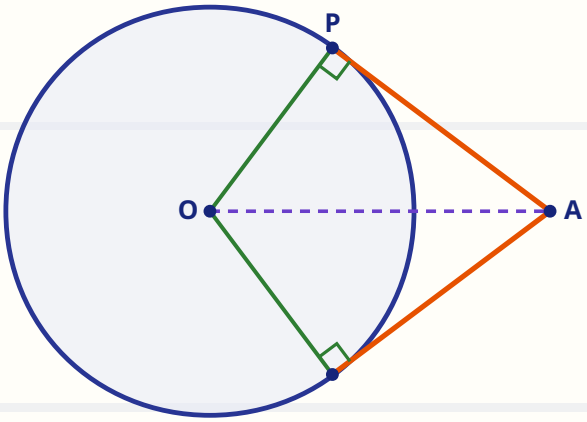
2. $\angle OPA = 90°$, so $\triangle OPA$ is right-angled at P with hypotenuse OA = 5 cm.
   Why: Theorem 10.1. Note: the point here is named A — do not let the letters confuse you; the roles matter, not the names.

3. Pythagoras: $OA^2 = OP^2 + PA^2$, that is, $25 = OP^2 + 16$.
   Why: PA = 4 cm is the tangent length.

4. $OP^2 = 25 - 16 = 9$, so OP = 3 cm.

ANSWER: The radius of the circle is 3 cm.

CBSE-STYLE MARKING SCHEME: 2 marks: 1 for the right angle + Pythagoras setup + 1 for the value.



Exercise 10.2 Q7 — The Concentric Chord

EXERCISE 10.2 Q7

Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

THINK FIRST: The chord of the big circle is a TANGENT of the small circle. Drop a perpendicular from O to the chord: it is the small radius (3), it bisects the chord, and half the chord comes from Pythagoras with the big radius (5).

1. Let O be the common centre, and let AB be the chord of the larger circle touching the smaller circle at P.

   Why: Concentric = same centre, different radii.

2. Join OP and OA. OP = 3 cm (radius of smaller circle) and OA = 5 cm (radius of larger).

3. AB is a tangent to the smaller circle at P, so OP ⊥ AB.

   Why: Theorem 10.1 — this is why P is the foot of the perpendicular.

4. The perpendicular from the centre to a chord bisects the chord, so P is the midpoint of AB and AP = PB.

   Why: Class 9 theorem (Section 02, tool 1).

Exercise 10.2 Q7 (continued)

5 In right-angled $\triangle OAP$: $AP^2 = OA^2 - OP^2 = 25 - 9 = 16$, so $AP = 4$ cm.

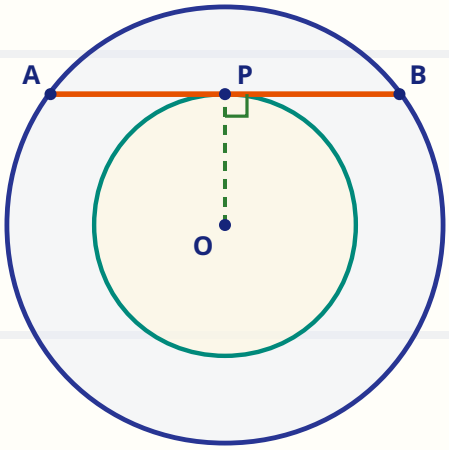
Why: Pythagoras, with OA as hypotenuse.

6 $AB = 2 \times AP = 2 \times 4 =$ **8 cm**.

Why: Do not stop at 4 – the question asks for the whole chord. Section 10 returns to this trap.

ANSWER: The chord is 8 cm long.

CBSE-STYLE MARKING SCHEME: 3 marks: 1 for $OP \perp AB$ (Theorem 10.1) + 1 for the bisection + 1 for Pythagoras and doubling.



Exercise 10.2 Q8 — The Circumscribing Quadrilateral

EXERCISE 10.2 Q8

A quadrilateral ABCD is drawn to circumscribe a circle (i.e., all four sides touch the circle). Prove that AB + CD = AD + BC.

THINK FIRST: Do not chase the sides — chase the VERTICES. From each vertex, two tangent segments leave for the circle, and Theorem 10.2 makes each pair equal. Name the four lengths and add.

1. Let the circle touch the sides at P, Q, R and S.

   Why: Mark the four points of contact.

2. From vertex A, the two tangent segments ending at A are equal (Theorem 10.2). Call each of them w.

   Why: A is outside the circle, so it owns an equal pair.

3. Similarly, from B the two tangent segments are equal; call them x. From C, call them y. From D, call them z.

   Why: Four vertices, four pairs, four letters.

4. Now read the sides: AB = w + x, BC = x + y, CD = y + z, DA = z + w.

   Why: Each side is split by its contact point into two tangent segments.

5. Add opposite sides: AB + CD = (w + x) + (y + z) = w + x + y + z.

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Exercise 10.2 Q8 (continued)

6 And AD + BC = (z + w) + (x + y) = w + x + y + z.

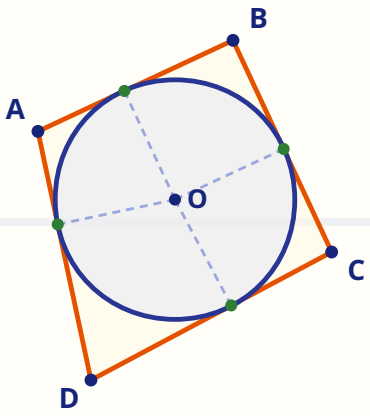
Why: The same four letters, in a different order.

7 Hence AB + CD = AD + BC. **Proved.**

Why: Sums of opposite sides of a circumscribing quadrilateral are equal – remember this as a ready-to-use result.

ANSWER: AB + CD = AD + BC — in a quadrilateral circumscribing a circle, the sums of opposite sides are equal.

CBSE-STYLE MARKING SCHEME: 3 marks: 1 for marking equal tangents from each vertex + 1 for expressing the four sides + 1 for the addition.



PIN POINT: Circumscribing figure? Give each VERTEX one letter for its two equal tangent pieces (w, x, y, z). Every side is then a sum of two letters — and opposite-side sums automatically match.

Exercise 10.2 Q9 — The 90° Theorem

EXERCISE 10.2 Q9

XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersects XY at A and X′Y′ at B. Prove that ∠AOB = 90°.

THINK FIRST: Join OP, OQ (radii to the parallel tangents' contact points) and OC. The line POQ is a straight diameter. Congruent triangles split the straight angle 180° into two pairs of equal angles — and ∠AOB is exactly one from each pair.

1. Let XY touch the circle at P and X′Y′ touch it at Q. Join OP, OQ and OC.
   Why: Name every radius in the figure.

2. OP ⊥ XY and OQ ⊥ X′Y′. Since XY ∥ X′Y′, the line POQ is a straight line — PQ is a diameter and ∠POQ = 180°.
   Why: Both radii are perpendicular to parallel lines through O, so they lie on one straight line.

3. From A, the two tangents are AP and AC. In △OAP and △OAC: OP = OC (radii), AP = AC (Theorem 10.2), OA common — congruent (SSS).
   Why: Or argue by RHS – either road reaches congruence.

4. Hence ∠AOP = ∠AOC (CPCT).
   Why: OA bisects ∠POC – Corollary A again.

5. From B, the two tangents are BQ and BC, and the same argument gives ∠BOQ = ∠BOC.
   Why: OB bisects ∠QOC.

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Exercise 10.2 Q9 (continued)

6 Now ∠POC + ∠QOC = 180° (they make the straight angle POQ).

Why: P–O–Q is a straight line.

7 But ∠POC = 2∠AOC and ∠QOC = 2∠BOC, so 2(∠AOC + ∠BOC) = 180°.

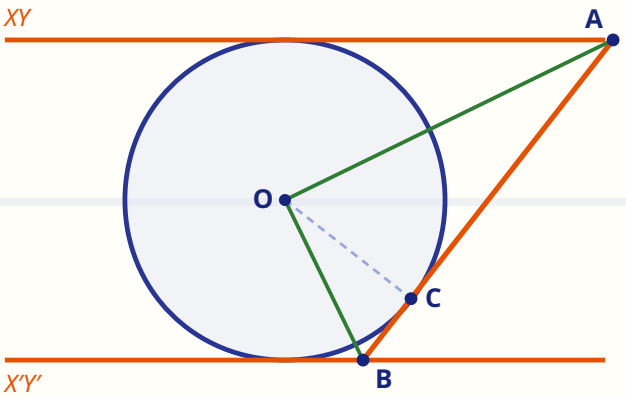
Why: Each angle is doubled by its bisector.

8 Therefore ∠AOC + ∠BOC = 90°, i.e., ∠AOB = 90°. **Proved.**

Why: And ∠AOC + ∠BOC is exactly ∠AOB.

ANSWER: ∠AOB = 90°.

CBSE-STYLE MARKING SCHEME: 3 marks: 1 for POQ being a straight line + 1 for the two congruences/bisections + 1 for the doubling argument.



The two parallel tangents XY and X′Y′, cut by the tangent AB at C. OA and OB meet at a perfect right angle at the centre.

Exercise 10.2 Q10 — The Supplementary Angle

EXERCISE 10.2 Q10

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

THINK FIRST: You proved this as Corollary B in Section 06 — here it is in full exam dress. Two right angles inside a quadrilateral leave exactly 180° for the other two angles to share.

1. Let the tangents from external point P touch the circle at A and B. Join OA, OB and OP; the 'angle between the tangents' is ∠APB and the 'angle subtended at the centre' is ∠AOB.

   Why: Translate the words of the question into the angles of the figure.

2. ∠OAP = 90° and ∠OBP = 90°.

   Why: Theorem 10.1 at A and at B.

3. In quadrilateral OAPB, the angles sum to 360°: ∠OAP + ∠OBP + ∠AOB + ∠APB = 360°.

   Why: Quadrilateral angle sum.

4. Substitute: 90° + 90° + ∠AOB + ∠APB = 360°.

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Exercise 10.2 Q10 (continued)

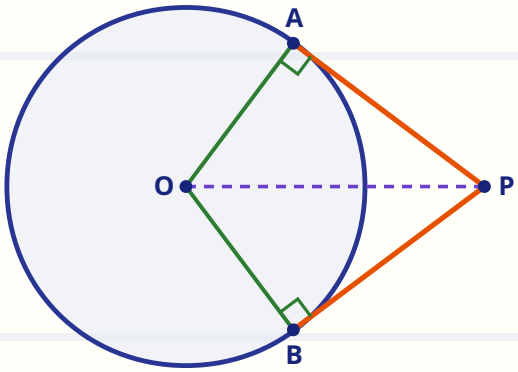
5 Hence ∠AOB + ∠APB = 360° − 180° = 180°.

6 So ∠APB and ∠AOB are supplementary. **Proved.**

Why: Two angles are supplementary exactly when they sum to 180°.

ANSWER: The angle between the two tangents + the angle at the centre = 180° — they are supplementary.

CBSE-STYLE MARKING SCHEME: 3 marks: 1 for the two right angles + 1 for the quadrilateral angle sum + 1 for the conclusion.



Exercise 10.2 Q11 — The Rhombus Test

EXERCISE 10.2 Q11

Prove that the parallelogram circumscribing a circle is a rhombus.

THINK FIRST: Feed the parallelogram into Q8's result. Opposite sides of a parallelogram are equal; opposite-side SUMS are equal by Q8. The two facts together force adjacent sides to be equal.

1. Let ABCD be a parallelogram circumscribing a circle. By Q8, AB + CD = AD + BC.
   Why: Q8 is a ready-made tool now — quote it, do not re-prove it.
2. In a parallelogram, opposite sides are equal: AB = CD and AD = BC.
   Why: Class 9 property of parallelograms.
3. Substitute into Q8's result: AB + AB = AD + AD, i.e., 2·AB = 2·AD.
4. So AB = AD.
   Why: Adjacent sides of the parallelogram are equal.
5. A parallelogram with a pair of adjacent sides equal has all four sides equal — it is a rhombus. **Proved.**
   Why: AB = AD, and opposite sides match in pairs, so AB = BC = CD = DA.

ANSWER: A parallelogram circumscribing a circle is a rhombus.

CBSE-STYLE MARKING SCHEME: 2 marks: 1 for quoting Q8 + 1 for combining with the parallelogram property.

TOPPER TIP: *One line of Q8 plus one property of parallelograms — that is the whole proof. Short questions reward students who keep a toolbox of ready results.*

Exercise 10.2 Q12 — The Circumscribed Triangle

EXERCISE 10.2 Q12

A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see figure). Find the sides AB and AC.

THINK FIRST: Three vertices, three pairs of equal tangents. Let the pair from A be x. Then every side is written in x, the semi-perimeter is x + 14, and the area is computed two ways — by Heron and as radius × semi-perimeter. Equating them hands you x.

1. Let the circle touch CA at E and AB at F. The tangents from A are AF and AE — equal; call each x cm.

   Why: Theorem 10.2 at vertex A.

2. Tangents from B: BF = BD = 8 cm. Tangents from C: CE = CD = 6 cm.

   Why: Theorem 10.2 at B and at C — the segments BD and DC are themselves tangent segments from B and C.

3. So the sides are: AB = AF + FB = x + 8, AC = AE + EC = x + 6, BC = BD + DC = 14.

   Why: Each side is a sum of two tangent segments.

4. Semi-perimeter $s = \frac{1}{2}[(x + 8) + (x + 6) + 14] = x + 14$.

5. Area by Heron's formula: $\sqrt{[s(s - a)(s - b)(s - c)]}$ with a = BC = 14, b = CA = x + 6, c = AB = x + 8. The factors are s − a = x, s − b = 8, s − c = 6, so Area = $\sqrt{[(x + 14) \cdot x \cdot 8 \cdot 6]} = \sqrt{[48x(x + 14)]}$.

   Why: Watch the beautiful cancellation — the factors 8 and 6 are exactly BD and DC.

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Exercise 10.2 Q12 (continued)

6 Area a second way: join O to A, B, C. The triangle splits into three triangles, each with height = radius 4 (radius ⊥ side at the contact point). So Area = ½ · 4 · (AB + BC + CA) = 4s = 4(x + 14).

Why: This 'area = r × s' for a circumscribed polygon is worth remembering in its own right.

7 Square the second expression and equate: $[4(x + 14)]^2 = 48x(x + 14)$, i.e., $16(x + 14)^2 = 48x(x + 14)$.

Why: Squaring removes the root from Heron's side.

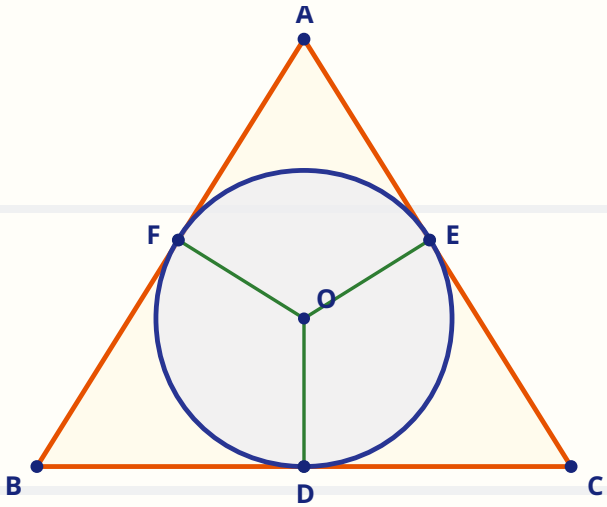
8 Divide both sides by 16(x + 14) (positive, so safe): x + 14 = 3x, giving 2x = 14 and x = 7.

9 Therefore AB = x + 8 = **15 cm** and AC = x + 6 = **13 cm**.

Why: Check: s = 21, Heron gives √(21 · 7 · 8 · 6) = √7056 = 84, and r · s = 4 × 21 = 84. The two areas agree.

ANSWER: AB = 15 cm and AC = 13 cm.

CBSE-STYLE MARKING SCHEME: 4 marks: 1 for the three tangent pairs + 1 for sides and s in terms of x + 1 for the two area expressions + 1 for solving and the final sides.



Exercise 10.2 Q13 — Supplementary Angles at the Centre

EXERCISE 10.2 Q13

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

THINK FIRST: 'Side AB subtends an angle at the centre' means ∠AOB — the angle between the lines from O to the side's endpoints. Split each such angle into two halves using tangent-pair congruences at the vertices; the four halves total 360°, and opposite-side angles collect the same halves in two groups of 180°.

1. Let ABCD circumscribe the circle with centre O, touching AB, BC, CD, DA at P, Q, R, S. Join OA, OB, OC, OD and OP, OQ, OR, OS.

   Why: Eight spokes from the centre: four to vertices, four to contact points.

2. At vertex A: $\triangle AOP \cong \triangle AOS$ (AP = AS tangents from A, OP = OS radii, OA common — SSS). So ∠AOP = ∠AOS.

   Why: OA bisects the angle between the spokes to A's two contact points.

3. Similarly at B, C, D: ∠BOP = ∠BOQ, ∠COR = ∠COQ, ∠DOR = ∠DOS.

   Why: The same argument at every vertex.

4. The eight half-angles around O sum to 360°. Group them as (∠AOP + ∠BOP + ∠COR + ∠DOR) + (∠AOS + ∠BOQ + ∠COQ + ∠DOS).

   Why: Every half-angle appears exactly once.

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Exercise 10.2 Q13 (continued)

5 By the equalities of Steps 2–3, the second group equals the first group, term by term. So each group = 180°.

Why: Two equal groups totalling 360° are 180° each.

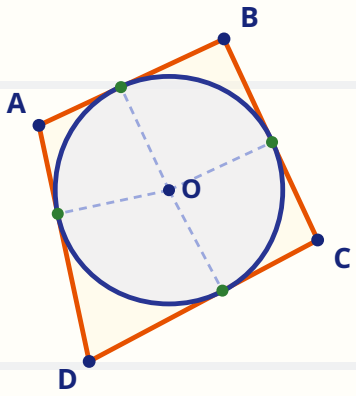
6 But the first group is (∠AOP + ∠BOP) + (∠COR + ∠DOR) = ∠AOB + ∠COD.

Why: Regroup neighbouring halves into the angles subtended by sides AB and CD.

7 Hence ∠AOB + ∠COD = 180° — the angles subtended by opposite sides AB and CD are supplementary. The same argument gives ∠BOC + ∠DOA = 180°. **Proved.**

ANSWER: Opposite sides of a circumscribing quadrilateral subtend supplementary angles at the centre.

CBSE-STYLE MARKING SCHEME: 3 marks: 1 for the vertex congruences + 1 for the 360° grouping + 1 for identifying ∠AOB + ∠COD.



TOPPER TIP: *Section 07 complete — all 17 NCERT questions. Q9, Q10 and Q13 were corollaries of Section 06, and Q11 was Q8 plus one line: derivations are prepaid solutions.*

08 Multiple Solution Methods

8D • CHALLENGE — ONE PROBLEM, TWO ROADS

A topper is never trapped by one method. Here are this chapter's two favourite problems, each solved by two genuinely different roads. In the exam, take whichever road opens first — but knowing both is your insurance.

Problem 1 — tangent 24 cm, distance 25 cm: find the radius

Method A — Pythagoras, head-on

$r^2 = 25^2 - 24^2 = 625 - 576 = 49$, so $r = 7$ cm.

Direct and safe. Two big squares to compute.

Method B — Difference of squares

$r^2 = 25^2 - 24^2 = (25 - 24)(25 + 24) = 1 \times 49 = 49$, so $r = 7$ cm.

No big squares at all — $a^2 - b^2 = (a - b)(a + b)$ does the heavy lifting. Beautiful when the two numbers are close.

Problem 2 — ∠POQ = 110°: find ∠PTQ

Method A — Quadrilateral angle sum

In OPTQ, two angles are 90° (Theorem 10.1). So $\angle PTQ = 360° - 90° - 90° - 110° = 70°$.

Method B — Bisect, then one triangle

OT bisects ∠POQ (Corollary A), so $\angle POT = 55°$. In right-angled $\triangle OPT$, $\angle PTO = 180° - 90° - 55° = 35°$. OT also bisects ∠PTQ, so $\angle PTQ = 2 \times 35° = 70°$.

TOPPER TIP: *Method B looks longer, but it is the one that survives when the quadrilateral is broken into pieces by extra lines in a harder problem. Learn both; use A for speed, B for structure.*

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Multiple Methods — When to Use Which?

Situation	Fastest road	Why
Length asked; radius & distance given (or any two of the three)	The length formula PA = $\sqrt{(d^2 - r^2)}$	One substitution. If the numbers are close together, use difference of squares.
Angle between tangents, or angle at centre, asked	Supplementary pair: they sum to 180°	Corollary B — no quadrilateral sum needed each time.
Half-angles like ∠POA asked	Bisector (Corollary A) after finding the full angle	∠POA = ½∠AOB, always.
A figure with sides touching a circle	Vertex letters w, x, y, z (Theorem 10.2)	Converts geometry into simple addition.
A chord touching a smaller concentric circle	Perpendicular from centre + Pythagoras + DOUBLE	The perpendicular bisects the chord; the answer is twice the half.
A proof about tangents	Quote Theorems 10.1/10.2 by number	Board examiners award the mark for the named theorem.

The third road — construction

Every formula in this chapter can also be reached with ruler and compass: construct the tangents (Section 09, Activity 1), measure with a scale, and compare with $\sqrt{(d^2 - r^2)}$. Construction is slow, but it is the ultimate doubt-killer — if your measured length and your computed length disagree, your figure or your arithmetic is wrong, and you find out *before* the examiner does.

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09 Interactive Simulations & Hands-on Activities

TRY IT YOURSELF — LEARN BY DOING

Activity 1 — Construct the two tangents (ruler & compass)

Steps: 1. Draw a circle of radius 3 cm, centre O. Mark a point P with OP = 5 cm. 2. Bisect OP at M. 3. With M as centre and MO as radius, draw a second circle; it cuts the first circle at A and B. 4. Join PA and PB.
Check: measure PA and PB (you should get 4 cm each — the 3-4-5 triangle), and test ∠OAP with a set square (exactly 90°). You have constructed Theorem 10.1 and Theorem 10.2 with your own hands. Why it works: ∠OAP stands in a semicircle of the second circle, and angles in a semicircle are right angles.

Activity 2 — The thread test for equal tangents

Take a bangle and a long thread. Fix one end of the thread at a point P on the table, wrap the thread so it grazes the bangle on the left and mark the grazing length; then graze it on the right from the same P. Compare the two marked lengths. Repeat with P nearer and farther. **Record:** are the two lengths ever different? What happens to the tangent length as P moves closer to the circle?

Activity 3 — Hunt the supplementary pair

Draw three circles with an external point P in three different positions. In each, draw both tangents, join OA and OB, and measure ∠APB and ∠AOB with a protractor. Fill the table:

Case	∠APB (measured)	∠AOB (measured)	Sum
1			
2			
3			

If every sum is 180° (within measurement error), you have experimentally discovered Corollary B.

KALVIYOGI DIGITAL COMPANION — CHAPTER 10: CIRCLES
Watch: Theorem 10.1 and 10.2 animated proofs • Tangent-length simulator (drag P, watch PA change) • Construction lab • Chapter test series.
QR code and link activate with the KalviYogi app/website at print.

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Activity Answers — Check After Trying

Activity 1 — what you should find

PA = PB = 4 cm exactly (3-4-5 triangle: $\sqrt{(5^2 - 3^2)} = 4$). The set square confirms ∠OAP = ∠OBP = 90°. If your measurements say 3.9 or 4.1, that is drawing error, not mathematics error — the theorems are exact.

Activity 2 — what you should find

The two grazing lengths from the same P are always equal, however you place P — Theorem 10.2 with thread instead of proof. As P moves closer, both tangent lengths shrink; when P reaches the circle, the 'two tangents' merge into one and the length becomes 0; inside the circle, grazing is impossible — matching Section 05's table exactly.

Activity 3 — what you should find

Every row sums to 180° (±2° of protractor error). Typical rows: 70° + 110°, 90° + 90°, 120° + 60°. The pair is always supplementary — Corollary B, measured.

TOPPER TIP: *Activities are not decoration. The construction in Activity 1 is itself a favourite board question ('construct the tangents from a point 5 cm away...') — you have already practised it.*

10 Common Mistakes

LEARN FROM OTHER STUDENTS' ERRORS — BEFORE THEY BECOME YOURS

Six errors account for nearly every mark lost in this chapter. Each one is shown the way students actually write it — then corrected.

COMMON MISTAKE: Mistake 1 — Subtracting instead of Pythagoras.
Wrong way: 'Radius 5 cm, point 13 cm away, so tangent = 13 − 5 = 8 cm.'
Why wrong: The tangent, radius and distance are not collinear pieces of one line — they are the three sides of a right triangle. Lengths inside a triangle obey Pythagoras, not subtraction.
Right way: Tangent = $\sqrt{(13^2 - 5^2)} = \sqrt{144} = 12$ cm.

COMMON MISTAKE: Mistake 2 — Pythagoras with the wrong hypotenuse.
Wrong way: '$PT^2 = OP^2 + OT^2$.'
Why wrong: OP joins the centre to the external point and sits opposite the right angle — OP is the hypotenuse, the LONGEST side. A leg can never equal the sum of squares of the hypotenuse and another side.
Right way: $OP^2 = OT^2 + PT^2$, so $PT^2 = OP^2 - OT^2$. Quick sanity check: the tangent length must come out SHORTER than OP.

COMMON MISTAKE: Mistake 3 — 'Equal' instead of 'supplementary'.
Wrong way: '$\angle POQ = 110°$, so $\angle PTQ = 110°$ (vertically opposite / equal angles).'
Why wrong: The two angles live in a quadrilateral with two right angles; they are not opposite equal angles of any standard configuration.
Right way: $\angle PTQ = 180° - 110° = 70°$. The pair always sums to 180° (Corollary B).

Common Mistakes 4–6

COMMON MISTAKE: Mistake 4 — Reporting the half-chord as the chord.
Wrong way: In the concentric-circles problem (radii 5 and 3): 'AP = √(25 − 9) = 4, so the chord is 4 cm.'
Why wrong: The perpendicular from the centre bisects the chord, so Pythagoras only ever gives you HALF of it.
Right way: Chord AB = 2 × AP = 8 cm. Whenever a solution ends with a chord, ask: 'did I double?'

COMMON MISTAKE: Mistake 5 — Equal tangents from DIFFERENT points.
Wrong way: 'PA is a tangent and QC is a tangent, so PA = QC.'
Why wrong: Theorem 10.2 compares the two tangents from the SAME external point. Tangents from different points have no reason to be equal.
Right way: Only pair up segments that leave from one common vertex or point.

COMMON MISTAKE: Mistake 6 — Imagining a third parallel tangent.
Wrong way: 'A circle can have 3 parallel tangents — one on top, one in the middle, one at the bottom.'
Why wrong: The 'middle' line passes within distance less than r of the centre — it cuts the circle at two points. It is a secant, not a tangent.
Right way: At most TWO parallel tangents, standing at the two ends of a diameter, exactly one diameter apart.

The mistake-proof checklist

- ☐ Did I join O to every point of contact and mark the right angles?
- ☐ Is OP (centre-to-point) my hypotenuse?
- ☐ Angle pair — did I make them supplementary, not equal?
- ☐ Chord answer — did I double the half?
- ☐ Equal tangents — do they really start from the same point?

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11 Graded Practice Problems

FIVE LEVELS — CLIMB ONE LEVEL AT A TIME • ANSWERS IN THE ANSWER KEY

Twenty-one questions, five floors. Do not skip floors: Level 1 checks the words, Level 2 the NCERT skills, Level 3 the same skills in the real world, Level 4 the reasoning, Level 5 the genius moves. Answers are in the Answer Key — attempt first, peek later.

LEVEL 1 — FOUNDATION

1.1 A tangent drawn from a point 10 cm from the centre touches a circle of radius 6 cm. Find the length of the tangent.

1.2 How many tangents can be drawn to a circle from a point lying outside it? From a point lying inside it?

1.3 The length of a tangent to a circle of radius 5 cm is 12 cm. How far is the point from the centre of the circle?

1.4 PA and PB are tangents to a circle with centre O. If $\angle AOB = 120°$, find $\angle APB$.

1.5 Fill in the blanks: A tangent intersects a circle in exactly _____ point(s). The radius through the point of contact is _____ to the tangent.

1.6 PA is a tangent of length 7 cm drawn from a point P to a circle. Find the length of the other tangent PB drawn from the same point P.

LEVEL 2 — SKILL BUILDER

2.1 Find the length of the tangent drawn from a point 15 cm from the centre of a circle of radius 9 cm.

2.2 Two tangents drawn from a point P to a circle with centre O are inclined to each other at 70°. Find $\angle AOB$ and $\angle POA$.

2.3 Two concentric circles have radii 13 cm and 5 cm. Find the length of the chord of the larger circle which touches the smaller circle.

2.4 PA and PB are tangents to a circle with centre O. If $\angle APB = 50°$, find $\angle AOB$.

2.5 A point is 25 cm from the centre of a circle of radius 7 cm. Find the length of the tangent drawn from the point to the circle.

2.6 A quadrilateral ABCD circumscribes a circle. If AB = 9 cm, CD = 7 cm and BC = 10 cm, find AD.

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Graded Practice — Levels 3, 4 and 5

LEVEL 3 — APPLICATION

3.1 A straight path just grazes a circular pond of radius 10 m. Ravi stands on the path at a point 26 m from the centre of the pond, and the line from Ravi to the grazing point is a tangent. Find the length of this tangent segment.

3.2 A straight belt leaves a pulley of radius 8 cm and runs in a straight line to a point on the next machine, 17 cm from the pulley's centre. The belt is a tangent to the pulley. Find the length of the straight part of the belt.

3.3 A quadrilateral cloth ABCD is stitched so that all four sides just touch a round table (the table's edge is a circle circumscribed by the cloth). If AB = 12 cm, BC = 14 cm and CD = 10 cm, find DA.

3.4 Two tangents drawn from a point P to a circle of radius 4 cm are perpendicular to each other. Find the length of each tangent and the distance of P from the centre.

LEVEL 4 — ADVANCED REASONING

4.1 Tangents PA and PB from P satisfy $\angle APB = 60°$ and OP = 12 cm. Find the radius of the circle and the length PA. (Hint: OP bisects $\angle APB$; use $\triangle OAP$.)

4.2 Two concentric circles have radii 17 cm and 8 cm. Find the length of the chord of the larger circle which touches the smaller circle, justifying each step in one line.

4.3 PA and PB are tangents from P, with OP = 10 cm and radius 6 cm. Find the length of the chord of contact AB. (Hint: compute the area of $\triangle OAP$ in two ways.)

LEVEL 5 — MATHEMATICAL GENIUS

5.1 AP and AQ are tangents from A to a circle, and the side BC of triangle ABC is also tangent to the same circle (the circle sits inside the triangle's angle at A, touching BC from the other side). If AB = 5 cm, AC = 6 cm and BC = 4 cm, find the length of AP.

5.2 Two circles of radii 9 cm and 4 cm touch each other externally. Find the length of a direct common tangent to the two circles. (Hint: the distance between the centres is 9 + 4; draw a line parallel to the common tangent through the smaller centre.)

> **TOPPER TIP:** *Stuck at a level? Section 07 contains a twin of almost every question here. Find the twin, study its steps, close the book, and try again.*

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12 Mathematical Reasoning

JUSTIFY AND PROVE — FULL SOLUTIONS

REASONING 1

PA and PB are tangents drawn from an external point P to a circle with centre O, touching it at A and B. Prove that OP is the perpendicular bisector of the chord of contact AB.

THINK FIRST: This is why the chord of contact is always perpendicular to the line joining the centre to the external point — a fact the Olympiad section will cash in.

1. Let M be the point where OP meets AB. We show AM = BM and OP ⊥ AB.
   Why: Perpendicular bisector = a line through the midpoint, at a right angle. Two things to prove.
2. PA = PB.
   Why: Theorem 10.2 — tangents from the same external point.
3. ∠APM = ∠BPM.
   Why: OP bisects ∠APB — Corollary A of Section 06.
4. In △APM and △BPM: PA = PB, ∠APM = ∠BPM, and PM is common. So △APM ≅ △BPM (SAS).
   Why: Side-Angle-Side, assembled from the two facts above.
5. Hence AM = BM (CPCT) — M is the midpoint of AB.
   Why: First of the two things: done.
6. Also ∠AMP = ∠BMP (CPCT). But they form a linear pair on the straight line AB, so ∠AMP + ∠BMP = 180°, giving ∠AMP = ∠BMP = 90°.
   Why: Equal angles that sum to 180° are 90° each.
7. Thus OP passes through the midpoint of AB at a right angle: OP is the perpendicular bisector of AB. **Proved.**

ANSWER: OP is the perpendicular bisector of the chord of contact AB.

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Reasoning 2 — Parallel Tangents Meet at the Centre

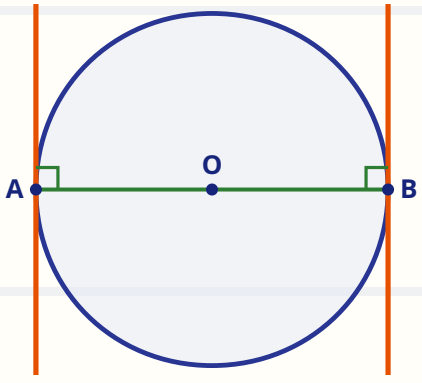
REASONING 2

Two tangents drawn to a circle are parallel to each other. Prove that the line joining their points of contact passes through the centre of the circle.

THINK FIRST: Compare with Exercise 10.2 Q4: there, a diameter produced parallel tangents; here, parallel tangents produce a diameter. The two results are converses — and both are true.

1. Let the parallel tangents touch the circle at A and B. Join OA and OB.
   Why: The claim is that A, O, B are collinear – AB is a diameter.
2. OA ⊥ tangent at A and OB ⊥ tangent at B.
   Why: Theorem 10.1, twice.
3. The two tangents are parallel, so the lines OA and OB — both perpendicular to this same direction — are parallel to each other.
   Why: Lines perpendicular to parallel lines are themselves parallel.
4. But OA and OB both pass through the point O. Two parallel lines through one point must be the same line.
   Why: Through O there is only one line in a given direction.
5. Hence A, O and B lie on one straight line, which passes through the centre O by construction. **Proved.**
   Why: Bonus: AB is therefore a diameter – the two contact points of parallel tangents are always diametrically opposite.

ANSWER: The line joining the points of contact of two parallel tangents passes through the centre (it is a diameter).



Two parallel tangents: their points of contact and the centre always lie on one straight line — a diameter.

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Reasoning 3 — The Bisected Chord

REASONING 3

In two concentric circles, prove that the chord of the larger circle which touches the smaller circle is bisected at the point of contact.

THINK FIRST: One segment, two theorems: OP is a radius for the small circle and a perpendicular for the large one. Most concentric-circle problems are this double identity in disguise.

1. Let the concentric circles have centre O, and let AB be a chord of the larger circle touching the smaller circle at P.
   Why: This is the theorem standing behind Exercise 10.2 Q7 — there you used it; here you prove it.
2. AB is a tangent to the smaller circle at P, so OP ⊥ AB.
   Why: Theorem 10.1 — OP is a radius of the smaller circle drawn to the point of contact.
3. Now view AB as a chord of the LARGER circle. OP is a line from the centre O perpendicular to this chord.
   Why: Same segment OP, second job.
4. The perpendicular from the centre of a circle to a chord bisects the chord. Hence AP = PB.
   Why: Class 9 theorem (Section 02).
5. So P, the point of contact, is the midpoint of AB. **Proved.**

ANSWER: The chord is bisected exactly at its point of contact with the smaller circle.

TOPPER TIP: *Reasoning questions are marked for the CHAIN, not the destination: given → theorem quoted → conclusion. Write the name of every theorem you use — Theorem 10.1, Theorem 10.2, CPCT, SAS — in the line where you use it.*

13 Real-Life Applications

8D • APPLY — THE CHAPTER AT WORK IN THE WORLD

APPLICATION 1 — THE ROUNDABOUT

A circular roundabout has radius 5 m. A straight road just grazes the roundabout (it is a tangent to the circle). A lamp post on the road stands at a point 13 m from the centre of the roundabout. Find the length of the road segment from the lamp post to the point where the road touches the roundabout.

THINK FIRST: Roads, rims and roundabouts: whenever a straight edge 'just grazes' a circle, you are holding a tangent.

1. Model the roundabout as a circle with centre O and radius 5 m; the road is a tangent line, the lamp post is the external point P with OP = 13 m, and the touching point is A.

   Why: Real-life problems are won or lost at the modelling step — name the circle, the point and the tangent.

2. Join OA. OA ⊥ road at A.

   Why: Theorem 10.1.

3. PA = $\sqrt{(OP^2 - OA^2)} = \sqrt{(169 - 25)} = \sqrt{144} = 12$ m.

   Why: The length formula of Section 06 — and a 5-12-13 triangle.

ANSWER: The road segment is 12 m long.

APPLICATION 2 — THE PULLEY BELT

In a flour mill, a flat belt leaves a pulley of radius 8 cm in a straight line and runs to the centre shaft of the next machine, which is 17 cm from the pulley's centre. The straight part of the belt is tangent to the pulley. Find its length.

THINK FIRST: Engineers genuinely size belt drives with this computation — the tangent length between pulleys is a standard machine-design formula.

1. The belt's straight part is a tangent segment from the external point (the shaft, 17 cm away) to the pulley (radius 8 cm).

   Why: Same skeleton as Application 1 — different costume.

2. Length = $\sqrt{(17^2 - 8^2)} = \sqrt{(289 - 64)} = \sqrt{225} = 15$ cm.

   Why: An 8-15-17 triangle this time.

ANSWER: The straight part of the belt is 15 cm long.

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Real-Life Applications 3 and 4

APPLICATION 3 — THE TRIANGULAR PARK

A triangular park ABC has a circular fountain of radius 3 m that just touches all three boundary fences. On fence BC, the touching point D divides it into BD = 6 m and DC = 4 m. Find the lengths of fences AB and AC.

THINK FIRST: A circular fountain touching all three fences is an incircle — and 'area = r × s' is the master key for every such problem.

1. Let the fountain touch CA at E and AB at F, and let the equal tangents from A be AF = AE = x.

   Why: Theorem 10.2 at each vertex, exactly as in Exercise 10.2 Q12.

2. Tangents from B: BF = BD = 6. Tangents from C: CE = DC = 4. So AB = x + 6, AC = x + 4, BC = 10, and s = x + 10.

3. Area two ways: Heron gives $\sqrt{[(x + 10) \cdot x \cdot 6 \cdot 4]} = \sqrt{[24x(x + 10)]}$; the radius way gives $r \cdot s = 3(x + 10)$.

   Why: Area = r × s for any circumscribed triangle (Section 07, Q12).

4. Equate after squaring: $9(x + 10)^2 = 24x(x + 10)$, so $9(x + 10) = 24x$, giving $90 = 15x$ and $x = 6$.

5. AB = 6 + 6 = **12 m** and AC = 6 + 4 = **10 m**.

   Why: Check: s = 16, Heron: √(16 · 6 · 6 · 4) = 48; r · s = 3 × 16 = 48. Agreed.

ANSWER: AB = 12 m and AC = 10 m.

Application 4 — The Ring Road Footpath

APPLICATION 4 — THE RING ROAD FOOTPATH

A circular park of radius 9 m sits inside a ring road whose outer edge is a concentric circle of radius 15 m. A straight footpath is built across the ring, joining two points of the outer edge and just touching the park's boundary. Find the length of the footpath.

THINK FIRST: If you can solve Exercise 10.2 Q7, you can already design this footpath. That is what 'application' means: the same theorem, a new address.

1. The footpath is a chord of the outer circle (radius 15) that is tangent to the inner circle (radius 9).

   Why: This is Exercise 10.2 Q7 wearing a city-planning costume.

2. Drop the perpendicular from the common centre O to the footpath: its length is the inner radius, 9 m, and it bisects the footpath.

   Why: Theorem 10.1 + the chord-bisection theorem (Reasoning 3).

3. Half the footpath = $\sqrt{(15^2 - 9^2)} = \sqrt{(225 - 81)} = \sqrt{144} = 12$ m.

   Why: Pythagoras in the right triangle with hypotenuse = outer radius.

4. Footpath = 2 × 12 = **24 m**.

   Why: Remember Mistake 4 — double the half!

ANSWER: The footpath is 24 m long.

Section 13 in one breath

Grazing road → tangent length formula. Belt on a pulley → tangent length formula. Fountain in a triangular park → equal tangents + area = r × s. Footpath across a ring → concentric chord (double the half). Four costumes, two theorems.

14 Competency-Based Questions

CASE STUDIES & ASSERTION–REASON — BOARD EXAM STYLE, FULLY SOLVED

Five case studies, twenty questions — the board's competency format. Each case gives a real situation; all twenty questions are solved step by step.

Case Study 1 — The Ferris Wheel

A Ferris wheel has radius 14 m and centre O. From a control point P, in the plane of the wheel, two straight steel ropes are tied so that each just touches the rim. OP = 50 m. The ropes touch the wheel at A and B.

CASE 1 (i)

Find the length of each rope.

1. Join OA. OA ⊥ PA (Theorem 10.1); △OAP is right-angled at A, hypotenuse OP = 50 m.
2. PA = $\sqrt{(50^2 - 14^2)} = \sqrt{(2500 - 196)} = \sqrt{2304} = 48$ m.
   Why: Recognise 14-48-50 as the 7-24-25 triple doubled.

ANSWER: Each rope is 48 m long.

CASE 1 (ii)

The second rope PB was never measured. State its length and justify.

1. PB is the second tangent from the same external point P.
2. By Theorem 10.2, PB = PA = 48 m.
   Why: Tangents from one external point are equal — measurement is unnecessary.

ANSWER: PB = 48 m, by Theorem 10.2.

Case Study 1 (continued) — The Ferris Wheel

CASE 1 (iii)

Find the angle between the rope PA and the radius OA at the point of contact.

1. The rope is a tangent and OA is the radius through its point of contact.
2. By Theorem 10.1 the angle is exactly 90°.

ANSWER: The angle is 90°.

CASE 1 (iv)

On a festival day the wheel is lit so that ∠AOB = 100°. Find the angle between the two ropes at P.

1. ∠APB + ∠AOB = 180°.
   Why: Corollary B — the angles are supplementary.
2. ∠APB = 180° − 100° = 80°.

ANSWER: The ropes meet at 80° at P.

TOPPER TIP: *Notice the pattern across all five case studies: model the situation as a circle, an external point and a tangent — then the chapter's two theorems and one formula finish the job.*

Case Study 2 — The Bicycle Chain

Case Study 2 — The Bicycle Chain

A bicycle chain leaves the front sprocket along a tangent. Model the sprocket as a circle of radius 6 cm, centre O. The straight part of the chain runs from its point of contact A to a guide point P, 10 cm from O.

CASE 2 (i)

Find the length of the straight part of the chain from A to P.

1. OA ⊥ chain at A (Theorem 10.1); △OAP is right-angled at A.
2. PA = $\sqrt{(10^2 - 6^2)} = \sqrt{64} = 8$ cm.
   Why: A 6-8-10 triangle.

ANSWER: The straight part is 8 cm.

CASE 2 (ii)

The chain returns along the other side of the same sprocket, again as a tangent from the same guide point P, touching at B. Find PB and name the theorem used.

1. PB is the second tangent from P, so PB = PA = 8 cm by Theorem 10.2.

ANSWER: PB = 8 cm (Theorem 10.2).

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Case Study 2 (continued) — The Bicycle Chain

CASE 2 (iii)

The chain also wraps the rear sprocket, which has the same radius and whose centre is on the line OP extended. The straight runs of the chain on the two sprockets are parallel. Give one reason.

1. Each straight run is perpendicular to the radius at its own point of contact (Theorem 10.1), and the two radii point in the same direction in this model.

2. Two lines perpendicular to the same direction are parallel to each other.

   Why: The Section 02 parallel-lines tool.

ANSWER: The straight runs are parallel.

CASE 2 (iv)

A mechanic moves the guide point to a new position 26 cm from O. Find the new straight length of chain from the point of contact.

1. New length = $\sqrt{(26^2 - 6^2)} = \sqrt{(676 - 36)} = \sqrt{640} = 8\sqrt{10} \approx 25.3$ cm.

   Why: Same formula, new distance — the method does not change when the numbers do.

ANSWER: $8\sqrt{10}$ cm ≈ 25.3 cm.

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Case Study 3 — The Traffic Roundabout

Case Study 3 — The Traffic Roundabout

A roundabout is a circle of radius 12 m, centre O. Two straight roads touch it and meet at a junction P, 13 m from O, touching the circle at A and B.

CASE 3 (i)

Find the length of each road from the junction to its touching point.

1. PA = $\sqrt{(13^2 - 12^2)} = \sqrt{(169 - 144)} = \sqrt{25} = 5$ m.
   Why: Length formula; a 5-12-13 triangle.
2. PB = PA = 5 m (Theorem 10.2).

ANSWER: Each road segment is 5 m.

CASE 3 (ii)

A third road touches the roundabout at the point diametrically opposite to a fourth road's touching point, and the two roads are parallel. How far apart are these two parallel roads?

1. Parallel tangents stand at the ends of a diameter (Exercise 10.2 Q4 and Reasoning 2).
2. Distance between them = length of the diameter = 2 × 12 = 24 m.

ANSWER: The parallel roads are 24 m apart.

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Case Study 3 (continued) — The Traffic Roundabout

CASE 3 (iii)

A gardener stands at a point inside the roundabout's central island. How many tangent roads can be drawn from the gardener's point to the circle? Justify.

1. The gardener's point is inside the circle (distance from O less than 12 m).
2. Every line through an interior point cuts the circle at two points, so no tangent exists: the answer is 0.

Why: Section 05's position table.

ANSWER: Zero tangents can be drawn from an interior point.

CASE 3 (iv)

Find the angle between the two roads at P if the angle between the radii OA and OB is 130°.

1. ∠APB = 180° − ∠AOB (Corollary B).
2. ∠APB = 180° − 130° = 50°.

ANSWER: The roads meet at 50°.

Case Study 4 — The Round Table Cloth

Case Study 4 — The Round Table Cloth

A quadrilateral cloth ABCD is stitched so that all four sides just touch a round table; the table's edge forms a circle circumscribed by the cloth. AB = 11 cm, BC = 9 cm and CD = 8 cm. The side AB is split by its point of contact into two pieces of 5 cm (next to A) and 6 cm (next to B).

CASE 4 (i)

Find the length of DA.

1. For a circumscribing quadrilateral, AB + CD = AD + BC (Exercise 10.2 Q8).
2. 11 + 8 = AD + 9, so AD = 10 cm.

ANSWER: DA = 10 cm.

CASE 4 (ii)

Find the perimeter of the cloth.

1. Perimeter = AB + BC + CD + DA = 11 + 9 + 8 + 10 = 38 cm.
2. Shortcut worth noticing: perimeter = 2(AB + CD) = 2 × 19 = 38 cm, because the two opposite-side sums are equal.

   Why: A free corollary of Q8.

ANSWER: The perimeter is 38 cm.

Case Study 4 (continued) — The Round Table Cloth

CASE 4 (iii)

Find the length of the piece of AD that lies next to vertex A.

1. The piece of AB next to A is a tangent segment from A, of length 5 cm.
2. The piece of AD next to A is the OTHER tangent segment from the same vertex A.
3. By Theorem 10.2 the two are equal: the piece is 5 cm.

ANSWER: The piece of AD next to A is 5 cm.

CASE 4 (iv)

If the cloth ABCD were a parallelogram, what special name must it have? Justify in one line.

1. A parallelogram circumscribing a circle is a rhombus (Exercise 10.2 Q11).
   Why: Q8's equal sums + the parallelogram's equal opposite sides force adjacent sides equal.

ANSWER: It must be a rhombus.

Case Study 5 — Two Circles in Orbit

Case Study 5 — Two Circles in Orbit

A science model shows two circular discs of radii 8 cm and 2 cm whose rims just touch each other (the circles touch externally). A straight scale (a common external tangent) rests across the tops of both discs, touching the big disc at A and the small disc at B.

CASE 5 (i)

Find the distance between the centres of the two discs.

1. Circles touching externally: the distance between centres equals the sum of the radii.
   Why: The touching point lies on the line of centres, between them.
2. Distance = 8 + 2 = 10 cm.

ANSWER: The centres are 10 cm apart.

CASE 5 (ii)

Find the length of the scale segment AB between the two points of contact.

1. Let the centres be O (radius 8) and O′ (radius 2). Draw O′C parallel to AB, meeting OA at C.
   Why: The standard trick: slide the small radius across to the big one.
2. O′CAB is a rectangle, so CA = O′B = 2 and OC = 8 − 2 = 6; also O′C = AB.
   Why: Both radii are perpendicular to the same tangent line.
3. In right-angled △OCO′: $O'C = \sqrt{(OO'^2 - OC^2)} = \sqrt{(100 - 36)} = \sqrt{64} = 8$ cm.
   Why: Hypotenuse = distance between centres = 10.
4. Hence AB = 8 cm.

ANSWER: AB = 8 cm.

Case Study 5 (continued) — Two Circles in Orbit

CASE 5 (iii)

How many common tangents do the two touching circles have in all?

1. Two direct common tangents (like the scale, one on each side) and one transverse common tangent at the point where the circles touch each other.

   Why: At the touching point, both circles share the same tangent line.

2. Total = 3 common tangents.

ANSWER: Three common tangents.

CASE 5 (iv)

The discs are now pulled apart so that they no longer touch or overlap. How many common tangents are possible now? What changes when one circle lies completely inside the other without touching?

1. Separate circles: two direct + two transverse common tangents = 4.

   Why: Pulling apart 'opens' a second transverse tangent.

2. One circle inside the other (no touching): 0 common tangents — every line touching the inner circle cuts the outer one, and vice versa no line can touch both.

ANSWER: Separate: 4 common tangents. One inside the other: 0.

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Competency — Assertion–Reason 1–5

Assertion–Reason (choose: (a) both true, R explains A (b) both true, R does not explain A (c) A true, R false (d) A false, R true)

AR 1. Assertion (A): The tangent at any point of a circle is perpendicular to the radius through the point of contact. **Reason (R):** The perpendicular distance from a point to a line is the shortest distance from that point to the line.

AR 2. Assertion (A): The lengths of tangents drawn from an external point to a circle are equal. **Reason (R):** The two right triangles formed by the centre, the external point and the points of contact are congruent by RHS.

AR 3. Assertion (A): A circle can have at most two parallel tangents. **Reason (R):** A line parallel to a given tangent and touching the circle must touch it at the end of the same diameter as the given tangent.

AR 4. Assertion (A): No tangent can be drawn to a circle from a point inside it. **Reason (R):** Every line through an interior point of a circle intersects the circle at two points.

AR 5. Assertion (A): If PA and PB are tangents from P to a circle with centre O, then ∠APB + ∠AOB = 180°. **Reason (R):** ∠OAP = ∠OBP = 90° and the angles of a quadrilateral sum to 360°.

TOPPER TIP: *In Assertion–Reason, judge A and R SEPARATELY first. Only if both are true, ask the third question: does R actually explain A? Answers are in the Answer Key.*

Assertion–Reason 6–10

Assertion–Reason (continued)

AR 6. Assertion (A): The length of the tangent drawn from a point 13 cm from the centre to a circle of radius 5 cm is 12 cm. **Reason (R):** If PT is the tangent, then $PT^2 = OP^2 + OT^2$, where O is the centre and T the point of contact.

AR 7. Assertion (A): If a quadrilateral ABCD circumscribes a circle, then AB + CD = AD + BC. **Reason (R):** The lengths of tangents drawn from an external point to a circle are equal.

AR 8. Assertion (A): The chord of the larger of two concentric circles which touches the smaller circle is bisected at the point of contact. **Reason (R):** The perpendicular drawn from the centre of a circle to a chord bisects the chord.

AR 9. Assertion (A): Exactly two tangents can be drawn to a circle from a point lying on the circle. **Reason (R):** Through a point on a circle, only one line can be drawn perpendicular to the radius at that point.

AR 10. Assertion (A): If the two tangents from P are perpendicular to each other, then $OP = r\sqrt{2}$, where r is the radius. **Reason (R):** In $\triangle OAP$, $\angle OAP = 90°$ and $\angle APO = 45°$, so OA = PA and $OP = \sqrt{2} \cdot OA$.

Section 14 in one breath

Every competency question in this chapter is Theorem 10.1, Theorem 10.2, or the supplementary pair — dressed in a story. Strip the story, name the theorem, and the question is already half solved.

15 Olympiad & Foundation Challenges

8D • CHALLENGE — FOR STUDENTS WHO FINISH EARLY AND THINK DEEP

CHALLENGE 1

Two tangents drawn from a point P to a circle of radius 3 cm are inclined to each other at 60°. Find the length of each tangent.

1. OP bisects the angle between the tangents (Corollary A), so ∠APO = 30°.
   Why: Halve first — the triangle OAP holds only half the story.
2. In right-angled △OAP, tan(∠APO) = opposite/adjacent = OA/PA.
   Why: The angle at P faces the radius OA.
3. tan 30° = 3/PA, so PA = 3 / tan 30° = 3 / (1/√3) = 3√3 cm ≈ 5.20 cm.
4. PB = PA (Theorem 10.2), so each tangent is 3√3 cm.

ANSWER: Each tangent is 3√3 cm (≈ 5.20 cm).

CHALLENGE 2

Two concentric circles have radii 13 cm and 5 cm. Prove that the chord of the larger circle which touches the smaller circle is bisected at the point of contact, and find its length.

1. The bisection is Reasoning 3 of Section 12: the radius to the contact point is perpendicular to the chord, and a perpendicular from the centre bisects a chord.
   Why: Quote the proved result — challenges are allowed to stand on earlier proofs.
2. Half the chord = $\sqrt{(13^2 - 5^2)} = \sqrt{(169 - 25)} = \sqrt{144} = 12$ cm.
   Why: Pythagoras with the larger radius as hypotenuse.
3. Chord = 2 × 12 = 24 cm.
   Why: Double the half — Mistake 4 avoided.

ANSWER: The chord is bisected at the contact point and measures 24 cm.

CHALLENGE 3

AP and AQ are tangents drawn from a point A to a circle. A third tangent BC cuts AP at B and AQ at C. If AB = 5 cm, AC = 6 cm and BC = 4 cm, find AP.

1. Let the tangent BC touch the circle at R. From B, the two tangent segments are the piece of AP from B to its contact point, and BR. Call the first u; then BR = u.

Olympiad Challenges 4–6

CHALLENGE 4

Two circles of radii 9 cm and 4 cm touch each other externally. Find the length of a direct common tangent to the two circles.

1. The centres, say O and O′, are 9 + 4 = 13 cm apart.
   Why: Touching externally ⇒ distance between centres = sum of radii (Case Study 5).
2. Slide the smaller radius onto the larger (the rectangle trick of Case Study 5): the direct common tangent is one side of a right triangle with hypotenuse 13 and one leg 9 − 4 = 5.
   Why: The leg is the DIFFERENCE of radii for a direct tangent.
3. Length = $\sqrt{(13^2 - 5^2)} = \sqrt{(169 - 25)} = \sqrt{144} = 12$ cm.
   Why: A 5-12-13 triangle in hiding.

ANSWER: The direct common tangent is 12 cm long.

CHALLENGE 5

PA and PB are tangents from P to a circle of radius 5 cm, with PA = 12 cm. Find the area of quadrilateral OAPB, where O is the centre.

1. Join OP. The quadrilateral splits into △OAP and △OBP, which are congruent (Theorem 10.2's proof).
   Why: Compute one triangle, then double.
2. △OAP is right-angled at A, with legs OA = 5 and PA = 12. Its area = ½ × 5 × 12 = 30 cm².
3. Area of OAPB = 2 × 30 = 60 cm².
   Why: General pattern worth memorising: area = radius × tangent length.

ANSWER: The area of OAPB is 60 cm².

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Olympiad Challenge 6 — The Chord of Contact

CHALLENGE 6

From a point P at distance 25 cm from the centre O of a circle of radius 7 cm, tangents PA and PB are drawn. Find the length of the chord of contact AB.

1. PA = $\sqrt{(25^2 - 7^2)} = \sqrt{(625 - 49)} = \sqrt{576} = 24$ cm.

   Why: The length formula first.

2. Let M be the point where OP meets AB. By Reasoning 1, OP is the perpendicular bisector of AB, so AM = ½AB and OP ⊥ AB.

   Why: Section 12's proof, cashed in.

3. Compute the area of △OAP two ways: ½ × OA × PA = ½ × 7 × 24 = 84 cm².

   Why: Legs as base and height.

4. Also area = ½ × OP × AM = ½ × 25 × AM. So ½ × 25 × AM = 84, giving AM = 168/25 = 6.72 cm.

   Why: Hypotenuse as base, AM as height — the two-ways area trick.

5. AB = 2 × 6.72 = 13.44 cm.

ANSWER: The chord of contact AB is 13.44 cm long.

TOPPER TIP: *The two-ways area trick — legs as base-height, then hypotenuse as base — is one of the most reusable weapons in this chapter. It also powers Exercise 10.2 Q12.*

Olympiad Challenges 7–10

CHALLENGE 7

PA and PB are tangents from P to a circle with centre O, and ∠APB = 80°. Find ∠OAB.

1. ∠AOB = 180° − ∠APB = 100°.
   Why: Corollary B.
2. In △OAB, OA = OB (radii), so the base angles are equal: ∠OAB = ∠OBA.
   Why: Angles opposite equal sides of an isosceles triangle.
3. ∠OAB = (180° − 100°) / 2 = 40°.
   Why: Angle sum of a triangle.

ANSWER: ∠OAB = 40°.

CHALLENGE 8

A circle is drawn inside a right-angled triangle, touching all three sides. The sides of the triangle are 6 cm, 8 cm and 10 cm, with the right angle at C. Find the radius of the inscribed circle.

1. Let the circle touch the sides and let the tangent lengths from vertices A, B, C be x, y, z respectively (each vertex contributes an equal pair, by Theorem 10.2).
   Why: Three vertices, three letters — the Q8/Q12 machine again.
2. Reading the sides: x + y = 10 (hypotenuse AB), x + z = 8 (side CA), y + z = 6 (side BC).
   Why: Each side is a sum of two tangent lengths.
3. Add all three: 2(x + y + z) = 24, so x + y + z = 12. Subtract x + y = 10: z = 2.
4. At the right-angle corner C, the two radii to the sides CA and CB are perpendicular to those sides, forming a square with the two tangent segments from C. So the radius equals z.
   Why: A square of side r sits in the corner; its side along each leg is exactly the tangent length from C.
5. Radius = z = 2 cm.
   Why: Worth remembering: for a right triangle, r = (leg1 + leg2 − hypotenuse)/2 = (6 + 8 − 10)/2 = 2.

ANSWER: The inradius is 2 cm.

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Olympiad Challenges 9 and 10

CHALLENGE 9

Two circles, each of radius 5 cm, have centres 26 cm apart. Find the length of a transverse (crossing) common tangent to the two circles.

1. For a transverse tangent, the tangent line passes between the circles; the slide trick now produces a right triangle whose one leg is the SUM of the radii, 5 + 5 = 10.
   Why: Direct tangent → difference of radii; transverse tangent → sum. The geometry of the slide decides which.
2. The hypotenuse is the distance between centres, 26.
3. Length = $\sqrt{(26^2 - 10^2)} = \sqrt{(676 - 100)} = \sqrt{576} = 24$ cm.

ANSWER: The transverse common tangent is 24 cm long.

CHALLENGE 10

A triangle ABC circumscribes a circle of radius 4 cm. The point of contact D on BC divides it into BD = 12 cm and DC = 4 cm. Find AB and AC.

1. Let the equal tangents from A be x. Then AB = x + 12, AC = x + 4, BC = 16, and s = x + 16.
   Why: The Q12 machine with new numbers.
2. Heron: $\text{Area}^2 = s(s - a)(s - b)(s - c) = (x + 16) \cdot x \cdot 12 \cdot 4 = 48x(x + 16)$.
   Why: The factors 12 and 4 are BD and DC again.
3. Radius way: Area = r · s = 4(x + 16), so $\text{Area}^2 = 16(x + 16)^2$.
4. Equate: $16(x + 16)^2 = 48x(x + 16) \Rightarrow x + 16 = 3x \Rightarrow x = 8$.
5. AB = 8 + 12 = **20 cm**, AC = 8 + 4 = **12 cm**.
   Why: Check: s = 24; Heron: $\sqrt{(24 \cdot 8 \cdot 12 \cdot 4)} = \sqrt{9216} = 96$; r · s = 4 × 24 = 96. Agreed.

ANSWER: AB = 20 cm and AC = 12 cm.

TOPPER TIP: *Every challenge here was assembled from Section 06/07 parts: bisection, supplementary angles, vertex letters, the area-two-ways trick, and the radius slide. Olympiad problems are not new mathematics — they are old mathematics in a trench coat.*

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16

Chapter Mind Map

THE WHOLE CHAPTER ON ONE PAGE — REVISE FROM HERE

CIRCLES — TANGENTS

Heart of the chapter: **two theorems** — the tangent is perpendicular to the radius at the point of contact, and tangents from an external point are equal.

Tangent & Secant

Tangent: touches at ONE point (the point of contact). Secant: cuts at TWO points. A circle has infinitely many tangents; at most 2 parallel ones.

→

Theorem 10.1

Tangent ⊥ radius at contact. Proof idea: the radius is the SHORTEST segment from O to the line. Gives a right angle → Pythagoras.

Theorem 10.2

Tangents from an external point are EQUAL. Proof: RHS congruence of the two right triangles. OP bisects both angles.

The Golden Right Triangle

PA = $\sqrt{(d^2 - r^2)}$. Triples to spot: 3-4-5, 5-12-13, 6-8-10, 7-24-25, 8-15-17.

The Supplementary Pair

∠APB + ∠AOB = 180°. Two right angles + quadrilateral sum 360°. (Ex 10.2 Q2, Q3, Q10.)

From a Point

Inside (d < r): 0 tangents. On (d = r): 1 tangent. Outside (d > r): 2 tangents.

Circumscribing Figures

Vertex letters w, x, y, z (equal tangent pairs). Quadrilateral: AB + CD = AD + BC. Parallelogram ⇒ rhombus. Triangle: area = r × s.

→

Concentric Circles

Chord of larger touching smaller: bisected at contact; length = $2\sqrt{(R^2 - r^2)}$. Never forget to double.

Key Results to Quote

√119 cm (Ex 10.1 Q3) • radius 7 from 24/25 (Ex 10.2 Q1) • chord 8 from radii 5, 3 (Q7) • AB = 15, AC = 13 (Q12) • ∠AOB = 90° for a tangent between parallel tangents (Q9)

TOPPER TIP: *Revise actively: cover a box, speak its contents aloud from memory, then uncover and check. A mind map you can REPRODUCE from memory is worth ten readings.*

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17 Pin-Point Flash Cards

THE WHOLE CHAPTER IN YOUR POCKET — CUT THEM OUT, CARRY THEM, MASTER THEM

How to use these cards

Cut along the dashed borders and keep the 16 cards in your pocket or a small box. On Smart Memory **Day 3, Day 7 and Day 21**, shuffle the cards and test yourself: read the title, say the pin-point aloud from memory, then check. Any card you miss goes back into the box for tomorrow. Two minutes a day — that is all it takes to keep this chapter alive all year.

CARD 1 — Tangent

A tangent touches the circle at exactly ONE point — the point of contact. It never enters the circle.

CARD 2 — Theorem 10.1

The tangent at any point is PERPENDICULAR to the radius through the point of contact. Join O to the contact point — you own a right angle.

CARD 3 — The length formula

Tangent length = $\sqrt{(d^2 - r^2)}$, d = distance from centre. The distance is ALWAYS the hypotenuse.

CARD 4 — Theorem 10.2

The two tangents drawn from an external point to a circle are EQUAL in length. Same point ⇒ same length.

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Flash Cards 5–10

CARD 5 — OP bisects twice

The line from the centre to the external point bisects BOTH the angle between the tangents AND the angle at the centre.

CARD 6 — Supplementary pair

∠APB + ∠AOB = 180°, always. Given one, write the other instantly.

CARD 7 — Counting tangents

From a point: inside → 0 tangents; on the circle → 1; outside → 2. Compare d with r to decide.

CARD 8 — Parallel tangents

At most TWO tangents can be parallel — at the ends of a diameter, exactly one diameter (2r) apart.

CARD 9 — Diameter ends

Tangents drawn at the ends of a diameter are PARALLEL — both are perpendicular to the same diameter.

CARD 10 — The perpendicular at contact

The perpendicular to a tangent at its point of contact passes through the CENTRE. (Ex 10.2 Q5)

TOPPER TIP: *Say each card aloud once before you test yourself on it — hearing your own voice say the fact makes the memory stick twice as fast.*

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Flash Cards 11–16

CARD 11 — Circumscribing quadrilateral

AB + CD = AD + BC — sums of OPPOSITE sides are equal. Bonus: perimeter = 2(AB + CD).

CARD 12 — Vertex letters

From each vertex, the two tangent pieces are equal. Label them w, x, y, z — every side becomes a sum of two letters.

CARD 13 — Concentric chord

Chord of the larger circle touching the smaller = $2\sqrt{(R^2 - r^2)}$, and it is BISECTED at the point of contact. Double the half!

CARD 14 — Parallelogram rule

A parallelogram circumscribing a circle is a RHOMBUS — Q8's sums force adjacent sides equal.

CARD 15 — The 90° theorem

A tangent drawn between two parallel tangents meets the centre at 90°: ∠AOB = 90°. (Ex 10.2 Q9)

CARD 16 — Area keys

Circumscribed triangle: area = r × s. Quadrilateral OAPB: area = r × (tangent length). Two area formulas that unlock the hardest questions.

MAKE YOUR OWN CARD

Write one pin-point YOU keep forgetting, in your own words. A card you write yourself is the card you remember best.

_____

_____

MAKE YOUR OWN CARD

One more: the mistake you made most often in this chapter, and the one-line rule that fixes it.

_____

18 Active Recall

8D • RECALL — CLOSE THE BOOK, THEN ANSWER

How this works: close every other page. Answer all ten questions on rough paper, from memory alone. Only then open the Answer Key. Anything you could not recall is exactly what your Day-3 revision must target — write it in the Error Tracker (Section 19).

Ten rapid questions

R1. State Theorem 10.1 in full.

R2. State Theorem 10.2 in full.

R3. Find the length of the tangent drawn from a point 17 cm from the centre of a circle of radius 8 cm.

R4. Tangents from P touch a circle at A and B, and ∠AOB = 130°. Find ∠APB.

R5. PA is a tangent of length 13 cm from the point P. Find the length of the other tangent from P.

R6. Two concentric circles have radii 10 cm and 6 cm. Find the length of the chord of the larger circle which touches the smaller circle.

R7. How many tangents can be drawn to a circle from a point inside it? From a point on it?

R8. A quadrilateral ABCD circumscribes a circle. If AB = 9 cm, BC = 11 cm and CD = 6 cm, find DA.

R9. What is the distance between the two parallel tangents of a circle of radius 9 cm?

R10. If ∠APB = 55°, find ∠AOB and ∠POA.

Five recall prompts — say each answer aloud, in a full sentence

P1. Define a tangent to a circle.

P2. Define the point of contact.

P3. Define a secant, and say how it differs from a tangent.

P4. State the formula for the length of a tangent from an external point, naming every symbol.

P5. State the two corollaries of Theorem 10.2 about the line OP.

> **TOPPER TIP:** *Scoring: 13–15 correct — chapter mastered, move on. 9–12 — revise the missed topics, retake in 3 days. Below 9 — re-study Sections 05–07, then retake. Record your score in the Mastery Tracker.*

19 Self-Assessment & Mastery Tracker

KALVIYOGI SMART MEMORY SYSTEM — LEARN IT, THEN KEEP IT

Memory fades on a predictable curve — unless you revisit at the right moments. This schedule is your revisit plan for THIS chapter. Tick each day off when done.

Day	Your activity for Chapter 10	Done
Day 0	Pre-learning assessment: attempt the Quick Check (Section 02) and Rapid Questions R1–R10 (Section 18) BEFORE studying. Note your score.	☐
Day 1	Learn the chapter fully (Sections 01–10) and complete Level 1–3 practice.	☐
Day 3	Without-book recall: reproduce the Mind Map (Section 16) on a blank page; redo Rapid Questions; finish Level 4.	☐
Day 7	Independent problem-solving: Level 5 + Olympiad Challenges, timed, book closed. Then take the **Day 7 Mini-Test** (Practice Plus) and run the Flash Cards (Section 17).	☐
Day 14	Mixed practice: re-attempt every question in your Error Tracker; solve Ex 10.2 Q7 and Q12 fresh.	☐
Day 21	Long-term recall test: write both theorem proofs (10.1 and 10.2) from memory, then verify line by line. Then take the **Day 21 Mini-Test** (Practice Plus).	☐
Day 30	Chapter mastery assessment: take the **Day 30 Mastery Test** (Practice Plus) under exam conditions; update your Concept Mastery Score below.	☐

Formula Recall Tracker — tick when you can say it with the book closed

Formula / Statement	Day 1	Day 3	Day 7	Day 21
Theorem 10.1 (tangent ⊥ radius) + its shortest-distance proof idea	☐	☐	☐	☐
Theorem 10.2 (equal tangents) + RHS proof	☐	☐	☐	☐
Tangent length = $\sqrt{(d^2 - r^2)}$	☐	☐	☐	☐
∠APB + ∠AOB = 180° and OP bisects both angles	☐	☐	☐	☐
Circumscribing quadrilateral: AB + CD = AD + BC	☐	☐	☐	☐
Concentric chord = $2\sqrt{(R^2 - r^2)}$, bisected at contact	☐	☐	☐	☐

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Mastery Tracker — Errors and Concept Scores

Error Tracker — every mistake becomes a lesson

Date	Question	What went wrong	The correct idea

Concept Mastery Score — rate yourself honestly, out of 5

Concept	Score / 5	Status
Tangent, secant and point of contact	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Theorem 10.1 and its proof	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Theorem 10.2 and its proof	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Length and angle numericals	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Circumscribing quadrilaterals and triangles	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Board-style proofs (Q4, Q5, Q9, Q13)	___ / 5	☐ Weak ☐ Okay ☐ Mastered

Learning Time Tracker

Day / Date	Minutes studied	What I studied

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Practice Plus — 25 Concept MCQs (1–13)

One mark each, board style. Cover the options, answer aloud, then check the key. Explanations for every MCQ are in the Answer Key.

Q1. A tangent to a circle intersects the circle in
(a) 0 points (b) 1 point (c) 2 points (d) infinitely many points

Q2. A line intersecting a circle in two distinct points is called a
(a) tangent (b) chord (c) secant (d) radius

Q3. The maximum number of parallel tangents a circle can have is
(a) 1 (b) 2 (c) 3 (d) infinite

Q4. The angle between a tangent and the radius through the point of contact is
(a) 30° (b) 60° (c) 90° (d) 180°

Q5. The number of tangents that can be drawn to a circle from an external point is
(a) 0 (b) 1 (c) 2 (d) 3

Q6. The length of the tangent drawn from a point 10 cm from the centre to a circle of radius 6 cm is
(a) 4 cm (b) 8 cm (c) 16 cm (d) $\sqrt{136}$ cm

Q7. The length of a tangent to a circle is 12 cm and the point is 13 cm from the centre. The radius is
(a) 1 cm (b) 5 cm (c) 25 cm (d) $\sqrt{313}$ cm

Q8. PA and PB are tangents from P to a circle with centre O. If $\angle AOB = 110°$, then $\angle APB$ is
(a) 110° (b) 55° (c) 70° (d) 90°

Q9. If $\angle APB = 60°$ for tangents PA, PB to a circle with centre O, then $\angle AOB$ is
(a) 60° (b) 120° (c) 30° (d) 90°

Q10. PA is a tangent of length 11 cm from point P. The length of the other tangent PB from P is
(a) 22 cm (b) 5.5 cm (c) 11 cm (d) cannot be found

Q11. Two concentric circles have radii 5 cm and 3 cm. The chord of the larger circle which touches the smaller circle has length
(a) 4 cm (b) 6 cm (c) 8 cm (d) 10 cm

Q12. The distance between the two parallel tangents of a circle of radius 4 cm is
(a) 4 cm (b) 6 cm (c) 8 cm (d) 16 cm

Q13. The common point of a tangent to a circle and the circle is called the
(a) centre (b) point of contact (c) end point (d) vertex

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25 Concept MCQs (14–25)

Q14. A point is at a distance exactly equal to the radius from the centre. The number of tangents from it is

(a) 0 (b) 1 (c) 2 (d) infinite

Q15. A point lies inside a circle. The number of tangents that can be drawn from it is

(a) 0 (b) 1 (c) 2 (d) infinite

Q16. From a point Q with OQ = 25 cm, a tangent is drawn to a circle of radius 7 cm. Its length is

(a) 18 cm (b) 24 cm (c) 32 cm (d) $\sqrt{674}$ cm

Q17. If the two tangents from P are perpendicular to each other, then $\angle POA$ equals

(a) 90° (b) 60° (c) 45° (d) 30°

Q18. A quadrilateral ABCD circumscribes a circle. If AB = 8 cm, CD = 6 cm and BC = 9 cm, then AD is

(a) 7 cm (b) 5 cm (c) 11 cm (d) 3 cm

Q19. If d is the distance of an external point from the centre and r the radius, the length of the tangent is

(a) $\sqrt{(d^2 + r^2)}$ (b) $\sqrt{(d^2 - r^2)}$ (c) $d - r$ (d) $d + r$

Q20. Two tangents drawn from P to a circle of radius 4 cm are perpendicular to each other. The length of each tangent is

(a) 2 cm (b) 4 cm (c) 8 cm (d) $4\sqrt{2}$ cm

Q21. The tangents drawn at the ends of a diameter of a circle are

(a) perpendicular (b) intersecting (c) parallel (d) coincident

Q22. TP and TQ are tangents from T to a circle with centre O. If $\angle OPQ = 25°$ (P, Q being the points of contact), then $\angle PTQ$ is

(a) 25° (b) 50° (c) 65° (d) 130°

Q23. Two concentric circles have radii 10 cm and 6 cm. The chord of the larger circle which touches the smaller circle is

(a) 8 cm (b) 12 cm (c) 16 cm (d) 20 cm

Q24. A triangle circumscribes a circle, and the two tangents from vertex A are each 5 cm. If AB = 9 cm, then the part of AB next to B measures

(a) 5 cm (b) 4 cm (c) 9 cm (d) 14 cm

Q25. Which of the following statements is FALSE?

(a) A tangent is perpendicular to the radius at the point of contact (b) Tangents from an external point are equal (c) A circle has infinitely many tangents (d) A tangent intersects the circle at two points

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Find the Error — Questions 1–5

Each piece of work below contains exactly one error. Find it, name it, and write the correct solution. Corrections are in the Answer Key.

ERROR 1

Radius = 5 cm, distance of point from centre = 13 cm. Student writes: 'Length of tangent = 13 − 5 = 8 cm.'

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 2

∠POQ = 100°. Student writes: '∠PTQ = 100°, because the angle at the centre and the angle between the tangents stand on the same chord and are equal.'

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 3

PA = 9 cm is one tangent from P. Student writes: 'There are two tangents, so the other tangent PB = 2 × 9 = 18 cm.'

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 4

OP = 10 cm, OT = 6 cm (T = point of contact). Student writes: '$PT^2 = OP^2 + OT^2 = 100 + 36 = 136$, so PT ≈ 11.7 cm.'

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 5

Concentric circles of radii 13 cm and 5 cm. Student writes: 'Chord of the larger circle touching the smaller = $\sqrt{(13^2 - 5^2)} = 12$ cm.'

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

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Find the Error — Questions 6–10

ERROR 6

Student writes: 'A circle can have three parallel tangents — one at the top, one through the middle and one at the bottom.'

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 7

Student writes: 'From a point on the circle, two tangents can be drawn — one going left and one going right along the same line.'

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 8

For a quadrilateral circumscribing a circle, student writes: 'AB + BC = CD + DA, because sums of adjacent sides of a circumscribing quadrilateral are equal.'

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 9

Tangents PA, PB are inclined at 80°. Student writes: '∠POA = 80°, because the angle at the centre equals the angle between the tangents.'

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 10

Student writes: 'The tangents at the ends of a diameter are perpendicular to each other, because each of them is perpendicular to the diameter.'

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

TOPPER TIP: Every error here is one of the six Common Mistakes of Section 10 in a new costume. If you can spot all ten, Section 10 has done its job.

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Previous Year Questions — Real CBSE Board Questions (1)

CBSE 2024 — MCQ (1 MARK)

In the figure, PT is a tangent to a circle with centre O and ∠TPO = 35°. The measure of ∠x, where x is the exterior angle at O in triangle OTP (between OT and the extension of PO beyond O), is: (a) 110° (b) 115° (c) 120° (d) 125°

1. ∠PTO = 90°.
   Why: Theorem 10.1 — radius OT is perpendicular to the tangent PT at the point of contact T.
2. x is an exterior angle of △OTP at O, so x = sum of the two interior opposite angles = ∠TPO + ∠PTO.
   Why: Exterior angle theorem.
3. x = 35° + 90° = 125°.

ANSWER: Option (d) — x = 125°.

CBSE 2023 — MCQ (1 MARK)

In the figure, PT is a tangent at T to the circle with centre O. If ∠TPO = 25°, then the exterior angle x at O in triangle OTP is equal to: (a) 25° (b) 65° (c) 90° (d) 115°

1. ∠PTO = 90° (Theorem 10.1).
2. x = ∠TPO + ∠PTO = 25° + 90° = 115°.
   Why: Same skeleton as the 2024 question — boards repeat skeletons; only the number changed.

ANSWER: Option (d) — x = 115°.

CBSE 2022 — MCQ (1 MARK)

From an external point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is: (a) 7 cm (b) 12 cm (c) 15 cm (d) 24.5 cm

1. $r = \sqrt{(25^2 - 24^2)} = \sqrt{(625 - 576)} = \sqrt{49} = 7$ cm.
   Why: This is NCERT Exercise 10.2 Q1 itself, asked as a board MCQ — proof that the NCERT exercises ARE the board syllabus.

ANSWER: Option (a) — radius = 7 cm.

Previous Year Questions — Real CBSE Board Questions (2)

CBSE 2023 — ASSERTION–REASON (1 MARK)

Assertion (A): A tangent to a circle is perpendicular to the radius through the point of contact. Reason (R): The lengths of tangents drawn from an external point to a circle are equal. Choose: (a) both true, R explains A (b) both true, R does not explain A (c) A true, R false (d) A false, R true

1. A is true — it is Theorem 10.1.
2. R is true — it is Theorem 10.2.
3. Does R explain A? R talks about lengths of two tangents; A talks about a right angle. The equality of tangents is proved USING A, not the other way round — so R does not explain A.

   Why: Judge truth first, explanation second — the Section 14 method.

ANSWER: Option (b) — both true, R does not explain A.

CBSE 2023 — PROOF (3 MARKS)

Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2∠OPQ.

1. In △OPQ, OP = OQ (radii), so ∠OQP = ∠OPQ. Call this common value x.

   Why: Base angles of an isosceles triangle.
2. Angle sum in △OPQ: ∠POQ = 180° − 2x.
3. By Corollary B (Exercise 10.2 Q10), ∠PTQ + ∠POQ = 180°, so ∠PTQ = 180° − (180° − 2x) = 2x.
4. Hence ∠PTQ = 2∠OPQ. **Proved.**

ANSWER: ∠PTQ = 2∠OPQ.

Previous Year Questions — Real CBSE Board Questions (2, continued)

CBSE 2023 — PROOF (3 MARKS)

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

1

This is Exercise 10.2 Q10, asked verbatim. The proof: two right angles (Theorem 10.1) inside quadrilateral OAPB leave ∠APB + ∠AOB = 360° − 180° = 180°.

Why: See Section 07, Q10 for the full step-by-step version — in the exam, write all six steps.

ANSWER: The two angles sum to 180° — they are supplementary.

TOPPER TIP: *Pattern alert: this exact proof has appeared as Exercise 10.2 Q10, as Corollary B of Section 06, and as a 3-mark board question. Learn it once, in full steps, and it pays you three times.*

Previous Year Questions — Real CBSE Board Questions (3)

CBSE 2025 — PROOF (2 MARKS)

Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

1. Each tangent is perpendicular to the diameter at its own end (Theorem 10.1).
2. Two lines perpendicular to the same line are parallel. **Proved.**

Why: This is Exercise 10.2 Q4 — the board asked the NCERT question directly. Full version: Section 07, Q4.

ANSWER: The tangents at the ends of a diameter are parallel.

CBSE 2025 — PROOF (3 MARKS)

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

1. Split each angle at the centre into two halves using the congruent tangent triangles at each vertex (Section 07, Q13).
2. The four half-angles around O total 360°; opposite-side angles ∠AOB and ∠COD collect the same halves in two groups, so ∠AOB + ∠COD = 180°. **Proved.**

Why: Write the full eight-step version from Section 07, Q13 in the exam.

ANSWER: Opposite sides subtend supplementary angles at the centre.

TOPPER TIP: *Pattern to notice: five of these eight board questions ARE NCERT Exercise 10.1/10.2 questions, word for word. The board is not hiding a secret syllabus — it is the exercise you just finished.*

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Chapter Model Test A — Basics (20 Marks)

Time: 45 minutes. Definitions, direct theorem use, friendly numbers. Target: full marks — this paper checks whether the foundations are automatic.

Q1. A tangent to a circle intersects it in _____ point(s). (a) 0 (b) 1 (c) 2 (d) infinite **[1 marks]**

Q2. The length of the tangent from a point 10 cm from the centre to a circle of radius 6 cm is: (a) 4 cm (b) 8 cm (c) 16 cm (d) √136 cm **[1 marks]**

Q3. State Theorem 10.2 (lengths of tangents from an external point). **[2 marks]**

Q4. PA and PB are tangents to a circle with centre O. If ∠AOB = 140°, find ∠APB. **[2 marks]**

Q5. Find the length of the tangent drawn from a point 15 cm from the centre of a circle of radius 9 cm. **[3 marks]**

Q6. Prove that the tangents drawn at the ends of a diameter of a circle are parallel. **[3 marks]**

Q7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle. **[4 marks]**

Q8. Tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80°. Find ∠AOB and ∠POA. **[4 marks]**

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Chapter Model Test B — Standard (20 Marks)

Time: 45 minutes. Board-paper level. Write the theorem you use in every solution — unnamed theorems earn no style marks here.

Q1. TP and TQ are tangents from T to a circle with centre O. If ∠OPQ = 25°, then ∠PTQ is: (a) 25° (b) 50° (c) 65° (d) 130° **[1 marks]**

Q2. A quadrilateral ABCD circumscribes a circle. If AB = 8 cm, CD = 6 cm and BC = 9 cm, then AD is: (a) 7 cm (b) 5 cm (c) 11 cm (d) 3 cm **[1 marks]**

Q3. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius of the circle. **[2 marks]**

Q4. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre. **[3 marks]**

Q5. Two tangents drawn from a point P to a circle of radius 4 cm are perpendicular to each other. Find the length of each tangent and the distance OP. **[3 marks]**

Q6. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC. **[5 marks]**

Q7. A triangle ABC is drawn to circumscribe a circle of radius 3 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 6 cm and 4 cm respectively. Find the sides AB and AC. **[5 marks]**

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Chapter Model Test C — Advanced (20 Marks)

Time: 45 minutes. Proof-heavy, HOTS-level — this is the paper that separates 95 from 100.

Q1. From a point P with OP = 25 cm, tangents are drawn to a circle of radius 7 cm. The length of each tangent is: (a) 24 cm (b) 18 cm (c) 32 cm (d) √674 cm **[1 marks]**

Q2. Two circles, each of radius 5 cm, have centres 26 cm apart. The length of a transverse common tangent is: (a) 12 cm (b) 24 cm (c) 26 cm (d) 20 cm **[1 marks]**

Q3. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2∠OPQ. **[3 marks]**

Q4. AP and AQ are tangents drawn from A to a circle. A third tangent BC cuts AP at B and AQ at C. If AB = 5 cm, AC = 6 cm and BC = 4 cm, find AP. **[4 marks]**

Q5. From a point P at distance 25 cm from the centre O of a circle of radius 7 cm, tangents PA and PB are drawn. Find the length of the chord of contact AB. **[5 marks]**

Q6. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle. **[6 marks]**

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Smart Memory Mini-Tests — Day 7 and Day 21

Day 7 Mini-Test (10 questions, book closed, 10 minutes):

1. State Theorem 10.1 in one line.

2. Find the length of the tangent from a point 13 cm from the centre of a circle of radius 5 cm.

3. If ∠APB = 70°, find ∠AOB.

4. PA is a tangent of length 9 cm from P. Find the length of the second tangent from P.

5. What is the maximum number of parallel tangents a circle can have?

6. Two concentric circles have radii 13 cm and 5 cm. Find the chord of the larger circle touching the smaller.

7. A point is at distance less than the radius from the centre. How many tangents can be drawn from it?

8. If ∠APB = 90°, find ∠POA.

9. A quadrilateral circumscribes a circle with AB = 7 cm, CD = 9 cm, BC = 12 cm. Find DA.

10. What is the distance between the parallel tangents of a circle of radius 11 cm?

Day 21 Mini-Test (10 questions, book closed, 10 minutes):

1. State Theorem 10.2, and name the congruence rule used in its proof.

2. The tangent from a point is 15 cm and the radius is 8 cm. How far is the point from the centre?

3. If ∠AOB = 95°, find ∠APB.

4. Two concentric circles have radii 10 cm and 6 cm. Find the chord of the larger circle touching the smaller.

5. If ∠OPQ = 30°, find ∠PTQ (P, Q are points of contact, T the external point).

6. Define a secant in one line.

7. Two tangents from P are perpendicular to each other and the radius is 6 cm. Find OP.

8. A circumscribing quadrilateral has AB = 10 cm and CD = 10 cm. Find its perimeter.

9. How many tangents can be drawn from a point on the circle?

10. If ∠APB = 80°, find ∠OAB.

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Smart Memory Mini-Test — Day 30 Mastery Test

Day 30 Mastery Test (10 questions, exam conditions, 20 minutes). Score 9–10: the chapter is yours. 6–8: revisit the missed concept in Section 19's tracker. Below 6: re-study Sections 06–07 and retake after 3 days.

1. State the key idea of the proof of Theorem 10.1 (one line).

2. State the congruence rule and the two equal quantities used in the proof of Theorem 10.2.

3. From a point Q, the tangent length is 24 cm and OQ = 25 cm. Find the radius.

4. State the result of Exercise 10.2 Q9 (the tangent between two parallel tangents) in one line.

5. A triangle circumscribes a circle of radius 4 cm; BD = 8 cm and DC = 6 cm. Find AB and AC.

6. Find the chord of contact for tangents from a point 25 cm from the centre of a circle of radius 7 cm.

7. Two circles of radii 9 cm and 4 cm touch externally. Find the direct common tangent.

8. Find the inradius of a right triangle with sides 6 cm, 8 cm, 10 cm.

9. In Exercise 10.2 Q9's figure, what is ∠AOB?

10. Find the area of quadrilateral OAPB when the radius is 5 cm and the tangent length is 12 cm.

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Answer Key — Graded Practice, Levels 1 and 2

Level 1 — Foundation

1.1 $\sqrt{(10^2 - 6^2)} = \sqrt{64} =$ **8 cm**. **1.2 Two** from an outside point; **none** from an inside point. **1.3** $\sqrt{(12^2 + 5^2)} = \sqrt{169} =$ **13 cm** (here the distance is the hypotenuse to be FOUND, so the squares add). **1.4** $180° - 120° =$ **60°**. **1.5 one**; **perpendicular**. **1.6 7 cm** (Theorem 10.2).

Level 2 — Skill Builder

2.1 $\sqrt{(15^2 - 9^2)} = \sqrt{144} =$ **12 cm**. **2.2** $\angle AOB = 180° - 70° =$ **110°**; $\angle POA = ½ \times 110° =$ **55°** (OP bisects). **2.3** $2\sqrt{(13^2 - 5^2)} = 2 \times 12 =$ **24 cm**. **2.4** $180° - 50° =$ **130°**. **2.5** $\sqrt{(25^2 - 7^2)} = \sqrt{576} =$ **24 cm**. **2.6** $AD = AB + CD - BC = 9 + 7 - 10 =$ **6 cm**.

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Answer Key — Levels 3, 4, 5 and Active Recall

Level 3 — Application

3.1 $\sqrt{(26^2 - 10^2)} = \sqrt{576} =$ **24 m**. **3.2** $\sqrt{(17^2 - 8^2)} = \sqrt{225} =$ **15 cm**. **3.3** DA = 12 + 10 − 14 = **8 cm**. **3.4** OAPB is a square (three right angles and OA = OB force PA = r): each tangent = **4 cm**; OP = $4\sqrt{2} \approx$ **5.66 cm**.

Level 4 — Advanced Reasoning

4.1 ∠APO = 30° (bisection); in △OAP, sin 30° = r/12 gives r = **6 cm**; PA = $\sqrt{(144 - 36)} = \sqrt{108} = 6\sqrt{3} \approx$ **10.39 cm**. **4.2** Half-chord = $\sqrt{(17^2 - 8^2)} = 15$ (perpendicular from centre, Pythagoras), so chord = **30 cm**. **4.3** PA = $\sqrt{(100 - 36)} = 8$. Area of △OAP two ways: ½ × 6 × 8 = 24 = ½ × 10 × AM, so AM = 4.8 and AB = **9.6 cm**.

Level 5 — Mathematical Genius

5.1 7.5 cm — full solution at Olympiad Challenge 3 (Section 15). **5.2 12 cm** — full solution at Olympiad Challenge 4: $\sqrt{(13^2 - 5^2)}$ with centre distance 13 and radius difference 5.

Active Recall (Section 18)

R1 Tangent ⊥ radius through the point of contact. **R2** Tangents from an external point are equal. **R3** $\sqrt{(17^2 - 8^2)} =$ **15 cm**. **R4** 180° − 130° = **50°**. **R5 13 cm**. **R6** $2\sqrt{(100 - 36)} =$ **16 cm**. **R7 0** from inside; **1** from on the circle. **R8** DA = 9 + 6 − 11 = **4 cm**. **R9** 2 × 9 = **18 cm**. **R10** ∠AOB = **125°**; ∠POA = **62.5°**.

P1 A line intersecting the circle at exactly one point. **P2** The single common point of the tangent and the circle. **P3** A secant intersects at TWO points; a tangent at ONE. **P4** $\sqrt{(d^2 - r^2)}$; d = distance of the point from the centre, r = radius. **P5** OP bisects ∠APB; OP bisects ∠AOB.

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Answer Key — Practice Plus

25 Concept MCQs — answers with one-line reasons

1 (b) definition of tangent. **2 (c)** definition of secant. **3 (b)** only at the ends of a diameter. **4 (c)** Theorem 10.1. **5 (c)** one on each side. **6 (b)** $\sqrt{(100 - 36)} = 8$. **7 (b)** $\sqrt{(169 - 144)} = 5$. **8 (c)** supplementary: $180 - 110$. **9 (b)** $180 - 60$. **10 (c)** Theorem 10.2. **11 (c)** $2\sqrt{(25 - 9)} = 8$. **12 (c)** one diameter = 8. **13 (b)** point of contact. **14 (b)** the point is ON the circle. **15 (a)** every line through it is a secant. **16 (b)** $\sqrt{(625 - 49)} = 24$. **17 (c)** $\angle AOB = 90°$, halved by OP. **18 (b)** $8 + 6 - 9 = 5$. **19 (b)** leg = $\sqrt{(hyp^2 - leg^2)}$. **20 (b)** OAPB is a square, PA = r. **21 (c)** both $\perp$ the diameter. **22 (b)** CBSE 2023's result: $\angle PTQ = 2\angle OPQ$. **23 (c)** $2\sqrt{(100 - 36)} = 16$. **24 (b)** $9 - 5$ (A's piece is 5). **25 (d)** a tangent meets the circle at ONE point, not two.

Find the Error — corrections

E1 Subtraction is wrong — the three lengths form a right triangle: $\sqrt{(13^2 - 5^2)}$ = **12 cm**. **E2** The angles are supplementary, not equal: $\angle PTQ = 180° - 100°$ = **80°**. **E3** Tangents from the same point are EQUAL, not double: PB = **9 cm**. **E4** OP is the hypotenuse: $PT^2 = OP^2 - OT^2 = 64$, PT = **8 cm**. **E5** 12 cm is the HALF-chord: chord = **24 cm**. **E6** The 'middle' line is a secant: at most **two** parallel tangents. **E7** Left and right along one line is ONE line: exactly **one** tangent from a point on the circle. **E8** OPPOSITE sums are equal: AB + CD = AD + BC. **E9** $\angle POA = ½(180° - 80°)$ = **50°**. **E10** Two lines perpendicular to the same line are **parallel**, not perpendicular.

Assertion–Reason — answers

AR1 (a) Theorem 10.1 is proved exactly by the shortest-distance fact R. **AR2 (a)** RHS congruence is the proof of A. **AR3 (a)** any parallel tangent must sit at the other end of the same diameter — only two ends exist. **AR4 (a)** R is precisely why no tangent exists. **AR5 (a)** R is the computation behind A. **AR6 (c)** A is true (12 cm), but R is false: $PT^2 = OP^2 - OT^2$. **AR7 (a)** A is proved by adding R's equal pairs. **AR8 (a)** the radius to the contact point is the perpendicular of R. **AR9 (d)** A is false — exactly ONE tangent from a point on the circle; R is true and is the reason. **AR10 (a)** R's triangle computation gives $OP = \sqrt{2} \cdot r$.

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Answer Key — Model Tests and Day Tests

Model Test A — Basics

Q1 (b). **Q2** (b) 8 cm. **Q3** The lengths of tangents drawn from an external point to a circle are equal. **Q4** 40°. **Q5** 12 cm. **Q6** Proof — Section 07, Q4. **Q7** 8 cm. **Q8** ∠AOB = 100°, ∠POA = 50°.

Model Test B — Standard

Q1 (b) 50°. **Q2** (b) 5 cm. **Q3** 7 cm. **Q4** Proof — Section 07, Q5. **Q5** Each tangent 4 cm; OP = $4\sqrt{2}$ cm. **Q6** Proof — Section 07, Q8. **Q7** AB = 12 cm, AC = 10 cm (same as Application 3).

Model Test C — Advanced

Q1 (a) 24 cm. **Q2** (b) 24 cm. **Q3** Proof — PYQ section (CBSE 2023). **Q4** AP = 7.5 cm. **Q5** 13.44 cm. **Q6** Proof — Section 07, Q13.

Day 7 Mini-Test

1 Tangent ⊥ radius at contact. **2** 12 cm. **3** 110°. **4** 9 cm. **5** Two. **6** 24 cm. **7** None. **8** 45°. **9** DA = 7 + 9 − 12 = 4 cm. **10** 22 cm.

Day 21 Mini-Test

1 Equal tangents; RHS. **2** $\sqrt{(15^2 + 8^2)} = 17$ cm. **3** 85°. **4** 16 cm. **5** 60°. **6** A line intersecting the circle at two points. **7** $6\sqrt{2}$ cm. **8** 2(10 + 10) = 40 cm. **9** One. **10** 40°.

Day 30 Mastery Test

1 The radius is the shortest segment from O to the tangent line, hence the perpendicular. **2** RHS; common hypotenuse OP and equal radii OA = OB. **3** 7 cm. **4** ∠AOB = 90°. **5** AB = 15 cm, AC = 13 cm. **6** 13.44 cm. **7** 12 cm. **8** 2 cm. **9** 90°. **10** 60 cm^2.

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Revision Card — Chapter 10 on One Page

The two theorems

Theorem 10.1: the tangent at any point of a circle is perpendicular to the radius through the point of contact (proof: the radius is the shortest segment from O to the line). **Theorem 10.2:** the lengths of tangents drawn from an external point to a circle are equal (proof: RHS congruence).

The golden right triangle

Radius ⊥ tangent ⇒ Pythagoras: **tangent = $\sqrt{(d^2 - r^2)}$**, where d = distance from the centre. The distance is always the hypotenuse. Spot the triples: 3-4-5, 5-12-13, 6-8-10, 7-24-25, 8-15-17.

The angle pair

∠APB + ∠AOB = 180° (supplementary), and **OP bisects both** angles. Also ∠PTQ = 2∠OPQ (CBSE 2023). Tangents from a point: inside 0, on 1, outside 2. At most 2 parallel tangents — at the ends of a diameter, which are themselves parallel.

Circumscribing figures

Give every vertex one letter for its equal tangent pair. Quadrilateral: **AB + CD = AD + BC**; perimeter = 2(AB + CD); a circumscribing parallelogram is a rhombus. Triangle: **area = r × s** (Heron = the second way). Concentric chord: **$2\sqrt{(R^2 - r^2)}$**, bisected at the contact point — double the half!

Exam-eve checklist

☐ Centre joined to every point of contact, right angles marked ☐ Hypotenuse = centre-to-point distance ☐ Angle pairs made supplementary, never equal ☐ Chord answers doubled ☐ Theorems quoted BY NUMBER in every proof

Chapter 10 complete. You did not memorise Circles — you discovered, visualised, derived and mastered them.

Next: Chapter 11 — Areas Related to Circles, in the same 8D format. • KalviYogi • Mathematics – Beyond the Formula • 369 Tesla × KalviYogi • © Edunivra Pvt Ltd

CBSE · CLASS 10 · MATHEMATICS

MATHEMATICS BEYOND THE FORMULA

CHAPTER 11 · AREAS RELATED TO CIRCLES

Don't Memorize Mathematics. Understand, Visualise and Discover It.

The KalviYogi 8D Way: Discover · Visualise · Understand · Derive · Solve · Apply · Challenge · Recall

Every solution with COMPLETE steps — one by one, nothing skipped

Sectors · Segments · Areas of Combinations — every formula DERIVED from the full circle

Five-Level Practice · Smart Memory System · Mastery Tracker

Written like a topper's own notebook — so you can learn

even when no teacher is beside you.

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01 Chapter Introduction

8D • DISCOVER — WHY SHOULD I LEARN THIS?

A clock hand sweeps across the dial. A cow tied to a corner post munches grass — but only in a curve-edged patch. A lighthouse throws a wedge of red light over the sea to warn ships. A car wiper cleans a fan-shaped window. None of these shapes is a full circle — each is a **part** of a circle. This chapter teaches you to measure exactly those parts.

The beautiful secret of this chapter: **you need no new area formulas at all.** Every sector is a honest fraction of the full circle, every segment is a sector with a triangle removed, and every complicated shaded design is just circles, squares and triangles added or subtracted. If you truly understand the full circle, this whole chapter is already yours — we will **derive** each formula from that one idea, step by step.

What you will master in this chapter

- Arc length and sector area — **derived** by proportion from the full circle, never memorised blindly.
- Minor and major sectors, quadrants, and the perimeter of a sector (the two radii everyone forgets!).
- Segment area = sector − triangle — derived with a diagram, for angles 60°, 90° and 120°.
- Major segment in two ways: circle − minor segment, and major sector + triangle.
- Real machines of the chapter: the clock hand, the grazing horse, the silver brooch, the umbrella, the wipers, the lighthouse, the table cover — all of NCERT Exercise 11.1, every step shown.
- Areas of combinations: shade, split, add, subtract — a strategy that solves every design problem.

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Your Chapter Map — 19 Stops, One Journey

This chapter is a journey with 19 stops. Tick each stop as you finish it.

Discover & Visualise	Understand & Derive	Solve, Apply, Recall
☐ 01 Chapter Introduction ☐ 02 Prerequisite Knowledge ☐ 03 Real-Life Story ☐ 04 Visual Concepts (CPA)	☐ 05 Mathematical Foundations ☐ 06 Formula Derivations ☐ 08 Multiple Solution Methods ☐ 10 Common Mistakes	☐ 07 Detailed Solved Examples ☐ 09 Hands-on Activities ☐ 11 Graded Practice (5 Levels) ☐ 12 Mathematical Reasoning
☐ 13 Real-Life Applications ☐ 14 Competency-Based Questions	☐ 15 Olympiad Challenges ☐ 16 Chapter Mind Map	☐ 17 Pin-Point Flash Cards ☐ 18 Active Recall ☐ 19 Mastery Tracker

How to use this chapter (read this first!)

1. **Draw the figure before you calculate.** In this chapter, the student who sketches the sector or segment first almost never picks the wrong formula.
2. Every solution shows **Step 1, Step 2, Step 3…** with a small *Why* note in purple — the reason behind the step. Examiners award marks for these steps, not only for the final answer.
3. **Obey the π instruction.** If the question says use π = 3.14, use 3.14; otherwise use π = 22/7. Mixing them is the most common mark-loser in this chapter.
4. After Section 07, test yourself in Sections 11–15 **before** opening the Answer Key. Finish with the Mind Map (16), Flash Cards (17), closed-book Recall (18) and your Mastery Tracker (19) — and return on Day 3, 7, 14, 21 and 30.

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02 Prerequisite Knowledge

CLASS 9 RECAP — THE TOOLS YOU ALREADY OWN

This chapter stands on five ideas from earlier classes. Refresh them here — two minutes now will save you twenty minutes later.

1. The full circle — your two master formulas

Circumference (the boundary length) = $2\pi r$. **Area** (the surface covered) = πr^2. Every formula in this chapter is a fraction of one of these two. If you remember nothing else, remember these.

2. Radius and diameter

Diameter = 2 × radius, so r = d/2. NCERT loves giving the diameter (the brooch is 35 mm across!) and watching students use 35 as the radius. Halve first, always.

3. Angles at a point

Angles around a point add to **360°**. A straight line is 180°, a right angle is 90°, and a **quadrant** of a circle is the sector with angle 90° — exactly one-fourth of the circle. A **semicircle** is the sector with angle 180°.

4. Triangles you will meet inside circles

Two radii and a chord always form an **isosceles triangle** (two sides = r). If the angle at the centre is 60°, the triangle is **equilateral** with area $(\sqrt{3}/4)r^2$. If it is 90°, the area is $\frac{1}{2}r^2$. If it is 120°, the area is again $(\sqrt{3}/4)r^2$ — Section 06 shows why.

5. The clock fact

A minute hand turns a full 360° in 60 minutes, so it turns **6° every minute**. In 5 minutes it sweeps 30°; in 10 minutes, 60°. An hour hand is twelve times slower: ½° per minute. NCERT's favourite question (Exercise 11.1 Q3) is built entirely on this one fact.

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Quick Check — Are You Ready? (5 Questions)

Try all five without looking back. Answers are given right below — cover them with your hand first!

Q1. Find the circumference and the area of a circle of radius 7 cm. (Use π = 22/7)

Q2. A circle has diameter 28 cm. What is its radius?

Q3. Through what angle does a minute hand turn in 15 minutes?

Q4. What fraction of a circle is a quadrant? What is the area of a quadrant of radius 14 cm? (Use π = 22/7)

Q5. Two radii and a chord form a triangle with the angle at the centre equal to 60°. What kind of triangle is it, and what is its area if r = 10 cm? (Use √3 = 1.73)

Answers — check only after trying

Q1. Circumference = 2 × (22/7) × 7 = **44 cm**. Area = (22/7) × 7^2 = **154 cm²**.

Q2. r = 28/2 = **14 cm**.

Q3. 15 × 6° = **90°**.

Q4. A quadrant is **1/4** of the circle. Area = (1/4) × (22/7) × 14^2 = (1/4) × 616 = **154 cm²**.

Q5. The two radii are equal and the included angle is 60°, so the base angles are (180° − 60°)/2 = 60° each — the triangle is **equilateral**. Area = (√3/4) × 10^2 = 25 × 1.73 = **43.25 cm²**.

TOPPER TIP: *Got 4 or 5 right? You are fully ready. Got fewer? Re-read the five cards on the previous page once, then come back — do not skip this step.*

03 Real-Life Mathematical Story

DISCOVER • VISUALISE — THE ROUND GARDEN OF GREEN VALLEY SCHOOL

Before any formula, live the problem. Read this story slowly — the mathematics will walk out of the story by itself.

*Green Valley School has a beautiful **circular garden** of radius **21 metres**, with a small fountain at its centre O. This year the gardening club, led by **Anitha** of Class 10, is redesigning it.*

*First, the gardener fixes a **rotating sprinkler** at the centre. Its arm is exactly 21 m long — it reaches the boundary — but a tap controls how far it rotates. For the rose beds, Anitha sets it to sweep only **60°** before stopping. "How much area will the sprinkler water?" she wonders.*

*Second, a **straight stone path** is laid across one edge of the garden as a chord, joining two points on the boundary so that the chord makes a 60° angle at the centre. Between this path and the boundary arc lies a small crescent-shaped bed where Anitha wants to plant marigolds. "What is the area of that little bed? It is not a sector — the path cuts the sector's corner off!"*

*Third, the club plans a **walking ring** all around the garden: a path of uniform width that takes the radius from 21 m out to **28 m**. "How much paving stone do we need for the ring?"*

Three questions. Three heroes of this chapter: the **sector** (the sprinkler's wedge), the **segment** (the marigold bed — a sector with its triangle cut away), and the **ring** (a big circle with a smaller circle removed). Let us solve all three, one step at a time.

Solving the Story — Step by Step

STORY PROBLEM 1

The sprinkler arm is 21 m and it sweeps an angle of 60° at the centre of the circular garden. Find the area it waters. (Use π = 22/7)

THINK FIRST: Two radii and an arc — whenever you see this boundary, think SECTOR, and take the fraction θ/360 of the full circle.

1. The watered region is a wedge bounded by two radii and an arc — a **sector** of the circle with r = 21 m and angle θ = 60°.
   Why: Naming the shape correctly is half the solution: two radii + arc = sector.
2. The full circle (360°) has area $\pi r^2 = (22/7) \times 21^2 = 1386$ m². Our sector is only 60° out of 360°, i.e. the fraction 60/360 = 1/6 of the circle.
   Why: A sector is a fraction of the circle: fraction = θ/360. Section 06 derives this properly.
3. Watered area = (1/6) × 1386 = **231 m²**.

ANSWER: 231 m²

STORY PROBLEM 2

The straight path is a chord that subtends 60° at the centre. Find the area of the marigold bed between the chord and the arc (the minor segment). (Use π = 22/7 and √3 = 1.73)

THINK FIRST: Segment = Sector − Triangle. Say it like a mantra; every segment question in this chapter is this one line.

1. Look at the picture: the segment = the 60° sector MINUS the triangle formed by the two radii and the chord. Segment = Sector − Triangle.
   Why: The chord slices the triangle out of the sector; what remains is the segment. This is the master idea of the chapter.
2. From Problem 1, the sector area is 231 m².
   Why: Reuse your own results — examiners design questions this way on purpose.
3. The triangle has two sides 21 m (radii) with 60° between them, so it is equilateral of side 21 m. Its area = (√3/4) × 21² = (1.73/4) × 441 = 762.93/4 = 190.73 m² (approx.).
   Why: Angle 60° between two equal radii forces an equilateral triangle — Section 05 proves this.
4. Segment area = 231 − 190.73 = **40.27 m²** (approx.).

Solving the Story — Problem 3: The Walking Ring

STORY PROBLEM 3

A walking path of uniform width surrounds the garden, increasing the radius from 21 m to 28 m. Find the area of the ring path. (Use π = 22/7)

THINK FIRST: A ring (annulus) is a subtraction: big circle minus small circle. Factor π out before multiplying — the arithmetic becomes tiny.

1 The ring is what remains when the inner circle (r = 21) is removed from the outer circle (R = 28). Ring area = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2)$.

Why: Combinations are always: draw, split into known shapes, then add or subtract. Here it is one subtraction.

2 Compute $R^2 - r^2 = 28^2 - 21^2 = 784 - 441 = 343$.

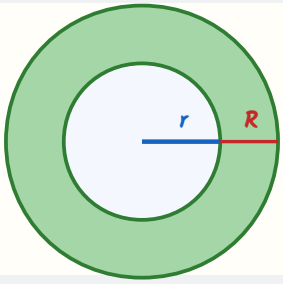
3 Ring area = $(22/7) \times 343 = 22 \times 49 =$ **1078 m²**.

Why: 343 = 7 × 49, so the 7 cancels beautifully — NCERT chooses numbers that cancel when you factor π out first.

ANSWER: 1078 m²

The three story shapes — pin these in your memory

Two radii + an arc → SECTOR → take the fraction θ/360 of the circle.
A chord + an arc → SEGMENT → sector − triangle.
One shape inside another → COMBINATION → draw, split, add or subtract.
Every question in Exercise 11.1 — clock, horse, brooch, umbrella, wiper, lighthouse — is one of these three shapes wearing a costume.



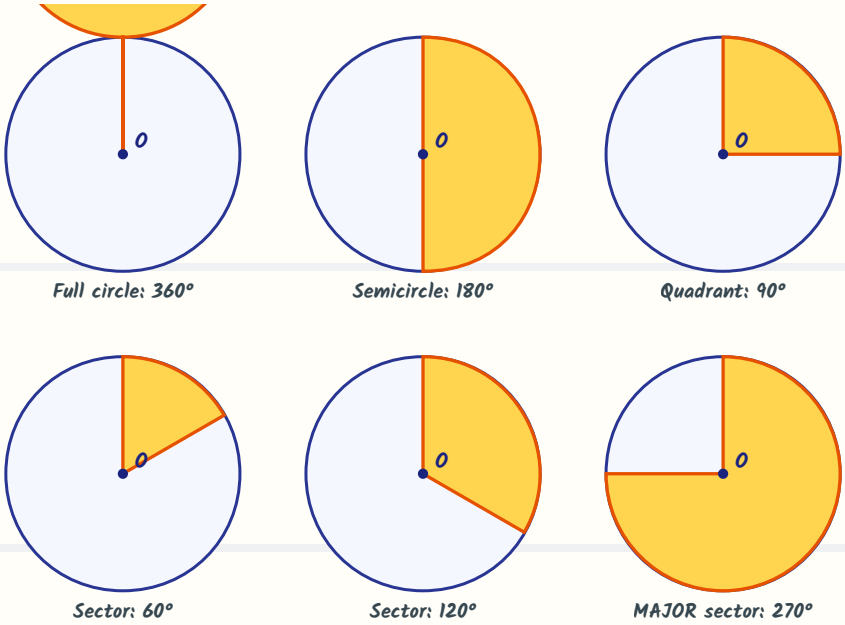
Ring (annulus) = $\pi R^2 - \pi r^2$

04 Visual Concept Explanation

SINGAPORE CPA — CONCRETE → PICTORIAL → ABSTRACT

Concrete first. Imagine a round pizza on the table. The whole pizza is the full circle: angle 360°, area πr^2. Cut it in half and each piece is a sector of 180°. Cut it into quarters: sectors of 90°. The smaller the angle at the centre, the smaller the slice — the area shrinks in exactly the same proportion as the angle.

Now the picture. Watch the shaded wedge as the angle changes:

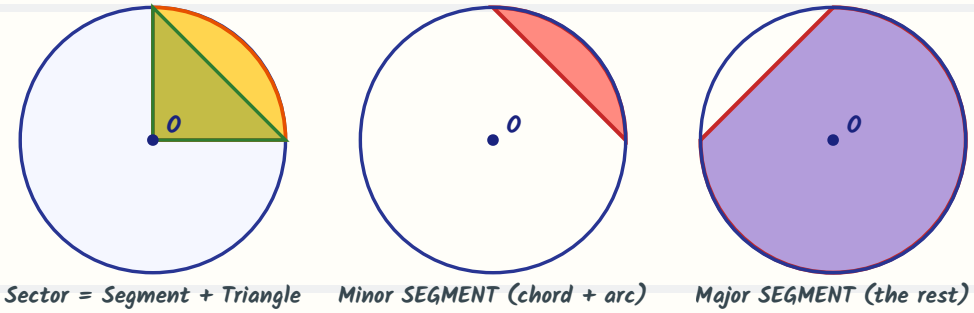


Then the abstract. If 360° owns the whole area πr^2, then θ degrees own the fraction θ/360 of it: **Area of sector = (θ/360) × πr^2**. The same fraction of the boundary gives the arc: **Length of arc = (θ/360) × $2\pi r$**. Symbols are just the picture, compressed. In Section 06 we will derive both formulas slowly, with full reasoning.

> **TOPPER TIP:** *A sector with angle MORE than 180° is called a MAJOR sector; less than 180° is a MINOR sector. If a question says just 'sector', it means the minor one — unless it says major.*

Visual Concepts — Sector, Segment and the Triangle Inside

Now cut the pizza slice with one straight cut — a **chord**. The small cap between the chord and the arc is the **segment**. Look carefully at the middle picture: the sector is exactly the segment PLUS the triangle. Turn that sentence around and you own the segment formula forever.



Read the three pictures like a story

Picture 1: the yellow sector is split by the chord into the green triangle (two radii + chord) and the small cap on top of the chord. So **Sector = Segment + Triangle**.
Picture 2: the cap alone — the **minor segment**. Segment = Sector − Triangle.
Picture 3: everything on the other side of the same chord — the **major segment**. Major segment = Area of circle − Minor segment. Never compute it with a fresh sector formula and a minus triangle — that is the classic trap of this chapter (Mistake 3, Section 10).

PIN POINT: Sector = Segment + Triangle. This ONE picture-sentence gives you the segment formula, the major-segment rule, and half the chapter's marks.

05 Mathematical Foundations

UNDERSTAND — THE PRECISE IDEAS, WORD BY WORD

Arc

An **arc** is a part of the circumference (the boundary) of a circle. The smaller piece is the **minor arc**; the larger piece is the **major arc**. When no qualification is given, 'the arc' means the minor arc.

Sector

The region bounded by **two radii and an arc** is a **sector**. The sector with the minor arc is the **minor sector**; the one with the major arc is the **major sector**. Special cases: angle 90° → **quadrant**; angle 180° → **semicircle**. The angle at the centre is written θ (theta) and is measured in degrees.

Segment

The region bounded by a **chord and an arc** is a **segment**. The segment on the minor-arc side is the **minor segment**; the rest of the circle is the **major segment**. Minor segment + Major segment = the whole circle — always.

The triangle inside every segment

Join the ends of the chord to the centre. You get a triangle with two sides equal to r and included angle θ. Three cases rule this chapter:

- θ = 60°: base angles are (180° − 60°)/2 = 60°, so the triangle is **equilateral**; area = $(\sqrt{3}/4)\ r^2$.
- θ = 90°: the radii are perpendicular; area = ½ × r × r = **½ r^2**.
- θ = 120°: dropping a perpendicular from O to the chord splits it into two 30°-60°-90° triangles; the area works out to **$(\sqrt{3}/4)\ r^2$** again (proved in Section 12).

Perimeter of a sector

The boundary of a sector has THREE pieces: arc + radius + radius. **Perimeter of a sector = length of arc + 2r.** Forgetting the two radii is the single most common error in board questions on this chapter — CBSE asked it directly as a 1-mark MCQ in 2024 (see Section: Previous Year Questions).

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06 Formula Derivations

8D • DERIVE — NEVER MEMORISE WHAT YOU CAN DERIVE

Derivation A — Length of an Arc, by Proportion

We derive the arc formula from one honest question: *what fraction of the boundary does an arc of θ degrees cover?*

1. When the angle at the centre is 360°, the 'arc' is the **whole circumference** = 2πr.
   Why: Start from the full turn — the one case you know for certain.
2. When the angle is 1°, the arc is 1/360 of the boundary = 2πr/360.
   Why: Equal angles cut equal arcs — the arc grows in direct proportion to the angle.
3. So when the angle is θ degrees, multiply by θ: **Length of arc = (θ/360) × 2πr**.
   Why: Direct proportion: arc ∝ θ. This is the whole derivation — three honest steps.
4. **Check with a known case:** θ = 90°, r = 7 cm, π = 22/7: arc = (90/360) × 2 × (22/7) × 7 = (1/4) × 44 = **11 cm**.
   Why: A quadrant's boundary curve should be one-fourth of 44 cm — it is. The formula passes its first test.

Derivation B — Area of a Sector, by the Same Proportion

1. When θ = 360°, the 'sector' is the whole circle: area = πr^2.
   Why: Same starting point: the full turn.
2. When θ = 1°, the sector is 1/360 of the circle: area = $\pi r^2/360$.
   Why: Equal angles cut equal sectors — sector area also grows in direct proportion to θ.
3. So for angle θ: **Area of sector = (θ/360) × πr^2**.
   Why: One fraction, θ/360, rules both formulas. Learn the fraction, and you own both.
4. **Check:** θ = 90°, r = 7: area = (1/4) × (22/7) × 49 = **38.5 cm^2** — exactly one-fourth of 154 cm^2.
   Why: Matches the Quick Check Q4 answer. A derived formula you have tested is a formula you will never forget.

PIN POINT: Both formulas wear the same crown: the fraction θ/360. Sector area = fraction × πr^2; arc length = fraction × 2πr.

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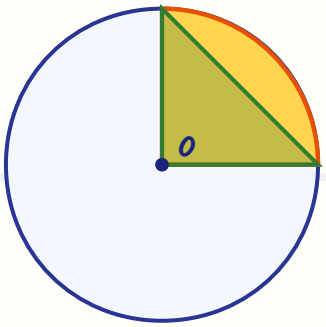
Derivation C — Area of a Segment = Sector − Triangle

There is no new formula here at all — only the picture from Section 04, read carefully.



OAB: segment (top cap) + triang

1. Draw the chord AB of a circle with centre O. The radii OA, OB and the chord AB form a triangle inside the sector OAB.

   Why: Every segment problem begins by drawing these two radii — they create the triangle.

2. By the picture: Area of sector OAB = Area of segment + Area of △OAB.

   Why: The sector is exactly the cap plus the triangle — no overlap, no gap.

3. Turn it around: **Area of (minor) segment = Area of sector − Area of △OAB**.

   Why: Subtraction, because the segment is what is LEFT after the triangle is removed.

4. **Worked check:** r = 10 cm, θ = 60°, π = 3.14, √3 = 1.73. Sector = (60/360) × 3.14 × 100 = 52.33 cm². Triangle is equilateral: (√3/4) × 100 = 25 × 1.73 = 43.25 cm². Segment = 52.33 − 43.25 = **9.08 cm²**.

   Why: Notice the segment (9.08) is much smaller than the sector (52.33) — a thin cap. If your segment answer ever exceeds your sector answer, you added instead of subtracting.

5. **The major segment:** Major segment = Area of circle − Minor segment. (Equivalently: major sector + triangle — Section 08 proves both give the same number.)

   Why: For the major side, ALWAYS go through the whole circle. It is shorter and trap-free.

PIN POINT: Segment = Sector − Triangle (minor side). Major segment = Circle − Minor segment. Two lines; the whole segment family is done.

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Derivation D — Areas of Combinations: The Master Strategy

A 'combination' figure is several known shapes joined together. There is no combination formula — there is a **strategy**, and it never changes:

1. **Draw and label.** Mark every radius, side and centre on the figure. Most combination problems die here — an unlabelled figure hides its own solution.
2. **Split.** Decide how the shaded region is built from circles, sectors, semicircles, squares, rectangles and triangles.
   Why: Ask: is the shade a SUM of pieces, or a whole shape with pieces REMOVED?
3. **Compute each piece separately**, writing its formula and substitution in full.
   Why: One piece per line. Crowded arithmetic is where marks leak.
4. **Add or subtract** exactly as your split decided, and finish with units (cm^2, m^2).

Worked Combination 1 — Square with an Inscribed Circle

COMBINATION 1

A circle of radius 5 cm is drawn inside a square of side 10 cm, touching all four sides. Find the area of the part of the square outside the circle. (Use π = 3.14)

1. Shaded (outside) area = Area of square − Area of circle.
   Why: The shade is what REMAINS – so it is a subtraction.
2. Area of square = 10 × 10 = 100 cm^2.
3. Area of circle = 3.14 × 5^2 = 3.14 × 25 = 78.5 cm^2.
   Why: The radius is 5 because the circle's diameter equals the square's side, 10 cm.
4. Shaded area = 100 − 78.5 = **21.5 cm^2**.

ANSWER: 21.5 cm^2

Derivation D (continued) — Two More Combinations

COMBINATION 2

A design is made of a rectangle 20 cm × 10 cm with a semicircle of radius 5 cm attached outward on each of the two shorter sides. Find the total area of the design. (Use π = 3.14)

1. Total area = Area of rectangle + 2 × Area of semicircle.
   Why: Here the shade is a SUM — pieces are added on, not removed.
2. Area of rectangle = 20 × 10 = 200 cm^2.
3. Two semicircles of radius 5 make ONE full circle: area = 3.14 × 25 = 78.5 cm^2.
   Why: Two equal semicircles always join into a full circle — a favourite NCERT shortcut (remember it for the Answer Key of Exercise Q13-style designs).
4. Total area = 200 + 78.5 = **278.5 cm^2**.

ANSWER: 278.5 cm^2

Worked Combination 3 — The Four Corners of a Square

COMBINATION 3

A circle of radius 6 cm is inscribed in a square of side 12 cm, touching all four sides. Find the total area of the four corner regions of the square that lie outside the circle. (Use π = 3.14)

1. Corner regions together = Area of square − Area of circle.
   Why: Four separate corners, ONE subtraction — never compute a corner alone.
2. Area of square = 12 × 12 = 144 cm^2. Area of circle = 3.14 × 6^2 = 3.14 × 36 = 113.04 cm^2.
   Why: The inscribed circle's diameter equals the square's side, so r = 6.
3. Corner regions = 144 − 113.04 = **30.96 cm^2**.

ANSWER: 30.96 cm^2

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07 Detailed Solved Examples

8D • SOLVE — NCERT EXAMPLES & EXERCISE 11.1, EVERY STEP SHOWN

First, the three solved examples from the NCERT textbook itself — read these before the exercise. Then every question of **Exercise 11.1** (the only exercise of this chapter in the rationalised textbook), solved with complete steps. Remember the house rule: unless a question says otherwise, use **π = 22/7**.

NCERT EXAMPLE 1

Find the area of the sector of a circle with radius 4 cm and of angle 30°. Also, find the area of the corresponding major sector. (Use π = 3.14)

THINK FIRST: The angle given (30°) belongs to the MINOR sector. The major sector's angle is what remains of the full turn: 360° − 30° = 330°.

1. Minor sector: area = (θ/360) × πr² = (30/360) × 3.14 × 4² = (1/12) × 3.14 × 16 = 50.24/12 = 4.19 cm² (approx.).
   Why: Substitute θ = 30, r = 4, π = 3.14 exactly as instructed.
2. Major sector angle = 360° − 30° = 330°.
   Why: The two sector angles at a centre always add to 360°.
3. Major sector: area = (330/360) × 3.14 × 16 = (11/12) × 50.24 = 46.05 cm² (approx.).
   Why: Same formula, bigger angle.
4. Cross-check: 4.19 + 46.05 = 50.24 = 3.14 × 16 = area of the whole circle. The two sectors rebuild the circle.
   Why: Always run this free check when a question asks for both sectors.

ANSWER: Minor sector ≈ 4.19 cm²; major sector ≈ 46.05 cm²

CBSE-STYLE MARKING SCHEME: Correct sector formula with θ = 30 [1] • Minor sector value [1] • Major angle 330° and major sector value [1]

NCERT Example 2 — The Segment, Worked by NCERT

NCERT EXAMPLE 2

Find the area of the segment AYB, if the radius of the circle is 21 cm and ∠AOB = 120°. (Use π = 22/7)

THINK FIRST: Segment = sector − triangle. The only delicate part is the triangle with a 120° angle at O — its area is $(\sqrt{3}/4)r^2$, which Section 12 proves.

1. Area of sector OAYB = $(120/360) \times (22/7) \times 21^2 = (1/3) \times (22/7) \times 441 = (1/3) \times 1386 = 462$ cm^2.

2. Area of $\triangle$OAB = $(\sqrt{3}/4) \times r^2 = (\sqrt{3}/4) \times 441$ cm^2.
   Why: For θ = 120° (and also 60°), the triangle formed by two radii has area $(\sqrt{3}/4)r^2$. NCERT leaves √3 as it is in this example.

3. Area of segment AYB = $462 - (441\sqrt{3}/4)$ cm^2.
   Why: Sector minus triangle — the master line of this chapter.

ANSWER: $462 - (441\sqrt{3}/4)$ cm^2

CBSE-STYLE MARKING SCHEME: Sector area 462 [1] • Triangle area $(441\sqrt{3}/4)$ [1] • Subtraction and final exact answer [1]

PIN POINT: When NCERT leaves an answer in exact form — $462 - (441\sqrt{3}/4)$ — copy the form exactly. Do not 'help' by approximating unless the question supplies a value for √3.

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NCERT Example 3 — A Combination Design

NCERT EXAMPLE 3

Find the area of the shaded region in the figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles drawn inside the square with diameters AD and BC respectively. (Use π = 22/7)

THINK FIRST: Shaded = the square with two semicircles REMOVED. And the two semicircles together make one full circle — watch for that gift.

1. Area of square ABCD = 14 × 14 = 196 cm^2.

2. Each semicircle has diameter 14 cm, so radius = 7 cm. Two semicircles = one full circle of radius 7: area = (22/7) × 7^2 = 154 cm^2.

   Why: Never compute two equal semicircles separately — join them mentally into a circle.

3. Shaded area = 196 − 154 = **42 cm^2**.

   Why: Whole minus removed parts — the combination strategy of Derivation D.

ANSWER: 42 cm^2

CBSE-STYLE MARKING SCHEME: Square area [1] • Two semicircles = one circle, 154 [1] • Subtraction and answer [1]

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Exercise 11.1 Q1 and Q2 — First Steps with the Formulas

EXERCISE 11.1 — Q1

Find the area of a sector of a circle with radius 6 cm, if the angle of the sector is 60°.

THINK FIRST: Direct sector formula. The textbook's standing instruction applies: use π = 22/7.

1. Write the data: r = 6 cm, θ = 60°. Area of sector = (θ/360) × πr².
   Why: Always begin by naming r and θ — it prevents diameter-radius mix-ups.
2. Substitute: = (60/360) × (22/7) × 6^2 = (1/6) × (22/7) × 36.
3. Simplify: 36/6 = 6, so area = (22/7) × 6 = 132/7 = **18.86 cm²** (approx.).
   Why: Cancel before you multiply — small numbers, fewer errors.

ANSWER: 132/7 cm² ≈ 18.86 cm²

CBSE-STYLE MARKING SCHEME: Formula [1] • Substitution and simplification [1]

EXERCISE 11.1 — Q2

Find the area of a quadrant of a circle whose circumference is 22 cm.

THINK FIRST: The radius is NOT given — steal it from the circumference first. A quadrant is the sector with θ = 90°.

1. Circumference 2πr = 22, so r = 22/(2π) = 22 × 7/(2 × 22) = 7/2 = 3.5 cm.
   Why: Use π = 22/7 and solve for r before touching the area.
2. Area of quadrant = (90/360) × πr² = (1/4) × (22/7) × $(7/2)^2$.
   Why: A quadrant is exactly one-fourth of the circle — the fraction 1/4 replaces θ/360.
3. = (1/4) × (22/7) × (49/4) = 77/8 = **9.625 cm²**.

ANSWER: 77/8 cm² = 9.625 cm²

CBSE-STYLE MARKING SCHEME: Radius from circumference [1] • Quadrant as one-fourth of circle area [1]

PIN POINT: Q1 and Q2 are the chapter in miniature: name r and θ first, choose the fraction θ/360 (or 1/4 for a quadrant), and let the formula do the rest. When r is missing, steal it from the circumference.

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Exercise 11.1 Q3 — The Clock Hand

EXERCISE 11.1 — Q3

The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

THINK FIRST: The hand is the radius; the swept region is a sector. First find the angle: the minute hand turns 6° per minute.

1. Angle swept in 1 minute = 360°/60 = 6°. So in 5 minutes, θ = 5 × 6° = 30°.

   Why: This angle-finding step is the hidden heart of the question — most errors happen here, not in the area formula.

2. The hand's length is the radius: r = 14 cm. Swept area = area of sector with θ = 30°.

   Why: A moving hand sweeps a sector — two positions of the hand are the two radii.

3. Area = (30/360) × (22/7) × 14^2 = (1/12) × (22/7) × 196 = (1/12) × 616 = 154/3 = **51.33 cm^2** (approx.).

ANSWER: 154/3 cm^2 ≈ 51.33 cm^2

CBSE-STYLE MARKING SCHEME: Angle in 5 minutes = 30° [1] • Sector formula and value [1]

PIN POINT: The clock's hidden step: convert TIME into ANGLE first (6° per minute). Every clock question in every exam begins with that one conversion.

Exercise 11.1 Q4 — Segment and Major Sector

EXERCISE 11.1 — Q4

A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding: (i) minor segment (ii) major sector. (Use π = 3.14)

THINK FIRST: 'Right angle' means θ = 90°, so the triangle inside is the easy one: area = ½r². The major sector uses the reflex angle 270°.

1. Area of the (minor) sector = (90/360) × 3.14 × 10^2 = (1/4) × 314 = 78.5 cm^2.

2. Area of △OAB = ½ × OA × OB = ½ × 10 × 10 = 50 cm^2.
   Why: At 90° the two radii are perpendicular, so they serve as base and height directly.

3. (i) Minor segment = sector − triangle = 78.5 − 50 = **28.5 cm^2**.

4. (ii) Major sector angle = 360° − 90° = 270°. Major sector = (270/360) × 3.14 × 100 = (3/4) × 314 = **235.5 cm^2**.
   Why: Or: whole circle (314) − minor sector (78.5) = 235.5. Two roads, one answer.

ANSWER: (i) 28.5 cm^2 (ii) 235.5 cm^2

CBSE-STYLE MARKING SCHEME: Sector 78.5 and triangle 50 [1] • Minor segment 28.5 [1] • Major sector via 270° [1]

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Exercise 11.1 Q5 — One Arc, Three Questions

EXERCISE 11.1 — Q5

In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find: (i) the length of the arc (ii) the area of the sector formed by the arc (iii) the area of the segment formed by the corresponding chord.

THINK FIRST: One figure, three quantities — and each answer feeds the next. The triangle at 60° is equilateral.

1. (i) Length of arc = $(\theta/360) \times 2\pi r = (60/360) \times 2 \times (22/7) \times 21 = (1/6) \times 132 =$ **22 cm**.

   Why: Arc uses the CIRCUMFERENCE family ($2\pi r$); sector will use the AREA family (πr^2). Keep the families separate.

2. (ii) Area of sector = $(60/360) \times (22/7) \times 21^2 = (1/6) \times 1386 =$ **231 cm²**.

3. For (iii), find the triangle: OA = OB = 21 and ∠AOB = 60°, so △OAB is equilateral of side 21. Area = $(\sqrt{3}/4) \times 21^2 = (441\sqrt{3}/4)$ cm².

   Why: Base angles = $(180° - 60°)/2 = 60°$ each — all angles 60°, hence equilateral.

4. (iii) Area of segment = sector − triangle = **$231 - (441\sqrt{3}/4)$ cm²**.

   Why: NCERT accepts this exact form — do not spoil it with an approximate √3 unless the question gives one.

ANSWER: (i) 22 cm (ii) 231 cm² (iii) $231 - (441\sqrt{3}/4)$ cm²

CBSE-STYLE MARKING SCHEME: Arc length [1] • Sector area [1] • Equilateral triangle identified [1] • Segment by subtraction [1]

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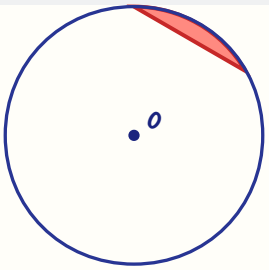
Exercise 11.1 Q6 — Minor AND Major Segments



Minor segment: r = 15, θ = 60°

EXERCISE 11.1 — Q6

A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use π = 3.14 and √3 = 1.73)

THINK FIRST: Find the minor segment first (sector − triangle). The major segment is then ONE subtraction from the whole circle — never a new sector calculation.

1. Area of sector = (60/360) × 3.14 × 15^2 = (1/6) × 3.14 × 225 = (1/6) × 706.5 = 117.75 cm^2.

2. △OAB is equilateral (angle 60° between equal radii). Area = (√3/4) × 15^2 = (1.73/4) × 225 = 389.25/4 = 97.3125 cm^2.

   Why: Use √3 = 1.73 exactly as the question orders.

3. Minor segment = 117.75 − 97.3125 = **20.4375 cm^2**.

4. Area of the whole circle = 3.14 × 225 = 706.5 cm^2. Major segment = 706.5 − 20.4375 = **686.0625 cm^2**.

   Why: Major = circle − minor. In Section 08 you will verify this same answer by the second road: major sector + triangle.

ANSWER: Minor segment = 20.4375 cm^2; major segment = 686.0625 cm^2

CBSE-STYLE MARKING SCHEME: Sector and triangle [1] • Minor segment [1] • Major segment as circle − minor [1]

PIN POINT: For the MAJOR segment, never write 'major sector − triangle'. The correct roads are: circle − minor segment, or major sector + triangle.

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Exercise 11.1 Q7 — The 120° Segment

EXERCISE 11.1 — Q7

A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use π = 3.14 and √3 = 1.73)

THINK FIRST: Same master line: segment = sector − triangle. At 120° the triangle's area is $(\sqrt{3}/4)r^2$ — resist the temptation to use $\frac{1}{2}r^2$, which belongs only to 90°.

1. Area of sector = $(120/360) \times 3.14 \times 12^2 = (1/3) \times 3.14 \times 144 = (1/3) \times 452.16 = 150.72$ cm².

2. Area of $\triangle OAB = (\sqrt{3}/4) \times 12^2 = (1.73/4) \times 144 = 249.12/4 = 62.28$ cm².

   Why: Drop a perpendicular from O to the chord: it splits the 120° triangle into two 30-60-90 triangles whose total area is $(\sqrt{3}/4)r^2$ — the proof is Reasoning 2 in Section 12.

3. Area of segment = 150.72 − 62.28 = **88.44 cm²**.

ANSWER: 88.44 cm²

CBSE-STYLE MARKING SCHEME: Sector 150.72 [1] • Triangle $(\sqrt{3}/4)r^2 = 62.28$ [1] • Segment 88.44 [1]

Exercise 11.1 Q8 — The Grazing Horse

EXERCISE 11.1 — Q8

A horse is tied to a peg at one corner of a square-shaped grass field of side 15 m by means of a 5 m long rope. Find: (i) the area of that part of the field in which the horse can graze. (ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use π = 3.14)

THINK FIRST: The horse is tied at a CORNER of the square, so the walls block it to a quarter-turn: the grazing region is a sector of angle 90° (a quadrant) whose radius is the rope's length. The field's side (15 m) is only there to confirm the rope is shorter than the field.

1. The corner angle of a square is 90°, and the rope (5 m) is shorter than the side (15 m), so the horse grazes a quadrant of radius 5 m.

   Why: If the rope were LONGER than the side, the story would change completely — always compare rope and side first.

2. (i) Grazing area = (90/360) × 3.14 × 5^2 = (1/4) × 3.14 × 25 = 78.5/4 = **19.625 m²**.

3. (ii) With a 10 m rope: grazing area = (1/4) × 3.14 × 10^2 = (1/4) × 314 = 78.5 m².

   Why: Same quadrant shape, radius doubled — note the area became 4 times, because area grows with r^2.

4. Increase = 78.5 − 19.625 = **58.875 m²**.

   Why: The question asks for the INCREASE, not the new area — the final subtraction is a step, not an afterthought.

ANSWER: (i) 19.625 m² (ii) increase = 58.875 m²

CBSE-STYLE MARKING SCHEME: Quadrant identified with rope as radius [1] • Area with 5 m rope [1] • Area with 10 m rope and increase [1]

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Exercise 11.1 Q9 — The Silver Brooch

EXERCISE 11.1 — Q9

A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors. Find: (i) the total length of the silver wire required. (ii) the area of each sector of the brooch.

THINK FIRST: Two traps guard this question: 35 mm is the DIAMETER, and the wire is used twice over — once around the circle and once across it, five times.

1. Radius r = 35/2 mm. Circumference = 2πr = 2 × (22/7) × (35/2) = 110 mm.
   Why: Halve the diameter before anything else.

2. Wire in the diameters = 5 × 35 = 175 mm.
   Why: Each of the 5 straight pieces is a full diameter, 35 mm.

3. (i) Total wire = circumference + diameters = 110 + 175 = **285 mm**.

4. (ii) The 10 sectors are equal, so each sector's area = (1/10) × area of circle = (1/10) × (22/7) × $(35/2)^2$ = (1/10) × (22/7) × (1225/4) = 385/4 = **96.25 mm²**.
   Why: Equal sectors share the circle's area equally — no need for the angle (36°) at all, though it agrees: (36/360) = 1/10.

ANSWER: (i) 285 mm (ii) 96.25 mm²

CBSE-STYLE MARKING SCHEME: Radius halved and circumference 110 [1] • Diameters wire 175 and total 285 [1] • Each sector = 1/10 of circle area [1]

PIN POINT: Diameter given? HALVE IT FIRST. The brooch (35 mm) and the table cover (28 cm radius, given directly) are NCERT's paired trap and anti-trap.

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Exercise 11.1 Q10 & Q11 — Umbrella and Wipers

EXERCISE 11.1 — Q10

An umbrella has 8 ribs which are equally spaced. Assuming the umbrella to be a flat circle of radius 45 cm, find the area between two consecutive ribs of the umbrella.

THINK FIRST: Eight equal ribs cut the circle into 8 equal sectors. You may use the angle (360°/8 = 45°) or simply take 1/8 of the circle — Section 08 compares both roads.

1. Area between two consecutive ribs = (1/8) × area of the circle = (1/8) × (22/7) × 45^2.
   Why: Equal parts of a circle: the fraction is 1/(number of parts).
2. = (1/8) × (22/7) × 2025 = 22275/28 = **795.54 cm²** (approx.).

ANSWER: 22275/28 cm² ≈ 795.54 cm²

CBSE-STYLE MARKING SCHEME: One-eighth of circle area set up [1] • Value [1]

EXERCISE 11.1 — Q11

A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

THINK FIRST: Each blade sweeps a sector (r = 25, θ = 115°). The wipers do not overlap, so the total is simply DOUBLE one sector.

1. Area cleaned by ONE wiper = (115/360) × (22/7) × 25^2 = (115/360) × (22/7) × 625.
2. = (115 × 22 × 625)/(360 × 7) = 1581250/2520 = 627.48 cm² (approx.).
   Why: Keep the fraction until the last step; rounding early is how 627.48 becomes a wrong 630.
3. Total area = 2 × 627.48 = **1254.96 cm²** (approx.).
   Why: 'Do not overlap' is the licence to double. If they overlapped, the shared part would have to be subtracted.

ANSWER: 1254.96 cm² (approx.)

CBSE-STYLE MARKING SCHEME: One sector set up [1] • One sector value [1] • Doubling for two wipers [1]

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Exercise 11.1 Q12 — The Lighthouse

EXERCISE 11.1 — Q12

To warn ships of underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use π = 3.14)

THINK FIRST: The light's reach is the radius (16.5 km); the warned region is a sector of 80°. Obey the instruction: π = 3.14 here, NOT 22/7.

1. Area warned = area of sector = (80/360) × 3.14 × $(16.5)^2$.
   Why: Distance reached = radius. 'Sector of angle 80°' is stated in the question itself.
2. $(16.5)^2$ = 272.25. So area = (2/9) × 3.14 × 272.25 = (2 × 854.865)/9 = 1709.73/9.
   Why: 80/360 simplifies to 2/9 before multiplying — small fractions first.
3. = **189.97 km²**.

ANSWER: 189.97 km²

CBSE-STYLE MARKING SCHEME: Sector with r = 16.5 identified [1] • Substitution with π = 3.14 [1] • Value [1]

EXERCISE 11.1 — Q14 (MCQ)

Tick the correct answer: Area of a sector of angle p (in degrees) of a circle with radius R is: (A) (p/180) × 2πR (B) (p/360) × πR² (C) (p/180) × πR² (D) (p/360) × 2πR

THINK FIRST: Eliminate by family: an AREA must contain R^2 and π — options (A) and (D) contain 2πR, which is a LENGTH family. Between (B) and (C), the fraction must be p/360.

1. Options (A) and (D) are built from 2πR — they measure lengths, not areas. Reject both.
   Why: Dimensional sense is a free MCQ weapon: area answers must carry R^2.
2. Option (C) has the area family but the wrong fraction (p/180). The full turn is 360°, so the fraction is p/360.
3. Correct: **(B) (p/360) × πR²** — exactly the formula derived in Section 06.

ANSWER: (B)

CBSE-STYLE MARKING SCHEME: Correct option with the fraction p/360 and πR^2 identified [1]

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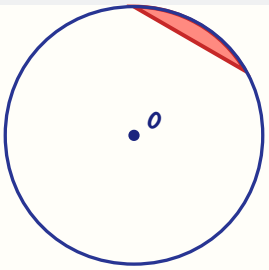
Exercise 11.1 Q13 — The Round Table Cover



e design = one 60° segment, r =

EXERCISE 11.1 — Q13

A round table cover has six equal designs as shown in the figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹0.35 per cm². (Use √3 = 1.7)

THINK FIRST: Each design is a SEGMENT: the six designs meet at the centre, so each central angle is 360°/6 = 60°, and each triangle is equilateral. Find ONE design, multiply by six, then charge it.

1. Each design's central angle = 360°/6 = 60°. Area of one sector = (60/360) × (22/7) × 28^2 = (1/6) × (22/7) × 784 = 1232/3 = 410.67 cm^2 (approx.).

2. The triangle is equilateral of side 28 (angle 60° between radii). Area = (√3/4) × 28^2 = (1.7/4) × 784 = 196 × 1.7 = 333.2 cm^2.
   Why: Use √3 = 1.7 exactly as instructed — not 1.73.

3. Area of ONE design (segment) = 410.67 − 333.2 = 77.47 cm^2 (approx.).

4. Area of SIX designs = 6 × 77.47 = 464.8 cm^2 (approx.).
   Why: Multiply AFTER subtracting — one segment first, then scale.

5. Cost = 464.8 × 0.35 = **₹162.68** (approx.).
   Why: The money step is part of the question. Many students lose the final mark by stopping at the area.

ANSWER: ₹162.68 (approx.)

CBSE-STYLE MARKING SCHEME: One sector 410.67 [1] • Triangle 333.2 and one segment 77.47 [1] • Six designs 464.8 [1] • Cost ₹162.68 [1]

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08 Multiple Solution Methods

8D • CHALLENGE — ONE PROBLEM, TWO ROADS

A topper does not know more formulas — a topper knows that most answers can be reached by two roads, and uses the second road as a free check. Here are this chapter's two great double-roads.

Double-Road 1 — The Umbrella (Exercise 11.1 Q10)

Road A — The angle road

8 ribs → angle between ribs = 360°/8 = 45°. Area = (45/360) × (22/7) × 45^2 = (1/8) × (22/7) × 2025 = **22275/28 ≈ 795.54 cm²**.

=

Road B — The fraction road

8 EQUAL sectors → each is 1/8 of the circle. Area = (1/8) × (22/7) × 2025 = **22275/28 ≈ 795.54 cm²**. Same answer, no angle needed.

When to use which? If the parts are EQUAL and counted (ribs, slices, sectors of a brooch), the fraction road is faster. If an angle is given directly, the angle road is the only road.

Double-Road 2 — The Major Segment (Exercise 11.1 Q6)

Road A — Circle minus minor

Circle = 706.5; minor segment = 20.4375. Major segment = 706.5 − 20.4375 = **686.0625 cm²**.

=

Road B — Major sector PLUS triangle

Major sector (angle 300°) = (300/360) × 3.14 × 225 = 588.75. The major segment is the major sector with the triangle ADDED back: 588.75 + 97.3125 = **686.0625 cm²**.

Why does Road B add? The minor segment was sector MINUS triangle. The major segment is everything else — so the triangle, which was removed from the minor side, belongs to the major side. If you ever write 'major sector − triangle', you have subtracted the triangle from BOTH sides and lost it entirely.

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09 Interactive Simulations & Hands-on Activities

TRY IT YOURSELF — LEARN BY DOING

Activity 1 — Build πr² from Sectors (the rearrangement lab)

Draw a circle of radius 7 cm on chart paper. Divide it into 16 equal sectors (22.5° each) and cut them out. Arrange the sectors alternately point-up and point-down in a row — they form a near-parallelogram. Measure it: its base is about half the circumference (πr ≈ 22 cm) and its height is r (7 cm). Area ≈ 22 × 7 = 154 cm² = πr². You have just SEEN why the circle's area is πr² — and why a sector, a slice of that arrangement, must be the fraction θ/360 of it.

Activity 2 — The Thread-and-Protractor Arc Lab

Draw circles of radius 10 cm. Mark sectors of 30°, 60°, 90° and 120° with a protractor. Lay a thread along each arc, straighten it, and measure. Then compute (θ/360) × 2 × 3.14 × 10 for each angle and compare with your thread. Fill the table:

θ	Thread length (measured)	Formula: (θ/360) × 2πr	Match?
30°	about 5.2 cm	5.24 cm	☐
60°	about 10.5 cm	10.47 cm	☐
90°	about 15.7 cm	15.70 cm	☐
120°	about 20.9 cm	20.93 cm	☐

Activity 3 — Segment Hunt in Your Own Room

Find three objects at home shaped like sectors or segments (a folding hand-fan, a pizza slice, a wall-clock's swept region, a crescent cut from a paper plate). For each: measure or estimate r and θ, compute the area, and record which formula you used and why. Bring your best example to class.

KALVIYOGI DIGITAL COMPANION — CHAPTER 11: AREAS RELATED TO CIRCLES
Video Lessons • Interactive Simulations (drag the θ slider and watch the sector grow) • Online Tests • Study Shorts • AI Math Tutor
QR code and link for this chapter activate with the KalviYogi app/website at print. Watch this space in your digital edition.

DIGITAL CHALLENGE: In the Sector Simulator (see the Digital Companion box above), set r = 10 and slowly drag θ from 0° to 360°. At what angle does the sector area cross exactly half the circle? At what angle does the SEGMENT (sector − triangle) stop growing the way you expected? Discuss with a friend before checking Section 12.

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10 Common Mistakes

LEARN FROM OTHER STUDENTS' ERRORS — BEFORE THEY BECOME YOURS

COMMON MISTAKE: 1. Using the diameter as the radius. Wrong: brooch area with r = 35 mm. **Right:** 35 mm is the diameter; r = 17.5 mm. Halve first — always, out loud, in writing.

COMMON MISTAKE: 2. Forgetting the two radii in a sector's perimeter. Wrong: perimeter of a quadrant (r = 7) = arc = 11 cm. **Right:** perimeter = arc + 2r = 11 + 14 = 25 cm. The boundary includes the two straight edges.

COMMON MISTAKE: 3. Major segment = major sector − triangle. Wrong: 588.75 − 97.3125 = 491.4375 cm^2 for Q6. **Right:** major segment = circle − minor segment = 686.0625 cm^2 (or major sector + triangle). The triangle belongs to the major side.

COMMON MISTAKE: 4. Using ½r^2 for every triangle. Wrong: in Q7 (120°), triangle = ½ × 144 = 72, giving segment 78.72. **Right:** ½r^2 is ONLY for 90°. For 60° and 120°, triangle = (√3/4)r^2 = 62.28, and the segment is 88.44 cm^2.

COMMON MISTAKE: 5. Mixing π values. Wrong: using 22/7 in the lighthouse question, which orders π = 3.14 (answer drifts to 190.29). **Right:** obey the question's instruction; if none is given, use 22/7. Circle the instruction before you begin.

COMMON MISTAKE: 6. Adding the triangle for a segment. Wrong: segment = 78.5 + 50 = 128.5 cm^2 in Q4 — bigger than the sector itself! **Right:** segment = sector − triangle = 28.5 cm^2. Sanity check: a segment is always SMALLER than its sector.

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Common Mistakes 7 — and the Antidote

COMMON MISTAKE: 7. Giving the clock hand the wrong angle. Wrong: in Q3, taking θ = 360° (a full turn) for 5 minutes, or 5°. **Right:** the minute hand turns 6° per minute; 5 minutes → 30°. Write the angle step FIRST, as its own line, before any area formula.

The 20-second antidote — run it before every answer

☐ Did I halve the diameter? ☐ Is my θ the angle the question really gives (or hides)? ☐ Did I use the π the question ordered? ☐ Segment smaller than sector? Major bigger than minor? ☐ Units written — cm^2, m^2, km^2, mm^2? ☐ If money is asked, did I do the money step?

TOPPER TIP: *Examiners design one trap per question — the diameter, the hidden angle, the π instruction, the final money step. Naming the trap in your THINK FIRST line disarms it before it can bite.*

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11 Graded Practice Problems

FIVE LEVELS — CLIMB ONE LEVEL AT A TIME • ANSWERS IN THE ANSWER KEY

Twenty-one original problems, climbing from direct formula use (Level 1) to mathematical genius (Level 5). Do them in order, without the key. Unless stated otherwise, use π = 22/7.

L1 — FOUNDATION — direct concepts and formulas

L1.1 Find the area of a sector of radius 7 cm with angle 90°.

L1.2 Find the length of an arc of a circle of radius 14 cm, if the arc subtends 45° at the centre.

L1.3 Find the area of a quadrant of a circle of radius 10.5 cm.

L1.4 Find the area of a semicircle of radius 7 cm.

L1.5 Find the perimeter of a sector of radius 6 cm and angle 60°.

L1.6 Find the length of an arc of a circle of radius 21 cm subtending 120° at the centre.

L2 — SKILL BUILDER — standard textbook problems

L2.1 Find the area of a sector of radius 10.5 cm with central angle 120°.

L2.2 A chord of a circle of radius 14 cm subtends a right angle at the centre. Find the area of the minor segment.

L2.3 The minute hand of a clock is 10.5 cm long. Find the area it sweeps in 10 minutes.

L2.4 Find the area of the major sector of a circle of radius 6 cm, if the corresponding minor sector has angle 60°.

L2.5 A chord of a circle of radius 7 cm subtends 60° at the centre. Find the area of the minor segment. (Use √3 = 1.73)

L2.6 Find the area of a ring whose outer and inner radii are 16 cm and 10 cm. (Use π = 3.14)

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Graded Practice — Levels 3, 4 and 5

L3 — APPLICATION — real-life and competency problems

L3.1 A cow is tied to a peg at one corner of a square field of side 20 m with a 7 m rope. Find the area of the field in which the cow can graze.

L3.2 A lighthouse warns ships over a sector of angle 90° to a distance of 14 km. Find the area of the sea warned. (Use π = 3.14)

L3.3 A car wiper blade is 21 cm long and sweeps through 105°. Find the area cleaned by the blade in one sweep.

L3.4 An umbrella has 12 equally spaced ribs and radius 35 cm. Find the area between two consecutive ribs.

L4 — ADVANCED REASONING — multi-concept problems

L4.1 A chord of a circle of radius 14 cm subtends 120° at the centre. Find the area of the corresponding minor segment. (Use $\sqrt{3} = 1.73$)

L4.2 Four quadrants, each of radius 10 cm, are drawn at the four corners of a square of side 20 cm, inside the square. Find the area of the part of the square not covered by the quadrants. (Use π = 3.14)

L4.3 The area of a sector of angle 90° is 616 cm^2. Find the radius of the circle, and hence the length of the arc of the sector.

L5 — MATHEMATICAL GENIUS — Olympiad-style creative challenges

L5.1 The perimeter of a sector of a circle is 25 cm and its radius is 7 cm. Find the angle of the sector and its area.

L5.2 A chord of a circle of radius r subtends 60° at the centre. Prove that the area of the major segment is $(5\pi/6 + \sqrt{3}/4)\ r^2$.

> **TOPPER TIP:** *Stuck at a level? Go back exactly one section: Level 3 troubles usually mean Section 07 needs a re-read; Level 5 troubles usually mean the derivations of Section 06 have not yet become yours. Derive, don't memorise.*

12 Mathematical Reasoning

JUSTIFY AND PROVE — FULL SOLUTIONS

REASONING 1

Prove that the area of a sector is half the product of its arc length and radius: Area of sector = ½ × l × r, where l is the length of the arc.

THINK FIRST: Whenever two formulas share the same fraction (θ/360), suspect a one-line bridge between them.

1. Write both formulas with the same fraction: $l = (\theta/360) \times 2\pi r$ and Area = $(\theta/360) \times \pi r^2$.
   Why: Proofs in this chapter are won by placing the two formulas side by side.
2. Compute $\frac{1}{2} \times l \times r = \frac{1}{2} \times (\theta/360) \times 2\pi r \times r$.
3. Simplify: the ½ and the 2 cancel, and $r \times r = r^2$: $= (\theta/360) \times \pi r^2$ = Area of sector. Hence proved.
   Why: This is why a sector behaves like a 'triangle' with base l and height r — a beautiful way to remember both formulas as one.

ANSWER: Area of sector = ½ l r — proved.

REASONING 2

A chord subtends 120° at the centre of a circle of radius r. Prove that the area of the triangle formed by the two radii and the chord is $(\sqrt{3}/4)\, r^2$.

THINK FIRST: Do not memorise the 120° case separately: derive it once with a perpendicular and it is yours forever.

1. Let the radii be OA and OB with ∠AOB = 120°. Drop a perpendicular OM from O to the chord AB.
   Why: The perpendicular from the centre bisects both the chord and the central angle — the standard opening move for chord problems.
2. In △OMA: ∠AOM = 60°, OA = r, so $AM = r \sin 60° = r\sqrt{3}/2$ and $OM = r \cos 60° = r/2$.
3. Hence $AB = 2 \times AM = r\sqrt{3}$, and area of $\triangle OAB = \frac{1}{2} \times AB \times OM = \frac{1}{2} \times r\sqrt{3} \times (r/2) = (\sqrt{3}/4)\, r^2$. Hence proved.
   Why: The same perpendicular trick with θ = 60° gives the equilateral shortcut — one method covers both angles.

ANSWER: Area of △OAB = $(\sqrt{3}/4)\, r^2$ for θ = 120° — proved.

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Reasoning 3 — The 90° Segment in Closed Form

REASONING 3

A chord subtends a right angle at the centre of a circle of radius r. Prove that the area of the minor segment is $(\pi/4 - ½)\, r^2$, and verify the formula against Exercise 11.1 Q4.

THINK FIRST: Closed forms like this are the difference between computing and understanding — AoPS-style reasoning inside an NCERT chapter.

1. Area of sector ($\theta = 90°$) = $(90/360) \times \pi r^2 = (\pi/4)\, r^2$.
   Why: Keep π symbolic — the proof is about structure, not arithmetic.
2. Area of triangle = $½ \times r \times r = (½)\, r^2$, since the radii are perpendicular.
3. Segment = sector − triangle = $(\pi/4 - ½)\, r^2$. Hence proved.
4. Verify with Q4 ($r = 10$, $\pi = 3.14$): $(3.14/4 - 0.5) \times 100 = (0.785 - 0.5) \times 100 = 0.285 \times 100 = 28.5$ cm² — exactly the Q4 answer.
   Why: A proved formula that reproduces a known answer is a formula you can trust in the exam hall.

ANSWER: Minor segment (90°) = $(\pi/4 - ½)\, r^2$ — proved and verified (28.5 cm² at r = 10).

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13 Real-Life Applications

8D • APPLY — THE CHAPTER AT WORK IN THE WORLD

APPLICATION 1

A pizza of radius 20 cm is cut into 6 equal slices. Find the area of the top surface of one slice. (Use π = 3.14)

THINK FIRST: A pizza slice IS a sector — the chapter began at the dining table.

1. Each slice is a sector with angle 360°/6 = 60°.
   Why: Equal slices – the fraction road of Section 08.
2. Area of one slice = (60/360) × 3.14 × 20^2 = (1/6) × 3.14 × 400 = 1256/6 = 209.33 cm^2 (approx.).

ANSWER: 209.33 cm^2 (approx.)

APPLICATION 2

A rotating garden sprinkler waters a sector of angle 120° with radius 12 m. Find the area watered in one rotation setting. (Use π = 3.14)

THINK FIRST: Anitha's sprinkler from the story, with new numbers — same sector, same fraction.

1. Watered area = (120/360) × 3.14 × 12^2 = (1/3) × 3.14 × 144.
   Why: The sprinkler's reach is the radius; its swing is the angle.
2. = (1/3) × 452.16 = **150.72 m^2**.

ANSWER: 150.72 m^2

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Real-Life Applications 3 and 4

APPLICATION 3

A bus windscreen wiper has a blade of length 30 cm and sweeps through an angle of 120°. Find the area of glass cleaned in one sweep.

THINK FIRST: One blade only is asked here — read whether the question wants one wiper or two.

1. Cleaned region = sector with r = 30 cm, θ = 120°.
2. Area = (120/360) × (22/7) × 30^2 = (1/3) × (22/7) × 900 = 19800/21 = 6600/7 = **942.86 cm^2** (approx.).
   Why: Compare with NCERT Q11 – same machine, longer blade, wider sweep.

ANSWER: 6600/7 cm^2 ≈ 942.86 cm^2

APPLICATION 4

A circular traffic roundabout has an outer radius of 35 m and an inner grass island of radius 28 m. The road between them is to be re-tarred. Find the area to be tarred. (Use π = 22/7)

THINK FIRST: Ring = big circle minus small circle. Factor π out first and the numbers stay friendly.

1. Road area = ring area = π(R^2 − r^2) with R = 35, r = 28.
   Why: The story's walking ring, wearing a traffic costume.
2. = (22/7) × (35^2 − 28^2) = (22/7) × (1225 − 784) = (22/7) × 441.
3. = 22 × 63 = **1386 m^2**.

ANSWER: 1386 m^2

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14 Competency-Based Questions

CASE STUDIES & ASSERTION-REASON — BOARD EXAM STYLE, FULLY SOLVED

Five case studies, four questions each — twenty competency questions in all, each fully solved. Read the case, draw the figure, then solve.

Case Study 1 — The School Clock Tower

The clock tower of Green Valley School has a minute hand 21 cm long (a model clock in the mathematics laboratory is built to the same design).

CASE 1 (i)

Through what angle does the minute hand turn in 20 minutes?

1 The hand turns 6° per minute, so in 20 minutes it turns 20 × 6° = **120°**.

Why: The clock fact from Section 02 does all the work.

ANSWER: 120°

CASE 1 (ii)

Find the area swept by the minute hand in 20 minutes. (Use π = 22/7)

1 Swept area = sector with r = 21, θ = 120°: = (120/360) × (22/7) × 21^2 = (1/3) × 1386 = **462 cm^2**.

ANSWER: 462 cm^2

Case Study 1 (continued)

CASE 1 (iii)

How far does the tip of the hand travel in 20 minutes?

1 The tip travels along the arc: = (120/360) × 2 × (22/7) × 21 = (1/3) × 132 = **44 cm**.

Why: Distance travelled by the tip is an ARC LENGTH – circumference family, not area family.

ANSWER: 44 cm

CASE 1 (iv)

Find the area swept by the hand in just 5 minutes.

1 θ = 5 × 6° = 30°. Area = (30/360) × (22/7) × 441 = (1/12) × 1386 = **115.5 cm²**.

Why: Notice: 462/115.5 = 4 – quadrupling the time quadruples the angle and the area. Proportion everywhere.

ANSWER: 115.5 cm²

Case Study 2 — The Pizza Parlour

A pizza parlour sells a large pizza of radius 14 cm, cut into 8 equal slices. (Use π = 22/7)

CASE 2 (i)

What is the angle at the centre of each slice?

1 360°/8 = **45°**.

ANSWER: 45°

CASE 2 (ii)

Find the area of the top of one slice.

1 Area = (1/8) × (22/7) × 14^2 = (1/8) × 616 = **77 cm²**.

Why: Fraction road — the slices are equal.

ANSWER: 77 cm²

CASE 2 (iii)

Find the length of the crust (outer edge) of one slice.

1 Crust = arc of the slice = (45/360) × 2 × (22/7) × 14 = (1/8) × 88 = **11 cm**.

ANSWER: 11 cm

CASE 2 (iv)

A student eats 3 slices. What area of pizza has she eaten?

1 3 × 77 = **231 cm²**.

Why: Or directly: 3/8 of 616 = 231. Both roads agree.

ANSWER: 231 cm²

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Case Study 3 — The Farmer's Cow

A farmer ties his cow to a peg at one corner of a square field of side 25 m. (Use π = 3.14)

CASE 3 (i)

With a 10 m rope, find the area in which the cow can graze.

1 Corner angle 90° → quadrant of radius 10: = (1/4) × 3.14 × 100 = **78.5 m²**.

Why: Same shape as NCERT Q8 — recognise the costume.

ANSWER: 78.5 m²

CASE 3 (ii)

The rope is lengthened to 14 m. Find the new grazing area.

1 = (1/4) × 3.14 × 14^2 = (1/4) × 3.14 × 196 = **153.86 m²**.

ANSWER: 153.86 m²

CASE 3 (iii)

Find the increase in grazing area.

1 153.86 − 78.5 = **75.36 m²**.

ANSWER: 75.36 m²

CASE 3 (iv)

With the 10 m rope, what percentage of the field can the cow graze?

1 Field area = 25 × 25 = 625 m^2. Percentage = (78.5/625) × 100 = **12.56%**.

Why: A competency twist: the chapter's answer becomes data for a percentage question.

ANSWER: 12.56%

Case Study 4 — The Circular Fountain

A circular fountain of radius 10.5 m stands in a park. One sector of the fountain basin, of angle 120°, is converted into a flower bed. The rest remains water. (Use π = 22/7)

CASE 4 (i)

Find the area of the flower bed.

1 Bed = sector: (120/360) × (22/7) × $(10.5)^2$ = (1/3) × (22/7) × 110.25 = (1/3) × 346.5 = **115.5 m^2**.

ANSWER: 115.5 m^2

CASE 4 (ii)

Find the area that remains as water.

1 Whole basin = (22/7) × 110.25 = 346.5 m^2. Water = 346.5 − 115.5 = **231 m^2**.

Why: The water is the MAJOR sector (240°) — check: (240/360) × 346.5 = 231. Two roads agree again.

ANSWER: 231 m^2

CASE 4 (iii)

A decorative railing is placed along the arc of the flower bed. Find its length.

1 Arc = (120/360) × 2 × (22/7) × 10.5 = (1/3) × 66 = **22 m**.

ANSWER: 22 m

CASE 4 (iv)

The railing is extended to include the two straight edges of the bed as well. Find the total railing length.

1 Total = arc + 2r = 22 + 21 = **43 m**.

Why: The sector perimeter rule — Mistake 2 in reverse: here forgetting the radii would cost real money.

ANSWER: 43 m

Case Study 5 — The Petal Logo

A designer makes a logo inside a circle of radius 7 cm: six identical petal designs, each a segment cut off by a chord subtending 60° at the centre (the design of NCERT Q13, in miniature). (Use π = 22/7 and √3 = 1.73)

CASE 5 (i)

Find the area of one petal.

1 Sector = (60/360) × (22/7) × 49 = 77/3 ≈ 25.67 cm². Triangle (equilateral) = (1.73/4) × 49 = 21.19 cm². Petal = 25.67 − 21.19 = **4.47 cm²** (approx.).

ANSWER: 4.47 cm² (approx.)

CASE 5 (ii)

Find the total area of the six petals.

1 6 × 4.47 = **26.84 cm²** (approx.).

ANSWER: 26.84 cm² (approx.)

CASE 5 (iii)

Find the area of the circle that is NOT covered by petals.

1 Circle = (22/7) × 49 = 154 cm². Uncovered = 154 − 26.84 = **127.16 cm²** (approx.).

ANSWER: 127.16 cm² (approx.)

CASE 5 (iv)

The petals are printed in gold ink costing ₹2 per cm². Find the printing cost of the petals.

1 Cost = 26.84 × 2 = **₹53.69** (approx.).

Why: The money step — always finish the story the question actually asks.

ANSWER: ₹53.69 (approx.)

Assertion–Reason Questions 1–5

For each question, choose: **(a)** Both A and R are true and R is the correct explanation of A. **(b)** Both true, but R does not explain A. **(c)** A is true, R is false. **(d)** A is false, R is true. Answers are in the Answer Key.

AR1. **A:** The area of a segment of a circle is the area of the corresponding sector minus the area of the triangle formed by the two radii and the chord. **R:** The sector is composed of the segment and that triangle, with no overlap.

AR2. **A:** The area of the major segment is the area of the circle minus the area of the minor segment. **R:** The minor and major segments together make the whole circle.

AR3. **A:** If the central angle of a sector is doubled (radius unchanged), the area of the sector doubles. **R:** The area of a sector is directly proportional to its central angle.

AR4. **A:** The length of the arc of a semicircle of radius r is πr. **R:** For $\theta = 180°$, arc length = $(180/360) \times 2\pi r = \pi r$.

AR5. **A:** If a chord subtends 60° at the centre, the triangle formed by the two radii and the chord is equilateral. **R:** The two radii are equal and the base angles are $(180° - 60°)/2 = 60°$ each.

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Assertion–Reason Questions 6–10

AR6. **A:** The perimeter of a quadrant of radius 7 cm is 25 cm. **R:** The perimeter of a sector is the length of its arc plus the two radii: 11 + 7 + 7 = 25.

AR7. **A:** A minute hand sweeps twice the area in 10 minutes that it sweeps in 5 minutes. **R:** The angle swept by a minute hand is proportional to time, at 6° per minute.

AR8. **A:** For a chord subtending 120° at the centre, the area of the triangle formed with the two radii is $(\sqrt{3}/4)r^2$. **R:** The area of that triangle is $\frac{1}{2}r^2 \sin 120° = \frac{1}{2}r^2 \times (\sqrt{3}/2) = (\sqrt{3}/4)r^2$.

AR9. **A:** The area of a segment can be found by ADDING the area of the triangle to the area of the sector. **R:** A chord divides a sector into a segment and a triangle.

AR10. **A:** If the radius of a sector is doubled (angle unchanged), its area becomes four times. **R:** The area of a sector is proportional to the square of the radius.

> **TOPPER TIP:** *Assertion–Reason questions reward the student who asks, of every true pair: 'does the reason EXPLAIN the assertion, or merely sit beside it?' That one question separates option (a) from option (b).*

15 Olympiad & Foundation Challenges

8D • CHALLENGE — FOR STUDENTS WHO FINISH EARLY AND THINK DEEP

CHALLENGE 1

A sector of a circle has area 25π cm² and central angle 36°. Find the radius of the circle.

THINK FIRST: When π appears on both sides, cancel it first and the problem shrinks.

1. Area = (36/360) × πr² = (1/10) πr².
   Why: Work with π symbolically — it will cancel, as it so often does in olympiad questions.
2. (1/10) πr² = 25π ⇒ r² = 250 ⇒ r = √250 = 5√10 ≈ 15.81 cm.
   Why: Take the positive root — a radius cannot be negative.

ANSWER: 5√10 cm ≈ 15.81 cm

CHALLENGE 2

Two concentric circles have radii 14 cm and 7 cm. Find the area of the part of the 90° sector of the larger circle that lies outside the smaller circle (a 'ring sector'). (Use π = 22/7)

THINK FIRST: Every 'part of a sector' problem is the fraction θ/360 applied to a combination.

1. Ring sector = big sector − small sector (same angle, 90°).
   Why: A combination in sector clothing — subtract, as with the ring.
2. Big sector = (1/4) × (22/7) × 196 = 154 cm². Small sector = (1/4) × (22/7) × 49 = 38.5 cm².
3. Ring sector = 154 − 38.5 = **115.5 cm²**.
   Why: Or in one line: (90/360) × π × (14² − 7²) — the ring formula wearing the θ/360 crown.

ANSWER: 115.5 cm²

Olympiad Challenges 3 and 4

CHALLENGE 3

A chord of a circle of radius 10 cm has length 10√3 cm. Find the area of the minor segment cut off by the chord. (Use π = 3.14 and √3 = 1.73)

THINK FIRST: Chord length → half-chord → sin(θ/2) → θ. Then the chapter's standard machinery takes over.

1. First find the hidden angle. Drop a perpendicular from O to the chord: it bisects the chord, so half-chord = 5√3. In the right triangle, sin(θ/2) = (5√3)/10 = √3/2, so θ/2 = 60° and θ = 120°.

   Why: The angle is never given — it must be EXTRACTED from the chord length. This reverse direction is what makes the problem olympiad-grade.

2. Sector = (120/360) × 3.14 × 100 = (1/3) × 314 = 104.67 cm² (approx.).

3. Triangle (120° case) = (√3/4) × 100 = 25 × 1.73 = 43.25 cm².

4. Segment = 104.67 − 43.25 = **61.42 cm²** (approx.).

ANSWER: 61.42 cm² (approx.)

CHALLENGE 4

A triangle is inscribed in a semicircle of radius 14 cm, with the diameter as its base and the third vertex on the arc at the highest point. Find the area between the semicircle and the triangle. (Use π = 22/7)

THINK FIRST: A semicircle problem is a sector problem with θ = 180° — nothing new to learn, everything to apply.

1. Area of semicircle = (1/2) × (22/7) × 14² = (1/2) × 616 = 308 cm².

2. The triangle's base is the diameter (28 cm) and its height is the radius (14 cm), since the highest point of the arc is exactly r above the base. Area = (1/2) × 28 × 14 = 196 cm².

   Why: The angle in a semicircle is a right angle (Thales) — but for AREA we need only base and height.

3. Area between = 308 − 196 = **112 cm²**.

ANSWER: 112 cm²

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Olympiad Challenges 5 and 6

CHALLENGE 5

Three circles, each of radius 7 cm, touch each other externally. Find the area of the small curvilinear region enclosed between them. (Use π = 22/7 and √3 = 1.732)

THINK FIRST: Triangle minus sectors: the combination strategy at its most elegant.

1. Join the three centres: they form an equilateral triangle of side 7 + 7 = 14 cm.
   Why: Tangent circles: the distance between centres is the sum of the radii — the key that unlocks the figure.
2. Area of triangle = $(\sqrt{3}/4) \times 14^2 = 49 \times 1.732 = 84.87$ cm^2 (approx.).
3. Inside the triangle, each circle contributes a sector of 60° (the triangle's angles). Three sectors of 60° = 180° worth = half a circle: $(1/2) \times (22/7) \times 49 = 77$ cm^2.
   Why: The three angles of ANY triangle sum to 180° — so for equal circles the sectors inside always total a semicircle, whatever the triangle's shape!
4. Enclosed region = 84.87 − 77 = **7.87 cm^2** (approx.).

ANSWER: 7.87 cm^2 (approx.)

Olympiad Challenge 6 — The Lunes of Hippocrates

CHALLENGE 6

A right triangle has legs 6 cm and 8 cm and hypotenuse 10 cm. Semicircles are drawn outward on the two legs, and a semicircle is drawn on the hypotenuse on the same side as the triangle. The two 'lune' (moon) regions between the leg-semicircles and the hypotenuse-semicircle have total area equal to a famous number. Find it.

THINK FIRST: When curved areas cancel to a whole number, Pythagoras is hiding nearby.

1. Area of semicircle on leg 6 (r = 3) = $(1/2)\pi(3^2) = 4.5\pi$. On leg 8 (r = 4): $(1/2)\pi(4^2) = 8\pi$. Sum = 12.5π.

   Why: Keep π symbolic — watch what happens.

2. Area of semicircle on hypotenuse 10 (r = 5) = $(1/2)\pi(25) = 12.5\pi$ — exactly the same!

   Why: This equality is Pythagoras in disguise: $3^2 + 4^2 = 5^2$, scaled by π/2.

3. The lunes = (semicircles on legs) + (triangle) − (semicircle on hypotenuse) = $12.5\pi + 24 - 12.5\pi$ = **24 cm²** — exactly the area of the triangle.

   Why: The π terms cancel completely. Hippocrates discovered this in the 5th century BCE — the first curved region in history whose exact area was found. You have just repeated his proof.

ANSWER: 24 cm² — the lunes equal the triangle.

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Olympiad Challenges 7 and 8

CHALLENGE 7

For a sector of a circle, the numerical value of the area (in cm^2) is exactly equal to the numerical value of its arc length (in cm). Find the radius of the circle. (The angle is not given — and, surprisingly, is not needed.)

THINK FIRST: If a problem withholds a value, perhaps the answer does not depend on it — set up the equation and let the algebra speak.

1. Set the two formulas equal: $(\theta/360)\,\pi r^2 = (\theta/360) \times 2\pi r$.
2. Cancel the shared factors $(\theta/360)$ and πr from both sides — they are all non-zero: $r = 2$.
   Why: Everything cancels except r itself. The angle was a decoy.
3. So the radius is **2 cm**, for EVERY angle θ.
   Why: Compare Reasoning 1: Area = ½ × l × r. Area equals l exactly when r/2 = 1 — the identity explains the surprise.

ANSWER: r = 2 cm (for any θ)

CHALLENGE 8

The area of a ring (annulus) is exactly equal to the area of its inner circle. If the inner radius is 7 cm, find the outer radius and the area of the ring. (Use π = 22/7)

THINK FIRST: 'Equal areas' is an equation asking to be written down. Translate words to symbols immediately.

1. Ring area = $\pi(R^2 - r^2)$ and inner circle area = πr^2. Set them equal: $R^2 - r^2 = r^2$, so $R^2 = 2r^2$.
2. $R^2 = 2 \times 49 = 98$, so $R = \sqrt{98} = 7\sqrt{2} \approx 9.9$ cm.
3. Ring area = $\pi r^2 = (22/7) \times 49 =$ **154 cm^2**.
   Why: By construction it equals the inner circle's area — the answer checks itself.

ANSWER: R = 7√2 cm ≈ 9.9 cm; ring area = 154 cm^2

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Olympiad Challenges 9 and 10

CHALLENGE 9

A sector of angle 120° is cut from a circle of radius 6 cm. Its area is exactly equal to the area of a smaller full circle. Find the radius of that smaller circle.

THINK FIRST: Equate the areas, cancel π, take the square root.

1. Sector area = (120/360) × π × 6^2 = (1/3) × 36π = 12π cm^2.
2. Let the small circle have radius R: πR^2 = 12π, so R^2 = 12 and R = 2√3 ≈ 3.46 cm.

Why: π cancels again — in olympiad land, π is a guest who often leaves early.

ANSWER: 2√3 cm ≈ 3.46 cm

CHALLENGE 10

The perimeter of a sector is 50 cm and its central angle is 90°. Find the radius of the circle and the area of the sector. (Use π = 22/7)

THINK FIRST: Perimeter questions are algebra questions: arc + 2r, one unknown, solve.

1. Perimeter = arc + 2r. With θ = 90°: arc = (90/360) × 2 × (22/7) × r = (1/4) × (44/7) r = (11/7) r.

   Why: Write the perimeter as a single expression in r — one unknown, one equation.
2. So (11/7) r + 2r = (25/7) r = 50, giving r = 50 × 7/25 = **14 cm**.
3. Area of sector = (90/360) × (22/7) × 14^2 = (1/4) × 616 = **154 cm^2**.

   Why: A quadrant in disguise — the answer 154 is an old friend by now.

ANSWER: r = 14 cm; area = 154 cm^2

16 Chapter Mind Map

THE WHOLE CHAPTER ON ONE PAGE — REVISE FROM HERE

AREAS RELATED TO CIRCLES

Heart of the chapter: **every part of a circle is the fraction θ/360 of the whole** — and every segment is a sector with a triangle removed.

Arc Length

$l = (\theta/360) \times 2\pi r$. The fraction of the CIRCUMFERENCE. Sector perimeter = $l + 2r$ — never forget the radii!

→

Sector Area

$A = (\theta/360) \times \pi r^2 = \frac{1}{2} \times l \times r$. Quadrant = 90°, semicircle = 180°, major sector uses 360° − θ.

Segment Area

Minor segment = sector − triangle. Major segment = circle − minor segment (= major sector + triangle).

The Triangle Inside

Two radii + chord. θ = 60° → equilateral, $(\sqrt{3}/4)r^2$. θ = 90° → $\frac{1}{2}r^2$. θ = 120° → $(\sqrt{3}/4)r^2$.

Combinations

Draw → split → add or subtract. Ring = $\pi(R^2 - r^2)$. Two semicircles = one circle. Shaded = whole − unshaded.

Clock & Corner Machines

Minute hand: 6° per minute, hand = radius. Corner-tied animal: quadrant, rope = radius (if rope ≤ side).

Traps — The Examiner's Five

Diameter used as radius • π instruction ignored (3.14 vs 22/7) • major segment by sector − triangle • radii dropped from perimeter • money/unit step skipped at the end.

→

Key Results to Quote

Horse: 19.625 m^2, increase 58.875 • Brooch wire 285 mm • Wipers 1254.96 cm^2 • Lighthouse 189.97 km^2 • Table cover ₹162.68

TOPPER TIP: Revise actively: cover a box, speak its contents aloud from memory, then uncover and check. A mind map you can REPRODUCE from memory is worth ten readings.

17 Pin-Point Flash Cards

THE WHOLE CHAPTER IN YOUR POCKET — CUT THEM OUT, CARRY THEM, MASTER THEM

How to use these cards

Cut along the dashed borders and keep the 16 cards in your pocket or a small box. On Smart Memory **Day 3, Day 7 and Day 21**, shuffle the cards and test yourself: read the title, say the pin-point aloud from memory, then flip the card to check. Any card you miss goes back into the box for tomorrow. Two minutes a day — that is all it takes to keep this chapter alive all year.

CARD 1 — The Master Fraction

Every part of a circle wears the fraction θ/360. Sector area = (θ/360)πr²; arc length = (θ/360) × 2πr.

CARD 2 — Segment

Minor segment = SECTOR − TRIANGLE. Always subtract. A segment is smaller than its sector.

CARD 3 — Major Segment

Major segment = CIRCLE − minor segment. (Also = major sector + triangle.) NEVER major sector − triangle.

CARD 4 — Sector Perimeter

Perimeter of a sector = arc + 2r. The two radii are part of the boundary — CBSE 2024 asked this for 1 mark.

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Flash Cards 5–10

CARD 5 — The ½lr Identity

Area of sector = ½ × arc length × radius. A sector is a 'triangle' with base = arc and height = r.

CARD 6 — Triangle at 60°

Two radii + chord at 60° form an EQUILATERAL triangle. Area = (√3/4) r^2.

CARD 7 — Triangle at 90° and 120°

At 90°: area = ½r^2. At 120°: area = (√3/4)r^2 (perpendicular from O proves it).

CARD 8 — The Clock Fact

Minute hand: 6° per minute; hand length = radius. 5 min → 30°, 10 min → 60°, 15 min → 90°.

CARD 9 — Quadrant and Semicircle

Quadrant (θ = 90°) = ¼ of the circle. Semicircle (θ = 180°) = ½. Major sector angle = 360° − θ.

CARD 10 — The Diameter Trap

Diameter given? HALVE IT FIRST. Brooch 35 mm → r = 17.5 mm. Write r = d/2 as your literal first line.

TOPPER TIP: Say each card aloud once before you test yourself on it — hearing your own voice say the fact makes the memory stick twice as fast.

Flash Cards 11–16

CARD 11 — Obey the π Order

Question says π = 3.14? Use 3.14. Says √3 = 1.7? Use 1.7, not 1.73. No instruction? Default π = 22/7. Circle the instruction first.

CARD 12 — The Ring

Ring (annulus) area = $\pi(R^2 - r^2) = \pi(R - r)(R + r)$. Factor π out before multiplying.

CARD 13 — Combinations Strategy

Draw → label → split into known shapes → add or subtract. Two equal semicircles = one full circle.

CARD 14 — Corner Animal

Animal tied at a field's corner: grazing region = QUADRANT with radius = rope length — provided the rope is shorter than the side.

CARD 15 — Equal Parts Shortcut

n equal sectors (umbrella ribs, pizza slices, brooch parts): each = 1/n of the circle. No angle needed.

CARD 16 — Chord Equal to Radius

If chord = radius, the central angle is 60° (equilateral triangle). If chord = r√3, the angle is 120°; chord = r√2 → 90°.

MAKE YOUR OWN CARD

Write one pin-point YOU keep forgetting, in your own words. A card you write yourself is the card you remember best.

_____

_____

MAKE YOUR OWN CARD

One more: the mistake you made most often in this chapter, and the one-line rule that fixes it.

_____

18 Active Recall

8D • RECALL — CLOSE THE BOOK, THEN ANSWER

Close the chapter. Answer all ten rapid questions and five recall prompts from memory, in writing. Then open the Answer Key and mark yourself honestly — every miss becomes an entry in your Error Tracker (Section 19).

R1. Write the formulas for the length of an arc and the area of a sector, and state the fraction they share.

R2. A sector has radius 7 cm and angle 90°. What is its area?

R3. State the segment rule, and the major-segment rule.

R4. Through what angle does a minute hand turn in 12 minutes?

R5. A chord subtends 60° at the centre. What is special about the triangle formed by the radii and the chord?

R6. What is the perimeter of a sector of radius 10 cm and angle 90°? (Use $\pi = 22/7$)

R7. A brooch is made from a circle of diameter 20 mm, with 5 diameters of wire. What total length of wire is used? (Use $\pi = 22/7$)

R8. Find the area of a ring with outer radius 12 cm and inner radius 5 cm. (Use $\pi = 22/7$)

R9. In Q4 of Exercise 11.1, why is the triangle's area exactly $\frac{1}{2}r^2$?

R10. A sector's area equals ½ × (arc length) × (radius). Say why in one sentence.

Five Formula/Definition Recall Prompts

P1. Define a segment. How is it different from a sector?

P2. State the area of the triangle inside a segment for $\theta = 60°$, 90° and 120°.

P3. What is a quadrant? What fraction of the circle is it?

P4. State the combination strategy in four verbs.

P5. When a question gives a diameter, what is your first written line?

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19 Self-Assessment & Mastery Tracker

KALVIYOGI SMART MEMORY SYSTEM — LEARN IT, THEN KEEP IT

Learning once is not learning forever. The Smart Memory System revisits this chapter at exactly the moments your memory would otherwise fade. Follow the schedule — each visit is short, and each has a clear job.

Day	Learning Activity	Your Evidence
Day 0	Pre-learning assessment: attempt the Quick Check (Section 02) and L1 problems cold.	Score: ____ / 11 Date: _____
Day 1	Learn the chapter fully: Sections 03–08, then Graded Practice L1–L3.	Time spent: _____ min
Day 3	Without-book recall: Flash Cards (Section 17) + Active Recall (Section 18).	Cards missed: _____
Day 7	Independent problem-solving: **Day 7 Mini-Test** (Practice Plus) + L4 problems.	Score: ____ / 10
Day 14	Mixed practice: Model Test A or B (Practice Plus) under time.	Score: ____ / 20
Day 21	Long-term recall test: **Day 21 Mini-Test** + Flash Cards again.	Score: ____ / 10
Day 30	Chapter mastery assessment: **Day 30 Mastery Test** + Model Test C.	Score: ____ / 10 and ____ / 20

Formula Recall Tracker — can you write each from memory?

☐ Arc length = $(\theta/360) \times 2\pi r$ ☐ Sector area = $(\theta/360) \times \pi r^2$ ☐ Sector area = $\frac{1}{2}lr$ ☐ Segment = sector − triangle

☐ Major segment = circle − minor ☐ Sector perimeter = arc + 2r ☐ Ring = $\pi(R^2 - r^2)$ ☐ Triangle areas: 60°/120° → $(\sqrt{3}/4)r^2$, 90° → $\frac{1}{2}r^2$

Learning Time Tracker

Day 0: _____ min • Day 1: _____ min • Day 3: _____ min • Day 7: _____ min • Day 14: _____ min • Day 21: _____ min • Day 30: _____ min • **Total for this chapter: _____ min**

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Mastery Tracker — Errors and Concept Scores

Error Tracker — every mistake becomes a lesson

Date / Source	What went wrong	The rule that fixes it

Concept Mastery Score — rate yourself honestly (1 = shaky, 5 = can teach it)

Concept	Day 1	Day 7	Day 30
Arc length and sector area by proportion			
Segment = sector − triangle (60°, 90°, 120°)			
Major sector and major segment			
Clock, horse, brooch, umbrella, wiper, lighthouse machines			
Combinations: draw, split, add, subtract			

TOPPER TIP: *A concept still at 1 or 2 on Day 30 is not a failure — it is information. Re-derive it from Section 06, redo two problems of that type, and test again in three days. That loop IS the Smart Memory System.*

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Practice Plus — 25 Concept MCQs (1–13)

One mark each, board style. Answers with explanations are in the Answer Key — attempt all 25 before looking.

1. The area of a sector of angle θ in a circle of radius r is:
(A) (θ/360) × 2πr (B) (θ/360) × πr² (C) (θ/180) × πr² (D) (θ/360) × 2πr²

2. The length of an arc of a circle of radius 7 cm subtending 180° at the centre is:
(A) 11 cm (B) 22 cm (C) 44 cm (D) 7 cm

3. The area of a quadrant of a circle of radius 14 cm is: (Use π = 22/7)
(A) 616 cm² (B) 308 cm² (C) 154 cm² (D) 77 cm²

4. The area of a segment of a circle is equal to:
(A) sector + triangle (B) sector − triangle (C) triangle − sector (D) sector × triangle

5. The angle turned by the minute hand of a clock in 1 minute is:
(A) 1° (B) 5° (C) 6° (D) 30°

6. The perimeter of a sector of radius 7 cm and angle 90° is: (Use π = 22/7)
(A) 11 cm (B) 18 cm (C) 25 cm (D) 39.5 cm

7. If θ = 360°, the 'sector' of a circle of radius r has area:
(A) 2πr (B) πr² (C) 2πr² (D) ½πr²

8. The area of a semicircle of diameter 14 cm is: (Use π = 22/7)
(A) 154 cm² (B) 77 cm² (C) 38.5 cm² (D) 308 cm²

9. The minor sector of a circle has angle 70°. The angle of the corresponding major sector is:
(A) 110° (B) 250° (C) 290° (D) 70°

10. The area of a sector of radius 10 cm and angle 90° is: (Use π = 3.14)
(A) 314 cm² (B) 157 cm² (C) 78.5 cm² (D) 31.4 cm²

11. If a chord of a circle is equal to its radius, the angle subtended by the chord at the centre is:
(A) 30° (B) 60° (C) 90° (D) 120°

12. The area of a ring with outer radius 5 cm and inner radius 3 cm is:
(A) 2π cm² (B) 8π cm² (C) 16π cm² (D) 34π cm²

13. The length of an arc of radius 21 cm subtending 90° at the centre is: (Use π = 22/7)
(A) 22 cm (B) 33 cm (C) 44 cm (D) 66 cm

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25 Concept MCQs (14–25)

14. The area of the minor segment of a circle of radius 10 cm, where the chord subtends 90° at the centre, is: (Use π = 3.14)
(A) 78.5 cm^2 (B) 50 cm^2 (C) 28.5 cm^2 (D) 235.5 cm^2

15. If l is the length of an arc and r the radius, the area of the corresponding sector is:
(A) l × r (B) 2lr (C) ½lr (D) l/r

16. The perimeter of a quadrant of a circle of radius 7 cm is: (Use π = 22/7)
(A) 11 cm (B) 25 cm (C) 18 cm (D) 44 cm

17. If the circumference of a circle is 44 cm, its area is: (Use π = 22/7)
(A) 616 cm^2 (B) 308 cm^2 (C) 154 cm^2 (D) 77 cm^2

18. The area of a sector of radius 6 cm and angle 120° is: (Use π = 22/7)
(A) 264/7 cm^2 (B) 132/7 cm^2 (C) 66/7 cm^2 (D) 528/7 cm^2

19. A horse is tied at the corner of a square field with a rope of length 14 m. The area it can graze is: (Use π = 22/7)
(A) 616 m^2 (B) 308 m^2 (C) 154 m^2 (D) 77 m^2

20. A brooch of diameter 35 mm is divided into 10 equal sectors. The angle of each sector is:
(A) 10° (B) 18° (C) 36° (D) 45°

21. Two sectors of the same circle of radius 7 cm have angles 60° and 120°. The sum of their areas is: (Use π = 22/7)
(A) 154 cm^2 (B) 77 cm^2 (C) 51.33 cm^2 (D) 231 cm^2

22. The area of the major segment of a circle can be obtained as:
(A) major sector − triangle (B) circle − minor segment (C) minor sector + triangle (D) circle − major sector

23. An arc of a circle of radius 7 cm has length 11 cm. The angle subtended by the arc at the centre is: (Use π = 22/7)
(A) 45° (B) 60° (C) 90° (D) 180°

24. A chord subtends 60° at the centre of a circle. The triangle formed by the two radii and the chord is:
(A) right-angled (B) equilateral (C) scalene (D) obtuse

25. The area of a sector of angle p (in degrees) of a circle with radius R is:
(A) (p/180) × 2πR (B) (p/360) × πR^2 (C) (p/180) × πR^2 (D) (p/360) × 2πR

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Find the Error — Questions 1–5

Each question shows a student's wrong working. Find the error, name it, and write the correct answer. (Corrections explained fully here — this section teaches by diagnosis.)

E1. Find the area of a sector of radius 6 cm and angle 60°.

STUDENT'S WORK: Area = (60/360) × 2 × (22/7) × 6 = 44/7 cm².

The error: The student used the CIRCUMFERENCE formula (2πr) for an AREA. Correct: (60/360) × (22/7) × 36 = 132/7 ≈ 18.86 cm².

E2. Find the area of a quadrant of a circle whose circumference is 22 cm.

STUDENT'S WORK: Circumference 22 → 2πr = 22 → r = 7. Quadrant area = (1/4)(22/7)(49) = 38.5 cm².

The error: Arithmetic slip in solving for r: r = 22/(2π) = 22 × 7/44 = 3.5, not 7. Correct area = (1/4)(22/7)(3.5)² = 77/8 = 9.625 cm².

E3. Find the minor segment for a chord subtending 90° in a circle of radius 10 cm (π = 3.14).

STUDENT'S WORK: Segment = sector + triangle = 78.5 + 50 = 128.5 cm².

The error: Sign error — and an impossible answer (a segment bigger than its sector!). Correct: 78.5 − 50 = 28.5 cm².

E4. Find the total wire for the brooch of diameter 35 mm with 5 diameters (NCERT Q9).

STUDENT'S WORK: Circumference = 2 × (22/7) × 35 = 220 mm; total = 220 + 175 = 395 mm.

The error: The diameter 35 was used as the radius. Correct circumference = 2 × (22/7) × 17.5 = 110 mm; total = 110 + 175 = 285 mm.

E5. Find the area swept by a 14 cm minute hand in 5 minutes.

STUDENT'S WORK: The hand makes a full turn, so area = (22/7) × 196 = 616 cm².

The error: In 5 minutes the hand turns only 30°, not 360°. Correct: (30/360) × (22/7) × 196 = 154/3 ≈ 51.33 cm².

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Find the Error — Questions 6–10

E6. Find the MAJOR sector area for a circle of radius 6 cm whose minor sector has angle 60°.

STUDENT'S WORK: Major sector = (60/360) × (22/7) × 36 = 132/7 cm^2.

The error: The student used the MINOR angle for the major sector. Major angle = 360° − 60° = 300°. Correct: (300/360) × (22/7) × 36 = 660/7 ≈ 94.29 cm^2.

E7. Find the segment for a chord subtending 120° in a circle of radius 12 cm (π = 3.14).

STUDENT'S WORK: Triangle = ½ × 12 × 12 = 72; segment = 150.72 − 72 = 78.72 cm^2.

The error: ½r^2 is the triangle area ONLY at 90°. At 120° it is (√3/4)r^2 = 62.28. Correct segment = 150.72 − 62.28 = 88.44 cm^2.

E8. Find the lighthouse warning area of NCERT Q12 (sector 80°, distance 16.5 km, use π = 3.14).

STUDENT'S WORK: Area = (80/360) × (22/7) × $(16.5)^2$ = 190.29 km^2.

The error: The π instruction was ignored: the question orders π = 3.14. Correct: (80/360) × 3.14 × 272.25 = 189.97 km^2.

E9. Find the area between two consecutive ribs of an umbrella of radius 45 cm with 8 ribs.

STUDENT'S WORK: Angle = 45°. Area = (45/360) × 2 × (22/7) × 45 = 35.36 cm^2.

The error: The ARC formula was used where the AREA was asked. Correct: (1/8) × (22/7) × 45^2 = 22275/28 ≈ 795.54 cm^2.

E10. A horse is tied at the corner of a square field with a 10 m rope. Find the grazing area (π = 3.14).

STUDENT'S WORK: The horse can graze the full circle: 3.14 × 100 = 314 m^2.

The error: The field's walls block the horse to a QUADRANT — it is tied at a corner, not in open ground. Correct: (1/4) × 3.14 × 100 = 78.5 m^2.

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Previous Year Questions — Real CBSE Board Questions (1)

Real questions from CBSE Class 10 board examinations, with the year and marks as asked. Each is fully solved — study how the board's questions are this chapter's machines in new costumes.

CBSE 2024 — 5 MARKS

In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find: (i) the length of the arc (ii) the area of the minor segment cut off by the chord of this arc. (Use π = 22/7 and √3 = 1.732)

THINK FIRST: This is NCERT Q5 wearing a board-exam badge — the segment is asked directly, so the equilateral triangle must appear.

1. (i) Length of arc = (60/360) × 2 × (22/7) × 21 = (1/6) × 132 = **22 cm**.

2. Area of sector = (60/360) × (22/7) × 21^2 = 231 cm^2.
   Why: Needed for the segment — the board rewards this intermediate step.

3. △OAB is equilateral (60° between equal radii): area = (√3/4) × 441 = (1.732/4) × 441 = 763.812/4 = 190.95 cm^2.
   Why: Here CBSE supplied √3 = 1.732, so a decimal answer is expected — unlike NCERT Q5, which keeps the exact form.

4. (ii) Minor segment = 231 − 190.95 = **40.05 cm^2** (approx.).

ANSWER: (i) 22 cm (ii) 40.05 cm^2 (approx.)

CBSE MARKING SCHEME: Arc [1] • Sector [1] • Triangle with √3 = 1.732 [2] • Segment [1]

Previous Year Questions — Real CBSE Board Questions (2)

CBSE 2023 — 3 MARKS

A chord of a circle of radius 14 cm subtends an angle of 60° at the centre. Find the area of the corresponding minor segment of the circle. Also find the area of the major segment of the circle. (Use π = 22/7 and √3 = 1.73)

THINK FIRST: Minor first, then major = circle − minor. The two-mark split of this question is exactly Section 08's Double-Road 2.

1. Sector = (60/360) × (22/7) × 14^2 = (1/6) × 616 = 308/3 ≈ 102.67 cm^2.
2. Triangle (equilateral, side 14) = (√3/4) × 196 = 49 × 1.73 = 84.77 cm^2.
3. Minor segment = 102.67 − 84.77 = **17.9 cm^2** (approx.).
4. Circle area = (22/7) × 196 = 616 cm^2. Major segment = 616 − 17.9 = **598.1 cm^2** (approx.).

ANSWER: Minor segment ≈ 17.9 cm^2; major segment ≈ 598.1 cm^2

CBSE MARKING SCHEME: Sector and triangle [1] • Minor segment [1] • Major segment as circle − minor [1]

PIN POINT: This exact question appeared in THREE CBSE 2023 papers. When the board repeats a question across years, it is telling you what to master: minor segment, then major = circle − minor.

Previous Year Questions (2)

CBSE 2025 — 2 MARKS

A chord of a circle of radius 10 cm subtends a right angle at the centre of the circle. Find the area of the corresponding minor segment. (Use π = 3.14)

THINK FIRST: NCERT Q4(i) itself, asked again by the board in 2025 — proof that this chapter's exercise is the board's question bank.

1. Sector = (90/360) × 3.14 × 100 = 78.5 cm².
2. Triangle = ½ × 10 × 10 = 50 cm².
   Why: Right angle at the centre → radii are base and height.
3. Minor segment = 78.5 − 50 = **28.5 cm²**.

ANSWER: 28.5 cm²

CBSE MARKING SCHEME: Sector and triangle [1] • Segment [1]

CBSE 2020 — 2 MARKS

A piece of wire 22 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle. (Use π = 22/7)

THINK FIRST: The arc formula run BACKWARDS: the length is given, the radius is the unknown.

1. Arc length = (θ/360) × 2πr ⇒ 22 = (60/360) × 2 × (22/7) × r.
2. 22 = (1/6) × (44/7) r = (44/42) r = (22/21) r.
3. r = 22 × 21/22 = **21 cm**.

ANSWER: 21 cm

CBSE MARKING SCHEME: Arc formula set up with unknown r [1] • Solving to r = 21 [1]

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Previous Year Questions (3)

CBSE 2020 — 2 MARKS

A circular park is surrounded by a road 21 m wide. If the radius of the park is 105 m, find the area of the road. (Use π = 22/7)

THINK FIRST: A ring: outer radius = 105 + 21 = 126 m. One subtraction.

1. Outer radius R = 105 + 21 = 126 m; inner radius r = 105 m.
   Why: The road's width ADDS to the park's radius — the step students rush past.
2. Area of road = $\pi(R^2 - r^2) = (22/7) \times (126^2 - 105^2) = (22/7) \times (15876 - 11025) = (22/7) \times 4851$.
3. = 22 × 693 = **15246 m²**.

ANSWER: 15246 m²

CBSE MARKING SCHEME: Outer radius 126 [1] • Ring area 15246 [1]

CBSE 2024 — 1 MARK (MCQ)

The perimeter of a sector of a circle whose central angle is 90° and radius 7 cm is: (a) 35 cm (b) 11 cm (c) 22 cm (d) 25 cm

1. Arc = (90/360) × 2 × (22/7) × 7 = 11 cm.
2. Perimeter = arc + 2r = 11 + 14 = **25 cm — option (d)**.
   Why: Option (b) 11 cm is the trap for students who forget the radii — Mistake 2, set by CBSE itself.

ANSWER: Option (d) 25 cm

Previous Year Questions (4) — The 1-Mark Hall of Fame

Short questions, full reasoning — in the board these carry 1 mark each and are lost only by haste.

CBSE 2023 — 1 MARK (MCQ)

If the area of a circle is 154 cm², then its circumference is: (a) 44 cm (b) 22 cm (c) 28 cm (d) 56 cm

1. $\pi r^2 = 154 \Rightarrow r^2 = 154 \times 7/22 = 49 \Rightarrow r = 7$ cm.
2. Circumference = $2 \times (22/7) \times 7$ = **44 cm — option (a)**.

ANSWER: Option (a) 44 cm

CBSE 2022 — 1 MARK (MCQ)

The area of the sector of a circle of radius 12 cm and angle 120° is: (a) 48π cm² (b) 36π cm² (c) 144π cm² (d) 24π cm²

1. Area = $(120/360) \times \pi \times 12^2 = (1/3) \times 144\pi$ = **48π cm² — option (a)**.
   Why: No π value is needed — the options carry π symbolically. Recognise the style and save a minute.

ANSWER: Option (a) 48π cm²

CBSE 2025 — 1 MARK (MCQ)

If the length of a chord of a circle is equal to its radius, then the angle subtended by the chord at the centre is: (a) 60° (b) 30° (c) 120° (d) 90°

1. The two radii and the chord are all equal — the triangle is equilateral, so every angle is 60°.
2. Answer: **option (a) 60°**.
   Why: Flash Card 16 in reverse — the board tests the same pin-point from both directions.

ANSWER: Option (a) 60°

Chapter Model Test A — Basics (20 Marks)

Time: 40 minutes. Unless stated otherwise, use π = 22/7. Marking key is in the Answer Key.

Q1. Find the area of a sector of radius 7 cm with angle 60°. **[2]**

Q2. Find the length of an arc of a circle of radius 14 cm subtending 90° at the centre. **[2]**

Q3. Find the area of a quadrant of a circle whose circumference is 44 cm. **[3]**

Q4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the minor segment. (Use π = 3.14) **[3]**

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Chapter Model Test A (continued)

Q5. A horse is tied to a peg at one corner of a square field of side 20 m with a rope of length 6 m. Find (i) the area in which it can graze, and (ii) the increase in grazing area if the rope is doubled to 12 m. (Use π = 3.14) **[5]**

Q6. A brooch is made with silver wire in the form of a circle of diameter 28 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors. Find (i) the total length of wire required, (ii) the area of each sector. **[5]**

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Chapter Model Test B — Standard (20 Marks)

Time: 45 minutes. Unless stated otherwise, use π = 22/7.

Q1. In a circle of radius 21 cm, an arc subtends 60° at the centre. Find the length of the arc and the area of the sector formed by it. **[3]**

Q2. A chord of a circle of radius 15 cm subtends 60° at the centre. Find the areas of the corresponding minor and major segments. (Use π = 3.14 and √3 = 1.73) **[5]**

Q3. A car has two wipers which do not overlap. Each wiper has a blade of length 24 cm sweeping through an angle of 100°. Find the total area cleaned at each sweep. **[4]**

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Chapter Model Test B (continued)

Q4. A circular park of radius 35 m is surrounded by a path 7 m wide. Find the area of the path. **[4]**

Q5. The perimeter of a sector of a circle is 25 cm and its radius is 7 cm. Find the area of the sector. **[4]**

Chapter Model Test C — Advanced (20 Marks)

Time: 50 minutes. Unless stated otherwise, use π = 22/7.

Q1. A chord of a circle of radius 14 cm subtends 120° at the centre. Find the area of the corresponding minor segment. (Use √3 = 1.73) **[5]**

Q2. Three circles, each of radius 7 cm, touch each other externally. Find the area enclosed between them. (Use √3 = 1.732) **[5]**

Chapter Model Test C (continued)

Q3. A right triangle has legs 6 cm and 8 cm. Semicircles are drawn outward on the two legs and on the hypotenuse (on the triangle's side). Prove that the total lune area equals the area of the triangle, and state its value. **[5]**

Q4. The perimeter of a sector is 50 cm and its central angle is 90°. Find the radius and the area of the sector. **[5]**

Smart Memory Mini-Tests — Day 7 and Day 21

Day 7 Mini-Test — Independent Problem-Solving (10 Questions)

1. Find the area of a sector of radius 7 cm and angle 30°.

2. Find the length of an arc of radius 10.5 cm subtending 60° at the centre.

3. Find the area of a quadrant of radius 14 cm.

4. Find the minor segment area for a chord subtending 90° in a circle of radius 14 cm.

5. Through what angle does a minute hand turn in 15 minutes?

6. Find the area of a ring with outer radius 10 cm and inner radius 6 cm. (Use $\pi = 3.14$)

7. Find the area of a major sector of radius 4 cm with angle 270°. (Use $\pi = 3.14$)

8. Find the perimeter of a sector of radius 7 cm and angle 90°.

9. A chord is equal to the radius of its circle. What angle does it subtend at the centre?

10. The circumference of a circle is 88 cm. Find its area.

Day 21 Mini-Test — Long-Term Recall (10 Questions)

1. Find the length of an arc of radius 6 cm subtending 90° at the centre.

2. Find the area of a sector of radius 12 cm and angle 45°.

3. Find the minor segment area for a chord subtending 60° in a circle of radius 7 cm. (Use $\sqrt{3} = 1.73$)

4. Find the major segment area for a chord subtending 90° in a circle of radius 10 cm. (Use $\pi = 3.14$)

5. An umbrella has 10 equally spaced ribs and radius 21 cm. Find the area between two consecutive ribs.

6. One wiper blade of length 25 cm sweeps 115°. Find the area it cleans.

7. Find the area of the triangle formed by two radii of a circle of radius 6 cm with 120° between them. (Use $\sqrt{3} = 1.73$)

8. The area of a sector of angle 90° is 154 cm^2. Find the radius of the circle.

9. Find the perimeter of a semicircle of radius 7 cm.

10. A lighthouse warns over a sector of 80° to a distance of 16.5 km. Find the warned area. (Use $\pi = 3.14$)

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Smart Memory Mini-Test — Day 30 Mastery Test

Ten questions, no notes, no formula sheet. This is the chapter's final examination of YOU. Answers in the Answer Key.

1. One design of the NCERT table cover (radius 28 cm) has what area? (Use √3 = 1.7)

2. What total length of silver wire is needed for the NCERT brooch (diameter 35 mm, 5 diameters)?

3. Write, in exact form, the area of the segment cut off by a chord subtending 120° in a circle of radius 21 cm.

4. Three circles of radius 7 cm touch each other externally. Find the area enclosed between them. (Use √3 = 1.732)

5. An arc has length 22 cm in a circle of radius 7 cm. Using Area = ½lr, find the area of the corresponding sector.

6. Find the area of the ring sector (angle 90°) between circles of radii 14 cm and 7 cm.

7. A chord of a circle of radius 10 cm has length 10√3 cm. What angle does it subtend at the centre?

8. State the lune result of Challenge 6: for a 6-8-10 right triangle, the total lune area equals what?

9. State the major segment area of NCERT Q6 (radius 15 cm, angle 60°, π = 3.14, √3 = 1.73).

10. State the total cost of the six designs in NCERT Q13.

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Answer Key — Graded Practice, Levels 1 and 2

Level 1 — Foundation

L1.1 38.5 cm^2 **L1.2** 11 cm **L1.3** 86.625 cm^2 **L1.4** 77 cm^2 **L1.5** 128/7 cm ≈ 18.29 cm (arc 44/7 + radii 12) **L1.6** 44 cm

Level 2 — Skill Builder

L2.1 115.5 cm^2 **L2.2** Sector 154 − triangle 98 = 56 cm^2 **L2.3** θ = 60°; 57.75 cm^2 **L2.4** Major angle 300°; 660/7 ≈ 94.29 cm^2 **L2.5** Sector 77/3 ≈ 25.67 − triangle 21.19 = 4.47 cm^2 (approx.) **L2.6** 3.14 × (256 − 100) = 489.84 cm^2

Quick Check (Section 02)

Full answers are printed with the Quick Check itself — if you missed any, redo that card before continuing.

TOPPER TIP: *Mark yourself strictly: a right answer with a skipped step earns only part marks in the board. Train the steps, not just the answers.*

Answer Key — Levels 3, 4, 5 and Active Recall

Level 3 — Application

L3.1 Quadrant, rope 7: 38.5 m^2 **L3.2** (1/4) × 3.14 × 196 = 153.86 km^2 **L3.3** (105/360)(22/7)(441) = 404.25 cm^2 **L3.4** (1/12)(22/7)(1225) = 1925/6 ≈ 320.83 cm^2

Level 4 — Advanced Reasoning

L4.1 Sector 616/3 ≈ 205.33 − triangle 84.77 = 120.56 cm^2 (approx.) **L4.2** Four quadrants = one circle of radius 10: 400 − 314 = 86 cm^2 **L4.3** (1/4)(22/7)r^2 = 616 ⇒ r^2 = 784 ⇒ r = 28 cm; arc = 44 cm

Level 5 — Mathematical Genius

L5.1 Arc = 25 − 14 = 11 cm; (θ/360) × 44 = 11 ⇒ θ = 90°; area = 38.5 cm^2 **L5.2** Major segment = $\pi r^2 - [(60/360)\pi r^2 - (\sqrt{3}/4)r^2] = \pi r^2 - (\pi/6)r^2 + (\sqrt{3}/4)r^2 = (5\pi/6 + \sqrt{3}/4)\, r^2$. Proved.

Active Recall (Section 18)

R1 Arc = (θ/360) × 2πr; sector = (θ/360) × πr^2; shared fraction θ/360 **R2** 38.5 cm^2 **R3** Segment = sector − triangle; major segment = circle − minor segment **R4** 72° **R5** It is equilateral **R6** Arc = 110/7 ≈ 15.71; perimeter = 250/7 ≈ 35.71 cm **R7** Circumference 440/7 + diameters 100 = 1140/7 ≈ 162.86 mm **R8** (22/7)(144 − 25) = 374 cm^2 **R9** The radii are perpendicular, so they are the base and height: area = ½ × r × r **R10** Because πr^2 = ½ × (2πr) × r, and both formulas share the fraction θ/360

P1 A segment is bounded by a chord and an arc; a sector by two radii and an arc **P2** 60°: $(\sqrt{3}/4)r^2$; 90°: ½r^2; 120°: $(\sqrt{3}/4)r^2$ **P3** The 90° sector; one-fourth of the circle **P4** Draw, split, add, subtract **P5** r = d/2 — halve it, in writing

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Answer Key — Practice Plus

25 Concept MCQs — answers with one-line reasons

1 (B) area family: πr^2 with fraction $\theta/360$ **2** (B) half of 44 **3** (C) (1/4) × 616 **4** (B) the master line **5** (C) 360°/60 min **6** (C) arc 11 + radii 14 **7** (B) the whole circle **8** (B) (1/2)(22/7)(49) **9** (C) 360° − 70° **10** (C) (1/4) × 314 **11** (B) equilateral triangle **12** (C) $\pi(25 - 9)$ **13** (B) (1/4) × 132 **14** (C) 78.5 − 50 **15** (C) Reasoning 1 **16** (B) same as Q6 **17** (C) r = 7 from C = 44 **18** (A) (1/3)(22/7)(36) **19** (C) quadrant of radius 14 **20** (C) 360°/10 **21** (B) angles add to 180° → semicircle area 77 **22** (B) the trap-free road **23** (C) 11 = (θ/360) × 44 **24** (B) **25** (B) NCERT Q14 itself

Assertion–Reason (Section 14)

AR1 (a) **AR2** (a) **AR3** (a) **AR4** (a) **AR5** (a) **AR6** (a) **AR7** (a) **AR8** (a) **AR9** (d) — A is false (we SUBTRACT), R is true **AR10** (a)

Find the Error (Section: Practice Plus)

Each correction is printed in full with its question — re-read E1–E10 and tick off the error type you personally make most often in your Error Tracker.

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Answer Key — Model Tests and Day Tests

Model Test A — Basics (20)

Q1 77/3 ≈ 25.67 cm² [formula 1, value 1]. **Q2** 22 cm [1+1]. **Q3** r = 7 [1]; quadrant = 38.5 cm² [2]. **Q4** 78.5 − 50 = 28.5 cm² [sector 1, triangle 1, segment 1]. **Q5** (i) 28.26 m² [2]; (ii) 113.04 m², increase 84.78 m² [3]. **Q6** (i) 88 + 140 = 228 mm [3]; (ii) (1/10) × 616 = 61.6 mm² [2].

Model Test B — Standard (20)

Q1 Arc 22 cm [1½]; sector 231 cm² [1½]. **Q2** Minor 20.4375 cm² [3]; major 686.0625 cm² [2]. **Q3** One wiper 3520/7 ≈ 502.86 cm² [2]; total 7040/7 ≈ 1005.71 cm² [2]. **Q4** R = 42 [1]; (22/7)(1764 − 1225) = 1694 m² [3]. **Q5** Arc = 11 [1]; θ = 90° [1]; area 38.5 cm² [2].

Model Test C — Advanced (20)

Q1 Sector 616/3 ≈ 205.33 [2]; triangle 84.77 [1]; segment 120.56 cm² [2]. **Q2** Triangle 84.87 [2]; three sectors = 77 [1]; enclosed 7.87 cm² [2]. **Q3** Semicircles on legs total 12.5π = semicircle on hypotenuse [3]; lunes = triangle = 24 cm² [2]. **Q4** (25/7) r = 50 ⇒ r = 14 cm [3]; area 154 cm² [2].

Day 7 Test

1 77/6 ≈ 12.83 cm² **2** 11 cm **3** 154 cm² **4** 56 cm² **5** 90° **6** 200.96 cm² **7** 37.68 cm² **8** 25 cm **9** 60° **10** 616 cm²

Day 21 Test

1 66/7 ≈ 9.43 cm **2** 396/7 ≈ 56.57 cm² **3** 4.47 cm² **4** 285.5 cm² **5** 138.6 cm² **6** 627.48 cm² **7** 15.57 cm² **8** 14 cm **9** 36 cm **10** 189.97 km²

Day 30 Mastery Test

1 77.47 cm² **2** 285 mm **3** 462 − (441√3/4) cm² **4** 7.87 cm² **5** 77 cm² **6** 115.5 cm² **7** 120° **8** 24 cm² — the area of the triangle itself **9** 686.0625 cm² **10** ₹162.68

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Revision Card — Chapter 11 on One Page

The one idea

Every part of a circle is the fraction θ/360 of the whole. Arc = (θ/360) × 2πr. Sector = (θ/360) × πr^2 = ½ × arc × r. Sector perimeter = arc + 2r.

The segment rules

Minor segment = sector − triangle. **Major segment** = circle − minor segment (= major sector + triangle). Triangle areas: 60° → (√3/4)r^2 (equilateral); 90° → ½r^2; 120° → (√3/4)r^2.

The machines

Clock: minute hand 6°/min, hand = radius. **Corner animal:** quadrant, rope = radius. **Brooch/umbrella/pizza:** n equal parts → 1/n of the circle. **Wipers/lighthouse/sprinkler:** plain sectors — count the wipers! **Designs:** one segment × number of designs × rate.

Combinations

Draw → label → split → add or subtract. Ring = π(R^2 − r^2). Two semicircles = one circle. Shaded = whole − unshaded (or sum of pieces).

Exam-eve checklist

☐ Diameter halved ☐ Hidden angle found (clock, equal parts, reflex) ☐ π and √3 values as instructed ☐ Segment < sector, major > minor ☐ Radii in the perimeter ☐ Units and the final money step done

Chapter 11 complete. You did not memorise sectors and segments — you discovered, visualised, derived and mastered them.

Next: Chapter 12 — Surface Areas and Volumes, in the same 8D format. • KalviYogi • Mathematics – Beyond the Formula • 369 Tesla × KalviYogi • © Edunivra Pvt Ltd

CBSE · CLASS 10 · MATHEMATICS

MATHEMATICS BEYOND THE FORMULA

CHAPTER 12 · SURFACE AREAS AND VOLUMES

Don't Memorize Mathematics. Understand, Visualise and Discover It.

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01 Chapter Introduction

8D • DISCOVER — WHY SHOULD I LEARN THIS?

Pick up any object near you — a water bottle, a pencil, an ice-cream cone, a cricket ball. Every one of them is a **solid**, and two questions follow it everywhere: **How much skin does it have?** (surface area — the paint, paper, metal sheet or canvas needed to cover it) and **How much space does it fill?** (volume — the water it holds, the clay it is made of, the air inside it).

Real objects are almost never single, pure solids. A capsule is a cylinder wearing two hemispheres. A tent is a cylinder standing under a cone. A toy is a cone sitting on a hemisphere. This chapter teaches you to **take such combination solids apart in your mind**, handle each piece with a formula you will *derive* — not memorise — and put the pieces back together.

What you will master in this chapter

- The formulas of the cuboid, cube, cylinder, cone, sphere and hemisphere — and **where each formula comes from**: unrolling a cylinder, opening a cone into a sector, peeling a sphere.
- The Golden Rule of combinations: for surface area, **joined faces disappear**; for volume, pieces simply add (or subtract, for cavities).
- Surface areas and volumes of combination solids — all 17 NCERT questions, solved step by step.
- Conversion of solids: melting and recasting never changes volume — the master key to a whole family of problems.
- The frustum of a cone: its volume and curved surface area, **built from the big cone minus the small cone**.
- Cost, capacity and mass problems: canvas rates, litres of milk, kilograms of iron.

PIN POINT: SURFACE AREA answers wear square units (cm^2, m^2). VOLUME answers wear cubic units (cm^3, m^3) or litres. The unit is part of the answer — examiners deduct for it.

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Your Chapter Map — 19 Stops, One Journey

Tick each stop as you finish it. The chapter is a journey, not a formula list.

Discover & Visualise	Understand & Derive	Solve, Apply, Recall
☐ 01 Chapter Introduction ☐ 02 Prerequisite Knowledge ☐ 03 Real-Life Story ☐ 04 Visual Concepts (CPA)	☐ 05 Mathematical Foundations ☐ 06 Formula Derivations ☐ 08 Multiple Solution Methods ☐ 10 Common Mistakes	☐ 07 Detailed Solved Examples ☐ 09 Hands-on Activities ☐ 11 Graded Practice (5 Levels) ☐ 12 Mathematical Reasoning
☐ 13 Real-Life Applications ☐ 14 Competency-Based Questions	☐ 15 Olympiad Challenges ☐ 16 Chapter Mind Map	☐ 17 Pin-Point Flash Cards ☐ 18 Active Recall ☐ 19 Mastery Tracker

How to use this chapter (read this first!)

1. **Sketch before you calculate.** Every solution in this chapter begins by naming the pieces and marking the faces that touch — those faces are hidden and must not be counted.
2. Every solution shows **Step 1, Step 2, Step 3…** with a purple *Why* note. Cover the steps, try yourself, then uncover one step at a time.
3. Use the value of π the question commands: 22/7 or 3.14. Mixing them mid-question is a classic mark-loser.
4. After Section 07, test yourself in Sections 11–15 **before** opening the Answer Key. Finish with the Mind Map (16), Flash Cards (17), closed-book Recall (18) and your Mastery Tracker (19), and return on Days 3, 7, 14, 21 and 30.

KALVIYOGI DIGITAL COMPANION: Watch the video lesson for this chapter, try the interactive solid-builder simulation and take the online chapter test. Scan the QR code printed on the book cover / open the KalviYogi app. (Digital links are activated in the print edition.)

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02 Prerequisite Knowledge

EARLIER CLASSES RECAP — THE SIX SOLIDS YOU ALREADY OWN

This chapter stands on six solids from Classes 8 and 9. Here is each one with the formulas you already met — in Section 06 you will see *why* they are true.

1. Cuboid and cube

For a cuboid of length l, breadth b, height h: **Surface area = 2(lb + bh + hl)**, **Volume = l × b × h**. For a cube of edge a: **Surface area = $6a^2$**, **Volume = a^3**. A room's walls alone (lateral surface) = 2h(l + b).

2. Cylinder

Radius r, height h. **Curved surface area = $2\pi rh$**, **Total surface area = $2\pi r(h + r)$**, **Volume = $\pi r^2 h$**. Think of a rolled-up rectangle with two circular lids.

3. Cone

Radius r, height h, slant height l, where **$l = \sqrt{(r^2 + h^2)}$** (Pythagoras!). **Curved surface area = πrl**, **Total surface area = $\pi r(l + r)$**, **Volume = $(1/3)\pi r^2 h$** — exactly one-third of the cylinder on the same base and height.

4. Sphere

Radius r. **Surface area = $4\pi r^2$** — exactly four great circles. **Volume = $(4/3)\pi r^3$**.

5. Hemisphere

Half a sphere, radius r. **Curved surface area = $2\pi r^2$**, **Total surface area = $3\pi r^2$** (curved part + the flat circular face), **Volume = $(2/3)\pi r^3$**.

6. Units and conversions

1 m = 100 cm • 1 m^2 = 10,000 cm^2 • 1 m^3 = 10,00,000 cm^3 • **1 litre = 1000 cm^3** • 1 m^3 = 1000 litres. Convert BEFORE you calculate, not after.

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Quick Check — Are You Ready? (5 Questions)

Try all five without looking back. Answers are below — cover them with your hand first!

Q1. A cone has radius 7 cm and height 24 cm. Find its slant height.

Q2. Find the volume of a cylinder of radius 7 cm and height 10 cm. (Use $\pi = 22/7$)

Q3. A solid hemisphere has radius 7 cm. Find its total surface area.

Q4. How many litres are there in 2.5 m^3?

Q5. A cube has surface area 294 cm^2. Find its edge.

Answers — check only after trying

Q1. $l = \sqrt{(7^2 + 24^2)} = \sqrt{(49 + 576)} = \sqrt{625} =$ **25 cm**. (The 7–24–25 triple — you will meet it again and again.)

Q2. $V = \pi r^2 h = (22/7) \times 49 \times 10 =$ **1540 cm^3**.

Q3. $TSA = 3\pi r^2 = 3 \times (22/7) \times 49 =$ **462 cm^2**. (Only the curved part, $2\pi r^2$, would be 308 cm^2.)

Q4. 1 m^3 = 1000 litres, so 2.5 m^3 = **2500 litres**.

Q5. $6a^2 = 294 \Rightarrow a^2 = 49 \Rightarrow a =$ **7 cm**.

TOPPER TIP: Got 4 or 5 right? You own the tools. The rest of this chapter is only about COMBINING these solids wisely.

03 Real-Life Mathematical Story

DISCOVER • VISUALISE — THE SCHOOL FETE ICE-CREAM STALL

Before any formula, live the problem. Read the story slowly — the mathematics will walk out of it by itself.

*It is the annual fete at **Green Valley School**, and Class 10 has been given the most popular stall of all — **ice cream**. **Meena** is in charge of the servings. Each serving is a crisp **cone**, and on top of it sits one round **scoop** of ice cream.*

*The cone has radius **3 cm** and height **8 cm**. The scoop is a perfect **hemisphere** of radius 3 cm, sitting exactly on the cone's mouth.*

*The ice-cream supplier sells the mixture in **1 litre tubs**. Meena's teacher asks the question that decides whether the stall runs out of stock by noon:*

"Meena, how much ice cream is there in ONE serving? If 120 students buy one serving each, how many litres must we order?"

*At the next stall, **Arun** is painting the school's old cylindrical water tank before the fete. The tank has radius **3.5 m** and height **5 m**. It is **open at the top**, and he must paint the **outside curved wall and the circular bottom**. Paint costs **₹20 per m²**. "How much will the paint cost?" he wonders.*

Two stalls, two questions — one about **volume** (how much ice cream?), one about **surface area** (how much paint?). Both objects are **combinations**: cone + hemisphere, and a cylinder with one lid missing. Solve them with Meena and Arun, one step at a time.

Solving the Story — Problem 1: One Serving of Ice Cream

STORY PROBLEM 1

One serving = a cone of radius 3 cm and height 8 cm, topped by a hemispherical scoop of radius 3 cm. Find the volume of ice cream in one serving, and the litres needed for 120 servings. (Use π = 22/7)

THINK FIRST: The scoop sits ON the cone, not inside it — so both volumes count fully. Ask 'do the pieces overlap?' before you add.

1. Split the serving into its two solids: a **cone** (r = 3 cm, h = 8 cm) and a **hemisphere** (r = 3 cm).

   Why: Combination solids are always handled piece by piece — never by one giant formula.

2. Volume of the cone = $(1/3)\pi r^2 h = (1/3) \times (22/7) \times 3^2 \times 8 = (1/3) \times (22/7) \times 72 = 24 \times (22/7) = 528/7 \approx 75.43$ cm^3.

   Why: The (1/3) belongs to every cone — a cone is exactly one-third of its cylinder.

3. Volume of the hemispherical scoop = $(2/3)\pi r^3 = (2/3) \times (22/7) \times 27 = 18 \times (22/7) = 396/7 \approx 56.57$ cm^3.

   Why: A hemisphere is half of a sphere, so its volume is half of $(4/3)\pi r^3$.

4. Add the pieces: total volume = 528/7 + 396/7 = 924/7 = **132 cm^3**.

   Why: For VOLUME, pieces simply add — no face is 'hidden' inside a volume. This is the great difference from surface area.

5. For 120 servings: 120 × 132 = 15,840 cm^3. Convert to litres: 15,840 ÷ 1000 = **15.84 litres**.

   Why: 1 litre = 1000 cm^3. The supplier sells whole tubs, so Meena must order 16 one-litre tubs.

ANSWER: One serving holds 132 cm^3 of ice cream; 120 servings need 15.84 litres, so order 16 litres.

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Solving the Story — Problem 2: Painting the Tank

STORY PROBLEM 2

A cylindrical tank of radius 3.5 m and height 5 m is open at the top. Find the cost of painting its outside curved surface and its circular bottom at ₹20 per m². (Use π = 22/7)

THINK FIRST: Had Arun blindly used the full TSA formula 2πr(h + r), he would have painted a lid that does not exist — and over-ordered by 38.5 m².

1. List the surfaces to be painted: the **curved surface** of the cylinder and **one** circular base (the bottom). The top is open — there is no lid to paint.

   Why: In surface-area problems, first list WHICH faces exist. A missing face is the examiner's favourite trap.

2. Curved surface area = 2πrh = 2 × (22/7) × 3.5 × 5 = 2 × 22 × 0.5 × 5 = **110 m²**.

   Why: The curved wall unrolls into a rectangle of length 2πr and width h — Section 06 derives this.

3. Area of the circular bottom = πr² = (22/7) × 3.5² = (22/7) × 12.25 = **38.5 m²**.

4. Total area to paint = 110 + 38.5 = **148.5 m²**.

   Why: Add only the faces on your list — nothing more, nothing less.

5. Cost = 148.5 × 20 = **₹2,970**.

   Why: Area first, money last. Never multiply by the rate before the area is final.

ANSWER: The paint will cost ₹2,970.

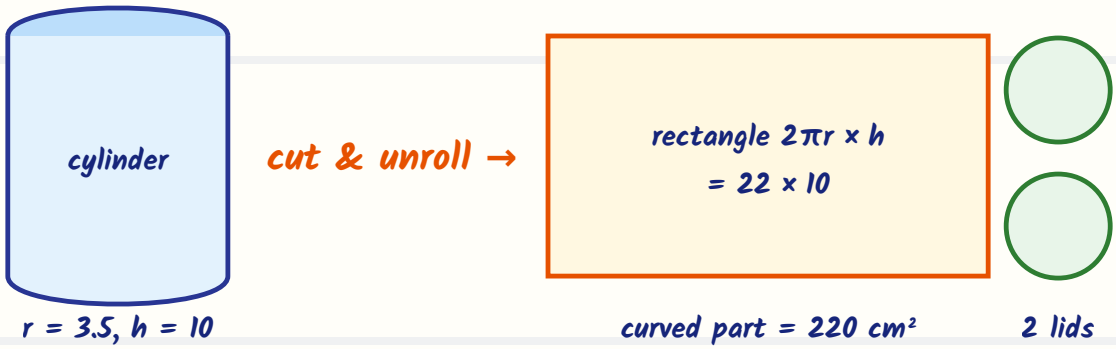
The two story signals — pin these in your memory

Paint / polish / canvas / sheet / cover / wrap → the question is about SKIN → **surface area**. List the faces that truly exist.
Holds / contains / fills / melts / is made of → the question is about SPACE or MATERIAL → **volume**. Add the pieces; subtract the cavities.
Nearly every problem in this chapter is one of these two signals wearing a costume.

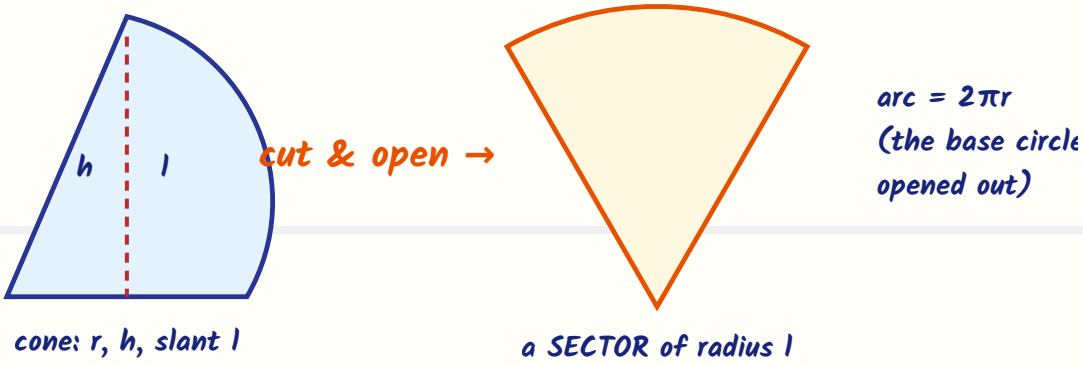
04 Visual Concept Explanation

SINGAPORE CPA — CONCRETE → PICTORIAL → ABSTRACT

Concrete first. Take a toilet-paper roll and slit it down its side: it opens into a flat **rectangle**. Take a paper party-hat (a cone) and cut along its slant: it opens into a **sector of a circle**. These two openings are the pictures behind the two hardest formulas of this chapter.

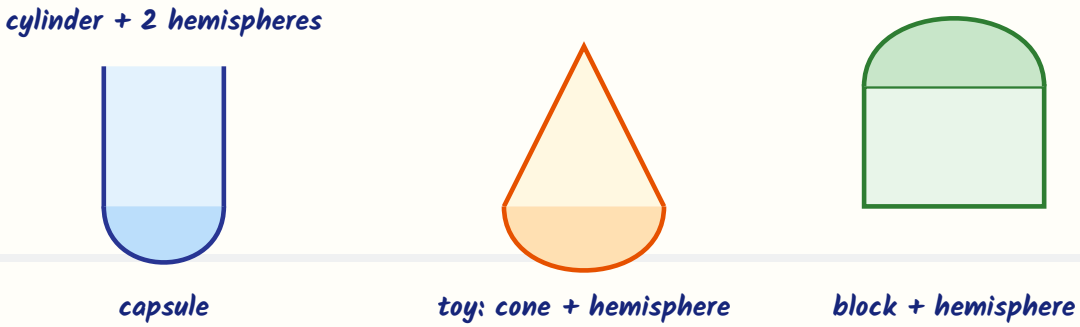


The cylinder's curved wall is a rectangle whose length is the **circumference of the base** (2πr) and whose width is the height h. So curved surface area = 2πr × h — the formula is the picture, compressed.



The cone's curved surface opens into a sector of radius l (the slant height). The sector's arc is exactly the base circle opened out: length 2πr. Section 06 turns this picture into the formula πrl, step by step.

Seeing Combinations — and the Faces That Vanish



Every combination solid in this chapter is built from the same six pieces. The artistic skill is not the formulas — it is **seeing which faces survive**:

The vanishing-faces picture

Press a hemisphere flat-side-down onto a cube. The circular face of the hemisphere and the patch of the cube beneath it are now **inside** the solid — no air touches them, no paint reaches them. They have **vanished from the surface**. Rule: when two solids are joined, **the touching face is subtracted TWICE** from the sum of the separate surface areas — once for each solid that owned it.

But volume never vanishes

Glue the same two solids: the total volume is simply the sum of the two volumes. Nothing is hidden inside a volume — the material is all still there. Cavities are the only exception: a scooped-out part is **subtracted**, and its inner surface is **added** to the surface area.

TOPPER TIP: *Before any calculation, draw the solid and shade the faces that touch another solid. Ten seconds of shading saves the whole question.*

05 Mathematical Foundations

UNDERSTAND — THE FORMULA TABLE AND THE THREE GOLDEN RULES

Here is the complete armoury of the chapter in one table. Do not memorise it yet — Section 06 derives the starred rows so that you can rebuild them even if you forget.

Solid	Curved Surface Area	Total Surface Area	Volume
Cuboid (l, b, h)	2h(l + b) (walls)	2(lb + bh + hl)	lbh
Cube (a)	$4a^2$ (walls)	$6a^2$	a^3
Cylinder (r, h)	2πrh *	2πr(h + r)	$\pi r^2 h$
Cone (r, h, l)	πrl *	πr(l + r)	$(1/3)\pi r^2 h$
Sphere (r)	—	$4\pi r^2$ *	$(4/3)\pi r^3$
Hemisphere (r)	$2\pi r^2$	$3\pi r^2$	$(2/3)\pi r^3$
Frustum (R, r, h, l)	πl(R + r) *	$\pi l(R + r) + \pi R^2 + \pi r^2$	$(1/3)\pi h(R^2 + r^2 + Rr)$ *

For the cone and frustum, $l = \sqrt{(h^2 + r^2)}$ and $l = \sqrt{(h^2 + (R - r)^2)}$ respectively. For a hollow cylinder (radii R and r): volume of material = $\pi h(R^2 - r^2)$.

Golden Rule 1 — Surface area of a combination

Add the surface areas of the pieces, then **subtract every touching face twice** (it vanished from both pieces). Add the inner surface of every cavity. Count only skin that air (or paint) can touch.

Golden Rule 2 — Volume of a combination

Volumes simply **add**. For a cavity or a scooped-out part, **subtract**. No other adjustment — ever.

Golden Rule 3 — Conversion of solids

Melting, recasting, reshaping, pouring: **the volume never changes**. Volume before = Volume after. This single equation solves every conversion problem, including "how many small solids from one big solid": n = (volume of big) ÷ (volume of one small).

PIN POINT: A hemisphere has TWO different areas: curved = $2\pi r^2$ (a bowl's inside), total = $3\pi r^2$ (a solid half-ball, including its flat face). Read the question to know which skin it wants.

06 Formula Derivations

DERIVE — WHERE EVERY FORMULA COMES FROM

A formula you can derive can never be truly forgotten. This section rebuilds the four starred formulas of Section 05 from pictures and simple logic.

Derivation 1 — Curved surface area of a cylinder: 2πrh

1. Cut the curved wall of the cylinder along a vertical line and unroll it flat (Section 04, first picture). The wall becomes a **rectangle**.
   Why: A curved surface that unrolls flat can be measured like any flat shape.
2. The rectangle's **length** is the distance around the circular base — the circumference 2πr. Its **width** is the height h of the cylinder.
   Why: Every point of the top rim travels exactly once around the base circle as you unroll.
3. Area of the rectangle = length × width = 2πr × h. Therefore **CSA of cylinder = 2πrh**.
4. Add the two circular lids (πr^2 each) and the total surface area follows: TSA = $2\pi rh + 2\pi r^2$ = **$2\pi r(h + r)$**.
   Why: Factoring out 2πr gives the compact form in the table.

Derivation 2 — Curved surface area of a cone: πrl

1. Cut the cone along its slant edge and open it flat. The curved surface becomes a **sector** of a circle whose radius is the slant height l (Section 04, second picture).
   Why: The slant edge becomes the two straight edges of the sector.
2. The sector's arc was the rim of the cone's base, so its length is the base circumference: **arc = 2πr**.
3. Compare with the FULL circle of radius l: its circumference is 2πl and its area is πl^2. Our sector is the fraction (2πr)/(2πl) = r/l of that full circle.
   Why: A sector with a given fraction of the circumference owns the same fraction of the area.
4. Area of sector = (r/l) × πl^2 = **πrl**. Therefore **CSA of cone = πrl**. Add the base circle and TSA = πrl + πr^2 = **$\pi r(l + r)$**.

TOPPER TIP: *Notice: both derivations are the same idea — unroll, measure the flat shape, roll back. If you forget πrl in the exam, redraw the sector and rebuild it in 20 seconds.*

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Derivation 3 — Surface area of a sphere: $4\pi r^2$ (the orange-peel idea)

A sphere cannot be unrolled flat — its skin curves in two directions at once. So this formula is built by a beautiful comparison, discovered by Archimedes. We present it as an **intuitive derivation**: it shows you why the answer must be $4\pi r^2$, even though a fully rigorous proof belongs to higher classes.

1. Wrap the sphere of radius r tightly inside a cylinder of radius r and height 2r (the cylinder just touches the sphere at the equator and at both poles).

   Why: This is the famous sphere-in-cylinder picture carved on Archimedes' tombstone.

2. Archimedes proved that every thin horizontal slice of the sphere has exactly the same area as the matching slice of the cylinder's wall — the sphere's slice is slanted, but it is also narrower, and the two effects cancel exactly.

   Why: This slice-by-slice equality is the heart of the argument; calculus later makes it fully rigorous.

3. Adding all the slices: surface area of sphere = curved surface area of the wrapping cylinder = $2\pi r \times$ (height $2r$) = **$4\pi r^2$**.

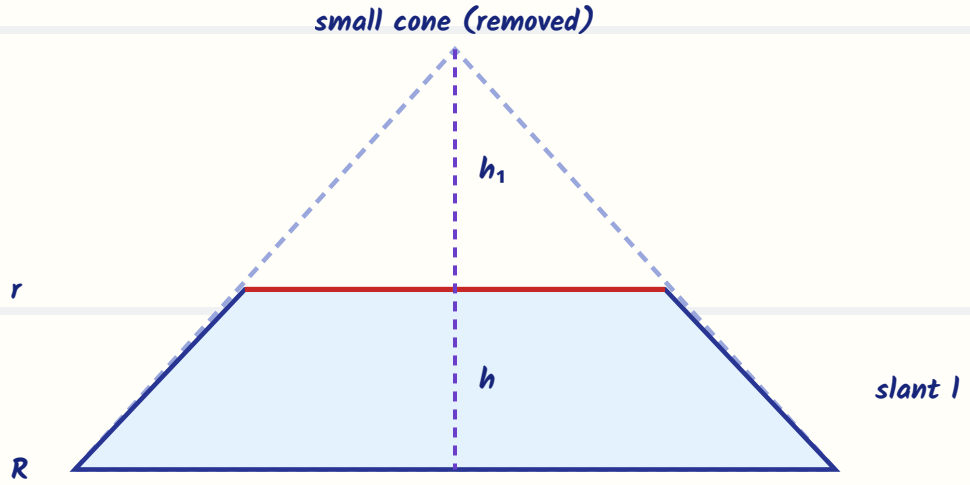
   Why: And we already DERIVED the cylinder's CSA by unrolling — so the sphere inherits a proved formula.

4. A great circle of the sphere has area πr^2. So the sphere's surface = **exactly 4 great circles** — a memorable check. Halving gives the hemisphere: curved area $2\pi r^2$, and adding the flat face πr^2 gives TSA $3\pi r^2$.

Derivation 4 — The frustum: big cone minus small cone

A **frustum** is what remains when the top of a cone is sliced off by a plane parallel to the base — a bucket, a lampshade, a drinking glass. Let the frustum have lower radius R, upper radius r, height h and slant height l. Complete the cone in your mind: the frustum is a **big cone with a small cone removed from the top**. Every frustum formula is just this subtraction, done carefully.

Deriving the Frustum Formulas — Step by Step



1. Let the small removed cone have height h_1. By similar triangles in the axial cross-section, $h_1 / (h_1 + h) = r / R$. Cross-multiplying: $h_1R = r\,h_1 + rh$, so $h_1(R - r) = rh$, giving **$h_1 = rh/(R - r)$**, and the full cone height $H = h_1 + h =$ **$Rh/(R - r)$**.

   Why: The radii grow in the same ratio as the heights — that is what 'similar' buys us.

2. Volume of frustum = Volume(big cone) − Volume(small cone) = $(1/3)\pi R^2H - (1/3)\pi r^2h_1$.

   Why: Golden Rule 2: the removed piece is subtracted.

3. Substitute H and h_1: $= (1/3)\pi \cdot h/(R - r) \cdot (R^3 - r^3)$.

   Why: Each term $R^2 \times Rh/(R-r)$ and $r^2 \times rh/(R-r)$ shares the factor $h/(R - r)$.

4. Use the identity $R^3 - r^3 = (R - r)(R^2 + Rr + r^2)$. The $(R - r)$ cancels, leaving **$V = (1/3)\pi h(R^2 + Rr + r^2)$**.

   Why: This is why the formula has that strange three-term bracket — it is a cancelled difference of cubes!

5. For the curved surface: CSA = $\pi R\,L - \pi r\,l_1$ (big cone minus small cone), where L and l_1 are the slant heights. Similar triangles give $L = lR/(R - r)$ and $l_1 = lr/(R - r)$, so CSA = $\pi l(R^2 - r^2)/(R - r) =$ **$\pi l(R + r)$**.

   Why: $R^2 - r^2 = (R - r)(R + r)$ does the same cancelling trick.

6. Finally, the slant height comes from the trapezium cross-section: the sloping side is the hypotenuse of a right triangle of height h and base $(R - r)$: **$l = \sqrt{(h^2 + (R - r)^2)}$**.

   Why: Pythagoras, as always, guards the slant height.

Check the derivation with numbers

Take R = 6, r = 3, h = 4. Then $l = \sqrt{(16 + 9)} = 5$. Small cone height $h_1 = (3 \times 4)/(6 - 3) = 4$, full height H = 8.
Big cone volume = $(1/3)\pi(36)(8) = 96\pi$; small cone = $(1/3)\pi(9)(4) = 12\pi$; frustum = $84\pi =$ **264**

07 Detailed Solved Examples

SOLVE — NCERT EXERCISE 12.1: SURFACE AREAS OF COMBINATIONS

Every NCERT question, with complete steps. Unless stated otherwise, use π = 22/7. Watch the first step of each solution — it is always the same habit: **name the pieces, list the surviving faces**.

EX 12.1 Q1

Two cubes, each of volume 64 cm³, are joined end to end. Find the surface area of the resulting cuboid.

THINK FIRST: The joining hides one face of each cube. Either compute the new cuboid directly, or start from the two cubes and remove the hidden faces (Section 08 shows both).

1. Find the edge of each cube: $a^3 = 64 = 4^3$, so **a = 4 cm**.
   Why: Surface area needs lengths, so the volume must first be turned back into an edge.
2. Joining end to end doubles one dimension: the cuboid has length l = 4 + 4 = **8 cm**, breadth b = 4 cm, height h = 4 cm.
   Why: Only the direction of joining grows; the other two dimensions stay 4 cm.
3. Surface area of cuboid = 2(lb + bh + hl) = 2(8×4 + 4×4 + 4×8) = 2(32 + 16 + 32) = 2 × 80 = **160 cm²**.

ANSWER: 160 cm²

CBSE-STYLE MARKING SCHEME: 1 mark for the edge a = 4 • 1 mark for the cuboid's dimensions • 1 mark for the surface area with unit.

EX 12.1 Q2

A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

THINK FIRST: The vessel is HOLLOW and we want the INNER surface — the skin the water touches: the inside of the cylinder plus the inside of the hemisphere. No flat faces at all.

1. Radius of both parts: r = 14/2 = **7 cm**.
   Why: Mounted parts share the same radius.
2. The hemisphere's depth equals its radius, 7 cm. So the cylinder's height h = 13 − 7 = **6 cm**.
   Why: Total height is shared between the two parts; the hemisphere claims exactly r of it.

Exercise 12.1 — Question 3

EX 12.1 Q3

A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of the same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

THINK FIRST: The toy's skin = cone's curved surface + hemisphere's curved surface. The circle where they join is inside the toy — it is counted ZERO times.

1. Radius of both parts: r = **3.5 cm**.

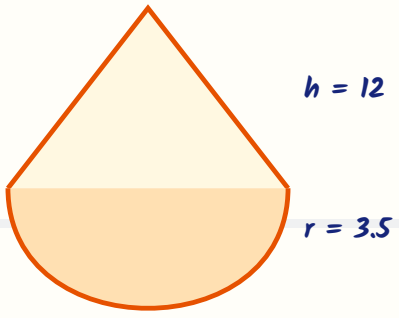
2. Height of the cone: total height − hemisphere's radius = 15.5 − 3.5 = **12 cm**.
   Why: The hemisphere contributes exactly its radius to the total height.

3. Slant height of the cone: $l = \sqrt{(r^2 + h^2)} = \sqrt{(3.5^2 + 12^2)} = \sqrt{(12.25 + 144)} = \sqrt{156.25} = $ **12.5 cm**.
   Why: Cone CSA needs the SLANT height, never the vertical height.

4. TSA of toy = CSA of cone + CSA of hemisphere = $\pi rl + 2\pi r^2 = (22/7) \times 3.5 \times 12.5 + 2 \times (22/7) \times 3.5^2 = 137.5 + 77 =$ **214.5 cm²**.
   Why: No base circle is added: the bottom of the hemisphere is its curved skin itself, and the joining circle is hidden.

ANSWER: 214.5 cm²

CBSE-STYLE MARKING SCHEME: 1 mark for the cone's height and slant height • 1 mark for choosing the two curved surfaces • 1 mark for the value with unit.



toy: cone on hemisphere
total height 15.5 cm

Exercise 12.1 — Questions 4 and 5

EX 12.1 Q4

A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

THINK FIRST: Start from the cube's full surface, REMOVE the circular patch covered by the hemisphere, ADD the hemisphere's curved skin.

1. The hemisphere sits on a 7 cm × 7 cm face, so the greatest possible diameter is the edge itself: **7 cm**, hence r = 3.5 cm.
   Why: A wider hemisphere would hang over the edges of the face.
2. Surface area = (surface area of cube) − (covered circle) + (CSA of hemisphere) = $6 \times 7^2 - \pi r^2 + 2\pi r^2 = 6 \times 49 + \pi r^2$.
   Why: The covered circle vanishes once (from the cube) and the hemisphere brings its own curved skin; net effect: + πr^2.
3. = $294 + (22/7) \times 3.5^2 = 294 + 38.5 =$ **$332.5\ cm^2$**.

ANSWER: Greatest diameter = 7 cm; surface area = $332.5\ cm^2$

CBSE-STYLE MARKING SCHEME: 1 mark for the greatest diameter with reason • 1 mark for the add-and-subtract structure • 1 mark for the value with unit.

EX 12.1 Q5

A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

THINK FIRST: A DEPRESSION is a cavity: the flat circle is removed from the face, but the bowl's inner curved surface is ADDED. The answer stays in terms of the edge l.

1. Let the edge of the cube be l. The hemisphere's diameter is l, so its radius is **l/2**.
2. Start with the cube: surface area = $6l^2$. The cut removes one circular patch of area $\pi(l/2)^2$ from a face.
   Why: The opening of the bowl is no longer wood surface.
3. The cut adds the inner curved surface of the hemisphere: $2\pi(l/2)^2$.
   Why: A cavity's inner skin is real surface — air and touch can reach it.
4. Total = $6l^2 - \pi l^2/4 + 2\pi l^2/4 = 6l^2 + \pi l^2/4 =$ **$(l^2/4)(24 + \pi)$** square units.

Exercise 12.1 — Question 6

EX 12.1 Q6

A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.

THINK FIRST: Two hemispheres of the same radius make one full sphere of skin. The capsule's surface = cylinder's curved wall + one sphere's surface.

1. Radius: r = 5/2 = **2.5 mm** for all three parts.

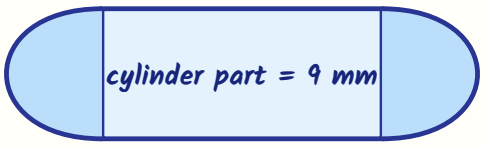
2. Length of the cylindrical part = total length − 2 radii = 14 − 5 = **9 mm**.
   Why: Each hemispherical cap claims one radius of the total length.

3. Surface area = CSA of cylinder + 2 × CSA of hemisphere = $2\pi rh + 2(2\pi r^2) = 2 \times (22/7) \times 2.5 \times 9 + 4 \times (22/7) \times 2.5^2$.

4. = 990/7 + 550/7 = 1540/7 = **220 mm²**.
   Why: Work in millimetres throughout — the question's unit is mm.

ANSWER: 220 mm²

CBSE-STYLE MARKING SCHEME: 1 mark for r and the cylinder's length • 1 mark for the surface structure • 1 mark for the value with unit mm².



capsule: length 14 mm, diameter 5 mm

Exercise 12.1 — Questions 7, 8 and 9

EX 12.1 Q7

A tent is in the shape of a cylinder surmounted by a conical top. The height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m. Find the area of the canvas used, and the cost of the canvas at ₹500 per m². (The base of the tent is not covered with canvas.)

THINK FIRST: Canvas covers only the two CURVED surfaces. No floor, no flat circles anywhere.

1. Radius: r = 4/2 = **2 m**.

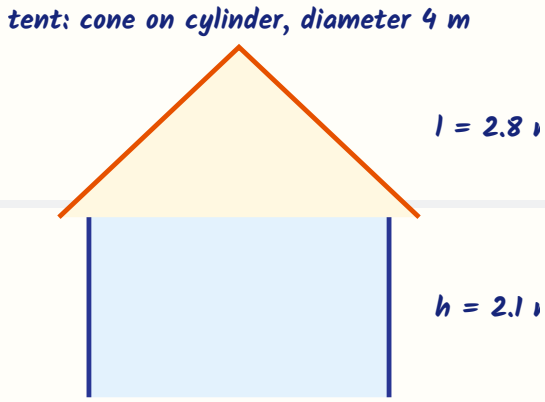
2. Canvas = CSA of cone + CSA of cylinder = πrl + 2πrh = (22/7) × 2 × 2.8 + 2 × (22/7) × 2 × 2.1 = 17.6 + 26.4 = **44 m²**.

   Why: The cone's slant height is GIVEN (2.8 m) — do not waste time finding the cone's vertical height.

3. Cost = 44 × 500 = **₹22,000**.

ANSWER: Canvas = 44 m²; cost = ₹22,000.

CBSE-STYLE MARKING SCHEME: 1 mark for the two curved surfaces only • 1 mark for the area • 1 mark for the cost.



EX 12.1 Q8

From a solid cylinder of height 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and diameter is hollowed out. Find the total surface area of the remaining solid, to the nearest cm².

THINK FIRST: Three skins survive: the cylinder's outer wall, the cylinder's bottom circle, and the cone's inner curved wall. The top circle is gone — it became the cavity's mouth.

1. Radius r = 1.4/2 = **0.7 cm**; height h = 2.4 cm for both parts.

2. Slant height of the cavity: l = √(h² + r²) = √(2.4² + 0.7²) = √(5.76 + 0.49) = √6.25 = **2.5 cm**.

Exercise 12.2 — Volumes of Combinations (Questions 1 and 2)

EX 12.2 Q1

A solid is in the shape of a cone standing on a hemisphere with both their radii equal to 1 cm and the height of the cone equal to its radius. Find the volume of the solid in terms of π.

THINK FIRST: Volumes add — and the question wants the answer left in terms of π, so do not substitute any value for π.

1. Radius r = 1 cm; height of cone h = r = 1 cm.
2. Volume = Volume of cone + Volume of hemisphere = $(1/3)\pi r^2 h + (2/3)\pi r^3$.
   Why: Golden Rule 2: pieces add.
3. Substitute r = h = 1: = $(1/3)\pi(1)(1) + (2/3)\pi(1) = (1/3)\pi + (2/3)\pi =$ **π cm³**.
   Why: The two fractions complete exactly one whole π — a designed answer.

ANSWER: π cm³

CBSE-STYLE MARKING SCHEME: 1 mark for the sum structure • 1 mark for substitution • 1 mark for the exact answer in terms of π.

EX 12.2 Q2

Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends, using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model. (Assume the outer and inner dimensions are nearly the same.)

THINK FIRST: The air inside fills ALL THREE parts: cylinder + cone + cone. First rescue the cylinder's own length from the total length.

1. Radius of every part: r = 3/2 = **1.5 cm**.
2. Length of the cylindrical part = 12 − 2 − 2 = **8 cm**.
   Why: The two cones claim 2 cm each from the total length of 12 cm.
3. Volume of air = $\pi r^2 h + 2 \times (1/3)\pi r^2 h_1 = (22/7)(1.5^2)(8) + 2 \times (1/3)(22/7)(1.5^2)(2)$.
4. = 396/7 + 66/7 = 462/7 = **66 cm³**. (Equivalently: 18π + 3π = 21π = 66.)

Exercise 12.2 — Question 3

EX 12.2 Q3

A gulab jamun contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends, of length 5 cm and diameter 2.8 cm.

THINK FIRST: A gulab jamun is a capsule shape: cylinder in the middle, a hemisphere at each end. Find ONE jamun, multiply by 45, then take 30%.

1. Radius: r = 2.8/2 = **1.4 cm**. Length of the cylindrical part = 5 − 2(1.4) = **2.2 cm**.

   Why: The two hemispherical ends together claim one full diameter of the length.

2. Volume of one jamun = $\pi r^2 h + 2 \times (2/3)\pi r^3 = (22/7)(1.4^2)(2.2) + (4/3)(22/7)(1.4^3) \approx 13.55 + 11.50 =$ **25.05 cm³** (approximately).

   Why: Two hemispheres = one sphere's worth of volume, $(4/3)\pi r^3$.

3. Volume of 45 jamuns = 45 × 25.05 = **1127.25 cm³**.

4. Syrup = 30% of 1127.25 = 0.30 × 1127.25 = 338.175 ≈ **338 cm³**.

   Why: The word 'approximately' licenses the final rounding.

ANSWER: Approximately 338 cm³ of syrup

CBSE-STYLE MARKING SCHEME: 1 mark for one jamun's volume • 1 mark for 45 jamuns • 1 mark for the 30% with approximation.

Exercise 12.2 — Questions 4 and 5

EX 12.2 Q4

A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm × 10 cm × 3.5 cm. The radius of each depression is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand.

THINK FIRST: Wood = cuboid MINUS the four hollowed cones. Depressions are cavities: subtract.

1. Volume of the cuboid = 15 × 10 × 3.5 = **525 cm³**.
2. Volume of one conical depression = $(1/3)\pi r^2 h = (1/3) \times (22/7) \times 0.5^2 \times 1.4 = (1/3) \times (22/7) \times 0.35$ = **0.3667 cm³** (approximately).
   Why: Depth of the depression is the cone's height.
3. Four depressions remove 4 × 0.3667 = 1.4667 cm³.
4. Volume of wood = 525 − 1.4667 = **523.53 cm³** (approximately).
   Why: Sanity check: the depressions are tiny, so the answer must stay just below 525 — it does.

ANSWER: 523.53 cm³ (approximately)

CBSE-STYLE MARKING SCHEME: 1 mark for the cuboid's volume • 1 mark for one depression • 1 mark for the subtraction and final value.

EX 12.2 Q5

A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each a sphere of radius 0.5 cm, are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

THINK FIRST: The shots push out their own volume of water. Shots' total volume = the water that flowed out = one-fourth of the cone's volume.

1. Volume of the conical vessel = $(1/3)\pi R^2 H = (1/3) \times (22/7) \times 5^2 \times 8 = 4400/21 \approx$ **209.52 cm³**.
2. Water that flows out = (1/4) × 4400/21 = **1100/21 cm³**.
   Why: Only the overflow matters — the water remaining in the vessel is irrelevant.
3. Volume of one lead shot = $(4/3)\pi r^3 = (4/3) \times (22/7) \times 0.5^3$ = **11/21 cm³**.

Exercise 12.2 — Question 6

EX 12.2 Q6

A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. (Use π = 3.14)

THINK FIRST: Two cylinders stacked: add the volumes, then let the density (8 g per cm^3) turn volume into mass. This question commands π = 3.14.

1. Radii: lower cylinder r_1 = 24/2 = **12 cm**; upper cylinder r_2 = 8 cm.

2. Total volume = $\pi r_1^2 h_1 + \pi r_2^2 h_2 = 3.14 \times (12^2 \times 220 + 8^2 \times 60) = 3.14 \times (31{,}680 + 3{,}840) = 3.14 \times 35{,}520$ = **111,532.8 cm^3**.

   Why: Factor π out — one multiplication by 3.14 instead of two.

3. Mass = 111,532.8 × 8 = 892,262.4 g = **892.26 kg** (approximately).

   Why: Convert grams to kilograms at the end: divide by 1000.

ANSWER: Approximately 892.26 kg

CBSE-STYLE MARKING SCHEME: 1 mark for the total volume • 1 mark for using π = 3.14 as commanded • 1 mark for the mass in kg.

Exercise 12.2 — Question 7

EX 12.2 Q7

A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water, of radius 60 cm and height 180 cm. Find the volume of water left in the cylinder.

THINK FIRST: Water left = cylinder's volume − the submerged solid's volume. The solid is fully inside (its total height 120 + 60 = 180 cm exactly matches the cylinder).

1. Volume of the cylinder = $\pi R^2 H = (22/7) \times 60^2 \times 180 =$ **2,036,571.43 cm³**.

2. Volume of the solid = hemisphere + cone = $(2/3)\pi(60^3) + (1/3)\pi(60^2)(120) = 452,571.43 + 452,571.43 =$ **905,142.86 cm³**.

   Why: A designed coincidence: the cone and the hemisphere have EQUAL volumes here — halve your work.

3. Water left = 2,036,571.43 − 905,142.86 = 1,131,428.57 cm^3 = **1.131 m³** (approximately).

   Why: Divide by 10,00,000 to convert to m^3.

ANSWER: Approximately 1.131 m^3

CBSE-STYLE MARKING SCHEME: 1 mark for the cylinder's volume • 1 mark for the solid's volume • 1 mark for the difference with conversion to m^3.

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Exercise 12.2 — Question 8: The Checking Question

EX 12.2 Q8

A spherical glass vessel has a cylindrical neck 8 cm long and 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements and $\pi = 3.14$.

THINK FIRST: Compute the true volume exactly as commanded ($\pi = 3.14$) and compare. Never trust a printed or remembered answer — recompute. (Many guidebooks print this one wrongly; the correct value is 346.51 cm^3.)

1. Radius of the spherical part: $r_1 = 8.5/2 =$ **4.25 cm**. Radius of the neck: $r_2 = 2/2 =$ **1 cm**, length 8 cm.
2. Volume of the spherical part = $(4/3)\pi r_1^3 = (4/3) \times 3.14 \times 4.25^3 = (4/3) \times 3.14 \times 76.765625 =$ **321.39 cm^3** (approximately).
3. Volume of the neck = $\pi r_2^2 h = 3.14 \times 1^2 \times 8 =$ **25.12 cm^3**.
4. Total volume = 321.39 + 25.12 = **346.51 cm^3**.
5. $346.51 \neq 345$, so **the child is NOT correct**. The vessel actually holds about 1.51 cm^3 more than she found.

   Why: A 'check whether' question is complete only when the verdict is stated in words.

ANSWER: The child is not correct; the true volume is 346.51 cm^3

CBSE-STYLE MARKING SCHEME: 1 mark for the two radii • 1 mark for each part's volume • 1 mark for the total and the verdict.

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Foundation Builder — Conversion of Solids (Masterclass)

Honest note: conversion of solids and the frustum were removed from the current NCERT textbook's exercises, but they live on in the NCERT Exemplar, in Olympiads and in real life — and Golden Rule 3 is too powerful to skip. Master these six worked examples as your safety margin.

BUILDER C1

A metallic sphere of radius 4.2 cm is melted and recast into a cylinder of radius 6 cm. Find the height of the cylinder. (Use π = 22/7)

THINK FIRST: One unknown, one equation: Volume(before) = Volume(after). Every conversion problem on Earth is this line.

1. Volume of the sphere = $(4/3)\pi r^3 = (4/3) \times (22/7) \times 4.2^3$ = **310.464 cm³**.
   Why: Melting destroys the shape, never the volume.
2. Let the cylinder's height be h. Its volume = $\pi(6^2)h = (22/7) \times 36 \times h$.
3. Apply Golden Rule 3: $(22/7) \times 36 \times h = 310.464$, so $h = 310.464 \times 7 / (22 \times 36)$ = **2.74 cm** (approximately).
   Why: π appears on both sides of such equations surprisingly often — here it cancels only after substitution, so substitute carefully.

ANSWER: Height of the cylinder ≈ 2.74 cm

BUILDER C2

Three metallic cubes of edges 3 cm, 4 cm and 5 cm are melted and formed into a single cube. Find the edge of the new cube.

1. Total volume = $3^3 + 4^3 + 5^3 = 27 + 64 + 125$ = **216 cm³**.
2. Let the new edge be a: $a^3 = 216 = 6^3$, so **a = 6 cm**.
   Why: 216 is a perfect cube — contest and board setters choose such numbers on purpose.

ANSWER: 6 cm

BUILDER C3

How many spherical lead shots, each of diameter 3 cm, can be made from a cuboidal lead solid of dimensions 9 cm × 11 cm × 12 cm? (Use π = 22/7)

1. Volume of the cuboid = 9 × 11 × 12 = **1188 cm³**.

Foundation Builder — The Frustum at Work (Masterclass)

BUILDER F1

A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass. (Use π = 22/7)

1. Radii: R = 4/2 = **2 cm** (top), r = 2/2 = **1 cm** (bottom); h = 14 cm.
2. Capacity = volume of frustum = $(1/3)\pi h(R^2 + r^2 + Rr) = (1/3) \times (22/7) \times 14 \times (4 + 1 + 2)$.
   Why: The derived formula of Section 06, in action.
3. $= (1/3) \times 22 \times 2 \times 7 = 308/3 =$ **$102.67\ cm^3$** (approximately).

ANSWER: Capacity ≈ $102.67\ cm^3$

BUILDER F2

A fez (a cap) is shaped like a frustum of a cone. Its radius on the open side is 10 cm, the radius at the upper base is 4 cm and its slant height is 15 cm. Find the area of material used for making it. (Use π = 22/7)

1. The material covers the curved surface AND the closed upper base — but NOT the open side (the head goes there).
   Why: List the faces first, as always.
2. Curved surface = $\pi l(R + r) = (22/7) \times 15 \times (10 + 4) = (22/7) \times 210 =$ **$660\ cm^2$**.
3. Upper base = $\pi r^2 = (22/7) \times 16 =$ **$352/7 \approx 50.29\ cm^2$**.
4. Total material = 660 + 50.29 = **$710.29\ cm^2$** (approximately).

ANSWER: Approximately $710.29\ cm^2$

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Foundation Builder — Frustum Problems (continued)

BUILDER F3

A container, open at the top and made of metal sheet, is in the form of a frustum of a cone of height 16 cm, with radii of its lower and upper ends 8 cm and 20 cm respectively. Find (i) the cost of the milk that can completely fill it at ₹20 per litre, and (ii) the cost of the metal sheet at ₹8 per 100 cm². (Use π = 22/7)

1. Volume = $(1/3)\pi h(R^2 + r^2 + Rr) = (1/3) \times (22/7) \times 16 \times (400 + 64 + 160) = (1/3) \times (22/7) \times 16 \times 624 =$ **10,459.43 cm³** ≈ 10.46 litres.

   Why: Note which radius is R: 20 is the UPPER (open) end here — the formula is symmetric in R and r for volume, so the order does not matter.

2. Cost of milk = 10.459 × 20 ≈ **₹209.19**.

   Why: Convert cm³ to litres (divide by 1000) BEFORE pricing.

3. Slant height: $l = \sqrt{(h^2 + (R - r)^2)} = \sqrt{(16^2 + 12^2)} = \sqrt{(256 + 144)} = \sqrt{400} =$ **20 cm**.

   Why: The 12–16–20 triangle — the 3–4–5 family in disguise.

4. Metal sheet = CSA + lower base $= \pi l(R + r) + \pi r^2 = (22/7)(20)(28) + (22/7)(64) = 1760 +$ 201.14 = **1961.14 cm²**. The top is open, so no upper base is added.

5. Cost of sheet = (1961.14/100) × 8 ≈ **₹156.89, i.e. about ₹157**.

ANSWER: (i) Milk ≈ ₹209.19 (ii) Metal sheet ≈ ₹157

TOPPER TIP: *Frustum sanity check: the true capacity always lies between the two cylinders built on the end radii — if your answer escapes that range, a radius has slipped somewhere.*

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08 Multiple Solution Methods

CHALLENGE — THE SAME ANSWER, TWO ROADS

A master is not the student who knows one method, but the one who can **choose**. Here are the chapter's two most useful double-roads.

Problem A — Ex 12.1 Q1 again: two cubes of volume 64 cm³ joined end to end

Method 1: Build the new solid

Edge a = 4 cm. New cuboid = 8 × 4 × 4.
SA = 2(32 + 16 + 32) = **160 cm²**.
Best when the joined shape is a standard solid.

VS

Method 2: Subtract the hidden faces

Two separate cubes: 2 × 6 × 4^2 = 192 cm^2.
The join hides 2 faces of 16 cm^2: 192 − 32 = **160 cm²**.
Best when the joined shape is irregular.

Problem B — The capsule (Ex 12.1 Q6): surface area 220 mm²

Method 1: Three pieces

Cylinder wall 2πrh = 990/7; two hemispheres 2 × $2\pi r^2$ = 550/7.
With r = 2.5, h = 9: total = 1540/7 = **220 mm²**.

VS

Method 2: Merge the caps

Two hemispheres of equal radius = ONE sphere: $4\pi r^2$.
SA = $2\pi rh + 4\pi r^2$ = 990/7 + 550/7 = **220 mm²**, in one line less.
The merging trick also halves the gulab-jamun volume work.

TOPPER TIP: *In the exam hall, use Method 2-style merging whenever two hemispheres share a radius — it is faster and leaves fewer places for arithmetic to slip.*

09 Interactive Simulations & Hands-on Activities

VISUALISE — LEARN IT WITH YOUR HANDS

Activity 1 — The unrolling experiment (10 minutes)

Take an empty toilet-paper roll or a cylindrical can. Wrap a sheet of paper tightly around its curved wall, mark where the paper meets itself, and unroll. Measure: the paper's length equals the can's circumference (2πr) and its width equals the height. You have just **proved Derivation 1 with scissors**. Record: r = ___ cm, h = ___ cm, paper length = ___ cm, 2πr = ___ cm.

Activity 2 — Three cones fill a cylinder

Make (or imagine) a cone and a cylinder with the SAME base and height. Fill the cone with sand or water and pour into the cylinder. It takes **exactly 3 cones**. That is why the cone's volume carries the factor 1/3 — you have now seen it, not just read it.

Activity 3 — The orange-peel sphere

Peel an orange of radius r and press the peel flat over a drawn circle of radius r. The peel fills the circle about **four times** — the sphere's surface is 4 great circles, as Archimedes promised.

Activity 4 — Hidden-face hunt

Take two dice or two identical cubes. Note the total surface area separately (12 faces visible per pair of cubes... count carefully). Now press them together face-to-face and count the visible faces again: **exactly 2 faces have vanished**. Repeat with three cubes in a row: each join costs 2 faces. Predict, then verify: n cubes in a row show (4n + 2) faces — Reasoning R3 proves it.

Activity 5 — Measure your world (field task)

Choose one object at home — a bottle, a matka, a bucket (frustum!). Measure it, compute its capacity, then verify by filling it with a 1-litre bottle. Record the percentage error and one reason for it.

KALVIYOGI DIGITAL: The companion app lets you build combination solids piece by piece, rotate them, and watch joined faces vanish in 3D. Scan the QR code on the book cover / open the KalviYogi app. (Activated in the print edition.)

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10 Common Mistakes

THE SIX TRAPS — AND HOW TOPPERS DODGE THEM

COMMON MISTAKE: COUNTING HIDDEN FACES. **Wrong: adding the full TSA of both pieces of a combination. Right: add only surviving skin — subtract each touching face twice.** This is the single most expensive trap in the chapter.

COMMON MISTAKE: HEMISPHERE CONFUSION. **Wrong: using $2\pi r^2$ for a solid hemisphere's total surface. Right: curved = $2\pi r^2$, total = $3\pi r^2$ (the flat face counts for a solid).** Ask: 'is it a bowl (hollow) or a half-ball (solid)?'

COMMON MISTAKE: DIAMETER AS RADIUS. **Wrong: substituting d where r belongs — it quadruples areas and octuples volumes. Right: the FIRST line of every solution converts diameter to radius.**

COMMON MISTAKE: HEIGHT FOR SLANT HEIGHT. **Wrong: cone CSA = πrh. Right: cone CSA = πrl with $l = \sqrt{(r^2 + h^2)}$ — compute l before you begin.**

COMMON MISTAKE: CAVITY SIGN ERRORS. **Wrong: subtracting a cavity's volume but forgetting its inner surface, or adding its volume. Right: cavity → volume SUBTRACTS, inner surface ADDS.**

COMMON MISTAKE: π AND UNIT CARELESSNESS. **Wrong: mixing 22/7 and 3.14 in one question, or answering a volume in cm^2. Right: obey the π the question commands; write the unit in every answer line; convert litres ↔ cm^3 at the START.**

PIN POINT: THE 10-SECOND AUDIT before you leave any solution: (1) radius, not diameter? (2) hidden faces removed? (3) slant height for cones? (4) π as commanded? (5) unit correct? Run it every time.

Find the Error — 10 Broken Solutions (Spot the Slip, Fix It)

Each solution below contains exactly one planted error. Find it, name it, and write the correct answer. Full corrections are in the Answer Key.

E1. "TSA of a solid hemisphere of radius 7 cm = $2\pi r^2 = 2 \times (22/7) \times 49 = 308$ cm^2."

E2. "Capsule of total length 14 mm and diameter 5 mm: cylinder length = 14 mm, so SA = $2\pi(2.5)(14) + 4\pi(2.5^2) = 220 + 78.6 = 298.6$ mm^2."

E3. "Cone with r = 7 cm, h = 24 cm: CSA = $\pi rh = (22/7) \times 7 \times 24 = 528$ cm^2."

E4. "Two cubes of edge 4 cm joined end to end: SA = $2 \times 6 \times 16 = 192$ cm^2."

E5. "Volume of a cone of radius 3.5 cm and height 12 cm = $(1/2)\pi r^2 h = (1/2) \times (22/7) \times 12.25 \times 12 = 231$ cm^3."

E6. "Frustum with R = 10, r = 4, l = 15: CSA = $\pi l(R - r) = (22/7) \times 15 \times 6 = 282.86$ cm^2."

E7. "Volume of a sphere of radius 21 cm = $(4/3)\pi r^2 = (4/3) \times (22/7) \times 441 = 1848$ cm^3."

E8. "A sphere is melted into a cube. Since the shapes change completely, equate their surface areas to find the cube's edge."

E9. "Cylinder of diameter 14 cm and height 10 cm: CSA = $2\pi rh = 2 \times (22/7) \times 14 \times 10 = 880$ cm^2."

E10. "A tank holds 2310 litres. In cm^3 that is $2310 \times 100 = 231{,}000$ cm^3, since 1 litre = 100 cm^3."

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11 Graded Practice Problems

APPLY — THE FIVE-LEVEL SYSTEM (RD SHARMA GRADATION)

Climb the ladder in order. Do not peek at the Answer Key until a full level is attempted. Use π = 22/7 unless a question says otherwise.

LEVEL 1 — FOUNDATION: direct formulas

L1.1 Find the curved surface area of a cylinder of radius 7 cm and height 10 cm.

L1.2 Find the volume of a cone of radius 3.5 cm and height 12 cm.

L1.3 Find the total surface area of a solid hemisphere of radius 7 cm.

L1.4 Find the volume of a sphere of radius 10.5 cm.

L1.5 A cuboid measures 8 cm × 6 cm × 4 cm. Find its volume and total surface area.

L1.6 Find the curved surface area of a cone of radius 7 cm and slant height 25 cm.

LEVEL 2 — SKILL BUILDER: textbook standard

L2.1 An open-top cylindrical dustbin has radius 3.5 cm and height 8 cm. Find the area of the metal sheet used (curved surface + one base).

L2.2 A toy is a cone of radius 3 cm and height 4 cm mounted on a cylinder of radius 3 cm and height 5 cm. Find the volume of the toy.

L2.3 Find the inner surface area of a hemispherical bowl of radius 10.5 cm.

L2.4 The volume of a cube is 512 cm^3. Find its edge and its total surface area.

L2.5 Two spheres have radii 6 cm and 8 cm. Find the ratio of their volumes and the ratio of their surface areas.

L2.6 A cylinder of radius 7 cm has volume 1078 cm^3. Find its height.

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Graded Practice — Levels 3, 4 and 5

LEVEL 3 — APPLICATION: real life

L3.1 A capsule has total length 12 mm and diameter 4 mm. Find its surface area.

L3.2 An ice-cream cone has radius 3 cm and height 9 cm, topped with a hemispherical scoop of radius 3 cm. Find the volume of ice cream in one serving.

L3.3 A well of radius 2 m is dug 10 m deep. Find the volume of the earth dug out.

L3.4 A hemispherical dome of radius 10.5 m is to be painted from outside. Find the cost at ₹15 per m^2.

LEVEL 4 — ADVANCED REASONING: multi-concept

L4.1 From the top of a solid cylinder of radius 7 cm and height 20 cm, a conical cavity of radius 7 cm and height 12 cm is hollowed out. Find the volume of the remaining solid.

L4.2 A sphere of radius 9 cm fits exactly inside a cylinder (same radius, height equal to the diameter). Find the ratio of the volume of the sphere to the volume of the cylinder.

L4.3 A bucket is a frustum of a cone with radii 14 cm and 7 cm and height 6 cm. Find its capacity.

LEVEL 5 — MATHEMATICAL GENIUS

L5.1 A right triangle of sides 6 cm, 8 cm and 10 cm is revolved about its hypotenuse. Find the volume and the total surface area of the double cone so formed. (Use π = 3.14)

L5.2 A solid sphere is cut into two equal hemispheres. Prove that the total surface area increases by exactly 50%, whatever the radius.

> *TOPPER TIP: Stuck at a level? That level is telling you which section to re-read: L1 → Section 05, L2 → Section 07, L3 → Section 13, L4–L5 → Sections 06 and 12. The ladder is a diagnostic, not just a test.*

25 Concept MCQs — One Mark Each

Choose the correct option. The key, with one-line reasons for the traps, is in the Answer Key.

MCQ 1. Curved surface area of a cylinder of radius 7 cm and height 10 cm is
(a) 220 cm^2 (b) 440 cm^2 (c) 308 cm^2 (d) 154 cm^2

MCQ 2. Total surface area of a solid hemisphere of radius 7 cm is
(a) 308 cm^2 (b) 616 cm^2 (c) 462 cm^2 (d) 154 cm^2

MCQ 3. The volume of a cone is what fraction of the volume of a cylinder of the same base and height?
(a) 1/2 (b) 1/4 (c) 2/3 (d) 1/3

MCQ 4. If the surface area of a sphere is 616 cm^2, its radius is
(a) 7 cm (b) 14 cm (c) 3.5 cm (d) 21 cm

MCQ 5. The slant height of a cone of radius 6 cm and height 8 cm is
(a) 14 cm (b) 10 cm (c) 12 cm (d) 48 cm

MCQ 6. If the volumes of two cubes are in the ratio 8 : 125, the ratio of their surface areas is
(a) 8 : 125 (b) 2 : 5 (c) 4 : 25 (d) 16 : 25

MCQ 7. The volume and total surface area of a solid hemisphere are numerically equal. Its diameter is
(a) 6 cm (b) 12 cm (c) 3 cm (d) 9 cm

MCQ 8. The curved surface area of a frustum of a cone (radii R, r; slant height l) is
(a) $\pi l(R - r)$ (b) $\pi l(R + r)$ (c) $2\pi l(R + r)$ (d) πl^2

MCQ 9. When a solid is melted and recast into another shape, which quantity stays the same?
(a) Surface area (b) Height (c) Volume (d) Radius

MCQ 10. A medicine capsule is a combination of
(a) a cylinder and two cones (b) a sphere and a cylinder (c) a cylinder and two hemispheres (d) two cones

MCQ 11. Curved surface area of a cone of radius 7 cm and height 24 cm is
(a) 528 cm^2 (b) 1056 cm^2 (c) 500 cm^2 (d) 550 cm^2

MCQ 12. If the radius of a sphere is doubled, its volume becomes
(a) 2 times (b) 4 times (c) 8 times (d) 16 times

MCQ 13. 1 m^3 is equal to
(a) 100 litres (b) 1000 litres (c) 10,000 litres (d) 10 litres

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25 Concept MCQs (continued)

MCQ 14. The volume of material in a hollow cylinder (outer radius R, inner radius r, length h) is
(a) $\pi h(R - r)^2$ (b) $2\pi h(R - r)$ (c) $\pi h(R^2 - r^2)$ (d) $\pi h(R + r)^2$

MCQ 15. Two cubes of edge a are joined end to end. The surface area of the resulting cuboid is
(a) $12a^2$ (b) $10a^2$ (c) $8a^2$ (d) $14a^2$

MCQ 16. A cone and a cylinder have the same radius. The cone's height is 3 times the cylinder's height. The ratio of their volumes is
(a) 1 : 3 (b) 3 : 1 (c) 1 : 9 (d) 1 : 1

MCQ 17. A sphere and a cone have the same radius and equal volumes. The height of the cone is
(a) r (b) 2r (c) 4r (d) 3r

MCQ 18. Total surface area of a cone of radius r and slant height l is
(a) πrl (b) $\pi r(l + r)$ (c) $2\pi r(l + r)$ (d) $\pi r^2 l$

MCQ 19. For a hemisphere, CSA : TSA is
(a) 1 : 2 (b) 3 : 2 (c) 2 : 3 (d) 1 : 3

MCQ 20. A capsule's total length is L and its radius is r. The length of its cylindrical part is
(a) L (b) $L - r$ (c) $L - 4r$ (d) $L - 2r$

MCQ 21. A big solid is melted into n identical small solids. To find n, we divide
(a) surface areas (b) volumes (c) radii (d) heights

MCQ 22. A sphere is cut into two hemispheres. The total surface area of the two pieces together is
(a) $4\pi r^2$ (b) $5\pi r^2$ (c) $6\pi r^2$ (d) $8\pi r^2$

MCQ 23. If a cone's radius is halved and its height is doubled, its volume becomes
(a) the same (b) double (c) one-fourth (d) one-half

MCQ 24. An open bucket used for carrying water is usually in the shape of a
(a) cone (b) frustum of a cone (c) cylinder (d) hemisphere

MCQ 25. The volume of a hemisphere of radius 3.5 cm is (approximately)
(a) 89.83 cm^3 (b) 179.67 cm^3 (c) 44.92 cm^3 (d) 269.5 cm^3

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12 Mathematical Reasoning

REASON — PROVE IT, DON'T JUST COMPUTE IT

REASONING R1

A sphere and a cube have equal surface areas. Show that the ratio of the volume of the sphere to the volume of the cube is √6 : √π.

THINK FIRST: Equal-area comparisons are a favourite of olympiads and of nature. The whole proof is one bridge equation, patiently cubed.

1. Let the sphere have radius R and the cube edge a. Equal surface areas: $4\pi R^2 = 6a^2$, so $a^2 = (2\pi/3)R^2$ and $a = R\sqrt{(2\pi/3)}$.
   Why: Turn the condition into a bridge between R and a.
2. Volume of sphere = $(4/3)\pi R^3$. Volume of cube = $a^3 = R^3 \times (2\pi/3)\sqrt{(2\pi/3)}$.
   Why: Cube the bridge — carefully: $a^3 = (a^2) \times a$.
3. Ratio = $[(4/3)\pi R^3] / [R^3(2\pi/3)\sqrt{(2\pi/3)}] = [(4\pi/3)] / [(2\pi/3) \times \sqrt{(2\pi/3)}] = 2 / \sqrt{(2\pi/3)} = 2\sqrt{3} / \sqrt{(2\pi)} = \sqrt{(12)}/\sqrt{(2\pi)} = \sqrt{(6/\pi)}$.
   Why: Cancel $(2\pi/3)$ against $(4\pi/3)$ to get 2; then rationalise the root.
4. Therefore V(sphere) : V(cube) = **√6 : √π**. Since √6 > √π, the sphere holds MORE volume for the same skin — nature's reason for bubbles and drops being spherical.

ANSWER: V(sphere) : V(cube) = √6 : √π — proved

REASONING R2

Prove that if the radius of a sphere is doubled, its surface area becomes 4 times and its volume becomes 8 times. State the general law this reveals.

THINK FIRST: Exponents are multipliers in disguise. Once seen, a dozen 'ratio' MCQs become one-line answers.

1. Original: $SA = 4\pi r^2$, $V = (4/3)\pi r^3$.
2. New radius 2r: $SA' = 4\pi(2r)^2 = 4\pi \times 4r^2 = 4 \times (4\pi r^2) =$ **4 × SA**.
   Why: The exponent 2 in r^2 turns the multiplier 2 into $2^2 = 4$.
3. $V' = (4/3)\pi(2r)^3 = (4/3)\pi \times 8r^3 =$ **8 × V**.
   Why: The exponent 3 turns 2 into $2^3 = 8$.
4. General law: for ANY solid scaled by factor k, areas multiply by k^2 and volumes by k^3. The same proof works for cubes, cylinders and cones — only the exponents matter.

13 Real-Life Applications

APPLY — THE CHAPTER AT WORK

APPLICATION A1

A hall has 10 cylindrical pillars of radius 0.7 m and height 3 m. Find the total cost of painting their curved surfaces at ₹15 per m². (Use π = 22/7)

1. CSA of one pillar = 2πrh = 2 × (22/7) × 0.7 × 3 = **13.2 m²**.
   Why: Only the curved surface is painted — the ends touch the floor and the ceiling.
2. Ten pillars: 10 × 13.2 = **132 m²**.
3. Cost = 132 × 15 = **₹1,980**.

ANSWER: ₹1,980

APPLICATION A2

A cylindrical water tank has radius 70 cm and height 150 cm. How many litres of water can it hold when full? (Use π = 22/7)

1. Capacity = $\pi r^2 h = (22/7) \times 70^2 \times 150 = 22 \times 700 \times 150$ = **2,310,000 cm³**.
   Why: Cancel the 7 against $70^2 = 4900$ early — the arithmetic collapses.
2. Convert: 2,310,000 ÷ 1000 = **2310 litres**.
   Why: 1 litre = 1000 cm³ — Golden Rule: convert at the end here because the formula was fed in cm.

ANSWER: 2310 litres

APPLICATION A3

An ice-cream seller serves a cone of radius 3.5 cm and height 12 cm topped with a hemispherical scoop of radius 3.5 cm. How many litres of ice cream does he need for 50 servings? (Use π = 22/7)

1. Cone's volume = $(1/3)\pi r^2 h = (1/3) \times (22/7) \times 3.5^2 \times 12$ = **154 cm³**.
2. Scoop's volume = $(2/3)\pi r^3 = (2/3) \times (22/7) \times 3.5^3$ = **89.83 cm³** (approximately).
3. One serving = 154 + 89.83 = **243.83 cm³**; fifty servings = 50 × 243.83 = 12,191.67 cm³ ≈ **12.19 litres**.
   Why: He should buy at least 13 litres — servings are never the last drop of the tub.

ANSWER: Approximately 12.19 litres (buy 13 litres)

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Real-Life Applications (continued)

APPLICATION A4

A conical tent has base radius 7 m and slant height 25 m. Find the area of canvas required and its cost at ₹40 per m². (Use π = 22/7)

1. l = 25 is given, and √(7² + 24²) confirms the height would be 24 m — the 7–24–25 triple.
   Why: A free consistency check costs five seconds.

2. Canvas = CSA of cone = πrl = (22/7) × 7 × 25 = **550 m²**.
   Why: A tent has no floor canvas in this problem — only the sloping skin.

3. Cost = 550 × 40 = **₹22,000**.

ANSWER: 550 m² of canvas, costing ₹22,000

14 Competency-Based Questions

APPLY • CHALLENGE — CASE STUDIES FROM REAL LIFE

Read each case, then answer its four questions. All answers are in the Answer Key. Use π = 22/7.

CASE STUDY 1 — The Ice-Cream Parlour

A parlour serves ice cream in a cone of radius 3.5 cm and height 12 cm, crowned with a hemispherical scoop of radius 3.5 cm.
(a) Find the volume of the empty cone. **(b)** Find the volume of one scoop. **(c)** Find the total volume of one serving. **(d)** How many litres of ice cream are needed for 100 servings?

CASE STUDY 2 — The School Water Tank

A school installs a cylindrical tank of radius 70 cm and height 200 cm.
(a) Find its capacity in litres. **(b)** The outside curved surface and the top lid are painted. Find the painted area in m^2. **(c)** Find the cost of painting at ₹30 per m^2. **(d)** How many litres does the tank hold when it is half full?

CASE STUDY 3 — The Recycled Metal Ball

A solid metal sphere of radius 6 cm is melted and recast into 8 identical smaller spheres.
(a) Find the radius of each small sphere. **(b)** Find the surface area of the original sphere. **(c)** Find the total surface area of all 8 small spheres. **(d)** By what factor did the total surface area change? What does this teach about melting?

CASE STUDY 4 — The Camping Tent

A conical tent has base radius 6 m and height 8 m.
(a) Find the slant height and the area of canvas used for the sloping part. **(b)** Find the area of the ground covered by the tent. **(c)** Find the volume of air inside the tent. **(d)** Find the cost of the canvas at ₹35 per m^2.

CASE STUDY 5 — Digging the Well

A well of radius 2 m is dug 21 m deep. The earth taken out is spread in a ring of width 2 m around the well to form an embankment.
(a) Find the volume of earth dug out. **(b)** Find the area of the ring on which the earth is spread. **(c)** Find the height of the embankment. **(d)** Find the cost of digging at ₹50 per m^3.

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Assertion–Reason — 10 Questions

For each question choose: **(a)** Both A and R are true and R is the correct explanation of A. **(b)** Both are true but R is NOT the correct explanation of A. **(c)** A is true, R is false. **(d)** A is false, R is true.

AR1. Assertion (A): Total surface area of a solid hemisphere of radius r is $3\pi r^2$. Reason (R): Surface area of a sphere of radius r is $4\pi r^2$.

AR2. Assertion (A): The volume of a cone is one-third the volume of a cylinder of the same base and height. Reason (R): Three cones of sand exactly fill such a cylinder.

AR3. Assertion (A): CSA of a frustum of a cone is $\pi l(R + r)$. Reason (R): The slant height of a frustum is $l = \sqrt{(h^2 + (R + r)^2)}$.

AR4. Assertion (A): When two cubes are joined end to end, the surface area decreases by two faces. Reason (R): The joined faces become internal and are no longer part of the surface.

AR5. Assertion (A): Melting a sphere into a cube changes its surface area. Reason (R): The volume of a solid remains unchanged on melting.

AR6. Assertion (A): TSA of a capsule (cylinder length h, radius r) is $2\pi rh + 4\pi r^2$. Reason (R): The two hemispherical ends together form a complete sphere.

AR7. Assertion (A): Doubling the radius of a sphere makes its surface area four times. Reason (R): Surface area varies directly as the square of the radius.

AR8. Assertion (A): Volume of a hemisphere of radius r is $(2/3)\pi r^3$. Reason (R): A hemisphere is exactly half of a sphere of volume $(4/3)\pi r^3$.

AR9. Assertion (A): Volume of a frustum is $(1/3)\pi h(R^2 + r^2 - Rr)$. Reason (R): A frustum is the difference of two cones.

AR10. Assertion (A): 1 litre = 1000 cm^3. Reason (R): 1 litre is the volume of a cube of edge 10 cm.

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15 Olympiad & Foundation Challenges

CHALLENGE — FOR THE MATHEMATICAL THINKERS

CHALLENGE 1

A cone of radius 10 cm is divided into two parts by a plane through the mid-point of its axis, parallel to its base. Compare the volumes of the two parts.

THINK FIRST: Half the height is NOT half the volume — the cube law punishes that guess with 1:7.

1. The cut creates a smaller cone on top, similar to the whole cone with linear scale 1/2 (its height is half, so its radius is 10/2 = 5 cm).
   Why: A plane parallel to the base always cuts off a SIMILAR cone.
2. Volumes scale as the CUBE of the linear ratio (Reasoning R2): V(small) = $(1/2)^3$ = 1/8 of the whole cone.
3. The lower part (a frustum) holds the remaining 1 − 1/8 = 7/8.
4. Ratio small cone : frustum = 1/8 : 7/8 = **1 : 7**.
   Why: Notice: the answer does not depend on the cone's height at all — only on the midpoint cut.

ANSWER: Volumes are in the ratio 1 : 7 (small cone : frustum)

CHALLENGE 2

The largest possible sphere is carved out of a cube of edge 14 cm. Find the surface area of the sphere, and the ratio of the sphere's surface area to the cube's surface area. (Use π = 22/7)

1. The largest sphere inside a cube touches all six faces, so its diameter = the edge: 2r = 14, r = **7 cm**.
2. Surface area of sphere = $4\pi r^2$ = 4 × (22/7) × 49 = **616 cm²**.
3. Surface area of cube = 6 × 14^2 = 1176 cm². Ratio = 616 : 1176 = **11 : 21**.
   Why: Divide both by 56. The π-free ratio 11:21 hides the elegant fact sphere SA = (π/6) × ... — in fact 616/1176 = 11/21 exactly because $4\pi r^2/(6(2r)^2) = \pi/6 \approx 11/21$ when π = 22/7.

ANSWER: Sphere's surface area = 616 cm²; ratio = 11 : 21

CHALLENGE 3

A solid sphere of radius 9 cm is melted and recast into small cones of radius 3 cm and height 4 cm. How many cones are obtained? (Use π = 22/7)

Olympiad Challenges 4–7

CHALLENGE 4

A hollow metallic sphere has external radius 10 cm and internal radius 8 cm. Find the volume of the metal. (Use π = 22/7)

1. Metal = outer sphere MINUS inner sphere (the empty core is a cavity — Golden Rule 2).
2. $V = (4/3)\pi(R^3 - r^3) = (4/3) \times (22/7) \times (1000 - 512) = (4/3) \times (22/7) \times 488$.
3. $= 42{,}944/21 \approx$ **2044.95 cm³**.

   Why: Only 2 cm of thickness, yet the shell holds over 2 litres of metal — surface layers are surprisingly heavy.

ANSWER: Approximately 2044.95 cm³

CHALLENGE 5

A sphere of radius 7 cm sits exactly inside a cylinder (the cylinder's height equals the sphere's diameter). Find the volume of the empty space in the cylinder, and show it is exactly one-third of the cylinder's volume. (Use π = 22/7)

1. Cylinder: r = 7, h = 14. Volume = π(49)(14) = **686π = 2156 cm³**.
2. Sphere: $(4/3)\pi(343) = 4312/3 \approx$ **1437.33 cm³**.
3. Empty space = 2156 − 1437.33 = **718.67 cm³**.
4. Check the claim: $2156/3 = 718.67$. Exactly one-third! In symbols: $\pi r^2(2r) - (4/3)\pi r^3 = 2\pi r^3 - (4/3)\pi r^3 = (2/3)\pi r^3 = (1/3)(2\pi r^3)$ — true for EVERY r. This is Archimedes' beloved 3:2:1 harmony of cylinder, sphere and cone.

ANSWER: Empty space = 718.67 cm³ = exactly 1/3 of the cylinder

CHALLENGE 6

A bucket in the form of a frustum of a cone holds 28.490 litres of water. The radii of its top and bottom are 28 cm and 21 cm. Find the height of the bucket. (Use π = 22/7)

1. Convert first: capacity = 28.490 × 1000 = **28,490 cm³**.

   Why: Litres never enter a cm formula directly.
2. $(1/3)\pi h(R^2 + r^2 + Rr) = 28{,}490$ with R = 28, r = 21: the bracket is 784 + 441 + 588 = 1813.
3. $(1/3) \times (22/7) \times 1813 \times h = 28{,}490 \Rightarrow h = 28{,}490 \times 21 / (22 \times 1813) =$ **15 cm**.

Olympiad Challenges 8–10

CHALLENGE 8

A cone and a hemisphere have the same base radius r and equal volumes. Find the height of the cone in terms of r. Hence find the height when r = 7 cm.

1. Equate the volumes: $(1/3)\pi r^2 h = (2/3)\pi r^3$.
   Why: Golden Rule: same material, two shapes.
2. Cancel $(1/3)\pi r^2$ from both sides: h = **2r**.
   Why: Everything except h and one r cancels — the answer is a pure relationship.
3. For r = 7 cm: h = **14 cm**.

ANSWER: h = 2r; for r = 7 cm, h = 14 cm

CHALLENGE 9

Rain falls on a flat terrace of 20 m × 10 m to a depth of 5 cm. All the water is collected into a cylindrical tank of diameter 2 m. Find the depth of water in the tank. (Use π = 3.14)

1. Volume of rain = 20 × 10 × 0.05 = **10 m^3**.
   Why: Convert 5 cm to 0.05 m FIRST — the terrace is measured in metres.
2. Tank radius = 1 m. Let the depth be h: $\pi(1^2)h = 10 \Rightarrow h = 10/3.14$ = **3.18 m** (approximately).
   Why: Volume is conserved as the water changes home — Golden Rule 3 in the rain.

ANSWER: Approximately 3.18 m

CHALLENGE 10

A solid sphere of radius 6 cm is melted and drawn into a cylindrical wire of diameter 0.4 cm. Find the length of the wire. (Use π = 22/7)

1. Volume of sphere = $(4/3)\pi(6^3) = (4/3)\pi(216)$ = **288π cm^3**.
2. Wire radius = 0.4/2 = 0.2 cm. Cross-section area = $\pi(0.2^2) = 0.04\pi$ cm^2.
   Why: A wire is just a very long, thin cylinder.
3. Length = volume ÷ cross-section = 288π / 0.04π = 7200 cm = **72 m**.

Previous Year Questions — CBSE Board (Verified)

Real board questions, with the year and paper verified against published CBSE papers and marking archives. Solve them exactly as in the board: complete steps earn the step marks.

PYQ 1 — CBSE 2024 (1 mark)

The radii of a sphere and a cone are the same. If their volumes are also equal, then the height of the cone is: (a) r (b) 2r (c) 3r (d) 4r

1. $(4/3)\pi r^3 = (1/3)\pi r^2 h \Rightarrow$ cancel $(1/3)\pi r^2$: $4r = h$.

ANSWER: Option (d) — h = 4r

PYQ 2 — CBSE 2024 (1 mark)

If the volumes of two cubes are in the ratio 8 : 125, then the ratio of their surface areas is: (a) 8 : 125 (b) 4 : 25 (c) 2 : 5 (d) 16 : 25

1. Edges ratio = cube root of volumes ratio = 2 : 5 (Reasoning R2's scaling law).
2. Surface areas ratio = $(2/5)^2$ = **4 : 25**.
   Why: Areas scale as the SQUARE — the trap options are the unsquared and cubed ratios.

ANSWER: Option (b) — 4 : 25

PYQ 3 — CBSE 2023 (1 mark)

The curved surface area of a cone having height 24 cm and radius 7 cm is: (a) 528 cm² (b) 1056 cm² (c) 550 cm² (d) 500 cm²

1. $l = \sqrt{(24^2 + 7^2)} = \sqrt{625} = 25$ cm.
   Why: The 7–24–25 triple, on sight.
2. CSA = $\pi rl = (22/7) \times 7 \times 25$ = **550 cm²**.
   Why: Option (a) 528 is the πrh trap — CBSE plants it deliberately.

ANSWER: Option (c) — 550 cm²

PYQ 4 — CBSE 2020 (1 mark)

Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. The ratio of their volumes is: (a) 1 : 3 (b) 3 : 1 (c) 1 : 1 (d) 9 : 1

1. $V_1/V_2 = (r_1^2 h_1)/(r_2^2 h_2) = (3^2 \times 1)/(1^2 \times 3) = 9/3$ = **3 : 1**.

Previous Year Questions — Long Answer (Verified)

PYQ 6 — CBSE 2022, Term 2

A conical cavity of height 24 cm is hollowed out of a solid cylinder of height 30 cm and radius 7 cm. The cone has the same radius as the cylinder. Find the total surface area of the solid that is left. (Use π = 22/7)

1. Slant height of the cavity: $l = \sqrt{(24^2 + 7^2)} =$ **25 cm**.

2. Surviving skin = CSA of cylinder + base circle + CSA of cavity = $2\pi rh + \pi r^2 + \pi rl = \pi r(2h + r + l)$.

   Why: The same three-surface pattern as Ex 12.1 Q8 — CBSE recycles its own best ideas.

3. $= (22/7) \times 7 \times (60 + 7 + 25) = 22 \times 92 =$ **2024 cm²**.

   Why: A memorable answer — the year of the paper's family. Coincidence, but a useful checksum.

ANSWER: 2024 cm²

PYQ 7 — CBSE 2023

A solid is in the shape of a right circular cone surmounted on a hemisphere, the radius of each being 7 cm and the height of the cone equal to its diameter. Find the volume of the solid. (Use π = 22/7)

1. Radius r = 7 cm; height of cone = diameter = **14 cm**.

   Why: 'Height equal to its diameter' — read such phrases twice.

2. Volume $= (2/3)\pi r^3 + (1/3)\pi r^2 h = (2/3)(22/7)(343) + (1/3)(22/7)(49)(14) = 2156/3 + 2156/3$.

   Why: The two parts are again equal — 2156/3 each.

3. Total $= 4312/3 \approx$ **1437.33 cm³**.

ANSWER: Approximately 1437.33 cm³

Previous Years' Questions (continued)

PYQ 8 — CBSE 2022

A solid piece of metal in the form of a cuboid of dimensions 11 cm × 7 cm × 7 cm is melted to form n solid spheres of radius 3.5 cm each. Find n. (Use π = 22/7)

1. Volume of cuboid = 11 × 7 × 7 = **539 cm³**.
2. Volume of one sphere = (4/3)(22/7)(3.5^3) = 539/3 cm³.
   Why: Watch: 3.5^3 = 42.875 and (4/3)(22/7)(42.875) = 539/3 exactly.
3. n = 539 ÷ (539/3) = **3**.
   Why: The setter hid the answer inside the cuboid's own volume — 539 appears on both floors of the division.

ANSWER: n = 3

PYQ 9 — CBSE 2022 (Case Study)

A hemispherical bowl of internal radius 9 cm is full of water. The water is emptied into cylindrical jars of radius 1.5 cm and height 4 cm. (a) How many jars are needed? (b) If instead a conical funnel of the same height and same diameter as the jar is immersed in a full jar, how much water flows out? (Use π = 22/7)

1. Volume of water in the bowl = (2/3)π(9^3) = (2/3)π(729) = **486π cm³**.
2. Volume of one jar = π(1.5^2)(4) = **9π cm³**. Jars needed = 486π/9π = **54**.
3. Water displaced by the funnel = volume of the cone = (1/3)π(1.5^2)(4) = 3π = **66/7 ≈ 9.43 cm³**.
   Why: An immersed solid pushes out exactly its own volume — Archimedes in a jar.

ANSWER: (a) 54 jars (b) Approximately 9.43 cm³ flows out

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Chapter Mind Map

RECALL — THE WHOLE CHAPTER ON ONE PAGE

SURFACE AREAS AND VOLUMES

Two questions for every solid: How much SKIN? How much SPACE?

The Six Solids

Cuboid $2(lb+bh+hl)$, lbh • Cube $6a^2$, a^3 • Cylinder $2\pi rh$, πr^2h • Cone πrl, $(1/3)\pi r^2h$ • Sphere $4\pi r^2$, $(4/3)\pi r^3$ • Hemisphere $3\pi r^2$, $(2/3)\pi r^3$

Derivations (Sec 06)

Cylinder unrolls → rectangle $2\pi r \times h$ • Cone opens → sector → πrl • Sphere = wrapping cylinder = 4 great circles • Frustum = big cone − small cone

Golden Rule 1 — Surface

Add surviving skin only. Joined face: subtract TWICE. Cavity: ADD its inner skin. Bowl vs solid hemisphere: $2\pi r^2$ vs $3\pi r^2$.

Golden Rule 2 — Volume

Pieces ADD. Cavities SUBTRACT. Nothing else — ever.

Golden Rule 3 — Conversion

Melting/pouring/recasting: volume is CONSTANT. n = V(big) ÷ V(small). Displaced water = volume of what was dropped in.

Frustum Kit

$V = (1/3)\pi h(R^2 + r^2 + Rr)$ • $CSA = \pi l(R + r)$ • $l = \sqrt{(h^2 + (R-r)^2)}$ • From the difference of cubes $R^3 - r^3$

Exam Habits

Radius first • Sketch & shade hidden faces • Slant height before CSA • π as commanded • Units always • The 10-second audit

Close the book. Redraw this map from memory. Whatever you cannot redraw is what you revise tonight.

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17 Pin-Point Flash Cards

RECALL — CUT THEM OUT, CARRY THEM, CONQUER THEM

Sixteen cards, one pin-point each. Cut along the dashed borders, keep them in your pocket or pencil box, and revise on Smart Memory Days 3, 7 and 21: read the title, say the point aloud, then flip and check.

Cylinder CSA

$2\pi rh$ — the unrolled rectangle: circumference × height.

Cylinder TSA

$2\pi r(h + r)$ — wall plus two lids, factored.

Cone Slant Height

$l = \sqrt{(r^2 + h^2)}$. Pythagoras guards every cone.

Cone CSA

πrl — the opened-out sector. Never πrh!

Cone Volume

$(1/3)\pi r^2 h$ — three cones fill one cylinder.

Sphere Surface

$4\pi r^2$ — exactly four great circles (Archimedes).

Sphere Volume

$(4/3)\pi r^3$ — the 4/3 is not negotiable.

Hemisphere Trio

CSA $2\pi r^2$ • TSA $3\pi r^2$ • Volume $(2/3)\pi r^3$. Bowl or ball? Read the question.

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Pin-Point Flash Cards (continued)

Hidden Face Rule

Joined faces vanish from BOTH solids — subtract the touching face TWICE.

Volume Rule

Pieces add. Cavities subtract. Nothing else — ever.

Conversion Rule

Melting never changes volume. n = V(big) ÷ V(one small).

Frustum Volume

$(1/3)\pi h(R^2 + r^2 + Rr)$ — big square, small square, and their product.

Frustum CSA

$\pi l(R + r)$, with $l = \sqrt{(h^2 + (R - r)^2)}$. Sum of radii for area; difference inside the root.

Litre Bridge

1 litre = 1000 cm^3 • 1 m^3 = 1000 litres = 10,00,000 cm^3.

Scaling Law

Scale by k: areas × k^2, volumes × k^3. Doubled radius → SA × 4, V × 8.

π Discipline

Use the π the question commands — 22/7 or 3.14. Never mix both in one solution.

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18 Active Recall

RECALL — CLOSED BOOK, HONEST ANSWERS

Close the rest of the chapter. Answer from memory — struggling a little is exactly what builds the memory. Answers are in the Answer Key.

Rapid Fire — 10 Questions

R1. Find the curved surface area of a cylinder of radius 2 cm and height 5 cm.

R2. A cone has radius 9 cm and height 12 cm. Find its slant height.

R3. Find the surface area of a sphere of radius 3 cm.

R4. Write the total surface area of a hemisphere of radius r, in terms of r.

R5. Find the volume of a cone of radius 7 cm and height 9 cm.

R6. How many litres are there in 3.5 m^3?

R7. Write the curved surface area of a frustum with radii R, r and slant height l.

R8. Two identical solid hemispheres are joined along their flat faces. What solid is formed?

R9. Find the total surface area of a cube of edge 5 cm.

R10. Find the volume of a cylinder of radius 7 cm and height 10 cm.

Recall Prompts — Say Them Aloud

P1. State the Three Golden Rules of this chapter, in your own words.

P2. Explain, with the sector picture, why the cone's CSA is πrl and not πrh.

P3. Why is a joined face subtracted TWICE, not once?

P4. Recite the frustum volume formula and explain where the bracket ($R^2 + r^2 + Rr$) comes from.

P5. State the scaling law and use it: a sphere's radius is tripled — what happens to its surface area and volume?

> **TOPPER TIP:** *Score below 7/10 on Rapid Fire? Revisit Sections 05–06 today, and repeat this page tomorrow. Recall is a muscle — it grows by being used, not by re-reading.*

19 Self-Assessment & Mastery Tracker

RECALL — THE KALVIYOGI SMART MEMORY SYSTEM

Learning once is an event; remembering forever is a system. Follow the schedule — each visit is short, and each visit makes the memory permanent.

Day	Your Mission	Done
Day 0	Pre-learning: attempt the 5 Quick Check questions (Section 02) cold. Note your score.	☐
Day 1	Learn the chapter fully: Sections 01–08, then NCERT solutions of Section 07 with steps covered.	☐
Day 3	Without-book recall: Flash Cards (Section 17) + Rapid Fire (Section 18).	☐
Day 7	Independent solving: Graded Practice L1–L3 + **Day 7 Mini-Test** (next page).	☐
Day 14	Mixed practice: L4–L5, Find the Error, Case Studies.	☐
Day 21	Long-term recall: **Day 21 Mini-Test** + Assertion–Reason set, closed book.	☐
Day 30	Mastery: **Day 30 Mini-Test** + one full Model Test under exam timing.	☐

Error Tracker — every mistake, logged once, never repeated

Date: _____ • Question: _____ • My error: _____ • The rule I broke: _____ • Redone correctly: ☐

Date: _____ • Question: _____ • My error: _____ • The rule I broke: _____ • Redone correctly: ☐

Date: _____ • Question: _____ • My error: _____ • The rule I broke: _____ • Redone correctly: ☐

Formula Recall Tracker — tick only if you can WRITE it from memory

☐ Cylinder CSA/TSA/V ☐ Cone l, CSA, TSA, V ☐ Sphere SA/V ☐ Hemisphere CSA/TSA/V
☐ Frustum V/CSA/l ☐ Litre bridges

Concept Mastery Score

Derivations: ___/10 • NCERT Ex 12.1: ___/9 • NCERT Ex 12.2: ___/8 • Practice ladder: reached Level ___ • PYQs correct: ___/9 • **Overall: ___ %**

Smart Memory Mini-Tests — Day 7, Day 21, Day 30

Ten questions each, closed book, ten minutes. Answers are in the Answer Key.

Day 7 Mini-Test — Formulas Alive?

1. Write the curved surface area of a cylinder (r, h). **2.** Slant height of a cone with r = 5, h = 12? **3.** TSA of a solid hemisphere of radius 7 cm? **4.** Volume of a sphere of radius 3 cm? **5.** Volume of a cuboid 5 × 4 × 3 cm? **6.** 5000 cm^3 = how many litres? **7.** A cone's volume is ___ of its cylinder's. **8.** Surface area of a sphere of radius 10.5 cm? **9.** When two solids are joined, the touching face is subtracted ___ times. **10.** Write the slant height of a frustum in terms of h, R, r.

Day 21 Mini-Test — Still Sharp?

1. A capsule is a cylinder plus what? **2.** TSA of (cone on hemisphere, same r) = which two curved surfaces? **3.** A cylinder of radius 7 cm has volume 1078 cm^3. Its height? **4.** In 'how many shots' problems, n = ___ ÷ ___. **5.** CSA of a cone with r = 7, l = 25? **6.** CSA : TSA of a hemisphere = ? **7.** A cavity's volume is ___ (added/subtracted). **8.** The frustum volume bracket is R^2 + ___ + Rr. **9.** Radius doubled → sphere's surface area × ? **10.** The true volume of the Ex 12.2 Q8 glass vessel is ___ cm^3.

Day 30 Mini-Test — Mastery

1. Derive the cylinder's CSA in one line (what shape, what sides?). **2.** In Ex 12.2 Q7, what lucky equality halves the work? **3.** TSA of n cubes of edge a joined in a row = ? **4.** Sphere and cube with equal surface areas: ratio of volumes = ? **5.** A cone cut at the midpoint of its axis: volumes ratio of the two parts = ? **6.** 1 m^3 = how many litres? **7.** For the fez (frustum cap), which two surfaces form the material? **8.** In the lead-shots problem, the shots' total volume equals what? **9.** Scale factor 3 → areas × ___, volumes × ___. **10.** Recite the 10-second audit (five checks).

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Chapter Model Tests — 20 Marks Each

Sit down, set a timer (Test A: 30 min, B: 35 min, C: 40 min), and write full solutions — step marks are real marks. Keys are in the Answer Key. Use π = 22/7 unless stated.

MODEL TEST A — Basics

A1. CSA of a cone of radius 3.5 cm and slant height 12.5 cm: (a) 137.5 cm^2 (b) 43.75 cm^2 (c) 275 cm^2 (d) 154 cm^2 **[1]**

A2. Volume of a hemisphere of radius 7 cm is about: (a) 1437.33 cm^3 (b) 718.67 cm^3 (c) 359.33 cm^3 (d) 1078 cm^3 **[1]**

A3. 1 litre equals: (a) 100 cm^3 (b) 10 cm^3 (c) 1000 cm^3 (d) 10,000 cm^3 **[1]**

A4. Total surface area of a cube of edge 6 cm: (a) 144 cm^2 (b) 216 cm^2 (c) 36 cm^2 (d) 1296 cm^2 **[1]**

A5. Find the volume of a cylinder of radius 3.5 cm and height 10 cm. **[2]**

A6. Find the surface area of a sphere of radius 21 cm. **[2]**

A7. Find the volume of a cone of radius 6 cm and height 7 cm. **[2]**

A8. Find the total surface area of a cuboid 10 cm × 5 cm × 4 cm. **[2]**

A9. A vessel is a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 7 cm and the total height of the vessel is 10 cm. Find the inner surface area of the vessel. **[4]**

A10. Two cubes, each of volume 216 cm^3, are joined end to end. Find the surface area of the resulting cuboid. **[4]**

Chapter Model Test B — Standard (20 Marks)

B1. CSA of a frustum (radii R, r; slant l) is: (a) $\pi l(R - r)$ (b) $2\pi l(R + r)$ (c) $\pi l(R + r)$ (d) πl^2 **[1]**

B2. For a hemisphere, CSA : TSA = (a) 3 : 2 (b) 2 : 3 (c) 1 : 2 (d) 1 : 3 **[1]**

B3. To count how many small solids are cast from a big solid, we divide their: (a) radii (b) surface areas (c) heights (d) volumes **[1]**

B4. Slant height of a cone with r = 8 cm, h = 15 cm: (a) 23 cm (b) 17 cm (c) 12 cm (d) 20 cm **[1]**

B5. A capsule has total length 20 mm and diameter 10 mm. Find its surface area. **[2]**

B6. Find the volume of a cone of radius 7 cm and height 12 cm. **[2]**

B7. Find the area of sheet needed for an open-top cylinder of radius 7 cm and height 10 cm. **[2]**

B8. Find the volume of a sphere of radius 6 cm. **[2]**

B9. An inverted conical vessel has height 12 cm and top radius 6 cm, filled with water. Lead shots of radius 1 cm are dropped in until one-fourth of the water flows out. How many shots were dropped? **[4]**

B10. A wooden block of 20 cm × 15 cm × 5 cm has 6 conical depressions, each of radius 1 cm and depth 2.1 cm. Find the volume of the wood. **[4]**

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Chapter Model Test C — Advanced (20 Marks)

C1. A cone is cut at the midpoint of its axis by a plane parallel to the base. The volumes of the two parts are in the ratio: (a) 1 : 1 (b) 1 : 3 (c) 1 : 7 (d) 1 : 8 **[1]**

C2. If every dimension of a solid is doubled, its volume becomes: (a) 2 times (b) 4 times (c) 6 times (d) 8 times **[1]**

C3. The bracket in the frustum volume formula is: (a) $R^2 - r^2$ (b) $(R + r)^2$ (c) $R^2 + r^2 + Rr$ (d) $R^2 + r^2 - Rr$ **[1]**

C4. The ratio of the surface area of the largest sphere in a cube to the surface area of the cube is: (a) π : 6 (b) π : 3 (c) 1 : 2 (d) 2 : 3 **[1]**

C5. Find the volume of metal in a hollow sphere of external radius 5 cm and internal radius 3 cm. **[2]**

C6. A frustum glass has height 7 cm and end radii 3 cm and 2 cm. Find its capacity. **[2]**

C7. A sphere of radius 3 cm is drawn into a wire of diameter 0.2 cm. Find the length of the wire. **[2]**

C8. A cone and a hemisphere of radius 10.5 cm have equal volumes. Find the height of the cone. **[2]**

C9. A right triangle of sides 6 cm, 8 cm, 10 cm is revolved about its hypotenuse. Find the volume and total surface area of the double cone formed. (Use π = 3.14) **[4]**

C10. A frustum container has height 8 cm and end radii 10 cm and 6 cm. Find its capacity in litres, and the cost of filling it with milk at ₹30 per litre. **[4]**

> ***TOPPER TIP:*** *Finished a test? Mark it with the Answer Key, log every error in the Error Tracker (Section 19), and redo the failed questions after two days — that is the Smart Memory loop closing.*

Answer Key — Graded Practice and MCQs

Section 11 — Graded Practice

L1: 1) 440 cm^2 2) 154 cm^3 3) 462 cm^2 4) 4851 cm^3 5) 192 cm^3, 208 cm^2 6) 550 cm^2

L2: 1) 214.5 cm^2 2) 179.14 cm^3 3) 693 cm^2 4) 8 cm, 384 cm^2 5) 27 : 64, 9 : 16 6) 7 cm

L3: 1) 150.86 mm^2 2) 141.43 cm^3 (45π) 3) 125.71 m^3 4) ₹10,395

L4: 1) 2464 cm^3 2) 2 : 3 3) 2156 cm^3

L5: 1) Volume 241.15 cm^3, surface area 211.01 cm^2 (common radius 4.8 cm; heights sum to 10)
2) The two hemispheres have total SA $2 \times 3\pi r^2 = 6\pi r^2$ against the sphere's $4\pi r^2$; increase = $2\pi r^2$ = $(2\pi r^2/4\pi r^2) \times 100\% = 50\%$, for every r.

25 Concept MCQs

1 b • 2 c • 3 d • 4 a • 5 b • 6 c • 7 d • 8 b • 9 c • 10 c • 11 d • 12 c • 13 b • 14 c • 15 b • 16 d • 17 c • 18 b • 19 c • 20 d • 21 b • 22 c • 23 d • 24 b • 25 a

Watch the traps: Q2 — solid hemisphere means TSA $3\pi r^2$, not 308. Q6 — square the edge ratio, never copy the volume ratio. Q11 — find l = 25 first; 528 is the πrh trap. Q15 — $12a^2$ forgets the 2 hidden faces. Q23 — $(1/2)^2 \times 2 = 1/2$.

Section 14 — Assertion–Reason

AR1 b • AR2 a • AR3 c (R is false: $(R - r)^2$, not $(R + r)^2$) • AR4 a • AR5 b • AR6 a • AR7 a • AR8 a • AR9 d (A is false: the bracket is $R^2 + r^2 + Rr$) • AR10 a

Answer Key — Cases, Find the Error, Recall

Section 14 — Case Studies

Case 1: (a) 154 cm^3 (b) 89.83 cm^3 (c) 243.83 cm^3 (d) 24,383 cm^3 ≈ 24.38 litres

Case 2: (a) 3080 litres (b) 10.34 m^2 (8.8 curved + 1.54 lid) (c) ₹310.20 (d) 1540 litres

Case 3: (a) 3 cm (b) 452.57 cm^2 (c) 905.14 cm^2 (d) It DOUBLES (×2) — melting preserves volume, never surface area.

Case 4: (a) l = 10 m; canvas 188.57 m^2 (b) 113.14 m^2 (c) 301.71 m^3 (d) ₹6,600

Case 5: (a) 264 m^3 (b) 37.71 m^2 (ring between radii 4 m and 2 m) (c) 7 m (d) ₹13,200

Section 10 — Find the Error (corrections)

E1. A SOLID hemisphere needs its flat face too: TSA = $3\pi r^2$ = **462 cm^2**. **E2.** Cylinder length = 14 − 2(2.5) = 9 mm, not 14: SA = **220 mm^2**. **E3.** CSA uses slant height: l = 25, CSA = **550 cm^2**. **E4.** Two joined faces vanish: 192 − 32 = **160 cm^2**. **E5.** The cone factor is 1/3, not 1/2: V = **154 cm^3**. **E6.** It is (R + r), not (R − r): CSA = π(15)(14) = **660 cm^2**. **E7.** Volume needs r^3: (4/3)(22/7)(21^3) = **38,808 cm^3**. **E8.** On melting, equate VOLUMES, not surface areas. **E9.** Diameter was used as radius: r = 7, CSA = **440 cm^2**. **E10.** 1 litre = 1000 cm^3: 2310 litres = **2,310,000 cm^3**.

Section 18 — Active Recall

Rapid Fire: R1 62.86 cm^2 (20π) • R2 15 cm • R3 113.14 cm^2 (36π) • R4 $3\pi r^2$ • R5 462 cm^3 • R6 3500 litres • R7 πl(R + r) • R8 A sphere • R9 150 cm^2 • R10 1540 cm^3

Prompts: P5 — areas × 9, volume × 27 (k = 3: k^2, k^3). For P1–P4, compare your spoken answer with Sections 05, 06 and 12 — every phrase should match a Golden Rule or a derivation step.

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Answer Key — Mini-Tests and Model Tests

Day 7 Mini-Test

1) $2\pi rh$ 2) 13 cm 3) 462 cm^2 4) 113.14 cm^3 (36π) 5) 60 cm^3 6) 5 litres 7) one-third 8) 1386 cm^2 9) twice (2 times) 10) $l = \sqrt{(h^2 + (R - r)^2)}$

Day 21 Mini-Test

1) Two hemispheres 2) $\pi rl + 2\pi r^2$ 3) 7 cm 4) V(big) ÷ V(one small) 5) 550 cm^2 6) 2 : 3 7) Subtracted 8) r^2 9) 4 times 10) 346.51 cm^3

Day 30 Mini-Test

1) Unroll: rectangle of $2\pi r$ by h 2) The cone and hemisphere have equal volumes 3) $(4n + 2)a^2$ 4) $\sqrt{6} : \sqrt{\pi}$ 5) 1 : 7 6) 1000 litres 7) Frustum CSA + the closed (upper) base 8) The volume of water that flows out (one-fourth of the vessel) 9) 9, 27 10) Radius not diameter; hidden faces removed; slant height for cones; π as commanded; unit correct.

Model Test A

A1 a • A2 b • A3 c • A4 b • A5 385 cm^3 • A6 5544 cm^2 • A7 264 cm^3 • A8 220 cm^2 • A9 220 cm^2 (r = 3.5, cylinder h = 6.5; $2\pi r(h + r)$) • A10 360 cm^2 (edge 6; cuboid 12 × 6 × 6)

Model Test B

B1 c • B2 b • B3 d • B4 b • B5 628.57 mm^2 (200π) • B6 616 cm^3 • B7 594 cm^2 (440 + 154) • B8 905.14 cm^3 (288π) • B9 27 shots (cone 144π; overflow 36π; one shot $4\pi/3$) • B10 1486.8 cm^3 (1500 − 6 × 2.2)

Model Test C

C1 c • C2 d • C3 c • C4 a • C5 410.67 cm^3 • C6 139.33 cm^3 (418/3) • C7 36 m • C8 21 cm (h = 2r) • C9 Volume 241.15 cm^3, surface area 211.01 cm^2 • C10 Capacity 1642.67 cm^3 ≈ 1.64 litres; milk ≈ ₹49.28

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Revision Card — Chapter 12 on One Page

The formula armoury

Cylinder: CSA $2\pi rh$, TSA $2\pi r(h + r)$, V $\pi r^2 h$ • Cone: $l = \sqrt{(r^2 + h^2)}$, CSA πrl, TSA $\pi r(l + r)$, V $(1/3)\pi r^2 h$ • Sphere: $4\pi r^2$, $(4/3)\pi r^3$ • Hemisphere: CSA $2\pi r^2$, TSA $3\pi r^2$, V $(2/3)\pi r^3$ • Frustum: V $(1/3)\pi h(R^2 + r^2 + Rr)$, CSA $\pi l(R + r)$, $l = \sqrt{(h^2 + (R - r)^2)}$.

The three Golden Rules

Surface: surviving skin only; joined face subtracted twice; cavity skin added. **Volume:** pieces add, cavities subtract. **Conversion:** volume is constant; n = V(big) ÷ V(small); displaced water = the intruder's volume.

The derivations in one breath

Cylinder unrolls to a $2\pi r \times h$ rectangle. Cone opens to a sector: (r/l) of a full πl^2 circle = πrl. Sphere = its wrapping cylinder's wall = 4 great circles. Frustum = big cone − small cone; $R^3 - r^3$ factorises and $(R - r)$ cancels.

Story signals

Surface area: paint, polish, canvas, sheet, wrap, cover. **Volume:** holds, fills, melts, is made of, capacity. **Check-whether questions:** recompute — never trust the printed claim (remember 346.51, not 345).

Exam-eve checklist

☐ Diameter halved in line 1 ☐ Hidden faces shaded on the sketch ☐ Slant height computed before any cone CSA ☐ π as commanded ☐ Litres converted at the right end ☐ Units in every answer ☐ The 10-second audit run

Chapter 12 complete. You did not memorise the solids — you unrolled them, opened them, peeled them and rebuilt them.

Next: Chapter 13 — Statistics, in the same 8D format. • KalviYogi • Mathematics – Beyond the Formula • 369 Tesla × KalviYogi • © Edunivra Pvt Ltd

CBSE · CLASS 10 · MATHEMATICS

MATHEMATICS BEYOND THE FORMULA

CHAPTER 13 · STATISTICS

Don't Memorize Mathematics. Understand, Visualise and Discover It.

The KalviYogi 8D Way: Discover · Visualise · Understand · Derive · Solve · Apply · Challenge · Recall

Every solution with COMPLETE steps — one by one, nothing skipped, every table shown in full

Mean (3 methods, derived) · Mode · Median · Ogives Masterclass

Five-Level Practice · Smart Memory System · Mastery Tracker

In this chapter, three numbers learn to speak for a hundred:

MEAN — the balance point · MEDIAN — the middle value · MODE — the most common value

01 Chapter Introduction

8D • DISCOVER — WHY SHOULD I LEARN THIS?

A cricket commentator says, "The team scores about **52 runs per over-phase**." A weather app says, "Tomorrow will be around **31°C**." Your school office says, "The **typical** student in Class 10 is 152 cm tall." None of these people listed hundreds of numbers. They replaced a whole crowd of data with **one representative number** — and choosing that number wisely is the science of this chapter: **Statistics**.

In Class 9 you met the mean, median and mode of *ungrouped* data — short lists you could sort by hand. Real life is not so kind. A hospital records the ages of **80 patients**; a city counts the electricity used by **68 homes**; a survey measures **225 electric bulbs**. The data arrives already **grouped into classes** like 25–35 and 35–45, and the individual values are lost. Can we still find the mean, the median and the mode? Yes — and this chapter derives exactly how, from first principles.

Your 8D route through this chapter

Discover (Section 03): a tiffin service in Madurai creates the need for three different "typical values".
Visualise (Section 04): see the mean as a balance point, the median as the middle of a queue, the mode as the tallest bar.
Understand (Section 05): classes, class marks and cumulative frequency — the anatomy of grouped data.
Derive (Section 06): we BUILD the assumed-mean, step-deviation, median and mode formulas — nothing is handed down.
Solve (Section 07): all 22 NCERT questions with full working tables, step by step.
Apply (Sections 13–14): canteens, factories, weather stations and libraries.
Challenge (Section 15): unequal class widths, "more than" tables, and a beautiful zero: $\Sigma f_i(x_i - \bar{x}) = 0$.
Recall (Sections 17–19): flash cards, closed-book recall and the Smart Memory tracker.

TOPPER TIP: *Marks warning, honestly given: Statistics is one of the most scoring chapters in the board exam — but only if your TABLES are complete. Examiners award most marks inside the table, not in the final answer. In this chapter we never compress a table.*

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02 Prerequisite Knowledge

FROM CLASS 9 — THE TOOLS YOU ALREADY OWN

Five Class 9 ideas carry this whole chapter. Test yourself on them first — cover the answers at the bottom while you think.

Idea 1 — Mean of ungrouped data

Mean = (sum of all observations) ÷ (number of observations) — the "equal share" value: what everyone would get if the total were shared equally.

Idea 2 — Median of ungrouped data

Arrange the data in order: the median is the middle value (odd count) or the average of the two middle values (even count). Half the data lies on each side of it.

Idea 3 — Mode of ungrouped data

The mode is the observation with the highest frequency — the most common value; a data set may have one mode, several, or none.

Idea 4 — Frequency tables and class intervals

Data is organised into classes (0–10, 10–20, ...) with frequencies f_i counting the observations in each class. The **class mark** x_i = (lower + upper) ÷ 2 represents the whole class.

Idea 5 — Bar graphs and histograms

In a histogram, each bar's height shows its class frequency; the tallest bar sits over the modal class — Section 06 turns that picture into the mode formula.

Quick Check — five questions, two minutes

Q1. Find the mean of 5, 7, 9, 11, 13.

Q2. Find the median of 3, 8, 5, 12, 7.

Q3. Find the mode of 2, 4, 2, 6, 2, 9, 4.

Q4. Write the class mark of the class 10–20.

Q5. For the classes 0–10, 10–20, 20–30, state the class size h.

Quick Check — Answers

Q1 Sum = 45, count = 5, mean = **9**. **Q2** Sorted: 3, 5, 7, 8, 12; middle value = **7**. **Q3** 2 appears three times — mode = **2**. **Q4** (10 + 20)/2 = **15**. **Q5** h = 10 − 0 = **10**.

03 Real-Life Mathematical Story

DISCOVER — RANI'S TIFFIN SERVICE, MADURAI

Rani runs a tiffin service near Madurai railway station. Every morning she cooks idlis, pongal and lemon rice, packs them into boxes, and sends them out with two delivery boys. Business is good — but Rani has a problem she cannot solve by instinct.

"Some days I cook for 60 boxes and food is wasted," she tells her daughter Meena, who is in Class 10. "Some days I cook for 60 and by 9 a.m. everything is sold out and customers go away angry. How many boxes should I plan for?"

*Meena does what a mathematician does: she **collects data**. For 30 days she notes the number of boxes sold each day, and then groups the days into classes:*

Boxes sold per day	30–40	40–50	50–60	60–70	70–80
Number of days	4	7	10	6	3

Now watch — Rani's one question, "How many boxes should I plan for?", is really **three different questions** wearing one coat:

Question 1 — the sharing question (MEAN)

"If the total boxes sold in 30 days were spread equally over all days, how many per day?" This is the **mean** — the right number for buying rice and dal for the whole month, because totals are what shopping lists are made of. (Answer, from Section 07's methods: total = 4(35) + 7(45) + 10(55) + 6(65) + 3(75) = 140 + 315 + 550 + 390 + 225 = 1620 boxes, so the mean is 1620 ÷ 30 = **54 boxes a day**.)

Question 2 — the middle question (MEDIAN)

"On a typical middle day — not a festival rush, not a rainy flop — how many boxes sell?" Line the 30 days up from smallest to largest: the middle day is the 15th–16th day, and it falls in the class 50–60. The exact **median**, found in Section 06, is **54 boxes**.

Question 3 — the most-common question (MODE)

"Which day-size happens most often, so the kitchen rhythm should be tuned to it?" The busiest class is 50–60, with 10 days. The exact **mode**, from Section 06's formula, is **53.75 ≈ 54 boxes**.

Three questions, three formulas — and for Rani, happily, nearly one answer: **plan for about 54 boxes**. But notice: the three numbers answer *different* questions, and in other data they can be

04 Visual Concept Explanation

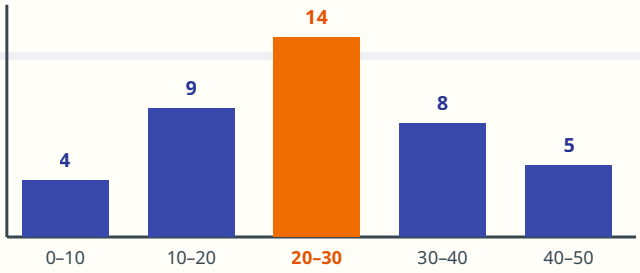
8D • VISUALISE — SINGAPORE CPA: CONCRETE → PICTORIAL → ABSTRACT

Concrete: tally marks on a notebook

A librarian groups a month of daily book-issue counts. Before any formula exists, there are piles of days in labelled boxes. **Grouping is the concrete act** — its price: the exact values are forgotten; only the counts (frequencies) survive.

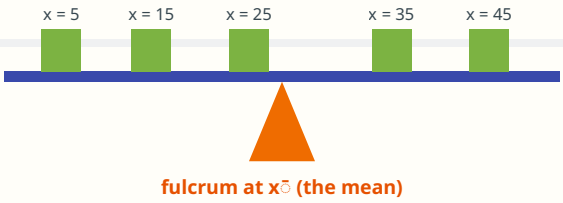
Pictorial 1: the mode is the tallest bar

Books read by 40 students in a month, drawn as a histogram. Your eye finds the tallest bar instantly — its class, 20–30, is the **modal class**, and the mode hides inside it.



Pictorial 2: the mean is a balance point

Place the class marks on a weightless beam, weighted by their frequencies. The beam balances at exactly one point — the **mean**. In symbols (proved in Challenge 10): $\Sigma f_i(x_i - \bar{x}) = 0$.



Pictorial 3: the median is the middle of the queue

Line all observations up in order: the median is the one standing exactly in the middle, half the queue on each side. For grouped data, count into the queue with **cumulative frequency** until the count crosses the halfway mark n/2 — that class is the **median class**.

Abstract: the three pictures become three formulas

Mean (balance) → $\bar{x} = \Sigma f_i x_i / \Sigma f_i$, with two shortcut forms derived in Section 06.

Median (queue) → $\text{Median} = l + [\,(n/2 - cf) / f\,] \times h$.

Mode (tallest bar) → $\text{Mode} = l + [\,(f_1 - f_0) / (2f_1 - f_0 - f_2)\,] \times h$.

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05 Mathematical Foundations

8D • UNDERSTAND — THE ANATOMY OF GROUPED DATA

Eight words carry the whole chapter — learn them precisely; every formula is written in them.

Term	Meaning	Example (classes 0–10, 10–20, 20–30)
Class interval	One row of the table; the range of values it collects	10–20
Lower / upper limit	The smallest / largest value a class can hold	10 and 20 for the class 10–20
Class size h	Upper limit − lower limit (same for all classes, in most problems)	h = 10
Class mark x_i	(Lower + upper) ÷ 2 — the single value that REPRESENTS the whole class	x = 15 for 10–20
Frequency f_i	How many observations fall in the class	If 8 students scored 10–20, f = 8
$\Sigma f_i = n$	Total number of observations	Sum of the whole frequency column
Cumulative frequency (cf)	Running total of frequencies, class by class	cf after 10–20 = f(0–10) + f(10–20)
Modal / median class	Class with the highest frequency / class where cf first reaches n/2	Found BEFORE any formula is used

The one honest warning of this chapter

Once data is grouped, the exact observations are **gone**. When we compute the mean, we *pretend* every observation in a class sits exactly at the class mark. So the mean, median and mode of grouped data are **estimates** — excellent ones, but estimates. Reasoning 2 (Section 12) shows two raw data sets sharing one grouped table but with different true means — a favourite board-exam discussion point.

The three formulas (derived next, not memorised): **Mean:** $\bar{x} = \Sigma f_i x_i / \Sigma f_i$ • **Median:** $l + [\,(n/2 - cf) / f\,] \times h$ • **Mode:** $l + [\,(f_1 - f_0) / (2f_1 - f_0 - f_2)\,] \times h$ • **Empirical relation:** 3 Median = Mode + 2 Mean.

PIN POINT: In the median formula, cf is the cumulative frequency of the class BEFORE the median class — never of the median class itself. The costliest single point in the chapter.

06 Formula Derivations

8D • DERIVE — NO FORMULA IS HANDED DOWN

Derivation 1: why the assumed-mean method works

The direct formula $\bar{x} = \Sigma f_i x_i / \Sigma f_i$ forces you to multiply big class marks (like 4750) by frequencies. The assumed-mean method is pure algebra — a change of measuring scale. Start from any fixed number a (the *assumed mean*) and measure every class mark from a:

1. Define $d_i = x_i - a$ for every class. Then $x_i = a + d_i$.
   Why: We have only renamed each class mark — nothing has changed yet.
2. Multiply by f_i and add over all classes: $\Sigma f_i x_i = \Sigma f_i(a + d_i) = a\Sigma f_i + \Sigma f_i d_i$.
   Why: a is a constant, so it comes out of the sum multiplied by the total frequency.
3. Divide both sides by Σf_i: $\bar{x} = \Sigma f_i x_i / \Sigma f_i = a + \Sigma f_i d_i / \Sigma f_i$.
   Why: The left side is exactly the direct mean. The formula is PROVED — for every choice of a.
4. Choose a = the middle class mark. Then the d_i are small, symmetric numbers (..., −20, −10, 0, 10, 20, ...) and the arithmetic shrinks.
   Why: a is a convenience, not a guess at the answer — any a gives the same final mean.

Derivation 2: why the step-deviation method works

1. When all classes have the same width h, the deviations d_i are all multiples of h. Divide them by h: $u_i = d_i/h = (x_i - a)/h$, so $d_i = h \cdot u_i$.
   Why: The u_i are the tiny numbers ..., −2, −1, 0, 1, 2, ...
2. Substitute into the assumed-mean formula: $\bar{x} = a + \Sigma f_i(h \cdot u_i)/\Sigma f_i = a + h \times (\Sigma f_i u_i / \Sigma f_i)$.
   Why: The constant h comes out of the sum.
3. Result: **$\bar{x} = a + (\Sigma f_i u_i / \Sigma f_i) \times h$**. Two warnings live inside this line: the method needs **equal class widths**, and you must **multiply by h at the end** — forgetting it is Error E6 in Find-the-Error.

TOPPER TIP: *Three methods, one mean. Direct for small numbers, assumed mean for medium, step deviation for large class marks with equal widths. The board accepts any of them — marks follow the table, not the method.*

Derivations — the Median and Mode Formulas

Derivation 3: the median formula is just a queue, measured fairly

Take a small distribution: classes 0–10, 10–20, 20–30, 30–40 with frequencies 6, 10, 14, 10 (n = 40). The cumulative frequencies are 6, 16, 30, 40. The middle of the queue is position n/2 = 20. The first 16 positions fill the first two classes, so position 20 stands inside the class 20–30 — the **median class**. Now reason:

1. Inside the median class there are f = 14 observations spread across the width h = 10. We need to advance (n/2 − cf) = 20 − 16 = **4 more observations** past the class's lower limit l = 20 to reach the middle.

   Why: cf = 16 is the count BEFORE the median class.

2. Grouped data has forgotten the exact positions, so we assume the 14 observations are spread **evenly** across the class. Each observation then 'owns' 10/14 of the width, and 4 observations own 4/14 × 10 = 2.86 of the width.

   Why: This even-spread assumption is called linear interpolation — it is exactly why the grouped median is an estimate.

3. Median = l + (fraction of the way through the class) × h = 20 + [(20 − 16) / 14] × 10 = 20 + 2.86 = **22.86**.

4. In general: **Median = l + [(n/2 − cf) / f] × h**, where l = lower limit of the median class, cf = cumulative frequency of the class before it, f = its frequency, h = its width.

   Why: Every symbol in the formula is now a picture in your head: l is the door of the class, cf is the crowd already inside the queue before the door, and the fraction is how far into the room the middle person stands.

Derivation 4: the mode formula — the tallest bar leans toward its taller neighbour

1. Look at the histogram in Section 04. The modal bar (height f_1) stands between a left neighbour (height f_0) and a right neighbour (height f_2). The true mode is pulled from the centre of the modal class toward the **taller** neighbour — more data piled on that side means the peak leans that way.

   Why: This is the standard geometric picture behind the formula: join the top corners of the neighbouring bars across the modal bar; the crossing point marks the mode.

2. The pull to the right is measured by ($f_1 - f_0$) — how much the modal bar towers over its LEFT neighbour; the pull to the left by ($f_1 - f_2$). The mode divides the class width in the ratio of these pulls, starting from l:

07 Detailed Solved Examples

8D • SOLVE — EXERCISE 13.1: MEAN OF GROUPED DATA • EVERY TABLE IN FULL

EXERCISE 13.1 — QUESTION 1

A survey was conducted by a group of students as part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house. Which method did you use, and why? [Plants: 0–2, 2–4, 4–6, 6–8, 8–10, 10–12, 12–14 • Houses: 1, 2, 1, 5, 6, 2, 3]

THINK FIRST: The class marks (1, 3, 5, ...) and frequencies are all small, so the DIRECT method is shortest — no shortcuts needed.

1 Write the class mark of each class: x_i = (lower + upper)/2, giving 1, 3, 5, 7, 9, 11, 13.

Why: Every house in a class is represented by its class mark.

2

Number of plants	Number of houses f_i	Class mark x_i	f_ix_i
0–2	1	1	1
2–4	2	3	6
4–6	1	5	5
6–8	5	7	35
8–10	6	9	54
10–12	2	11	22
12–14	3	13	39
Total	**$\Sigma f_i = 20$**		**$\Sigma f_ix_i = 162$**

3 Apply the direct formula: $\bar{x} = \Sigma f_ix_i / \Sigma f_i = 162 / 20 = \mathbf{8.1}$.

4 Method used: the **direct method**, because both the class marks and the frequencies are small enough to multiply directly.

Why: The question explicitly asks for the method and the reason — that sentence carries its own mark.

ANSWER: Mean number of plants per house = 8.1, by the direct method.

CBSE-STYLE MARKING SCHEME: Correct class marks [1] • Complete f_ix_i table with totals 20 and 162 [1] • Mean = 8.1 with method and reason stated [1]

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Exercise 13.1 — Question 2

EXERCISE 13.1 — QUESTION 2

Consider the following distribution of daily wages of 50 workers of a factory. Find the mean daily wages of the workers of the factory by using an appropriate method. [Daily wages (₹): 100–120, 120–140, 140–160, 160–180, 180–200 • Workers: 12, 14, 8, 6, 10]

THINK FIRST: Class marks run into hundreds and the class width is a constant 20 — this is a textbook case for the STEP-DEVIATION method with a = 150, h = 20.

1 Take the assumed mean a = 150 (the middle class mark) and h = 20. Compute $u_i = (x_i - 150)/20$ for each class.

Why: Section 06, Derivation 2 guarantees this gives the same mean as the direct method.

2

Daily wages (₹)	Workers f_i	x_i	$u_i = (x_i - 150)/20$	f_iu_i
100–120	12	110	−2	−24
120–140	14	130	−1	−14
140–160	8	150	0	0
160–180	6	170	1	6
180–200	10	190	2	20
Total	**$\Sigma f_i = 50$**			**$\Sigma f_iu_i = -12$**

3 Apply the step-deviation formula: $\bar{x} = a + (\Sigma f_iu_i / \Sigma f_i) \times h = 150 + (-12 / 50) \times 20$.

Why: Do not drop the × h at the end — see Error E6.

4 $\bar{x} = 150 - 4.80 =$ **145.20**.

ANSWER: Mean daily wage of a worker = ₹145.20.

CBSE-STYLE MARKING SCHEME: Choice of a and h with u_i column correct [1] • Complete table, $\Sigma f_iu_i = -12$ [1] • Formula with × h and mean ₹145.20 [1]

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Exercise 13.1 — Question 3

EXERCISE 13.1 — QUESTION 3

The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is ₹18. Find the missing frequency f. [Allowance (₹): 11–13, 13–15, 15–17, 17–19, 19–21, 21–23, 23–25 • Children: 7, 6, 9, 13, f, 5, 4]

THINK FIRST: The unknown sits inside BOTH totals, Σf_i and $\Sigma f_i x_i$. Build the table with f left as a letter, write the mean equation, and solve it.

1. Class marks are 12, 14, 16, 18, 20, 22, 24. Build the table keeping f symbolic:

2.

Pocket allowance (₹)	Children f_i	x_i	$f_i x_i$
11–13	7	12	84
13–15	6	14	84
15–17	9	16	144
17–19	13	18	234
19–21	f	20	20f
21–23	5	22	110
23–25	4	24	96
Total	**$\Sigma f_i = 44 + f$**		**$\Sigma f_i x_i = 752 + 20f$**

3. Use the given mean: $\bar{x} = (752 + 20f) / (44 + f) = 18$.

   Why: The known part of $\Sigma f_i x_i$ is 84 + 84 + 144 + 234 + 110 + 96 = 752.

4. Cross-multiply: $752 + 20f = 18(44 + f) = 792 + 18f$.

5. Collect the f terms: $20f - 18f = 792 - 752$, so $2f = 40$ and **$f = 20$**.

6. Check: $\Sigma f_i = 64$, $\Sigma f_i x_i = 752 + 400 = 1152$, and $1152 / 64 = 18$. (verified)

   Why: Always feed the answer back into the given mean — a ten-second check that catches most algebra slips.

ANSWER: The missing frequency is f = 20 (there are 20 children in the 19–21 class).

CBSE-STYLE MARKING SCHEME: Table with f carried symbolically, totals 44 + f and 752 + 20f [1] • Mean equation formed [1] • f = 20 [1]

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Exercise 13.1 — Question 4

EXERCISE 13.1 — QUESTION 4

Thirty women were examined in a hospital by a doctor and the number of heartbeats per minute were recorded and summarised as follows. Find the mean heartbeats per minute for these women, choosing a suitable method. [Heartbeats/min: 65–68, 68–71, 71–74, 74–77, 77–80, 80–83, 83–86 • Women: 2, 4, 3, 8, 7, 4, 2]

THINK FIRST: The class marks are decimals (66.5, 69.5, …) but the width is a constant h = 3, so step deviation with a = 75.5 keeps every number a small integer.

1. Take a = 75.5 (the middle class mark) and h = 3. Compute $u_i = (x_i - 75.5)/3$.

2.

Heartbeats per minute	Women f_i	x_i	u_i	f_iu_i
65–68	2	66.5	−3	−6
68–71	4	69.5	−2	−8
71–74	3	72.5	−1	−3
74–77	8	75.5	0	0
77–80	7	78.5	1	7
80–83	4	81.5	2	8
83–86	2	84.5	3	6
Total	**$\Sigma f_i = 30$**			**$\Sigma f_iu_i = 4$**

3. $\bar{x} = a + (\Sigma f_iu_i / \Sigma f_i) \times h = 75.5 + (4 / 30) \times 3 = 75.5 + 0.4$.

4. $\bar{x}$ = **75.9** heartbeats per minute.

ANSWER: Mean heartbeats per minute = 75.9, by the step-deviation method.

CBSE-STYLE MARKING SCHEME: Suitable method chosen with a = 75.5, h = 3 [1] • Complete u_i, f_iu_i table, $\Sigma f_iu_i = 4$ [1] • Mean = 75.9 [1]

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Exercise 13.1 — Question 5

EXERCISE 13.1 — QUESTION 5

In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying numbers of mangoes. The following was the distribution of mangoes according to the number of boxes. Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose? [Mangoes: 50–52, 53–55, 56–58, 59–61, 62–64 • Boxes: 15, 110, 135, 115, 25]

THINK FIRST: Look at the classes: 50–52, then 53–55 — there is a GAP of 1 between consecutive classes. Grouped formulas assume continuous classes, so first convert to true class limits by moving each boundary by half the gap (0.5). Then step deviation with a = 57, h = 3.

1 Convert to continuous classes: subtract 0.5 from every lower limit and add 0.5 to every upper limit: 49.5–52.5, 52.5–55.5, 55.5–58.5, 58.5–61.5, 61.5–64.5. The class marks do not change: 51, 54, 57, 60, 63.

Why: The gap is 53 − 52 = 1, and half of it is 0.5. Frequencies stay untouched.

2

Number of mangoes (continuous)	Boxes f_i	x_i	$u_i = (x_i - 57)/3$	f_iu_i
49.5–52.5	15	51	−2	−30
52.5–55.5	110	54	−1	−110
55.5–58.5	135	57	0	0
58.5–61.5	115	60	1	115
61.5–64.5	25	63	2	50
Total	**$\Sigma f_i = 400$**			**$\Sigma f_iu_i = 25$**

3 $\bar{x} = 57 + (25 / 400) \times 3 = 57 + 0.1875 =$ **57.19** (approx.).

4 Method chosen: **step deviation**, because the frequencies are large (up to 135) while the u_i stay tiny.

ANSWER: Mean number of mangoes in a packing box = 57.19 (approx.), by the step-deviation method after converting to continuous classes.

CBSE-STYLE MARKING SCHEME: Continuous classes formed with the 0.5 adjustment [1] • Complete step-deviation table, $\Sigma f_iu_i = 25$ [1] • Mean 57.19 with method named [1]

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Exercise 13.1 — Question 6

EXERCISE 13.1 — QUESTION 6

The table below shows the daily expenditure on food of 25 households in a locality. Find the mean daily expenditure on food by a suitable method. [Daily expenditure (₹): 100–150, 150–200, 200–250, 250–300, 300–350 • Households: 4, 5, 12, 2, 2]

THINK FIRST: Equal widths of 50 and class marks in the hundreds — step deviation with a = 225, h = 50.

1 Take a = 225 (middle class mark), h = 50, and compute $u_i = (x_i - 225)/50$.

2

Daily expenditure (₹)	Households f_i	x_i	u_i	f_iu_i
100–150	4	125	−2	−8
150–200	5	175	−1	−5
200–250	12	225	0	0
250–300	2	275	1	2
300–350	2	325	2	4
Total	**$\Sigma f_i = 25$**			**$\Sigma f_iu_i = -7$**

3 $\bar{x} = 225 + (-7 / 25) \times 50 = 225 - 14 = \mathbf{211}$.

Why: The negative total pulls the mean BELOW the assumed mean — the sign is information, not an error.

ANSWER: Mean daily expenditure on food = ₹211.

CBSE-STYLE MARKING SCHEME: a = 225, h = 50 with correct u_i column [1] • $\Sigma f_iu_i = -7$ [1] • Mean ₹211 [1]

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Exercise 13.1 — Question 7

EXERCISE 13.1 — QUESTION 7

To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below. Find the mean concentration of SO_2 in the air. [Concentration (ppm): 0.00–0.04, 0.04–0.08, 0.08–0.12, 0.12–0.16, 0.16–0.20, 0.20–0.24 • Localities: 4, 9, 9, 2, 4, 2]

THINK FIRST: Only six classes with small frequencies — the direct method is cleanest, decimals notwithstanding.

1 Class marks: 0.02, 0.06, 0.10, 0.14, 0.18, 0.22. Build the direct table:

2

Concentration of SO_2 (ppm)	Localities f_i	x_i	f_ix_i
0.00–0.04	4	0.02	0.08
0.04–0.08	9	0.06	0.54
0.08–0.12	9	0.10	0.90
0.12–0.16	2	0.14	0.28
0.16–0.20	4	0.18	0.72
0.20–0.24	2	0.22	0.44
Total	**$\Sigma f_i = 30$**		**$\Sigma f_ix_i = 2.96$**

3 $\bar{x} = 2.96 / 30 = 0.0987 \approx$ **0.099 ppm**.

Why: Keep all four decimal places until the final line — rounding inside the table is how small-data answers drift.

ANSWER: Mean concentration of SO_2 in the air = 0.099 ppm (approx.).

CBSE-STYLE MARKING SCHEME: Correct decimal class marks [1] • Complete table, $\Sigma f_ix_i = 2.96$ [1] • Mean 0.099 ppm [1]

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Exercise 13.1 — Question 8

EXERCISE 13.1 — QUESTION 8

A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent. [Days absent: 0–6, 6–10, 10–14, 14–20, 20–28, 28–38, 38–40 • Students: 11, 10, 7, 4, 4, 3, 1]

THINK FIRST: Measure the class widths first: 6, 4, 4, 6, 8, 10, 2 — they are NOT equal. Step deviation is built on a common h, so it cannot be used here; the DIRECT method it is. This question exists to test exactly that judgement.

1 Write each class mark carefully — with unequal classes there is no pattern to lean on: 3, 8, 12, 17, 24, 33, 39.

2

Number of days absent	Students f_i	x_i	$f_i x_i$
0–6	11	3	33
6–10	10	8	80
10–14	7	12	84
14–20	4	17	68
20–28	4	24	96
28–38	3	33	99
38–40	1	39	39
Total	**$\Sigma f_i = 40$**		**$\Sigma f_i x_i = 499$**

3 $\bar{x} = 499 / 40 = 12.475 \approx$ **12.48 days**.

ANSWER: Mean number of days a student was absent = 12.48 days (approx.), by the direct method — the class widths are unequal, so step deviation does not apply.

CBSE-STYLE MARKING SCHEME: Unequal widths recognised / direct method used [1] • Correct class marks for unequal classes and $\Sigma f_i x_i = 499$ [1] • Mean 12.48 days [1]

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Exercise 13.1 — Question 9

EXERCISE 13.1 — QUESTION 9

The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate. [Literacy rate (%): 45–55, 55–65, 65–75, 75–85, 85–95 • Cities: 3, 10, 11, 8, 3]

THINK FIRST: Equal width h = 10 with a central class mark of 70 — step deviation with a = 70.

1 Take a = 70, h = 10; $u_i = (x_i - 70)/10$.

2

Literacy rate (%)	Cities f_i	x_i	u_i	f_iu_i
45–55	3	50	−2	−6
55–65	10	60	−1	−10
65–75	11	70	0	0
75–85	8	80	1	8
85–95	3	90	2	6
Total	**$\Sigma f_i = 35$**			**$\Sigma f_iu_i = -2$**

3 $\bar{x} = 70 + (-2 / 35) \times 10 = 70 - 0.57 =$ **69.43%** (approx.).

ANSWER: Mean literacy rate = 69.43% (approx.).

CBSE-STYLE MARKING SCHEME: Step-deviation table with $\Sigma f_iu_i = -2$ [1] • Formula applied with × h [1] • Mean 69.43% [1]

Exercise 13.2 — Mode of Grouped Data

EXERCISE 13.2 — QUESTION 1

The following table shows the ages of the patients admitted in a hospital during a year. Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency. [Age (years): 5–15, 15–25, 25–35, 35–45, 45–55, 55–65 • Patients: 6, 11, 21, 23, 14, 5]

THINK FIRST: Two jobs: mode first (find the modal class, then the formula), then the mean (direct method is fine — class marks are round tens).

1. The highest frequency is 23, so the modal class is 35–45. Read off: l = 35, f_1 = 23, f_0 = 21 (class before), f_2 = 14 (class after), h = 10.

   Why: Section 06, Derivation 4: f0 and f2 are the two neighbours whose 'pulls' position the mode inside the class.

2. Mode = $l + [(f_1 - f_0) / (2f_1 - f_0 - f_2)] \times h = 35 + [(23 - 21) / (46 - 21 - 14)] \times 10 = 35 + (2/11) \times 10 = 35 + 1.82 =$ **36.82**.

   Why: NCERT's answer key prints 36.83 due to intermediate rounding; the exact value is 36.818...

3. For the mean, class marks are 10, 20, 30, 40, 50, 60 and $\Sigma f_i x_i = 6(10) + 11(20) + 21(30) + 23(40) + 14(50) + 5(60) = 60 + 220 + 630 + 920 + 700 + 300 = 2830$, with $\Sigma f_i = 80$.

4. Mean = 2830 / 80 = **35.38** years (approx.).

5. Interpretation: the **mode (36.82)** says the most common age of admission is about 37 years; the **mean (35.38)** says the average age of all 80 patients is about 35 years. The mode sits higher because the single busiest age-band is 35–45, while younger patients pull the overall average down.

   Why: The question's last sentence is a real instruction — an interpretation line is expected, in words.

ANSWER: Mode = 36.82 years (approx.; NCERT prints 36.83) and Mean = 35.38 years (approx.). The typical admitted patient is about 37 (mode), while the average over all patients is about 35 (mean).

CBSE-STYLE MARKING SCHEME: Modal class with f_1, f_0, f_2 identified [1] • Mode formula substituted correctly, 36.82 [1] • Mean 35.38 from $\Sigma f_i x_i = 2830$ [1] • Interpretation comparing the two [1]

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Exercise 13.2 — Question 2

EXERCISE 13.2 — QUESTION 2

The following data gives the information on the observed lifetimes (in hours) of 225 electrical components. Determine the modal lifetimes of the components. [Lifetimes (hours): 0–20, 20–40, 40–60, 60–80, 80–100, 100–120 • Components: 10, 35, 52, 61, 38, 29]

THINK FIRST: Only the mode is asked. Find the modal class, name the five values, substitute once, carefully.

1. The highest frequency is 61, so the modal class is 60–80.

2. Read off the five values: l = 60, f_1 = 61, f_0 = 52, f_2 = 38, h = 20.

   Why: Write these five values down BEFORE touching the formula — mixing up f0 and f2 is the classic slip here.

3. Mode = 60 + [(61 − 52) / (2(61) − 52 − 38)] × 20 = 60 + (9/32) × 20.

4. Mode = 60 + 5.625 = **65.63** hours (approx.).

ANSWER: Modal lifetime of the components = 65.63 hours (approx.).

CBSE-STYLE MARKING SCHEME: Modal class 60–80 [1] • Five values identified and substituted [1] • Mode 65.63 hours [1]

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Exercise 13.2 — Question 3

EXERCISE 13.2 — QUESTION 3

The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure. [Expenditure (₹): 1000–1500, 1500–2000, 2000–2500, 2500–3000, 3000–3500, 3500–4000, 4000–4500, 4500–5000 • Families: 24, 40, 33, 28, 30, 22, 16, 7]

THINK FIRST: Mode from the formula; mean by the assumed-mean method with a = 2750 — the direct products (like 40 × 1750) are heavy, and the deviations are clean hundreds.

1. Modal class: the highest frequency is 40, so the modal class is 1500–2000, with l = 1500, $f_1 = 40$, $f_0 = 24$, $f_2 = 33$, h = 500.

2. Mode = 1500 + [(40 − 24) / (80 − 24 − 33)] × 500 = 1500 + (16/23) × 500 = 1500 + 347.83 = **1847.83**.

3. For the mean, take a = 2750. The deviations $d_i = x_i - 2750$ are −1500, −1000, −500, 0, 500, 1000, 1500, 2000.

4.

Expenditure (₹)	Families f_i	x_i	$d_i = x_i - 2750$	$f_i d_i$
1000–1500	24	1250	−1500	−36000
1500–2000	40	1750	−1000	−40000
2000–2500	33	2250	−500	−16500
2500–3000	28	2750	0	0
3000–3500	30	3250	500	15000
3500–4000	22	3750	1000	22000
4000–4500	16	4250	1500	24000
4500–5000	7	4750	2000	14000
Total	**$\Sigma f_i = 200$**			**$\Sigma f_i d_i = -17500$**

5. Mean = $a + \Sigma f_i d_i / \Sigma f_i$ = 2750 + (−17500 / 200) = 2750 − 87.50 = **2662.50**.

ANSWER: Modal monthly expenditure = ₹1847.83 and mean monthly expenditure = ₹2662.50. (The mean is far above the mode: a minority of high-spending families pulls the average up — the distribution is skewed.)

CBSE-STYLE MARKING SCHEME: Mode: modal class + substitution + 1847.83 [2] • Mean: assumed-mean table with $\Sigma f_i d_i = -17500$ [1] • Mean 2662.50 [1]

Exercise 13.2 — Question 4

EXERCISE 13.2 — QUESTION 4

The following distribution gives the state-wise teacher–student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures. [Number of students per teacher: 15–20, 20–25, 25–30, 30–35, 35–40, 40–45, 45–50, 50–55 • States/UTs: 3, 8, 9, 10, 3, 0, 0, 2]

THINK FIRST: Note the two zero frequencies — they are real data (no state had that ratio) and they stay in the table. Mode first, then a direct mean.

1. The highest frequency is 10, so the modal class is 30–35: $l = 30$, $f_1 = 10$, $f_0 = 9$, $f_2 = 3$, $h = 5$.

2. Mode = $30 + [(10 - 9) / (20 - 9 - 3)] \times 5 = 30 + (1/8) \times 5 = 30 + 0.625 =$ **30.63**.

3. Mean: class marks are 17.5, 22.5, 27.5, 32.5, 37.5, 42.5, 47.5, 52.5, and $\Sigma f_i x_i = 52.5 + 180 + 247.5 + 325 + 112.5 + 0 + 0 + 105 = 1022.5$, with $\Sigma f_i = 35$.

   Why: The two zero rows contribute 0 to the sum but their classes still count when locating neighbours — here they do not touch the modal class.

4. Mean = $1022.5 / 35 =$ **29.21**.

5. Interpretation: the **mode (30.63)** is the most common teacher–student ratio — in the largest group of states, one teacher handles about 31 students. The **mean (29.21)** is the ratio if all students and teachers of all states were pooled and shared evenly — about 29 students per teacher.

ANSWER: Mode = 30.63 students per teacher and Mean = 29.21 students per teacher. The mode describes the typical state; the mean describes the pooled national average.

CBSE-STYLE MARKING SCHEME: Mode with correct neighbours, 30.63 [1] • Mean from $\Sigma f_i x_i = 1022.5$, giving 29.21 [1] • Interpretation of both measures [1]

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Exercise 13.2 — Questions 5 and 6

EXERCISE 13.2 — QUESTION 5

The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches. Find the mode of the data. [Runs scored: 3000–4000, 4000–5000, 5000–6000, 6000–7000, 7000–8000, 8000–9000, 9000–10000, 10000–11000 • Batsmen: 4, 18, 9, 6, 7, 2, 4, 1]

THINK FIRST: Big numbers, same machine: modal class, five values, one substitution.

1. The highest frequency is 18, so the modal class is 4000–5000: l = 4000, f_1 = 18, f_0 = 4, f_2 = 9, h = 1000.
2. Mode = 4000 + [(18 − 4) / (36 − 4 − 9)] × 1000 = 4000 + (14/23) × 1000.
3. Mode = 4000 + 608.70 = **4608.70** runs (approx.).

Why: The mode need not be an actual batsman's score — it is the centre of gravity of the busiest class.

ANSWER: Mode of the runs scored = 4608.7 runs (approx.).

CBSE-STYLE MARKING SCHEME: Modal class 4000–5000 [1] • Substitution [1] • Mode 4608.7 [1]

EXERCISE 13.2 — QUESTION 6

A student noted the number of cars passing through a spot on a road for 100 periods, each of 3 minutes, and summarised it in the table below. Find the mode of the data. [Cars: 0–10, 10–20, 20–30, 30–40, 40–50, 50–60, 60–70, 70–80 • Periods: 7, 14, 13, 12, 20, 11, 15, 8]

THINK FIRST: Same machine once more — by now your hand should find the modal class before your eye finishes the table.

1. The highest frequency is 20, so the modal class is 40–50: l = 40, f_1 = 20, f_0 = 12, f_2 = 11, h = 10.
2. Mode = 40 + [(20 − 12) / (40 − 12 − 11)] × 10 = 40 + (8/17) × 10 = 40 + 4.71 = **44.71**.

ANSWER: Mode = 44.71 cars (approx.) per 3-minute period.

CBSE-STYLE MARKING SCHEME: Modal class 40–50 [1] • Substitution [1] • Mode 44.71 [1]

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Exercise 13.3 — Median of Grouped Data (Question 1, Part 1: the Median)

EXERCISE 13.3 — QUESTION 1 (MEDIAN)

The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them. [Consumption (units): 65–85, 85–105, 105–125, 125–145, 145–165, 165–185, 185–205 • Consumers: 4, 5, 13, 20, 14, 8, 4]

THINK FIRST: Three measures, three machines. The median needs the cumulative frequency table — build it first, in full.

1. Build the cf column by running totals: 4, 4 + 5 = 9, 9 + 13 = 22, 22 + 20 = 42, 42 + 14 = 56, 56 + 8 = 64, 64 + 4 = 68.

2.

Monthly consumption (units)	Consumers f_i	Cumulative frequency (cf)
65–85	4	4
85–105	5	9
105–125	13	22
125–145	20	42
145–165	14	56
165–185	8	64
185–205	4	68
Total	**n = 68**	

3. n/2 = 68/2 = 34. The cf first reaches 34 inside the class 125–145 (cf jumps from 22 to 42 there), so the median class is 125–145.

Why: Median class is found from the cf column — NOT from the largest frequency. That class (also 125–145 here) coincidence is exactly what Error E4 exploits.

4. Read off: l = 125, cf = 22 (cf BEFORE the median class), f = 20, h = 20.

5. Median = l + [(n/2 − cf) / f] × h = 125 + [(34 − 22) / 20] × 20 = 125 + 12 = **137 units**.

ANSWER: Median monthly consumption = 137 units. (Mean and mode follow on the next page.)

CBSE-STYLE MARKING SCHEME: Complete cf table [1] • Median class located via n/2 = 34 [1] • cf = 22 (preceding class) used [1] • Median 137 units [1]

Exercise 13.3 — Question 1 (Part 2: Mean, Mode and the Comparison)

EXERCISE 13.3 — QUESTION 1 (MEAN AND MODE)

Same electricity data: 65–85, 85–105, 105–125, 125–145, 145–165, 165–185, 185–205 with 4, 5, 13, 20, 14, 8, 4 consumers. Find the mean and the mode, and compare all three measures.

THINK FIRST: Mean by step deviation (a = 135, h = 20); mode from the modal class 125–145.

1. Class marks: 75, 95, 115, 135, 155, 175, 195. With a = 135 and h = 20, the u_i are −3, −2, −1, 0, 1, 2, 3.

2.

Consumption (units)	f_i	x_i	$u_i = (x_i - 135)/20$	f_iu_i
65–85	4	75	−3	−12
85–105	5	95	−2	−10
105–125	13	115	−1	−13
125–145	20	135	0	0
145–165	14	155	1	14
165–185	8	175	2	16
185–205	4	195	3	12
Total	**$\Sigma f_i = 68$**			**$\Sigma f_iu_i = 7$**

3. Mean = 135 + (7/68) × 20 = 135 + 2.06 = **137.05 units** (approx.; exactly 9320/68).

4. Mode: modal class is 125–145 (f = 20), so Mode = 125 + [(20 − 13) / (40 − 13 − 14)] × 20 = 125 + (7/13) × 20 = 125 + 10.77 = **135.77 units**.

5. Comparison: Mean = 137.05, Median = 137, Mode = 135.77. All three are close, with Mean > Median > Mode — the distribution is *nearly symmetric*, very slightly skewed toward higher consumption. Check the empirical relation: (135.77 + 2 × 137.05)/3 = 136.63 ≈ 137 = the median — close, as Section 06 promised.

Why: This comparison sentence is part of the question — never leave it unwritten.

ANSWER: Mean = 137.05 units, Median = 137 units, Mode = 135.77 units — three close values; the data is nearly symmetric.

CBSE-STYLE MARKING SCHEME: Step-deviation table with $\Sigma f_iu_i = 7$ and mean 137.05 [2] • Mode 135.77 [1] • Comparison of the three measures [1]

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Exercise 13.3 — Question 2

EXERCISE 13.3 — QUESTION 2

If the median of the distribution given below is 28.5, find the values of x and y. [Class: 0–10, 10–20, 20–30, 30–40, 40–50, 50–60 • Frequency: 5, x, 20, 15, y, 5 • Total frequency = 60]

THINK FIRST: Two unknowns need two facts: the total frequency (60) and the median (28.5). The median 28.5 lies in the class 20–30 — that class will supply the second equation.

1. Total frequency: 5 + x + 20 + 15 + y + 5 = 60, so **x + y = 15** ... (i)
2. Build the cf column: 5, 5 + x, 25 + x, 40 + x, 40 + x + y, 60.
3. Since the median is 28.5, the median class is 20–30. For it: l = 20, f = 20, cf = 5 + x (the cf before it), h = 10, and n/2 = 30.
4. 28.5 = 20 + [(30 − (5 + x)) / 20] × 10 = 20 + (25 − x)/2.
5. So (25 − x)/2 = 8.5, giving 25 − x = 17 and **x = 8**.
6. From (i): y = 15 − 8 = **7**. Check: cf values become 5, 13, 33, 48, 55, 60 — n/2 = 30 first crossed in 20–30, consistent.

   Why: The check confirms the median class choice — if x had come out larger than 20, the assumed median class would have been impossible and we would revisit Step 3.

ANSWER: x = 8 and y = 7.

CBSE-STYLE MARKING SCHEME: Equation from total frequency [1] • cf column with x carried [1] • Median formula applied to class 20–30 [1] • x = 8, y = 7 [1]

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Exercise 13.3 — Question 3

EXERCISE 13.3 — QUESTION 3

A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 years. [Age below: 20, 25, 30, 35, 40, 45, 50, 55, 60 • Policy holders (cumulative): 2, 6, 24, 45, 78, 89, 92, 98, 100]

THINK FIRST: The table is already cumulative ('less than' type). First recover the ordinary frequencies by subtracting consecutive cf values, then run the median machine.

1 Form classes of width 5 ending at the given limits: 15–20, 20–25, ..., 55–60. Frequencies are the cf differences: 2, 6 − 2 = 4, 24 − 6 = 18, 45 − 24 = 21, 78 − 45 = 33, 89 − 78 = 11, 92 − 89 = 3, 98 − 92 = 6, 100 − 98 = 2.

Why: The first class starts at 15 because the classes march in steps of 5 up to 60, and policies begin at 18 — inside 15–20.

2

Age (years)	f_i	cf
15–20	2	2
20–25	4	6
25–30	18	24
30–35	21	45
35–40	33	78
40–45	11	89
45–50	3	92
50–55	6	98
55–60	2	100
Total	**n = 100**	

3 n/2 = 50. The cf first reaches 50 in the class 35–40 (cf jumps 45 → 78), so the median class is 35–40, with l = 35, cf = 45, f = 33, h = 5.

4 Median = 35 + [(50 − 45) / 33] × 5 = 35 + 25/33 = 35 + 0.76 = **35.76 years** (approx.).

ANSWER: Median age of the policy holders = 35.76 years (approx.).

CBSE-STYLE MARKING SCHEME: Less-than data converted to classes with frequencies as cf differences [1] • cf table complete [1] • Median class 35–40 via n/2 = 50 [1] • Median 35.76 years [1]

Exercise 13.3 — Question 4

EXERCISE 13.3 — QUESTION 4

The lengths of 40 leaves of a plant are measured correct to the nearest millimetre, and the data obtained is represented in the following table. Find the median length of the leaves. [Length (mm): 118–126, 127–135, 136–144, 145–153, 154–162, 163–171, 172–180 • Leaves: 3, 5, 9, 12, 5, 4, 2] (Hint: convert to continuous classes: 117.5–126.5, 126.5–135.5, ..., 171.5–180.5.)

THINK FIRST: Measurements 'to the nearest millimetre' leave gaps of 1 between classes (126 to 127). The median formula assumes continuous classes, so adjust every boundary by 0.5 first — the frequencies do not change.

1 Convert: subtract 0.5 from lower limits, add 0.5 to upper limits. New classes: 117.5–126.5, 126.5–135.5, 135.5–144.5, 144.5–153.5, 153.5–162.5, 162.5–171.5, 171.5–180.5, with the same frequencies 3, 5, 9, 12, 5, 4, 2.

2

Length (mm, continuous)	Leaves f_i	cf
117.5–126.5	3	3
126.5–135.5	5	8
135.5–144.5	9	17
144.5–153.5	12	29
153.5–162.5	5	34
162.5–171.5	4	38
171.5–180.5	2	40
Total	**n = 40**	

3 n/2 = 20. The cf first reaches 20 in 144.5–153.5 (cf jumps 17 → 29), so: l = 144.5, cf = 17, f = 12, h = 9.

Why: Note h = 9 now, not 8 — the adjustment widened every class by 1 in total (0.5 + 0.5). Using h = 8 is the planted trap of this question.

4 Median = 144.5 + [(20 − 17) / 12] × 9 = 144.5 + 2.25 = **146.75 mm**.

ANSWER: Median length of the leaves = 146.75 mm.

CBSE-STYLE MARKING SCHEME: Continuous classes formed correctly [1] • cf table complete [1] • Median class with h = 9 [1] • Median 146.75 mm [1]

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Exercise 13.3 — Question 5

EXERCISE 13.3 — QUESTION 5

The following table gives the distribution of the lifetime of 400 neon lamps. Find the median lifetime of a lamp. [Lifetime (hours): 1500–2000, 2000–2500, 2500–3000, 3000–3500, 3500–4000, 4000–4500, 4500–5000 • Lamps: 14, 56, 60, 86, 74, 62, 48]

THINK FIRST: A straight run of the median machine with big round numbers — accuracy in the cf column is everything.

1

Lifetime (hours)	Lamps f_i	cf
1500–2000	14	14
2000–2500	56	70
2500–3000	60	130
3000–3500	86	216
3500–4000	74	290
4000–4500	62	352
4500–5000	48	400
Total	**n = 400**	

2

n/2 = 200. The cf first reaches 200 in 3000–3500 (cf jumps 130 → 216), so the median class is 3000–3500: l = 3000, cf = 130, f = 86, h = 500.

3

Median = 3000 + [(200 − 130) / 86] × 500 = 3000 + (70 × 500)/86 = 3000 + 406.98 = **3406.98 hours** (approx.).

ANSWER: Median lifetime of a lamp = 3406.98 hours (approx.).

CBSE-STYLE MARKING SCHEME: Complete cf table [1] • Median class 3000–3500 via n/2 = 200 [1] • Median 3406.98 hours [1]

Exercise 13.3 — Question 6

EXERCISE 13.3 — QUESTION 6

100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabet in the surnames was obtained as follows. Determine the median number of letters in the surnames. Find the mean number of letters in the surnames. Also, find the modal size of the surnames. [Letters: 1–4, 4–7, 7–10, 10–13, 13–16, 16–19 • Surnames: 6, 30, 40, 16, 4, 4]

THINK FIRST: One table can serve all three measures if we give it every column at once: f, x_i, f_ix_i and cf. Build it once, read three answers off it.

1

Number of letters	Surnames f_i	x_i	f_ix_i	cf
1–4	6	2.5	15	6
4–7	30	5.5	165	36
7–10	40	8.5	340	76
10–13	16	11.5	184	92
13–16	4	14.5	58	96
16–19	4	17.5	70	100
Total	**$\Sigma f_i = 100$**		**$\Sigma f_ix_i = 832$**	

2 MEDIAN: n/2 = 50; cf first reaches 50 in 7–10, so Median = 7 + [(50 − 36) / 40] × 3 = 7 + 1.05 = **8.05**.

3 MEAN: $\bar{x}$ = 832 / 100 = **8.32**.

4 MODE: modal class is 7–10 (f = 40), so Mode = 7 + [(40 − 30) / (80 − 30 − 16)] × 3 = 7 + (10/34) × 3 = 7 + 0.88 = **7.88**.

5 Read the three together: Mode (7.88) < Median (8.05) < Mean (8.32) — a mild skew toward longer surnames, and the empirical relation checks out: (7.88 + 2 × 8.32)/3 = 8.17 ≈ 8.05.

ANSWER: Median = 8.05 letters, Mean = 8.32 letters, Mode = 7.88 letters (all approx.).

CBSE-STYLE MARKING SCHEME: Combined table with Σf_ix_i = 832 and cf column [1] • Median 8.05 [1] • Mean 8.32 [1] • Mode 7.88 [1]

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Exercise 13.3 — Question 7

EXERCISE 13.3 — QUESTION 7

The distribution below gives the weights of 30 students of a class. Find the median weight of the students. [Weight (kg): 40–45, 45–50, 50–55, 55–60, 60–65, 65–70, 70–75 • Students: 2, 3, 8, 6, 6, 3, 2]

THINK FIRST: The last run of the machine in this exercise — full cf table, then one substitution.

1

Weight (kg)	Students f_i	cf
40–45	2	2
45–50	3	5
50–55	8	13
55–60	6	19
60–65	6	25
65–70	3	28
70–75	2	30
Total	**n = 30**	

2 n/2 = 15. The cf first reaches 15 in 55–60 (cf jumps 13 → 19), so: l = 55, cf = 13, f = 6, h = 5.

3 Median = 55 + [(15 − 13) / 6] × 5 = 55 + 1.67 = **56.67 kg** (approx.).

ANSWER: Median weight of the students = 56.67 kg (approx.).

CBSE-STYLE MARKING SCHEME: Complete cf table [1] • Median class 55–60 via n/2 = 15 [1] • Median 56.67 kg [1]

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Masterclass — The Ogive: Finding the Median from a Graph

MASTERCLASS — BEYOND THE EXERCISE

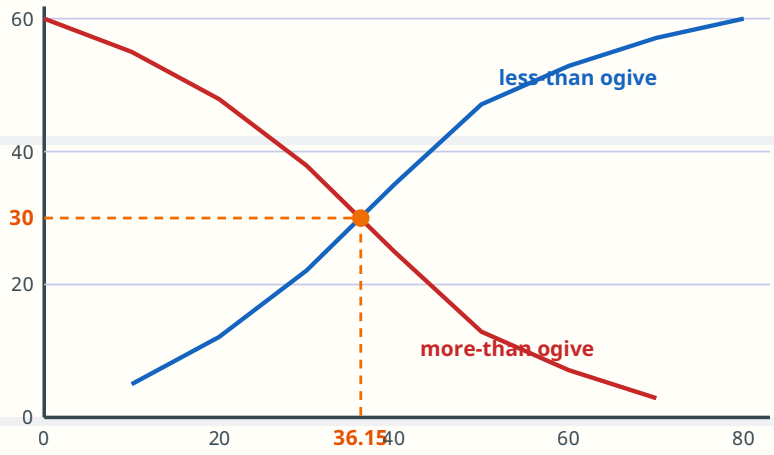
The ogives were removed from the rationalised NCERT exercise — but the *idea* is too beautiful, and too useful for checking, to skip. An **ogive** is a graph of cumulative frequency. The **less-than ogive** plots (upper limit, cf); the **more-than ogive** plots (lower limit, cf counted from the top). Where the two curves cross, exactly half the data lies on each side — that x-value **is** the median.

Worked example. Marks of 60 students: less than 10: 5, less than 20: 12, less than 30: 22, less than 40: 35, less than 50: 47, less than 60: 53, less than 70: 57, less than 80: 60.

1. Recover the frequencies: 5, 12 − 5 = 7, 22 − 12 = 10, 35 − 22 = 13, 47 − 35 = 12, 53 − 47 = 6, 57 − 53 = 4, 60 − 57 = 3. Total 60. (verified)
2. Plot the less-than ogive through (10, 5), (20, 12), (30, 22), (40, 35), (50, 47), (60, 53), (70, 57), (80, 60) and the more-than ogive through (0, 60), (10, 55), (20, 48), (30, 38), (40, 25), (50, 13), (60, 7), (70, 3).

   Why: The more-than cf at a lower limit = total − (less-than cf up to the previous limit).
3. The curves cross at cf = 30 = n/2. Drop a perpendicular to the marks axis: it lands at **36.15** (read off the graph: ≈ 36.2).
4. Verify by formula: median class is 30–40 (cf before = 22, f = 13), so Median = 30 + [(30 − 22) / 13] × 10 = 30 + 6.15 = **36.15**. The graph and the formula agree — each checks the other.

   Why: This is the professional habit: two independent routes to one answer.



TOPPER TIP: *In the exam hall, use the formula — but a rough ogive sketch is the fastest way to catch a wrong median class.*

08 Multiple Solution Methods

ONE MEAN, THREE ROADS — CHOOSE BY THE NUMBERS IN FRONT OF YOU

One data set, three complete solutions. Marks of 30 students: 0–20: 4, 20–40: 6, 40–60: 10, 60–80: 7, 80–100: 3.

Method 1 — Direct

Class	f_i	x_i	f_ix_i (Direct)
0–20	4	10	40
20–40	6	30	180
40–60	10	50	500
60–80	7	70	490
80–100	3	90	270
Total	**30**		**$\Sigma f_ix_i = 1480$**

$\bar{x}$ = 1480 / 30 = **49.33**. Honest, but the products (like 7 × 70) are already growing teeth.

Method 2 — Assumed mean (a = 50)

Class	f_i	x_i	$d_i = x_i - 50$	f_id_i
0–20	4	10	−40	−160
20–40	6	30	−20	−120
40–60	10	50	0	0
60–80	7	70	20	140
80–100	3	90	40	120
Total	**30**			**$\Sigma f_id_i = -20$**

$\bar{x}$ = 50 + (−20 / 30) = 50 − 0.67 = **49.33**. Same answer — Derivation 1 promised exactly this.

Method 3 — Step deviation (a = 50, h = 20)

Class	f_i	x_i	$u_i = (x_i - 50)/20$	f_iu_i
0–20	4	10	−2	−8
20–40	6	30	−1	−6
40–60	10	50	0	0
60–80	7	70	1	7

09 Interactive Simulations & Hands-on Activities

8D • VISUALISE BY DOING — YOUR HANDS LEARN TOO

Activity 1 — The class census (30 minutes)

Measure the height of every student in your class to the nearest cm. Group the heights into classes of width 5 (140–145, 145–150, ...). Compute the mean, median and mode of YOUR class. Then answer in your notebook: which of the three best deserves the title 'the typical height of our class', and why?

Activity 2 — Balance the ruler (the mean, physically)

Mark a metre ruler at 10, 30, 50, 70, 90 cm. Place stacks of coins at the marks: 4, 6, 10, 7, 3 coins (the Section 08 data!). Balance the ruler on a pencil. Where does it balance? It should balance near 49.3 cm — the mean of Section 08, found by physics. Move one coin and watch the balance point shift: that is exactly what one extreme observation does to a mean, and why the median resists it.

Activity 3 — Draw your own ogive

Using your Activity 1 data, draw the less-than ogive on graph paper and read the median off the graph. Then compute the median by formula. The two values should agree to within half a class width — if they do not, one of the two workings has an error, and finding it is the real exercise.

Activity 4 — The three-measures race

For the tiffin data of Section 03, time yourself: mean by direct method, then by step deviation. Record both times in the Learning Time Tracker (Section 19). Most students find step deviation 30–40% faster on large numbers — measure whether that is true for you, because in the exam hall you should use YOUR fastest reliable method, not the textbook's favourite.

KALVIYOGI DIGITAL COMPANION — CHAPTER 13: Video lessons for every derivation in Section 06 • Interactive ogive simulator (drag the data, watch the median move) • Online chapter test with instant marking • Flash-card app version of Section 17.
QR code and link activate with the KalviYogi app/website at print. (No code is printed here yet, so that no student ever scans a dead code.)

TOPPER TIP: *Activities are not decoration. Every formula in this chapter was invented by someone watching data behave — Activities 2 and 3 let you watch it too, and watched mathematics is remembered mathematics.*

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10 Common Mistakes

SIX WAYS STUDENTS LOSE MARKS HERE — AND THE FIXES

COMMON MISTAKE: MISTAKE 1 — Using the lower limit as the class mark.

Wrong way: For the class 10–20, taking $x_i = 10$ in the mean table.

Why wrong: The class mark is the MIDPOINT, (10 + 20)/2 = 15 — it is the value that represents the whole class.

Right way: x_i = (lower limit + upper limit) ÷ 2, always, written as its own column before any multiplication.

COMMON MISTAKE: MISTAKE 2 — Swapping f_0 and f_2 in the mode formula.

Wrong way: Taking f_0 as the frequency of the class AFTER the modal class.

Why wrong: f_0 belongs to the class BEFORE (below) the modal class and f_2 to the class AFTER (above) it; swapping them mirrors the mode to the wrong side of the class.

Right way: Write the five values (l, f_1, f_0, f_2, h) in a row before substituting — every time, even when you feel fast.

COMMON MISTAKE: MISTAKE 3 — Using the cf OF the median class instead of the cf BEFORE it.

Wrong way: In Ex 13.3 Q1, putting cf = 42 (the cf up to and including 125–145) into the formula, getting $125 + ((34 - 42)/20) \times 20 = 117$ — outside the median class itself, an obvious absurdity students still submit.

Why wrong: The formula measures how far INTO the median class the middle value stands; the crowd counted must be the one that entered BEFORE it.

Right way: cf = cumulative frequency of the class PRECEDING the median class (22 in Q1). Sanity check: your median must lie INSIDE the median class.

Common Mistakes — continued

COMMON MISTAKE: MISTAKE 4 — Choosing the median class by highest frequency.
Wrong way: 'The median class is the class with the biggest f.'
Why wrong: That is the MODAL class. The median class is found from the cf column, where the running total first reaches n/2.
Right way: Median questions always start with a cf column; mode questions never need one. Let the question choose the table, not your habit.

COMMON MISTAKE: MISTAKE 5 — Forgetting × h at the end of step deviation (or dropping a minus sign).
Wrong way: $\bar{x} = 150 + (-12/50) = 149.76$ for Ex 13.1 Q2.
Why wrong: The u_i are measured in units of h; the correction term is still in 'step units' until you multiply by h.
Right way: $\bar{x} = a + (\Sigma f_i u_i / \Sigma f_i) \times h$ — and keep the sign of $\Sigma f_i u_i$ attached through every line: $150 + (-12/50) \times 20 = 145.20$.

COMMON MISTAKE: MISTAKE 6 — Using step deviation on unequal classes.
Wrong way: Hunting for an h in Ex 13.1 Q8 (widths 6, 4, 4, 6, 8, 10, 2) and forcing one anyway.
Why wrong: $u_i = (x_i - a)/h$ needs ONE common h; with unequal widths the u_i stop being the neat integers the method exists for.
Right way: Check the widths first. Unequal → direct method, patiently, class mark by class mark.

PIN POINT: Three sanity checks catch almost everything: the mean and median must lie inside the data's range; the median must lie inside the MEDIAN class; the mode must lie inside the MODAL class. Ten seconds, three checks, before you submit.

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11 Graded Practice Problems

FIVE LEVELS — CLIMB ONE LEVEL AT A TIME • ANSWERS IN THE ANSWER KEY

Rules of the climb: finish a level with at most one error before moving up. From Level 2 onwards, show the full working table — in the board exam, the table IS the answer.

LEVEL 1 — FOUNDATION (direct concepts)

L1.1 Find the mean of the distribution: classes 0–10, 10–20, 20–30, 30–40, 40–50 with frequencies 5, 8, 12, 6, 4.

L1.2 Find the mean: classes 5–15, 15–25, 25–35, 35–45, 45–55 with frequencies 4, 6, 10, 7, 3.

L1.3 Find the mode: classes 0–5, 5–10, 10–15, 15–20, 20–25 with frequencies 3, 7, 12, 6, 2.

L1.4 Find the median: classes 10–20, 20–30, 30–40, 40–50, 50–60 with frequencies 4, 6, 9, 5, 3.

L1.5 The mean of a distribution is 30 and its median is 28. Find its mode using the empirical relation.

L1.6 The class marks of a distribution are 12, 22, 32, 42, 52. Find the class size and write the classes.

LEVEL 2 — SKILL BUILDER (textbook standard)

L2.1 Find the mean daily wages by the assumed-mean method (take a = 325): wages (₹) 200–250, 250–300, 300–350, 350–400, 400–450 with workers 6, 9, 12, 8, 5.

L2.2 Find the mean marks by the step-deviation method: classes 0–20, 20–40, 40–60, 60–80, 80–100 with students 5, 9, 14, 12, 10.

L2.3 Find the mean and the mode, and say which is larger: classes 100–110, 110–120, 120–130, 130–140, 140–150 with frequencies 4, 7, 12, 9, 5.

L2.4 A survey records: marks less than 20: 6 students, less than 40: 15, less than 60: 29, less than 80: 41, less than 100: 50. Find the median marks.

L2.5 The mean of the distribution 0–10: 5, 10–20: 8, 20–30: f, 30–40: 7, 40–50: 4 is 24. Find f.

L2.6 The median of the distribution 0–10: 4, 10–20: f, 20–30: 9, 30–40: 6, 40–50: 3 is 25. Find f.

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Graded Practice — Levels 3, 4 and 5

LEVEL 3 — APPLICATION (real-life problems)

L3.1 Daily rainfall (mm) over a month was grouped as: 0–5: 4 days, 5–10: 7, 10–15: 9, 15–20: 6, 20–25: 4. Find the modal daily rainfall.

L3.2 Passengers per bus trip over 40 trips: 10–20: 5, 20–30: 8, 30–40: 14, 40–50: 9, 50–60: 4. Find the median number of passengers per trip.

L3.3 Noon temperatures (in °C) for 40 days: 20–24: 3, 24–28: 8, 28–32: 12, 32–36: 9, 36–40: 5, 40–44: 3. Find the mean temperature.

L3.4 Customers entering a shop per hour, over 50 hours: 0–10: 6, 10–20: 11, 20–30: 15, 30–40: 10, 40–50: 8. Find the mean and the mode. Which better represents a 'typical hour'? Give one reason.

LEVEL 4 — ADVANCED REASONING (multi-concept)

L4.1 In the distribution 0–20: 8, 20–40: f_1, 40–60: 15, 60–80: f_2, 80–100: 7, the total frequency is 60 and the mean is 46. Find f_1 and f_2.

L4.2 (a) Section A of 40 students has mean marks 52; Section B of 60 students has mean marks 47. Find the combined mean. (b) The combined mean of 100 students is 49, and one group of 40 has mean 52. Find the mean of the other group.

L4.3 In the distribution 0–10: 3, 10–20: 7, 20–30: f, 30–40: 9, 40–50: 5, the median is 29. Find f.

LEVEL 5 — MATHEMATICAL GENIUS (olympiad flavour)

L5.1 Prove that the step-deviation formula gives the same mean for EVERY choice of the assumed mean a. (Hint: work with two assumed means a and a + k, or expand $\Sigma f_i u_i$ back into $\Sigma f_i x_i$.) Then explain, in two lines, why this independence is what makes the method safe to use.

L5.2 A distribution has classes 0–10, 10–20, 20–30, 30–40, 40–50 with frequencies 5, x, 12, y, 6. Its total frequency is 50, its mean is 25.2 and its median is 25. Find x and y, and verify your answer using all three given facts.

> **TOPPER TIP:** *Stuck at a level? Section 07 contains a worked twin of almost every question here. Find the twin, study its table, close the book, and rebuild the table from memory.*

12 Mathematical Reasoning

JUSTIFY AND PROVE — FULL SOLUTIONS

REASONING 1

Show, with a numerical example and then in general, what happens to the mean, median and mode when (a) every observation is increased by 5, and (b) every observation is doubled.

1. Take the data 10, 14, 14, 20, 22: mean = 80/5 = 16, median = 14, mode = 14.
2. (a) Add 5: the data becomes 15, 19, 19, 25, 27. New mean = 105/5 = 21 = 16 + 5; new median = 19 = 14 + 5; new mode = 19 = 14 + 5. ALL THREE increase by exactly 5.
3. (b) Double: the data becomes 20, 28, 28, 40, 44. New mean = 160/5 = 32 = 2 × 16; new median = 28 = 2 × 14; new mode = 28 = 2 × 14. ALL THREE double.
4. In general: adding k to every observation adds k to the total n·k, so the mean rises by k; the ORDER of the data is unchanged, so the middle value (median) and the most frequent value (mode) simply ride along by +k.

   Why: The median and mode depend on position and repetition, not on size — shifting every value shifts them identically.
5. Multiplying every observation by k multiplies the total by k, so the mean scales by k; order and repetition patterns are again unchanged, so the median and mode scale by k too.
6. The same conclusions hold for grouped data, because every class mark shifts or scales in exactly the same way.

ANSWER: Adding a constant k shifts the mean, median and mode by k; multiplying by k scales all three by k. (This is why converting temperatures from °C to °F shifts all three measures predictably.)

REASONING 2

Two students group the SAME raw data and get the same table: 0–10: 2 observations, 10–20: 1 observation. Student A claims the mean is exactly 8.33. Show that the grouped table cannot decide the true mean, and state the correct conclusion.

1. The grouped method gives (2 × 5 + 1 × 15)/3 = 25/3 = 8.33 — using class marks 5 and 15.
2. But the raw data might have been {2, 8, 12}: it groups to the same table, yet its true mean is 22/3 = 7.33.

Reasoning — continued

REASONING 3

A distribution has mean 25 and mode 31. (a) Estimate its median. (b) Explain why the word 'estimate' is compulsory, using the electricity data of Ex 13.3 Q1 as evidence.

1. (a) From the empirical relation, Mode = 3 Median − 2 Mean, so 31 = 3 Median − 50, giving 3 Median = 81 and Median = **27**.
2. (b) The relation is Pearson's OBSERVATION about many real data sets, not a proved identity. In Ex 13.3 Q1 the three measures were computed exactly: Mode = 135.77, Mean = 137.05, Median = 137.
3. Feed the exact mode and mean into the relation: predicted median = (135.77 + 2 × 137.05)/3 = 409.87/3 = 136.62.
4. The prediction (136.62) misses the true median (137) by 0.38. A theorem cannot miss; an empirical rule can. Hence 'estimate'.

   Why: For a perfectly symmetric distribution the relation is exact, because all three measures coincide.

ANSWER: (a) Median ≈ 27. (b) The empirical relation is approximate — verified by the 136.62 vs 137 gap in real NCERT data.

TOPPER TIP: *In board answers, the phrase 'by the empirical relation' must appear whenever you use Mode = 3 Median − 2 Mean. It tells the examiner you know the result is approximate — that awareness itself is examined.*

PIN POINT: The empirical relation is a measuring tape, not a law of nature — in real data expect small disagreements (136.62 predicted vs 137 actual in Ex 13.3 Q1), and never use it to 'correct' a value you computed exactly.

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13 Real-Life Applications

8D • APPLY — STATISTICS AT WORK, FULLY SOLVED

APPLICATION 1 — THE CLASS REPORT CARD

The marks (out of 60) of 40 students are grouped as: 0–10: 3, 10–20: 5, 20–30: 9, 30–40: 11, 40–50: 7, 50–60: 5. The teacher wants one number for the class's overall performance. Find the mean marks.

1. Class marks: 5, 15, 25, 35, 45, 55. Products f_ix_i: 15, 75, 225, 385, 315, 275.
2. $\Sigma f_ix_i = 15 + 75 + 225 + 385 + 315 + 275 = 1290$, and $\Sigma f_i = 40$.
3. Mean = 1290 / 40 = **32.25** marks out of 60.

   Why: Performance questions are sharing questions – totals matter (total marks obtained by the class), so the mean is the right measure.

ANSWER: Mean marks = 32.25 out of 60 — just above the halfway mark; the teacher's one number for the report.

APPLICATION 2 — THE FACTORY FLOOR

A factory counts the units produced per day over 50 days: 100–120: 6 days, 120–140: 9, 140–160: 14, 160–180: 11, 180–200: 6, 200–220: 4. The manager must set the 'normal day' target that the production line should be tuned to. Find the mode.

1. The manager asks for the MOST COMMON day — a mode question. Modal class: 140–160 (f = 14), with l = 140, $f_1 = 14$, $f_0 = 9$, $f_2 = 11$, h = 20.
2. Mode = $140 + [\,(14 - 9) / (28 - 9 - 11)\,] \times 20 = 140 + (5/8) \times 20 = 140 + 12.5 =$ **152.5**.

ANSWER: Modal production = 152.5 units per day — the line should be tuned to about 153 units as the normal day.

APPLICATION 3 — THE SCHOOL HEALTH CHECK

Heights of 60 students (in cm): 140–145: 4, 145–150: 10, 150–155: 18, 155–160: 15, 160–165: 9, 165–170: 4. The school orders sports jerseys in one 'middle' size meant to fit the central student. Find the median height.

1. A 'middle' size is a median question — half the students shorter, half taller. cf column: 4, 14, 32, 47, 56, 60.
2. n/2 = 30; cf first reaches 30 in 150–155, so: l = 150, cf = 14, f = 18, h = 5.
3.

14 Competency-Based Questions

FIVE CASE STUDIES — 20 QUESTIONS, FULLY SOLVED

Case Study 1 — The State Athletics Camp. *A coach records the weights (in kg) of 36 athletes selected for a state camp: 45–50: 4 athletes, 50–55: 8, 55–60: 11, 60–65: 8, 65–70: 5. The camp's diet plan, uniform order and training groups all depend on how these weights are summarised.*

CASE 1 (a)

Find the mean weight of the athletes.

1. Class marks: 47.5, 52.5, 57.5, 62.5, 67.5. Products: 190, 420, 632.5, 500, 337.5; $\Sigma f_i x_i$ = 2080.
2. Mean = 2080 / 36 = **57.78 kg** (approx.).

ANSWER: Mean weight = 57.78 kg.

CASE 1 (b)

Find the median weight.

1. cf column: 4, 12, 23, 31, 36. n/2 = 18; cf first reaches 18 in 55–60, so l = 55, cf = 12, f = 11, h = 5.
2. Median = 55 + [(18 − 12) / 11] × 5 = 55 + 2.73 = **57.73 kg** (approx.).

ANSWER: Median weight = 57.73 kg.

CASE 1 (c)

Find the modal weight.

1. Modal class is 55–60 (f = 11): Mode = 55 + [(11 − 8) / (22 − 8 − 8)] × 5 = 55 + (3/6) × 5 = **57.5 kg**.

ANSWER: Modal weight = 57.5 kg.

CASE 1 (d)

The coach says: 'For the diet plan, any of the three numbers will do.' Is he right? Comment in two lines.

1. The three values — mean 57.78, median 57.73, mode 57.5 — agree to within 0.3 kg, so the distribution is nearly symmetric and the coach is practically right.

Case Study 2 — The Weather Station

A hill-station weather office recorded the noon temperature (in °C) for 40 days of a summer: 15–20: 3 days, 20–25: 7, 25–30: 12, 30–35: 10, 35–40: 6, 40–45: 2. A travel magazine wants a one-line summary of the summer.

CASE 2 (a)

Find the mean noon temperature, using the step-deviation method with a = 27.5.

1. h = 5; u_i values: −2, −1, 0, 1, 2, 3. f_iu_i values: −6, −7, 0, 10, 12, 6, giving $\Sigma f_iu_i = 15$.
2. Mean = 27.5 + (15/40) × 5 = 27.5 + 1.875 = **29.38°C** (approx.).

ANSWER: Mean noon temperature = 29.38°C.

CASE 2 (b)

Find the modal noon temperature.

1. Modal class is 25–30 (f = 12): Mode = 25 + [(12 − 7) / (24 − 7 − 10)] × 5 = 25 + (5/7) × 5 = **28.57°C** (approx.).

ANSWER: Modal temperature = 28.57°C.

CASE 2 (c)

Find the median noon temperature.

1. cf column: 3, 10, 22, 32, 38, 40. n/2 = 20; median class is 25–30 (cf jumps 10 → 22), so Median = 25 + [(20 − 10) / 12] × 5 = 25 + 4.17 = **29.17°C** (approx.).

ANSWER: Median temperature = 29.17°C.

CASE 2 (d)

Arrange the three measures in order and state what the order tells you about the shape of this summer's weather.

1. Mode (28.57) < Median (29.17) < Mean (29.38).
2. Mean > median > mode is the fingerprint of a mild POSITIVE (right) skew: a minority of very hot days (the 40–45 class) drags the mean upward, while the most common day stays pleasant. The magazine's honest one-liner: ‘Most days sat near 29°C, with a few scorchers pulling the average up.’

ANSWER: Mode < Median < Mean — a mildly right-skewed summer.

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Case Study 3 — The Bakery Oven

A bakery sells loaves of bread daily. Over 32 days the sales were: 50–60 loaves: 3 days, 60–70: 6, 70–80: 10, 80–90: 8, 90–100: 5. Unsold loaves are a loss; sold-out shelves are lost customers. The baker must fix one daily baking quantity.

CASE 3 (a)

Find the mean daily sale of loaves.

1. Class marks: 55, 65, 75, 85, 95. Products: 165, 390, 750, 680, 475; $\Sigma f_i x_i$ = 2460 over 32 days.
2. Mean = 2460 / 32 = **76.88 loaves** (approx.).

ANSWER: Mean sale = 76.88 loaves.

CASE 3 (b)

Find the modal daily sale.

1. Modal class is 70–80 (f = 10): Mode = 70 + [(10 − 6) / (20 − 6 − 8)] × 10 = 70 + (4/6) × 10 = **76.67 loaves** (approx.).

ANSWER: Modal sale = 76.67 loaves.

CASE 3 (c)

Find the median daily sale.

1. cf column: 3, 9, 19, 27, 32. n/2 = 16; median class is 70–80 (cf jumps 9 → 19): Median = 70 + [(16 − 9) / 10] × 10 = **77 loaves**.

ANSWER: Median sale = 77 loaves.

CASE 3 (d)

Recommend a daily baking quantity and justify it with one of the three measures.

1. All three measures sit between 76.7 and 77 — bake **77 loaves**.
2. Justification by the MEDIAN: on half of all days, demand was at most 77, and on half it was at least 77 — so 77 splits the loss risk evenly between unsold loaves and sold-out shelves. The mode (76.67) backs the same shelf size: the single most likely demand band centres there.

ANSWER: Bake 77 loaves a day — the median splits the risk, and the mode agrees.

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Case Study 4 — The School Library

A librarian counts the books issued per day over 42 days: 0–5 books: 6 days, 5–10: 10, 10–15: 14, 15–20: 8, 20–25: 4. She is preparing a report for the principal.

CASE 4 (a)

Find the median number of books issued per day.

1. cf column: 6, 16, 30, 38, 42. n/2 = 21; median class is 10–15 (cf jumps 16 → 30): Median = 10 + [(21 − 16) / 14] × 5 = 10 + 1.79 = **11.79 books** (approx.).

ANSWER: Median = 11.79 books per day.

CASE 4 (b)

Find the mean number of books issued per day.

1. Class marks: 2.5, 7.5, 12.5, 17.5, 22.5. Products: 15, 75, 175, 140, 90; $\Sigma f_i x_i$ = 495 over 42 days.
2. Mean = 495 / 42 = **11.79 books** (approx.) — the same as the median to two decimal places, a happy coincidence of this data.

ANSWER: Mean = 11.79 books per day.

CASE 4 (c)

On how many days were fewer than 15 books issued? Which column of your table answers this instantly?

1. Fewer than 15 books means the classes 0–5, 5–10 and 10–15: 6 + 10 + 14 = **30 days**.
2. That total is exactly the CUMULATIVE FREQUENCY at the class 10–15 — 'less than' questions are read straight off the cf column. This is why cf exists.

ANSWER: 30 days — read directly from the cf column.

CASE 4 (d)

Find the mode, and write the principal's one-line summary.

1. Modal class is 10–15 (f = 14): Mode = 10 + [(14 − 10) / (28 − 10 − 8)] × 5 = 10 + (4/10) × 5 = **12 books**.

Case Study 5 — The Fitness App

A fitness app summarises the daily step counts (in thousands) of 50 users: 2–4: 5 users, 4–6: 9, 6–8: 14, 8–10: 12, 10–12: 7, 12–14: 3. The app wants to display a 'community typical day' on its home screen.

CASE 5 (a)

Find the mean daily step count.

1. Class marks: 3, 5, 7, 9, 11, 13 (thousand steps). Products: 15, 45, 98, 108, 77, 39; $\Sigma f_i x_i$ = 382 over 50 users.
2. Mean = 382 / 50 = **7.64 thousand steps** (7,640 steps).

ANSWER: Mean = 7,640 steps.

CASE 5 (b)

Find the median daily step count.

1. cf column: 5, 14, 28, 40, 47, 50. n/2 = 25; median class is 6–8 (cf jumps 14 → 28): Median = 6 + [(25 − 14) / 14] × 2 = 6 + 1.57 = **7.57 thousand steps** (approx.).

ANSWER: Median = 7,570 steps (approx.).

CASE 5 (c)

Find the modal daily step count.

1. Modal class is 6–8 (f = 14): Mode = 6 + [(14 − 9) / (28 − 9 − 12)] × 2 = 6 + (5/7) × 2 = **7.43 thousand steps** (approx.).

ANSWER: Mode = 7,430 steps (approx.).

CASE 5 (d)

Test the empirical relation on this data. How accurate is it here?

1. 3 × Median = 3 × 7.57 = 22.71. Mode + 2 × Mean = 7.43 + 2 × 7.64 = 7.43 + 15.28 = 22.71.
2. The two sides agree to two decimal places (22.714 vs 22.709 before rounding) — this distribution is very close to the mildly-skewed shape Pearson's rule describes. The app can safely display: 'Typical day: about 7,500 steps.'

ANSWER: 3 Median ≈ Mode + 2 Mean (22.71 ≈ 22.71) — the empirical relation is nearly exact for this data.

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15 Olympiad & Foundation Challenges

8D • CHALLENGE — TEN PROBLEMS THAT STRETCH THE CHAPTER

CHALLENGE 1

Find the mean of the first n odd natural numbers — as a formula in n — and evaluate it for n = 15.

1. The first n odd numbers are 1, 3, 5, ..., (2n − 1). Their sum is $\Sigma(2k - 1)$ for k = 1 to n = 2 × (1 + 2 + ... + n) − n = 2 × n(n + 1)/2 − n = n(n + 1) − n = n^2.

   Why: The sum of the first n odd numbers is the perfect square n^2 — draw 1, 3, 5, 7 as L-shaped layers around a square and you will SEE it.

2. Mean = n^2 / n = **n**.

3. For n = 15: mean = **15**. (The middle odd number itself — symmetry strikes again.)

ANSWER: The mean of the first n odd natural numbers is exactly n; for n = 15, it is 15.

CHALLENGE 2

A student computed $\Sigma f_i = 20$ and $\Sigma f_i x_i = 560$ for a distribution, but then discovered that one class mark had been wrongly taken as 45 instead of 54, for a class whose frequency is 2. Find the correct mean.

1. Only the wrong class's contribution changes: it was counted as 2 × 45 = 90 and should be 2 × 54 = 108.

2. Corrected $\Sigma f_i x_i$ = 560 − 90 + 108 = 578. (Equivalently: add back 2 × (54 − 45) = 18.)

   Why: Correct the SUM, not the mean — the mean is rebuilt from the repaired total.

3. Correct mean = 578 / 20 = **28.9**. (The wrong mean was 28.0 — one slip, nearly a full mark of error.)

ANSWER: Correct mean = 28.9.

CHALLENGE 3

A distribution has classes 0–10, 10–20, 20–30, 30–40 with frequencies 8, 12, 12, 9. Two classes tie for the highest frequency. What is the mode? Discuss carefully.

1. Blindly applying the formula to the class 10–20 (f_1 = 12, f_0 = 8, f_2 = 12): Mode = 10 + [(12 − 8) / (24 − 8 − 12)] × 10 = 10 + (4/4) × 10 = **20**.

2.

Olympiad Challenges 4–7

CHALLENGE 4

A distribution has UNEQUAL classes: 0–10: 5, 10–30: 16, 30–60: 21, 60–70: 6. A student announces: 'The modal class is 30–60, it has the highest frequency, 21.' Prove the student wrong and find the mode.

1. Frequency alone cannot compare bars of different widths — a class 30 wide collects more observations simply by being wider. Compare FREQUENCY DENSITY = frequency ÷ class width, rescaled to a standard width of 10:

2. Adjusted frequencies (per width 10): 0–10: 5 × (10/10) = 5; 10–30: 16 × (10/20) = 8; 30–60: 21 × (10/30) = 7; 60–70: 6 × (10/10) = 6.

3. The highest ADJUSTED frequency is 8, so the true modal class is **10–30**, not 30–60. The student's class wins only because it is three times as wide.

   Why: In histogram language: it is the AREA (height × width) that must represent frequency — with unequal widths, bar heights are densities.

4. Now apply the mode formula across the modal class 10–30, using the density-proportional (adjusted) frequencies $f_1 = 8$, $f_0 = 5$, $f_2 = 7$, with $l = 10$ and the class's own width $h = 20$: Mode = $10 + [\,(8 - 5) / (16 - 5 - 7)\,] \times 20 = 10 + (3/4) \times 20 =$ **25**.

   Why: The formula only ever compares RATIOS of the three bars, so feeding it density-proportional heights (8, 5, 7 instead of 0.8, 0.5, 0.7) changes nothing — while the interpolation still runs across the modal class's real width, 20.

ANSWER: The modal class is 10–30 (by frequency density), and the mode is 25. Raw frequency comparisons are meaningless across unequal widths.

CHALLENGE 5

A 'more than' table records: 0 or more: 80 students, 10 or more: 74, 20 or more: 60, 30 or more: 42, 40 or more: 25, 50 or more: 12, 60 or more: 0 (marks out of 60). Find the median marks.

1. Convert to ordinary classes by subtracting consecutive values: 0–10: 80 − 74 = 6; 10–20: 74 − 60 = 14; 20–30: 60 − 42 = 18; 30–40: 42 − 25 = 17; 40–50: 25 − 12 = 13; 50–60: 12 − 0 = 12. Total: 80. (verified)

   Why: 'More than' tables count DOWNWARD — each class frequency is the drop between two consecutive entries.

2.

Olympiad Challenges 8–10

CHALLENGE 8

The mean of 30 observations is 21. The mean of the first 18 of them is 19. Find the mean of the remaining 12 observations.

1. Total of all 30 = 30 × 21 = 630. Total of the first 18 = 18 × 19 = 342.

   Why: Means are just totals in disguise — convert, subtract, convert back.

2. Total of the remaining 12 = 630 − 342 = 288, so their mean = 288 / 12 = **24**.

ANSWER: Mean of the remaining 12 observations = 24.

CHALLENGE 9

The class marks of a distribution are 25, 35, 45, 55, 65 and the class size is 10. (a) Reconstruct the classes. (b) If the frequencies are 3, 7, 9, 5, 2, find the mean.

1. (a) Each class is centred on its mark with width 10, so it extends 5 on either side: 20–30, 30–40, 40–50, 50–60, 60–70.

   Why: Class mark = centre; class = centre ± h/2.

2. (b) $\Sigma f_i x_i$ = 3(25) + 7(35) + 9(45) + 5(55) + 2(65) = 75 + 245 + 405 + 275 + 130 = 1130, over Σf_i = 26.

3. Mean = 1130 / 26 = **43.46** (approx.).

ANSWER: (a) Classes: 20–30, ..., 60–70. (b) Mean = 43.46 (approx.).

CHALLENGE 10

Prove that for any grouped distribution, $\Sigma f_i(x_i - \bar{x}) = 0$. Then verify the result numerically for the distribution 0–10: 2, 10–20: 3, 20–30: 5.

1. Expand the sum: $\Sigma f_i(x_i - \bar{x}) = \Sigma f_i x_i - \bar{x}\Sigma f_i$.

2. But $\bar{x} = \Sigma f_i x_i / \Sigma f_i$ means $\Sigma f_i x_i = \bar{x} \cdot \Sigma f_i$. Substituting: $\Sigma f_i(x_i - \bar{x}) = \bar{x}\Sigma f_i - \bar{x}\Sigma f_i = \mathbf{0}$. ∎

   Why: The algebra behind Section 04's balance picture: the weighted deviations cancel exactly.

3. Numerical check: $\bar{x}$ = (2 × 5 + 3 × 15 + 5 × 25)/10 = 180/10 = 18. Deviations: 5 − 18 = −13, 15 − 18 = −3, 25 − 18 = 7. Weighted: 2(−13) + 3(−3) + 5(7) = −26 − 9 + 35 = **0**. (verified)

ANSWER: $\Sigma f_i(x_i - \bar{x}) = 0$ always — the mean is the exact balance point of the data. ∎

TOPPER TIP: *Challenges 4 and 10 are the examiners' favourites: one tests what a frequency means when widths change; the other is the s[illegible] of the mean. Revisit both on Day 21.*

16

Chapter Mind Map

THE WHOLE CHAPTER ON ONE PAGE — REPRODUCE IT FROM MEMORY

STATISTICS — Grouped Data

One data set, three representatives. Everything hangs from the frequency table.

MEAN — the balance point

$\bar{x} = \Sigma f_i x_i / \Sigma f_i$ (direct)

$= a + \Sigma f_i d_i / \Sigma f_i$ (assumed mean)

$= a + (\Sigma f_i u_i / \Sigma f_i) \times h$ (step deviation, equal widths only)

Balance law: $\Sigma f_i (x_i - \bar{x}) = 0$

MEDIAN — the middle value

Build the cf column → find n/2 → median class = first class whose cf reaches n/2

Median = $l + [\,(n/2 - cf) / f\,] \times h$

cf = cf of the class BEFORE the median class

Graphically: the ogives cross at the median

MODE — the most common value

Modal class = highest frequency (density, if widths differ)

Mode = $l + [\,(f_1 - f_0) / (2f_1 - f_0 - f_2)\,] \times h$

f_0 = class before, f_2 = class after the modal class

THE TABLE ITSELF

Class mark x_i = (lower + upper)/2 represents its class → every grouped answer is an ESTIMATE

Gaps between classes? Adjust boundaries by half the gap (Ex 13.1 Q5, Ex 13.3 Q4)

'Less than' / 'more than' data: subtract consecutive entries to recover f

EMPIRICAL RELATION

3 Median = Mode + 2 Mean

Mode = 3 Median − 2 Mean

Exact only when symmetric (all three equal); otherwise an estimate — Pearson's observation

CHOOSING THE MEASURE

Totals & sharing → MEAN

Middle / typical / resistant to extremes → MEDIAN

Most common / stocking / tuning → MODE

Shift by k: all three shift by k.

Scale by k: all three scale by k

TOPPER TIP: *Revise actively: cover this page, draw the map on a blank sheet from memory, then uncover and repair the gaps in red pen. A map you can reproduce is worth ten readings.*

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17 Pin-Point Flash Cards

THE WHOLE CHAPTER IN YOUR POCKET — CUT THEM OUT, CARRY THEM, MASTER THEM

How to use these cards

Cut along the dashed borders and keep the 16 cards in your pocket or a small box. On Smart Memory **Day 3, Day 7 and Day 21**, shuffle the cards and test yourself: read the title, say the pin-point aloud from memory, then flip the card to check. Any card you miss goes back into the box for tomorrow. Two minutes a day keeps this chapter alive all year.

CARD 1 — Class mark

x_i = (lower limit + upper limit) ÷ 2. The class mark REPRESENTS its whole class — that is why grouped answers are estimates.

CARD 2 — Direct mean

$\bar{x} = \Sigma f_i x_i \div \Sigma f_i$. Use it when the numbers are small.

CARD 3 — Assumed mean

$\bar{x} = a + \Sigma f_i d_i \div \Sigma f_i$, where $d_i = x_i - a$. Any a works — the middle class mark works best.

CARD 4 — Step deviation

$\bar{x} = a + (\Sigma f_i u_i \div \Sigma f_i) \times h$, $u_i = (x_i - a)/h$. EQUAL widths only — and never forget the final × h.

Flash Cards 5–10

CARD 5 — Mode formula

Mode = l + [$(f_1 - f_0)$ / $(2f_1 - f_0 - f_2)$] × h. Write the five values down BEFORE substituting.

CARD 6 — The five values of mode

l = lower limit of modal class; f_1 = its frequency; f_0 = class BEFORE; f_2 = class AFTER; h = class width.

CARD 7 — Median formula

Median = l + [(n/2 − cf) / f] × h. Every symbol is a picture: door of the class, crowd before it, share of the room.

CARD 8 — Finding the median class

Build cf. The median class is the FIRST class whose cf reaches n/2. NOT the class with the biggest frequency — that is the modal class.

CARD 9 — The cf in the formula

cf = cumulative frequency of the class BEFORE the median class. The costliest single point in the chapter — guard it.

CARD 10 — Empirical relation

3 Median = Mode + 2 Mean. Given any two measures, the third is one line away — but the answer is an ESTIMATE, exact only for symmetric data.

TOPPER TIP: *Say each card aloud once before testing yourself on it — hearing your own voice state the formula makes the memory stick twice as fast.*

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Flash Cards 11–16

CARD 11 — Symmetric data

If a distribution is symmetric, Mean = Median = Mode, all at the centre. Check the empirical relation there first — it must come out exact.

CARD 12 — The balance law

$\Sigma f_i(x_i - \bar{x}) = 0$. The weighted deviations from the mean cancel exactly — the mean is the balance point.

CARD 13 — Shift and scale

Add k to every value: mean, median, mode all rise by k. Multiply every value by k: all three multiply by k.

CARD 14 — Ogives

Less-than ogive: plot (upper limit, cf). More-than ogive: plot (lower limit, cf from the top). They cross at cf = n/2 — the x-value there is the MEDIAN.

CARD 15 — Gaps between classes

Classes like 118–126, 127–135 have a gap of 1: adjust every boundary by 0.5 (half the gap) BEFORE using the median formula. h becomes 9, not 8.

CARD 16 — Which measure?

Totals and sharing → MEAN. Middle and typical, extremes resisted → MEDIAN. Most common, stocking and tuning → MODE. Let the story choose.

MAKE YOUR OWN CARD

Write one pin-point YOU keep forgetting, in your own words. A card you write yourself is the card you remember best.

_____

_____

MAKE YOUR OWN CARD

One more: the mistake you made most often in this chapter, and the one-line rule that fixes it.

_____

_____

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18 Active Recall

8D • RECALL — CLOSE THE BOOK, THEN ANSWER

How this works: close every other page. Answer all ten questions on rough paper, from memory alone. Only then open the Answer Key. Anything you could not recall is exactly what your Day-3 revision must target — write it in the Error Tracker (Section 19).

Ten rapid questions

R1. Write the class mark of the class 30–40.

R2. If $\Sigma f_i = 25$ and $\Sigma f_i x_i = 600$, what is the mean?

R3. Find the mode of the ungrouped data: 5, 7, 5, 9, 5, 11.

R4. Find the median of the ungrouped data: 4, 8, 15, 16, 23, 42.

R5. A distribution has mean 40 and median 38. Estimate its mode.

R6. For the classes 0–10, 10–20, 20–30, what is h?

R7. In the median formula, exactly which cf is used?

R8. If a = 50, h = 10, $\Sigma f_i u_i = -15$ and $\Sigma f_i = 30$, find the mean.

R9. True or false: the mean of grouped data is always the exact mean of the raw data. Justify in one line.

R10. If the mean, median and mode of a distribution are all equal, what is special about its shape?

Five recall prompts — say each answer aloud, in a full sentence

P1. Define the class mark of a class interval.

P2. In the mode formula, state what f_1, f_0 and f_2 stand for.

P3. In the median formula, state what l, cf, f and h stand for.

P4. State the empirical relation among the three measures.

P5. Explain in one line why the assumed-mean method gives the same answer as the direct method.

> **TOPPER TIP:** *Scoring: 13–15 correct — chapter mastered, move on. 9–12 — revise the missed topics, retake in 3 days. Below 9 — re-study Sections 05–07, then retake. Record your score in the Mastery Tracker.*

PIN POINT: Recall first, check later. Peeking at the key before answering feels like revision but builds nothing — the struggle to remember IS the revision.

19 Self-Assessment & Mastery Tracker

KALVIYOGI SMART MEMORY SYSTEM — LEARN IT, THEN KEEP IT

Memory fades on a predictable curve — unless you revisit at the right moments. This schedule is your revisit plan for THIS chapter. Tick each day off when done.

Day	Your activity for Chapter 13	Done
Day 0	Pre-learning assessment: attempt the Quick Check (Section 02) and Rapid Questions R1–R10 (Section 18) BEFORE studying. Note your score.	☐
Day 1	Learn the chapter fully (Sections 01–10) and complete Level 1–3 practice, tables written in full.	☐
Day 3	Without-book recall: reproduce the Mind Map (Section 16) on a blank page; redo the Rapid Questions; run the Flash Cards; finish Level 4.	☐
Day 7	Independent problem-solving: Level 5 + Olympiad Challenges, timed, book closed. Then take the **Day 7 Mini-Test** (Practice Plus).	☐
Day 14	Mixed practice: re-attempt every question in your Error Tracker; solve Ex 13.3 Q1 and Q6 fresh, all three measures each.	☐
Day 21	Long-term recall test: write the median and mode formulas WITH the meaning of every symbol, from memory; redo Challenges 4 and 10. Then take the **Day 21 Mini-Test** (Practice Plus).	☐
Day 30	Chapter mastery assessment: take the **Day 30 Mastery Test** (Practice Plus) under exam conditions; update your Concept Mastery Score below.	☐

Formula Recall Tracker — tick when you can say it with the book closed

Formula / Statement	Day 1	Day 3	Day 7	Day 21
Direct mean: $\bar{x} = \Sigma f_i x_i / \Sigma f_i$	☐	☐	☐	☐
Assumed mean: $\bar{x} = a + \Sigma f_i d_i / \Sigma f_i$	☐	☐	☐	☐
Step deviation: $\bar{x} = a + (\Sigma f_i u_i / \Sigma f_i) \times h$	☐	☐	☐	☐
Mode: $l + [(f_1 - f_0)/(2f_1 - f_0 - f_2)] \times h$	☐	☐	☐	☐
Median: $l + [(n/2 - cf)/f] \times h$	☐	☐	☐	☐
Empirical relation: 3 Median = Mode + 2 Mean	☐	☐	☐	☐

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Mastery Tracker — Error Log, Concept Scores and Time

Error Tracker — every error you make, logged and killed

Date	Question	What went wrong	The rule that fixes it

Concept Mastery Score — rate yourself honestly, 0 (lost) to 5 (can teach it)

Concept	Day 1	Day 7	Day 21	Day 30
Mean — all three methods, and choosing between them				
Mode — modal class, five values, formula				
Median — cf table, median class, formula				
Less-than / more-than data and ogives				
Empirical relation and choosing the right measure				
Missing-frequency problems (mean and median)				

Learning Time Tracker

Session	Date	Minutes	What I finished
First study (Sections 01–07)			
Practice (Sections 11–15)			
Day 7 review + mini-test			
Day 21 review + mini-test			
Day 30 mastery test			

A rising line of scores — not any single day's mark — is the real certificate of this chapter. If a score dips, the Error Tracker tells you exactly which table column to repair.

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Practice Plus 25 CONCEPT MCQs

ONE MARK EACH — ANSWERS AND EXPLANATIONS IN THE KEY

Q1. The mean of the first five multiples of 3 is (a) 3(b) 9(c) 12(d) 15

Q2. If $\Sigma f_i = 40$ and $\Sigma f_i x_i = 880$, the mean is (a) 20(b) 22(c) 24(d) 44

Q3. The mode of 4, 6, 4, 8, 4, 9, 4 is (a) 4(b) 6(c) 8(d) 9

Q4. The median of 3, 7, 5, 9, 11 is (a) 3(b) 5(c) 7(d) 9

Q5. If the mean of a distribution is 34 and its mode is 40, its median is (a) 34(b) 36(c) 37(d) 38

Q6. The class mark of the class 20–30 is (a) 20(b) 30(c) 25(d) 50

Q7. In the step-deviation method, if a = 60, h = 10, $\Sigma f_i u_i = -18$ and $\Sigma f_i = 30$, the mean is (a) 46(b) 54(c) 58.2(d) 66

Q8. If the mode of a distribution is 36 and its median is 30, its mean is (a) 27(b) 30(c) 33(d) 36

Q9. For the classes 0–10: 5, 10–20: 9, 20–30: 14, 30–40: 8, 40–50: 4, the median class is (a) 10–20(b) 30–40(c) 20–30(d) 40–50

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MCQs 10–17

Q10. In the median formula for grouped data, cf denotes (a) the frequency of the median class(b) the cumulative frequency of the class preceding the median class(c) the total frequency(d) the cumulative frequency of the median class

Q11. The median of the distribution 0–10: 4, 10–20: 6, 20–30: 10, 30–40: 5 is (a) 20(b) 22.5(c) 25(d) 27.5

Q12. The empirical relation between the three measures is (a) Mode = 3 Median − 2 Mean(b) Mode = 2 Median − 3 Mean(c) Mode = 3 Mean − 2 Median(d) Mode = Median + 2 Mean

Q13. Using the assumed-mean method with a = 100, if $\Sigma f_i d_i = -50$ and $\Sigma f_i = 25$, the mean is (a) 98(b) 102(c) 100(d) 50

Q14. If every observation of a data set is increased by 7, then the mean (a) stays the same(b) increases by 7(c) decreases by 7(d) increases by 49

Q15. In the mode formula, the denominator is (a) $2f_1 - f_0 - f_2$(b) $f_1 - f_0 - f_2$(c) $f_0 + f_1 + f_2$(d) $2f_0 - f_1 - f_2$

Q16. The point where the less-than ogive and the more-than ogive intersect gives the (a) mean(b) median(c) mode(d) range

Q17. A ‘less than’ table directly gives, for each class boundary, the (a) cumulative frequency(b) class frequency(c) class mark(d) modal class

MCQs 18–25

Q18. The mean of 10 numbers is 15. If each number is multiplied by 3, the new mean is (a) 5(b) 15(c) 18(d) 45

Q19. For the classes 5–15, 15–25, 25–35, the class size h is (a) 10(b) 5(c) 15(d) 20

Q20. For any distribution, $\Sigma f_i(x_i - \bar{x})$ equals (a) 1(b) Σf_i(c) 0(d) $\bar{x}$

Q21. The median of 2, 5, 7, 9, 12 is (a) 5(b) 7(c) 9(d) 12

Q22. The mean of the distribution 0–10: 2, 10–20: x, 20–30: 4 is 16. The value of x is (a) 10(b) 12(c) 14(d) 16

Q23. Which measure of central tendency is affected MOST by extreme values? (a) Mean(b) Median(c) Mode(d) All equally

Q24. For a perfectly symmetric distribution, (a) mean > median > mode(b) mode > median > mean(c) mean < median(d) mean = median = mode

Q25. The modal class of a grouped distribution is the class with the (a) highest frequency(b) highest class mark(c) largest width(d) highest cumulative frequency

> **TOPPER TIP:** *Finished all 25? Time yourself on a second attempt after Day 7 — board MCQs allow about 90 seconds each, and Statistics MCQs are won or lost on the cf column and the five values of the mode.*

Assertion–Reason Questions — 10

Directions: (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is NOT the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.

AR1. Assertion: The mean of grouped data is only an estimate of the true mean. **Reason:** Every observation in a class is represented by the class mark of that class.

AR2. Assertion: The step-deviation method can be applied to the absentee record of Ex 13.1 Q8. **Reason:** The step-deviation method requires all classes to have the same width.

AR3. Assertion: If each observation is increased by 10, the median also increases by 10. **Reason:** Adding a constant shifts every value, including the middle value, by the same constant.

AR4. Assertion: The median class is the class with the highest frequency. **Reason:** The median class is the first class whose cumulative frequency reaches n/2.

AR5. Assertion: For symmetric data, the empirical relation gives the exact common value of the three measures. **Reason:** For symmetric data, mean = median = mode.

AR6. Assertion: The two ogives of a distribution intersect at the median. **Reason:** At the intersection point, the less-than cumulative frequency and the more-than cumulative frequency are both equal to n/2.

AR7. Assertion: Mode = 3 Median − 2 Mean is a theorem that holds exactly for every data set. **Reason:** Karl Pearson observed this relation approximately in moderately skewed distributions.

AR8. Assertion: In the mode formula, f_0 is the frequency of the class preceding the modal class. **Reason:** In the mode formula, f_2 is the frequency of the class following the modal class.

AR9. Assertion: $\Sigma f_i(x_i - \bar{x}) = 0$ for every grouped distribution. **Reason:** By definition, $\bar{x} = \Sigma f_i x_i / \Sigma f_i$, so $\Sigma f_i x_i = \bar{x} \cdot \Sigma f_i$.

AR10. Assertion: Choosing a different assumed mean a changes the value of the mean. **Reason:** The correction term $\Sigma f_i d_i / \Sigma f_i$ adjusts exactly for whichever a was chosen.

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Find the Error — 10 Student Solutions to Repair

FIND THE ERROR 1

Question: Find the mean of 10–20: 4, 20–30: 6, 30–40: 5. **Student's work:** 'x_i values are 10, 20, 30. $\Sigma f_i x_i$ = 40 + 120 + 150 = 310. Mean = 310/15 = 20.67.'

Your task: find the mistake, name it, and write the corrected solution in your notebook. (Correction in the Answer Key.)

FIND THE ERROR 2

Question: By step deviation, a = 50, h = 20, $\Sigma f_i u_i = -15$, $\Sigma f_i = 25$. Find the mean. **Student's work:** 'Mean = 50 + (−15/25) = 49.4.'

Your task: find the mistake, name it, and write the corrected solution in your notebook. (Correction in the Answer Key.)

FIND THE ERROR 3

Question: Find the median of 0–10: 5, 10–20: 8, 20–30: 7, 30–40: 4. **Student's work:** 'cf column: 5, 13, 20, 24. n/2 = 12, median class is 10–20. Median = 10 + ((12 − 13)/8) × 10 = 8.75.'

Your task: find the mistake, name it, and write the corrected solution in your notebook. (Correction in the Answer Key.)

FIND THE ERROR 4

Question: Find the median of 0–10: 12, 10–20: 5, 20–30: 8, 30–40: 5. **Student's work:** 'The median class is 0–10, the class with the highest frequency. Median = 0 + ((15 − 0)/12) × 10 = 12.5.'

Your task: find the mistake, name it, and write the corrected solution in your notebook. (Correction in the Answer Key.)

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Find the Error — continued

FIND THE ERROR 5

Question: By the assumed-mean method, $a = 100$, $\Sigma f_i d_i = -250$, $\Sigma f_i = 50$. Find the mean.

Student's work: 'Mean = 100 + 250/50 = 105.'

Your task: find the mistake, name it, and write the corrected solution in your notebook. (Correction in the Answer Key.)

FIND THE ERROR 6

Question: Find the mode when the modal class is 40–50 with $f_1 = 14$, $f_0 = 9$, $f_2 = 11$.

Student's work: 'Mode = 40 + (14 − 9)/(28 − 9 − 11) = 40.63.'

Your task: find the mistake, name it, and write the corrected solution in your notebook. (Correction in the Answer Key.)

FIND THE ERROR 7

Question: Find the mode when the modal class is 20–30 with $f_1 = 15$, $f_0 = 6$, $f_2 = 10$.

Student's work: 'Mode = 20 + ((15 − 10)/(30 − 10 − 6)) × 10 = 23.57.'

Your task: find the mistake, name it, and write the corrected solution in your notebook. (Correction in the Answer Key.)

FIND THE ERROR 8

Question: Find the median of 118–126: 3, 127–135: 5, 136–144: 9, 145–153: 4. **Student's work:** 'Median class is 136–144. Median = 136 + ((10.5 − 8)/9) × 8 = 138.22.'

Your task: find the mistake, name it, and write the corrected solution in your notebook. (Correction in the Answer Key.)

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Find the Error — last two

FIND THE ERROR 9

Question: Find the mode when the modal class is 30–40 with $f_1 = 18$, $f_0 = 12$, $f_2 = 8$.

Student's work: 'Mode = 30 + ((18 − 12)/(18 + 12 + 8)) × 10 = 31.58.'

Your task: find the mistake, name it, and write the corrected solution in your notebook. (Correction in the Answer Key.)

FIND THE ERROR 10

Question: A table says: marks less than 10: 3 students, less than 20: 8, less than 30: 15, less than 40: 20. Find the mean (classes 0–10, ..., 30–40). **Student's work:** 'f values are 3, 8, 15, 20. $\Sigma f_i x_i$ = 15 + 120 + 375 + 700 = 1210, Σf_i = 46, mean = 26.3.'

Your task: find the mistake, name it, and write the corrected solution in your notebook. (Correction in the Answer Key.)

TOPPER TIP: *Notice the family resemblance: Errors 2 and 6 are the same slip (× h forgotten) in two different formulas; Errors 3 and 4 are both cf confusions. Your Error Tracker should group errors by FAMILY, not by question.*

Previous Years' Board Questions (PYQs) — verified papers

Every question below is from an actual CBSE Class 10 board paper; the year and set were verified against published board papers and official-style solutions. Solve them under exam conditions first — then compare with the stepped solutions.

PYQ 1 — CBSE 2020 (Delhi Set 1) • 5 marks

The median of the following distribution is 525 and the total frequency is 100. Find the missing frequencies x and y: 0–100: 2, 100–200: 5, 200–300: x, 300–400: 12, 400–500: 17, 500–600: 20, 600–700: y, 700–800: 9, 800–900: 7, 900–1000: 4.

1. Total frequency: 2 + 5 + x + 12 + 17 + 20 + y + 9 + 7 + 4 = 76 + x + y = 100, so **x + y = 24**. … (1)
2. The cf column (in terms of x): 2, 7, 7 + x, 19 + x, 36 + x, 56 + x, … Since the median is 525, the median class is 500–600, with l = 500, cf = 36 + x, f = 20, h = 100, n/2 = 50.
3. Median formula: 525 = 500 + [(50 − (36 + x)) / 20] × 100.
4. 25 = (14 − x) × 5, so 14 − x = 5 and **x = 9**.
5. From (1): y = 24 − 9 = **15**. Check: the cf before the median class is 36 + 9 = 45 ≤ 50, and the median 525 lies inside 500–600. (verified)

ANSWER: x = 9 and y = 15.

CBSE MARKING SCHEME: relation x + y = 24 (1) • correct cf table and median class (1) • substitution in the median formula (1) • x = 9 (1) • y = 15 (1)

PYQ 2 — CBSE 2024 • 1 mark

The mean of the first n natural numbers is 15. Find n.

1. Mean of 1, 2, …, n is (n + 1)/2. Set (n + 1)/2 = 15, so n + 1 = 30 and **n = 29**.

ANSWER: n = 29.

PYQ 3 — CBSE 2024 (Set 30/5/1) • 1 mark

The observations 2, 9, x + 6, 2x + 3, 5, 10, 5 (arranged in ascending order) have mean 7. Find x.

1. Sum = 2 + 9 + (x + 6) + (2x + 3) + 5 + 10 + 5 = 40 + 3x. Mean = (40 + 3x)/7 = 7, so 40 + 3x = 49 and **x = 3**.

ANSWER: x = 3.

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PYQs — continued

PYQ 4 — CBSE 2023 • 1 mark

If the mean of a distribution is 21 and its median is 23, find its mode.

1. Mode = 3 Median − 2 Mean = 3(23) − 2(21) = 69 − 42 = **27** (by the empirical relation).

ANSWER: Mode = 27.

PYQ 5 — CBSE 2023 (Set 30/1/1) • 1 mark

Candidates were given the less-than cumulative frequency table of the marks of 80 students and asked for the modal class. State the official answer and the method.

1. First convert the less-than table into ordinary class frequencies by subtracting consecutive cumulative frequencies.
   Why: 'Less than' tables never show the modal class directly — the conversion is the whole question.
2. The class with the highest converted frequency is the modal class. Official answer: **30–40**.

ANSWER: Modal class = 30–40 (after converting the less-than table to class frequencies).

PYQ 6 — CBSE 2020 • 1 mark

If the mode of a distribution is 15x and its mean is 18x, find its median in terms of x.

1. 3 Median = Mode + 2 Mean = 15x + 36x = 51x, so Median = **17x**.

ANSWER: Median = 17x.

PYQ 7 — CBSE 2022 • 1 mark

If the mode and the mean of a distribution are both 8, find its median.

1. 3 Median = Mode + 2 Mean = 8 + 16 = 24, so Median = **8** — as expected when two of the three coincide, the third joins them.

ANSWER: Median = 8.

PYQ 8 — CBSE 2024 • Assertion–Reason, 1 mark

Assertion: If the mean and median of a distribution are 10 and 11 respectively, then its mode is 13. Reason: Mode = 3 Median + 2 Mean.

Chapter Model Test A — 20 marks · 45 minutes

Instructions: Attempt all questions. Show the full working table wherever a mean, median or mode is computed — table marks are real marks. No calculator shortcuts in the working: write every total.

Q1. (1 mark) If the mean of a distribution is 18 and its mode is 24, its median is (a) 18(b) 20(c) 21(d) 24

Q2. (1 mark) The class mark of the class 45–55 is (a) 45(b) 50(c) 55(d) 100

Q3. (2 marks) Find the mean of the distribution: 0–10: 2, 10–20: 5, 20–30: 8, 30–40: 3, 40–50: 2.

Q4. (3 marks) Find the median of the distribution: 0–20: 5, 20–40: 8, 40–60: 12, 60–80: 6, 80–100: 4.

Q5. (5 marks) For the distribution 30–40: 4, 40–50: 7, 50–60: 10, 60–70: 6, 70–80: 3, find (a) the mean, (b) the median, (c) the mode.

Q6. (3 marks) The mean of the distribution 0–10: 3, 10–20: f, 20–30: 6, 30–40: 4 is 20. Find f.

Q7. (5 marks) Daily wages (in ₹) of workers in a workshop: 100–150: 6 workers, 150–200: 10, 200–250: 14, 250–300: 8, 300–350: 2. Find (a) the median wage, (b) the modal wage.

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Chapter Model Test B — 20 marks · 45 minutes

Instructions: Attempt all questions. Show the full working table wherever a mean, median or mode is computed.

Q1. (1 mark) In the step-deviation method, a = 45, h = 5, $\Sigma f_i u_i = 20$ and $\Sigma f_i = 40$. The mean is (a) 42.5(b) 45(c) 47.5(d) 50

Q2. (1 mark) The median class of a grouped distribution is the first class for which the cumulative frequency (a) is maximum(b) reaches or crosses n/2(c) equals the frequency(d) is zero

Q3. (2 marks) Find the mean of the distribution 0–20: 4, 20–40: 6, 40–60: 10, 60–80: 5, using the assumed-mean method with a = 50.

Q4. (3 marks) Find the mode of the distribution: 5–15: 4, 15–25: 11, 25–35: 9, 35–45: 6.

Q5. (5 marks) A survey reports: marks less than 20: 7 students, less than 40: 19, less than 60: 32, less than 80: 40. (a) Convert this into an ordinary frequency distribution. (b) Find the median marks.

Q6. (3 marks) A group of 25 students has mean marks 40. Another group of 35 students has mean marks 46. Find the mean marks of all 60 students combined.

Q7. (5 marks) For the distribution 10–20: 3, 20–30: 5, 30–40: 2: (a) find the mean; (b) verify that $\Sigma f_i(x_i - \bar{x}) = 0$ for your answer.

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Chapter Model Test C — 20 marks · 45 minutes

Instructions: Attempt all questions. Show the full working table wherever a mean, median or mode is computed.

Q1. (1 mark) The abscissa (x-coordinate) of the point where the two ogives of a distribution intersect is the (a) mean(b) mode(c) median(d) class mark

Q2. (1 mark) If the mode of a distribution is 45 and its mean is 39, its median is (a) 39(b) 41(c) 42(d) 45

Q3. (2 marks) Find the median of the distribution: 100–200: 4, 200–300: 6, 300–400: 9, 400–500: 5.

Q4. (3 marks) Find the mean and the mode of the distribution: 0–10: 5, 10–20: 8, 20–30: 12, 30–40: 7, 40–50: 3.

Q5. (5 marks) In the distribution 0–20: 8, 20–40: f_1, 40–60: 15, 60–80: f_2, 80–100: 7, the total frequency is 60 and the mean is 46. Find f_1 and f_2.

Q6. (3 marks) In the distribution 0–10: 3, 10–20: 7, 20–30: f, 30–40: 9, 40–50: 5, the median is 29. Find f.

Q7. (5 marks) (a) The mean of 30 observations is 21 and the mean of the first 18 of them is 19. Find the mean of the remaining 12 observations. (b) A distribution has median 34 and mode 40. Estimate its mean.

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Smart Memory Mini-Tests — Day 7, Day 21, Day 30

Take each test on its scheduled day (see the Smart Memory timetable, Section 19), book closed, timed. Keys are in the Answer Key.

DAY 7 MINI-TEST — 5 marks

1. (1) Write the class mark of the class 60–70.

2. (2) Find the mean of: 0–10: 3, 10–20: 7, 20–30: 5.

3. (2) Find the mode of the data: 3, 5, 3, 7, 3, 9, 3, 11.

DAY 21 MINI-TEST — 5 marks

1. (2) Find the mode of: 10–20: 5, 20–30: 11, 30–40: 8.

2. (3) Find the median of: 0–10: 4, 10–20: 6, 20–30: 8, 30–40: 2. (Careful: watch what the cf column does at the second class.)

DAY 30 MASTERY TEST — 10 marks

1. (3) Find the mean of 0–20: 5, 20–40: 9, 40–60: 12, 60–80: 7, 80–100: 2 by the step-deviation method.

2. (3) Find the mode of: 100–120: 6, 120–140: 13, 140–160: 9, 160–180: 4.

3. (4) For the distribution 0–15: 4, 15–30: 10, 30–45: 8, 45–60: 6, 60–75: 2, find the mean, the median and the mode, and state whether the empirical relation holds exactly for this data.

My Test Record

Test	Date taken	Score	Main error family
Model Test A			
Model Test B			
Model Test C			
Day 7 Mini-Test			
Day 21 Mini-Test			
Day 30 Mastery Test			

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Answer Key — Quick Check, Graded Practice, Reasoning Practice

Section 02 — Quick Check

1. 9 2. 7 3. 2 4. 15 5. h = 10.

Section 11 — Level 1

L1.1 Mean = 835/35 = **23.86**. **L1.2** Mean = 890/30 = **29.67**. **L1.3** Mode = 10 + (5/11) × 5 = **12.27**. **L1.4** Median = 30 + (3.5/9) × 10 = **33.89**. **L1.5** Mode = 3(28) − 2(30) = **24**. **L1.6** h = 22 − 12 = **10**; classes: **7–17, 17–27, 27–37, 37–47, 47–57**.

Section 11 — Level 2

L2.1 Mean = 325 − 150/40 = **₹321.25**. **L2.2** Mean = 50 + (13/50) × 20 = **55.2**. **L2.3** Mean = 4665/37 = **126.08**; Mode = 120 + (5/8) × 10 = **126.25** — the mode is (slightly) larger. **L2.4** Classes from the less-than table: 0–20: 6, 20–40: 9, 40–60: 14, 60–80: 12, 80–100: 9; Median = 40 + (10/14) × 20 = **54.29**. **L2.5** (570 + 25f)/(24 + f) = 24 gives **f = 6**. **L2.6** Median class must be 20–30 with the middle value at its centre point: (4 + f + 9 + 6 + 3)/2 = 11 + f/2 and 20 + ((11 + f/2 − 4 − f)/9) × 10 = 25 gives **f = 5**.

Section 11 — Level 3

L3.1 Mode = 10 + (2/5) × 5 = **12 mm**. **L3.2** Median = 30 + (7/14) × 10 = **35 passengers**. **L3.3** Mean = 1256/40 = **31.4°C**. **L3.4** Mean = 1280/50 = **25.6**; Mode = 20 + (4/9) × 10 = **24.44**. The mode better represents a typical hour: it names the crowd level that actually occurs most often, while the mean is pulled upward by a few very busy hours. (Either answer earns the mark with a sound reason.)

Section 11 — Level 4

L4.1 $f_1 + f_2 = 30$ and $30f_1 + 70f_2 = 1400$ give $\mathbf{f_1 = 20, f_2 = 10}$. **L4.2** (a) (40 × 52 + 60 × 47)/100 = **49**. (b) (100 × 49 − 40 × 52)/60 = 2820/60 = **47**. **L4.3** Median 29 inside 20–30 forces 20 + ((12 + f/2 − 10)/f) × 10 = 29, giving **f = 5**.

Section 11 — Level 5

L5.1 Model proof: with $u_i = (x_i - a)/h$ we have $x_i = a + hu_i$; hence $\Sigma f_i x_i = a\Sigma f_i + h\Sigma f_i u_i$, and dividing by Σf_i gives $\bar{x} = a + h(\Sigma f_i u_i/\Sigma f_i)$ with the SAME $\bar{x}$ for every a — the a cancels identically, so the choice of a can never corrupt the answer; it only changes the arithmetic's size. **L5.2** Total frequency: x + y = 50 − 23 = 27. Mean: $\Sigma f_i x_i = 595 + 15x + 35y = 25.2 \times 50 = 1260$, so 15x + 35y = 665, i.e. 3x + 7y = 133. Solving with x + y = 27: **x = 14, y = 13**. Verification of all three facts: $\Sigma f_i = 5 + 14 + 12 + 13 + 6 = 50$ (verified); $\Sigma f_i x_i = 25 + 210 + 300 + 455 + 270 = 1260$, mean = 25.2 (verified); cf column 5, 19, 31, … gives median class 20–30 and median = 20 + ((25 − 19)/12) × 10 = 25 (verified).

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Answer Key — Active Recall and MCQs

Section 18 — Rapid Questions

R1 35. **R2** 600/25 = 24. **R3** 5 (it occurs three times). **R4** (15 + 16)/2 = 15.5. **R5** Mode = 3(38) − 2(40) = 34. **R6** h = 10. **R7** The cf of the class PRECEDING the median class. **R8** 50 + (−15/30) × 10 = 45. **R9** False — the grouped mean is an estimate, because class marks replace the unknown raw values. **R10** The distribution is symmetric.

P1 The class mark is the midpoint of a class, (lower limit + upper limit)/2, which represents all observations of the class. **P2** f_1 = frequency of the modal class; f_0 = frequency of the class before it; f_2 = frequency of the class after it. **P3** l = lower limit of the median class; cf = cumulative frequency of the class before the median class; f = frequency of the median class; h = class width. **P4** 3 Median = Mode + 2 Mean. **P5** Because $d_i = x_i - a$ shifts every value by the same a, and the formula adds back exactly the average shift — the algebra of Derivation 1.

Practice Plus — MCQ key with explanations

Q1 (b) 3, 6, 9, 12, 15 sum to 45; 45/5 = 9. **Q2 (b)** 880/40 = 22. **Q3 (a)** 4 occurs four times. **Q4 (c)** Sorted: 3, 5, 7, 9, 11; middle is 7. **Q5 (b)** Median = (40 + 2 × 34)/3 = 36. **Q6 (c)** (20 + 30)/2 = 25. **Q7 (b)** 60 + (−18/30) × 10 = 54. **Q8 (a)** Mean = (3 × 30 − 36)/2 = 27. **Q9 (c)** cf: 5, 14, 28; n/2 = 20 first crossed in 20–30. **Q10 (b)** By definition of the median formula. **Q11 (b)** n = 25, n/2 = 12.5; median class 20–30; 20 + (2.5/10) × 10 = 22.5. **Q12 (a)** The empirical relation. **Q13 (a)** 100 + (−50/25) = 98 — option (a); note (c) 100 is the trap for forgetting the correction. **Q14 (b)** Shift property. **Q15 (a)** $2f_1 - f_0 - f_2$. **Q16 (b)** The ogives cross at the median. **Q17 (a)** 'Less than' counts are cumulative frequencies. **Q18 (d)** Scale property: 15 × 3 = 45. **Q19 (a)** 15 − 5 = 10. **Q20 (c)** The balance law. **Q21 (b)** Middle value 7. **Q22 (c)** (110 + 15x)/(6 + x) = 16 gives x = 14. **Q23 (a)** The mean uses every value's size. **Q24 (d)** All three coincide at the centre. **Q25 (a)** By definition (for equal widths; density for unequal).

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Answer Key — Assertion–Reason and Find-the-Error

Assertion–Reason key

AR1 (a) — the class-mark representation is exactly why the grouped mean is an estimate. **AR2 (d)** — Q8's classes are unequal, so A is false; R is true. **AR3 (a)** — the shift property, explained by R. **AR4 (d)** — A confuses median class with modal class; R is the correct rule. **AR5 (a)** — with all three equal, 3M = M + 2M holds exactly. **AR6 (a)** — the crossing happens precisely where both cf's equal n/2, i.e., at the median. **AR7 (d)** — the relation is empirical, not a theorem; R states its true status. **AR8 (b)** — both statements are correct definitions, but the meaning of f_2 does not explain the meaning of f_0. **AR9 (a)** — R is exactly the step that proves A (Challenge 10). **AR10 (d)** — A is false: Derivation 1 proves the mean is independent of a; R is true.

Find-the-Error — corrections

E1. The student used the LOWER LIMITS as class marks. Correct x_i: 15, 25, 35; $\Sigma f_i x_i$ = 60 + 150 + 175 = 385; mean = 385/15 = **25.67**.

E2. The final × h was forgotten. Mean = 50 + (−15/25) × 20 = 50 − 12 = **38**.

E3. cf must be of the class BEFORE the median class (5), not of the median class itself (13). Median = 10 + ((12 − 5)/8) × 10 = **18.75**. (The student's 8.75 lies outside the median class — the sanity check would have caught it.)

E4. The student picked the MODAL class. cf: 12, 17, 25, 30; n/2 = 15, so the median class is 10–20. Median = 10 + ((15 − 12)/5) × 10 = **16**.

E5. The minus sign of $\Sigma f_i d_i$ was dropped. Mean = 100 + (−250/50) = **95**.

E6. The fraction was never multiplied by h = 10. Mode = 40 + (5/8) × 10 = **46.25**.

E7. f_0 and f_2 were swapped. Mode = 20 + ((15 − 6)/(30 − 6 − 10)) × 10 = 20 + (9/14) × 10 = **26.43**.

E8. The classes have a gap of 1, so the boundaries must be adjusted by 0.5: the median class is 135.5–144.5 and h = 9. Median = 135.5 + ((10.5 − 8)/9) × 9 = **138**.

E9. The denominator must be $2f_1 - f_0 - f_2$ = 16, not $f_0 + f_1 + f_2$. Mode = 30 + (6/16) × 10 = **33.75**.

E10. The student used the CUMULATIVE frequencies as class frequencies. True f values: 3, 8 − 3 = 5, 15 − 8 = 7, 20 − 15 = 5. $\Sigma f_i x_i$ = 15 + 75 + 175 + 175 = 440 over 20 students; mean = **22**.

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Answer Key — Model Tests and Mini-Tests (with marking splits)

Model Test A

Q1 (b) 20. **Q2 (b)** 50. **Q3** $\Sigma f_i x_i = 480$, mean = **24** (table 1, answer 1). **Q4** cf: 5, 13, 25, 31, 35; median class 40–60; median = 40 + (4.5/12) × 20 = **47.5** (cf table 1, substitution 1, answer 1). **Q5** (a) $\Sigma f_i x_i = 1620$, mean = **54** (2). (b) Median class 50–60; median = 50 + (4/10) × 10 = **54** (1.5). (c) Mode = 50 + (3/7) × 10 = **54.29** (1.5). **Q6** (305 + 15f)/(13 + f) = 20 gives **f = 9** (equation 1.5, answer 1.5). **Q7** (a) cf: 6, 16, 30, 38, 40; median class 200–250; median = 200 + (4/14) × 50 = **₹214.29** (3). (b) Mode = 200 + (4/10) × 50 = **₹220** (2).

CBSE-STYLE MARKING SCHEME applied throughout Test A.

Model Test B

Q1 (c) 47.5. **Q2 (b)**. **Q3** $\Sigma f_i d_i = -180$; mean = 50 − 180/25 = **42.8** (table 1, answer 1). **Q4** Mode = 15 + (7/9) × 10 = **22.78** (values 1, substitution 1, answer 1). **Q5** (a) Classes 0–20: 7, 20–40: 12, 40–60: 13, 60–80: 8 (2). (b) cf: 7, 19, 32, 40; median class 40–60; median = 40 + (1/13) × 20 = **41.54** (3). **Q6** (25 × 40 + 35 × 46)/60 = 2610/60 = **43.5** (totals 2, answer 1). **Q7** (a) $\Sigma f_i x_i = 240$, mean = **24** (2). (b) 3(15 − 24) + 5(25 − 24) + 2(35 − 24) = −27 + 5 + 22 = **0** (verified) (3).

CBSE-STYLE MARKING SCHEME applied throughout Test B.

Model Test C

Q1 (c). **Q2 (b)** 41. **Q3** cf: 4, 10, 19, 24; median class 300–400; median = 300 + (2/9) × 100 = **322.22** (cf 1, answer 1). **Q4** Mean = 825/35 = **23.57** (1.5); Mode = 20 + (4/9) × 10 = **24.44** (1.5). **Q5** $f_1 + f_2 = 30$ and $30f_1 + 70f_2 = 1400$ give $\mathbf{f_1 = 20, f_2 = 10}$ (equations 2 + 2, answers 1). **Q6 f = 5** (median-class reasoning 1.5, answer 1.5). **Q7** (a) (630 − 342)/12 = **24** (3). (b) Mean = (3 × 34 − 40)/2 = **31** (2).

CBSE-STYLE MARKING SCHEME applied throughout Test C.

Mini-Tests

Day 7: 1. 65. 2. $\Sigma f_i x_i = 245$, mean = 245/15 = **16.33**. 3. Mode = **3** (occurs four times). **Day 21:** 1. Mode = 20 + (6/9) × 10 = **26.67**. 2. cf: 4, 10, 18, 20; n/2 = 10 is reached EXACTLY at the end of 10–20, so the median sits on the boundary: Median = 10 + ((10 − 4)/6) × 10 = **20**. **Day 30:** 1. a = 50, h = 20: $\Sigma f_i u_i = -8$; mean = 50 − (8/35) × 20 = **45.43**. 2. Mode = 120 + (7/11) × 20 = **132.73**. 3. Mean = 1005/30 = **33.5**; Median = 30 + (1/8) × 15 = **31.88**; Mode = 15 + (6/8) × 15 = **26.25**. Check: 3 Median = 95.63 but Mode + 2 Mean = 93.25 — NOT equal: the relation is empirical and this data is strongly skewed, so it holds only roughly.

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Chapter 13 Revision Card — tear out for the exam week

The three formulas

Mean: $\bar{x} = \Sigma f_i x_i / \Sigma f_i = a + \Sigma f_i d_i / \Sigma f_i = a + (\Sigma f_i u_i / \Sigma f_i) \times h$ (equal widths only)

Median: $l + [\,(n/2 - cf) / f\,] \times h$ — cf of the class BEFORE; median class = first cf to reach n/2

Mode: $l + [\,(f_1 - f_0) / (2f_1 - f_0 - f_2)\,] \times h$ — f_0 before, f_2 after

Empirical: 3 Median = Mode + 2 Mean (an estimate, exact only when symmetric)

Method choice in ten seconds

Small numbers → direct. Big class marks → assumed mean. Big marks + equal widths → step deviation. Unequal widths → direct, patiently. 'Less than' data → subtract first. Gaps between classes → adjust boundaries by half the gap first.

The three sanity checks (run them on EVERY answer)

1. Mean and median inside the data's range. 2. Median inside the MEDIAN class. 3. Mode inside the MODAL class. A failed check means a table error — find it before the examiner does.

Exam-eve checklist

☐ I can write all three formulas with every symbol's meaning. ☐ I write the five mode values before substituting. ☐ I never use the cf of the median class itself. ☐ I check class widths before step deviation. ☐ I write 'by the empirical relation' when I use it. ☐ My Flash Cards (Section 17) are all green.

Chapter 13 complete. You now own the frequency table — every column of it, and every trap inside it. Keep the cards in your pocket; the Smart Memory days will do the rest.

Next: Chapter 14 — Probability, where counting outcomes replaces counting frequencies, and the same discipline — full tables, justified steps, verified answers — turns chance itself into arithmetic.

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CBSE · CLASS 10 · MATHEMATICS

MATHEMATICS BEYOND THE FORMULA

CHAPTER 14 · PROBABILITY

Don't Memorize Mathematics. Understand, Visualise and Discover It.

The KalviYogi 8D Way: Discover · Visualise · Understand · Derive · Solve · Apply · Challenge · Recall

Every solution with COMPLETE steps — one by one, nothing skipped

The Master Formula built by counting · Coins, Dice & Cards · Complementary Events

The full Two-Dice Grid · Five-Level Practice · Smart Memory System

Written like a topper's own notebook — so you can learn

even when no teacher is beside you.

Based on the latest rationalised NCERT textbook · Simple English · Full Colour

Next Generation Guide · First Edition

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01 Chapter Introduction

8D • DISCOVER — WHY SHOULD I LEARN THIS?

"There is a **70% chance of rain** today." "The captain wins the **toss** and chooses to bat." "You need a **six** to win the board game." Chance runs your day — but until now it was just luck and guessing. This chapter turns luck into **mathematics**: you will learn to measure chance with an exact number between 0 and 1.

Here is the beautiful secret of this chapter: there is only **ONE master formula**, and you will not memorise it — you will **build it yourself** in Section 06 by counting real coins and dice with your own hands. After that, every question in the chapter — coins, dice, cards, marbles, spinners — is the same two steps: **count all outcomes, count favourable outcomes, divide.**

What you will master in this chapter

- What an **outcome**, an **event** and **equally likely** really mean — and why equally likely is the licence that lets us count.
- The master formula P(E) = favourable ÷ total — **derived** from coins and dice, never just stated.
- **Sure events** (probability 1) and **impossible events** (probability 0) — and why every probability lives in between.
- The **complement rule**: P(not E) = 1 − P(E) — proved by counting, and used to crack "at least once" questions in seconds.
- Coins: 2, 4 and 8 outcomes. Dice: 6 outcomes, and the full **36-outcome two-dice grid**.
- A deck of 52 cards — suits written as words (Spades, Hearts, Diamonds, Clubs), 12 face cards — counted without fear.
- Draws **without replacement**, where the total changes for the second draw.

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Your Chapter Map — 19 Stops, One Journey

Tick each stop as you finish it. The last chapter of the book is also the most scoring one — Probability questions are short, exact and fully in your control.

Discover & Visualise	Understand & Derive	Solve, Apply, Recall
☐ 01 Chapter Introduction ☐ 02 Prerequisite Knowledge ☐ 03 Real-Life Story ☐ 04 Visual Concepts (CPA)	☐ 05 Mathematical Foundations ☐ 06 Formula Derivations ☐ 08 Multiple Solution Methods ☐ 10 Common Mistakes	☐ 07 Detailed Solved Examples ☐ 09 Hands-on Activities ☐ 11 Graded Practice (5 Levels) ☐ 12 Mathematical Reasoning
☐ 13 Real-Life Applications ☐ 14 Competency-Based Questions	☐ 15 Olympiad Challenges ☐ 16 Chapter Mind Map	☐ 17 Pin-Point Flash Cards ☐ 18 Active Recall ☐ 19 Mastery Tracker

How to use this chapter (read this first!)

1. **Never read a solution passively.** Cover the steps, try yourself, then uncover one step at a time.
2. Every solution shows **Step 1, Step 2, Step 3…** with a small *Why* note in purple — the reason behind the step. Examiners award marks for the counting steps, not just the final fraction.
3. After Section 07, test yourself in Sections 11–15 **before** looking at the Answer Key.
4. Finish with the Mind Map (16), the Flash Cards (17), the closed-book Recall (18) and your Mastery Tracker (19). Come back on Day 3, 7, 14, 21 and 30 — the Smart Memory schedule tells you exactly what to do each day.

KALVIYOGI DIGITAL COMPANION: In the print edition, a QR code at this spot links to the Chapter 14 video lessons, the two-dice simulation and the online test pack. (Placeholder — the QR code and link activate with the KalviYogi app/website at print. No link is printed in this digital preview.)

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02 Prerequisite Knowledge

CLASS 9 RECAP — THE TOOLS YOU ALREADY OWN

In Class 9 you already met probability — the **experimental** kind, found by actually repeating an experiment. This chapter needs that idea, plus honest fraction skills. Refresh five tools here.

1. Experimental (empirical) probability — Class 9

P(E) = (number of trials in which E happened) ÷ (total number of trials). Example: a coin tossed 500 times lands head 243 times, so the experimental P(head) = 243/500. This chapter builds the **theoretical** twin of this formula — predicted by counting, without doing the experiment 500 times.

2. Fractions in lowest terms

Every probability is a fraction, and board answers must be in **lowest terms**: divide numerator and denominator by their HCF. Example: 12/52 = 3/13 (dividing by 4). Practise this until it is automatic — half the "silly mistakes" in this chapter are unreduced fractions.

3. Decimals and percentages

Probabilities also wear decimal and percent clothes: 0.05 = 5% = 1/20, and 15% = 15/100 = 3/20. NCERT Q4 and Q5 test exactly this switching — be fluent in all three forms.

4. Counting outcomes of coins and dice

One coin: 2 outcomes (H, T). One die: 6 outcomes (1–6). Two coins tossed together: the outcomes are HH, HT, TH, TT — note carefully that **HT and TH are different outcomes**. This single observation decides many exam questions.

5. "Part out of whole" thinking

Probability is always **a part divided by the whole**: favourable outcomes are the part, all possible outcomes are the whole. Because a part can never exceed the whole, a probability can never exceed 1 — and it can never be negative. Hold on to this picture; Section 06 turns it into the formula.

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Quick Check — Are You Ready? (5 Questions)

Try all five without looking back. Answers are given right below — cover them with your hand first!

Q1. A coin is tossed 100 times and head appears 47 times. What is the experimental probability of getting a head?

Q2. Write the fraction 12/52 in its lowest terms.

Q3. Write 0.25 as a fraction in lowest terms.

Q4. A die is thrown 60 times and the number 6 appears 9 times. What is the experimental probability of getting a 6?

Q5. In 200 throws of a die, the number 3 appears 28 times. What is the experimental probability of getting a 3?

Answers — check only after trying

Q1. 47/100 = 0.47.

Q2. Divide numerator and denominator by 4: 12/52 = **3/13**.

Q3. 0.25 = 25/100 = **1/4**.

Q4. 9/60 = **3/20**.

Q5. 28/200 = **7/50**.

TOPPER TIP: *Notice: every experimental answer is close to the theoretical value you will meet soon (1/2 for a head, 1/6 for a die face). The more trials, the closer the two agree — that is why the theoretical formula works.*

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03 Real-Life Mathematical Story

DISCOVER • VISUALISE — THE FETE STALL AND GRANDFATHER'S DICE

Before any formula, live the problem. Read this story slowly — the mathematics will walk out of the story by itself.

*It is the annual **school fete**, and **Meera** of Class 10 is running the Lucky Dip stall. Her jar holds **20 marbles** — **8 red** and **12 blue** — all the same size, well mixed. A player closes their eyes, picks ONE marble, and wins a prize if it is red.*

*Meera's younger brother **Nikhil** watches three players pick blue, blue, blue. He whispers, "Akka, red almost never comes. Blue is luckier!" Meera smiles. "It is not luck, kanna. There are simply **more blue marbles**. Out of every 20 picks, about 8 should be red — the red share of the jar. Shall we count it exactly?"*

*That evening at home, their **grandfather** brings out two old dice for a game of Ludo. "In my sixty years of playing," he says, "one total has served me best. **Seven.** Seven comes again and again, more than any other total. That is not luck either — that is mathematics, and one day you will prove it."*

Meera decides to prove both claims tonight: the red share of her jar, and grandfather's mysterious seven.

Two claims. Two counting problems. Let us settle both, one step at a time.

Solving the Story — Problem 1: The Lucky Dip Jar

STORY PROBLEM 1

Meera's jar has 20 marbles: 8 red and 12 blue, all alike, well mixed. One marble is picked with eyes closed. What is the probability that it is red? And that it is blue?

THINK FIRST: Meera's sentence 'the red share of the jar' is the whole chapter in five words: probability = the share of outcomes an event owns.

1. Count ALL the possible outcomes first: any of the 20 marbles can be picked, so the total number of outcomes is **20**.

   Why: Probability always starts with the whole — never with the colour you are hoping for.

2. Count the outcomes favourable to 'red': there are **8** red marbles, so 8 outcomes favour red.

   Why: Each single marble is one outcome. We count marbles, not colours.

3. Divide favourable by total: P(red) = 8/20 = **2/5**.

   Why: This division IS the master formula of this chapter — Section 06 will build it properly.

4. In the same way, P(blue) = 12/20 = **3/5**.

5. Sanity check: 2/5 + 3/5 = 1. Every pick is either red or blue, so the two shares must add to the whole jar.

   Why: A probability split that does not add to 1 is a warning bell — always run this check.

ANSWER: P(red) = 2/5 and P(blue) = 3/5. Nikhil was right that blue comes more often — not because blue is lucky, but because blue owns a bigger share of the jar: 12 marbles against 8.

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Solving the Story — Problem 2: Grandfather's Seven

STORY PROBLEM 2

Two dice are thrown together. Grandfather claims the total 7 comes more often than any other total. Prove or disprove his claim.

THINK FIRST: Whenever a game feels 'lucky', ask the chapter's question: how many outcomes does each possibility own?

1. Count ALL outcomes: the first die has 6 faces and the second die has 6 faces, and every pairing can occur, so total outcomes = 6 × 6 = **36**.
   Why: The pairs are ordered: (2, 5) and (5, 2) are two different outcomes, because the dice are two different objects.
2. List the pairs that give a total of 7: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) — that is **6** favourable outcomes.
   Why: Work along the diagonal: as the first die climbs 1 to 6, the second falls 6 to 1. Six pairs, no more, no less.
3. So P(total 7) = 6/36 = **1/6**.
4. Compare with the extreme total 2: only (1, 1) works, so P(total 2) = 1/36. And total 6 has 5 pairs: (1, 5), (2, 4), (3, 3), (4, 2), (5, 1), so P(total 6) = 5/36.
   Why: No total can beat 7: the counts rise 1, 2, 3, 4, 5, 6 up to 7 and fall 5, 4, 3, 2, 1 after it. Section 04 shows the full grid.
5. Conclusion: 7 owns the largest share of the 36 outcomes — 6 of them. Grandfather's sixty years of Ludo agree exactly with the count.

ANSWER: Grandfather is RIGHT: P(total 7) = 1/6, the highest probability of any total. Seven is not lucky — it is simply the total with the most outcomes.

The two story lessons — pin these in your memory

Lesson 1: Probability is a SHARE. An event's probability = (outcomes it owns) ÷ (all outcomes).
Lesson 2: Count single outcomes, never types. 20 marbles are 20 outcomes (not "2 colours"); 36 dice pairs are 36 outcomes (not "11 totals"). Nearly every error in this chapter — and in the board exam — is a failure of Lesson 2.

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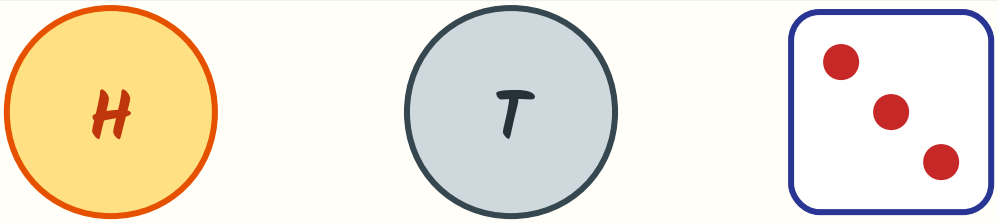
04 Visual Concept Explanation

SINGAPORE CPA — CONCRETE → PICTORIAL → ABSTRACT

CONCRETE — hold the experiment in your hand

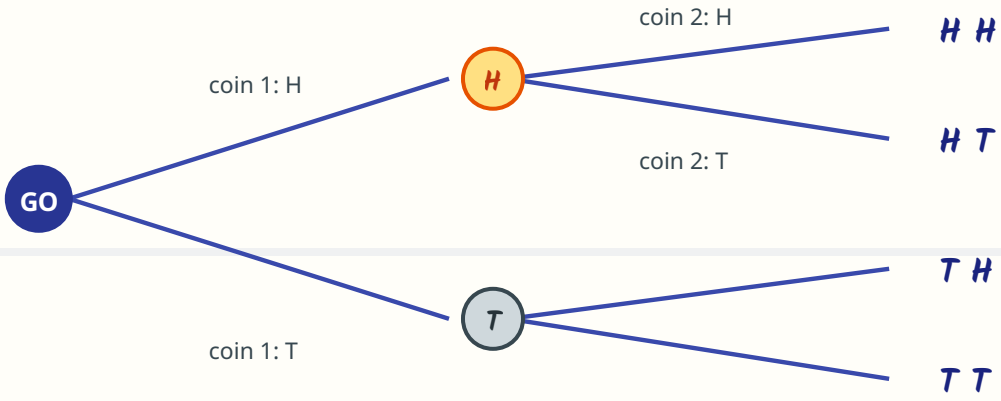
A fair coin has two faces, Head and Tail, and no reason to prefer either. A fair die has six faces, 1 to 6, all alike. “Fair” means: **every single outcome has the same chance as every other**. That sameness is the foundation of the whole chapter.



Head | Tail | a die face: 3

PICTORIAL 1 — two coins as a tree

Toss two coins together. Follow the branches: the first coin splits into H or T; from each, the second coin splits again. The tree ends in exactly **4 outcomes: HH, HT, TH, TT**. Read the tree once and you will never again merge HT and TH into one “mixed” outcome.



PICTORIAL 2 — The Two-Dice Grid (all 36 outcomes)

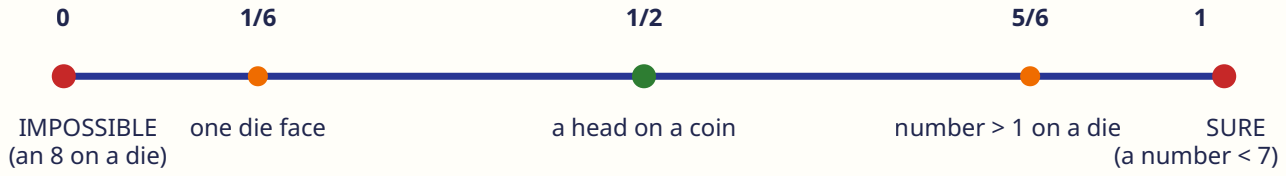
Each cell is ONE outcome: (row = first die, column = second die), and the number inside is the total. The yellow diagonal is grandfather's seven — count its cells: **6**, more than any other total. This grid is the picture behind every two-dice question in the chapter.

Die 1 \ Die 2	1	2	3	4	5	6
1	2	3	4	5	6	**7**
2	3	4	5	6	**7**	8
3	4	5	6	**7**	8	9
4	5	6	**7**	8	9	10
5	6	**7**	8	9	10	11
6	**7**	8	9	10	11	12

Total	2	3	4	5	6	**7**	8	9	10	11	12
Outcomes	1	2	3	4	5	**6**	5	4	3	2	1
Probability	1/36	2/36	3/36	4/36	5/36	**6/36**	5/36	4/36	3/36	2/36	1/36

The counts climb 1, 2, 3, 4, 5, 6 to the middle total 7, then fall symmetrically. The 11 totals are **NOT** equally likely — only the 36 cells are. NCERT Q22 is built entirely on this picture.

PICTORIAL 3 — every probability lives on one line



ABSTRACT — from pictures to the one formula

The master formula

P(E) = (Number of outcomes favourable to E) ÷ (Number of all possible outcomes)

Valid when — and only when — all outcomes are **equally likely**. (Laplace, 1795. You will DERIVE it in Section 06, not just copy it.)

Reading the formula off the pictures

Coin: event 'Head' owns 1 of the 2 outcomes → P = 1/2. **Die:** event 'even' owns 3 of the 6 faces {2, 4, 6} → P = 3/6 = 1/2. **Two dice:** event 'total 7' owns 6 of the 36 cells → P = 6/36 = 1/6. Same formula, three costumes.

The deck of cards, pictured as a table (learn this anatomy by heart)

Suit (name)	Colour	Cards in the suit (13)	Face cards in the suit (3)
Spades	Black	Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King	Jack, Queen, King — so 4 suits × 3 = **12 face cards** in the deck. Aces are NOT face cards.
Clubs	Black		
Hearts	Red		
Diamonds	Red		

Total: 4 suits × 13 cards = **52 cards**; 26 red and 26 black; 4 cards of every rank (4 Kings, 4 Aces, ...). Every card question in this chapter is a counting question about this table.

PIN POINT: Suit symbols are never needed — in this book the four suits are always written as WORDS: Spades, Hearts, Diamonds, Clubs. If a question says 'red card', it means Hearts or Diamonds: 26 cards.

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05 Mathematical Foundations

UNDERSTAND — THE PRECISE IDEAS, WORD BY WORD

1. Experiment, outcome, sample space

An **experiment** is an action whose result is not known in advance (tossing a coin). Each possible result is an **outcome**. The set of ALL outcomes is the **sample space**: for a die, {1, 2, 3, 4, 5, 6}; for two coins, {HH, HT, TH, TT}. The size of the sample space is the denominator of every probability in that experiment.

2. Event and elementary event

An **event** is a collection of outcomes we are watching for ('getting an even number' = {2, 4, 6}). An event with exactly ONE outcome is an **elementary event** ('getting a 4'). The number of outcomes inside the event is the numerator — the "favourable outcomes".

3. Equally likely outcomes

Outcomes are **equally likely** when each has the same chance as every other — true for a fair coin, a fair die, a well-shuffled deck, a well-mixed jar. It is NOT true for 'a car starts or does not start' or 'a basketball shot scores or misses' (skill and condition interfere). The master formula is licensed ONLY by equally likely outcomes.

4. Sure event and impossible event

An event that is **certain** to happen ('a number less than 7 on a die') is a **sure event**; its probability is **1**. An event that **cannot** happen ('an 8 on a die') is an **impossible event**; its probability is **0**. Every other event sits strictly between: **$0 \le P(E) \le 1$**, always.

5. Complementary events

The event **'not E'** (also written E′) collects every outcome that is NOT in E. E and 'not E' are **complementary events**: between them they cover the whole sample space with no overlap. Section 06 proves from this covering that **$P(E) + P(\text{not } E) = 1$**.

Foundations — The Counting Tables You Will Reuse All Chapter

Experiment	Total outcomes	Sample space / counting key
One coin	**2**	H, T
Two coins	**4**	HH, HT, TH, TT (HT and TH are different!)
Three coins	**8**	HHH, HHT, HTH, THH, HTT, THT, TTH, TTT
One die	**6**	1, 2, 3, 4, 5, 6 — primes {2, 3, 5}, composites {4, 6}, 1 is neither
Two dice	**36**	All pairs (a, b); totals counted from the grid in Section 04
One card from 52	**52**	4 suits × 13 ranks; 12 face cards; 26 red, 26 black
One item from a bag/box	**total items**	Count ITEMS, never types: 3 red + 1 blue = 4 outcomes, not 2

The pattern in the coin rows: each extra coin doubles the outcomes — 2, 4, 8, … = 2^n for n coins. Each extra die multiplies by 6: two dice give $6^2 = 36$.

PIN POINT: Before writing any probability, write the total outcomes FIRST and say why: '36, because 6 × 6 pairs'. Examiners give a mark for the total — and a wrong total poisons everything after it.

TOPPER TIP: *The phrase 'at random' in a question is the examiner whispering: 'every single outcome is equally likely — you may count.' Well-shuffled, well-mixed and fair mean the same thing.*

06 Formula Derivations

8D • DERIVE — NEVER MEMORISE WHAT YOU CAN DERIVE

Three derivations build the whole chapter: the master formula itself, the two boundary values 0 and 1, and the complement rule. Each is built by **counting something physical first** — that is the Singapore CPA promise of this book.

Derivation A — Building P(E) from equally likely outcomes

DERIVE

Why is P(E) = favourable ÷ total? Build it, do not quote it.

1. Start with a fair die in your hand. It has 6 faces, all alike, so in a long run of throws each face must claim the same share of the throws: each face owns **1/6** of all outcomes.

   Why: Fairness means equal shares. If one face owned more, the die would be loaded.

2. So a single outcome of a die has probability 1/6. In general, if an experiment has **n** equally likely outcomes, each single outcome owns **1/n** — because n equal shares must add up to the whole, 1.

   Why: n shares of size s give n × s = 1, so s = 1/n. This is also why the probabilities of all elementary events always sum to 1.

3. Now take an event E that contains **m** outcomes — say 'even number' on a die, with m = 3 outcomes {2, 4, 6}. The event owns m shares of size 1/n each, so P(E) = m × (1/n) = **m/n**. For the die: 3 × 1/6 = 3/6 = 1/2.

   Why: An event's probability is just the total share owned by its outcomes.

4. Replace m and n by their meanings and the master formula appears: **P(E) = (number of outcomes favourable to E) ÷ (number of all possible outcomes)**.

   Why: You have not memorised this formula – you have BUILT it from equal shares. It can never be forgotten, only re-derived.

Derivation B — Sure and impossible events, from the formula

1. Impossible event: E owns NO outcomes, so m = 0 and P(E) = 0/n = **0**. Example: an 8 on a die.

   Why: Zero favourable outcomes is not a trick – the formula handles it directly.

2. Sure event: E owns EVERY outcome, so m = n and P(E) = n/n = **1**. Example: a number less than 7 on a die.

3. Every other event owns some but not all outcomes: 0 < m < n, so 0 < P(E) < 1. Conclusion for life: **0 ≤ P(E) ≤ 1** — any answer outside this range is automatically wrong.

Derivation C — The Complement Rule, Proved by Counting

DERIVE

Prove that P(not E) = 1 − P(E), using nothing but counting.

1. Take any experiment with n equally likely outcomes, and an event E owning m of them. Example to hold in your hand: a die, E = 'even number', n = 6, m = 3.

   Why: Fixing one concrete example keeps the proof honest.

2. The event 'not E' owns every outcome that E does NOT own: that is n − m outcomes. For the die, 'not even' = odd = {1, 3, 5}: 6 − 3 = 3 outcomes.

   Why: E and not E together cover all n outcomes exactly once — no outcome is in both, none is left out.

3. Apply the master formula to 'not E': P(not E) = (n − m)/n.

4. Split the fraction: (n − m)/n = n/n − m/n = 1 − P(E). Hence **P(not E) = 1 − P(E)**, and equally **P(E) + P(not E) = 1**. ∎

   Why: The rule is not a new formula to memorise — it is the same counting, done on the other side of the fence.

5. Check on the die: P(odd) = 1 − P(even) = 1 − 1/2 = 1/2. Direct count: {1, 3, 5} gives 3/6 = 1/2. Both roads agree.

   Why: Always verify a derived rule on one concrete case before trusting it.

Why the complement rule is a weapon

Questions with the words **"at least once"** hide many favourable outcomes (a 5 on the first die, OR the second, OR both...). But the opposite event — "not even once" — is usually ONE clean count. Compute the easy side and subtract from 1. NCERT Q24 falls to this weapon in Section 07, and Section 08 shows both roads side by side.

PIN POINT: P(not E) = 1 − P(E). Hear 'at least one', think '1 minus none'. This single sentence is worth several marks every board year.

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07 Detailed Solved Examples

8D • SOLVE — NCERT EXERCISE 14.1 (ALL 25 QUESTIONS), EVERY STEP SHOWN

These are the complete NCERT Exercise 14.1 questions of the rationalised textbook, solved the KalviYogi way: total outcomes first, favourable outcomes next, divide, reduce, sanity-check. Q20 carries an asterisk in NCERT ("not from the examination point of view") — we solve it too, because it teaches a beautiful idea.

EX 14.1 Q1

Complete the following statements: (i) Probability of an event E + Probability of the event 'not E' = _____. (ii) The probability of an event that cannot happen is _____. Such an event is called _____. (iii) The probability of an event that is certain to happen is _____. Such an event is called _____. (iv) The sum of the probabilities of all the elementary events of an experiment is _____. (v) The probability of an event is greater than or equal to _____ and less than or equal to _____.

THINK FIRST: Every blank is one of the results derived in Section 06 — recall the derivation, not just the word.

1. (i) E and 'not E' cover all outcomes between them, so by the complement rule their probabilities add to **1**.

   Why: Derivation C, final line.

2. (ii) An event that cannot happen owns 0 outcomes, so its probability is 0/n = **0**; it is called an **impossible event**.

   Why: Derivation B, first step.

3. (iii) An event certain to happen owns all n outcomes, so its probability is n/n = **1**; it is called a **sure (certain) event**.

   Why: Derivation B, second step.

4. (iv) The elementary events are all the single outcomes; each has probability 1/n and there are n of them, so the sum is n × 1/n = **1**.

   Why: Derivation A, second step.

5. (v) Since 0 ≤ m ≤ n in P(E) = m/n, every probability satisfies **0** ≤ P(E) ≤ **1**.

   Why: A part can never be smaller than nothing or bigger than the whole.

ANSWER: (i) 1 (ii) 0; impossible event (iii) 1; sure (certain) event (iv) 1 (v) 0 and 1.

CBSE-STYLE MARKING SCHEME: [1 mark] Half a mark each for (i)–(iii) facts • the two 1's of (i) and (iv) are the most-tested blanks in this exercise.

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EX 14.1 Q2

Which of the following experiments have equally likely outcomes? Explain. (i) A driver attempts to start a car. The car starts or does not start. (ii) A player attempts to shoot a basketball. She/he shoots or misses the shot. (iii) A trial is made to answer a true-false question. The answer is right or wrong. (iv) A baby is born. It is a boy or a girl.

THINK FIRST: Ask one question of each experiment: is there any reason one outcome should happen more often than the other?

1. (i) **Not equally likely.** Whether a car starts depends on its battery, fuel and condition — a healthy car starts almost always, a faulty one almost never.

   Why: Two named outcomes do NOT automatically mean two equal chances.

2. (ii) **Not equally likely.** Scoring depends on the player's skill, distance and defence; a trained player scores far more often than she misses from close range.

   Why: Skill breaks equality.

3. (iii) **Equally likely** — under the textbook's assumption that the answer is a pure guess: right and wrong are symmetric, with no reason to favour either.

   Why: NCERT treats a blind true-false guess as a fair coin with two faces.

4. (iv) **Equally likely** — a baby being a boy or a girl is nature's fair coin; neither outcome has an advantage.

ANSWER: (i) Not equally likely (ii) Not equally likely (iii) Equally likely (iv) Equally likely.

CBSE-STYLE MARKING SCHEME: [1 mark] Half a mark per correct judgement with its reason • the reason earns the mark, not the label alone.

EX 14.1 Q3

Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a football game?

THINK FIRST: Fairness = neither team can gain an advantage. Connect this to equally likely outcomes.

1. A fair coin has exactly two outcomes, Head and Tail.

2. The coin is symmetric and the toss is random, so Head and Tail are **equally likely** — each has probability 1/2.

   Why: No player can influence or predict the toss.

EX 14.1 Q4

Which of the following cannot be the probability of an event? (A) 2/3 (B) −1.5 (C) 15% (D) 0.7

THINK FIRST: Run every option through the boundary rule $0 \le P(E) \le 1$ from Derivation B.

1. (A) $2/3 \approx 0.67$ lies between 0 and 1 — possible.
2. (B) −1.5 is **negative** — a probability counts a share of outcomes and can never be negative. This one is impossible.
   Why: Probability is a share: shares start at 0.
3. (C) 15% = 15/100 = 0.15 lies between 0 and 1 — possible.
   Why: Percentages are probabilities in a different dress: divide by 100.
4. (D) 0.7 lies between 0 and 1 — possible.

ANSWER: Option (B) −1.5 cannot be the probability of an event.

CBSE-STYLE MARKING SCHEME: [1 mark] One mark for option (B); the boundary rule $0 \le P(E) \le 1$ is the whole solution.

EX 14.1 Q5

If P(E) = 0.05, what is the probability of 'not E'?

THINK FIRST: A one-line question — but write the rule, not just the arithmetic; the rule is what the examiner is testing.

1. Write the complement rule: $P(\text{not } E) = 1 - P(E)$.
   Why: Derivation C.
2. Substitute: $P(\text{not } E) = 1 - 0.05 = \mathbf{0.95}$.
3. Sanity check: 0.05 + 0.95 = 1, and 0.95 lies in [0, 1].
   Why: The two complements must always add back to 1.

ANSWER: P(not E) = 0.95.

CBSE-STYLE MARKING SCHEME: [1 mark] Half a mark for the complement rule • half a mark for 0.95.

EX 14.1 Q6

A bag contains lemon-flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out (i) an orange-flavoured candy? (ii) a lemon-flavoured candy?

THINK FIRST: This is Derivation B wearing a story: one part is impossible, the other is sure.

1. (i) The bag contains NO orange-flavoured candy, so the number of favourable outcomes is 0. Whatever the total number of candies n is, P(orange) = 0/n = **0** — an impossible event.

   Why: You do not even need to know how many candies are in the bag.

2. (ii) EVERY candy in the bag is lemon-flavoured, so all n outcomes are favourable: P(lemon) = n/n = **1** — a sure event.

ANSWER: (i) 0 (impossible event) (ii) 1 (sure event).

CBSE-STYLE MARKING SCHEME: [1 mark] Half a mark each • naming the event type (impossible / sure) completes the answer.

EX 14.1 Q7

It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

THINK FIRST: 'Same birthday' and 'not the same birthday' are complements — the rule from Derivation C applies word for word.

1. Let E = '2 students have the same birthday'. Then 'not E' = '2 students do NOT have the same birthday', whose probability is given: P(not E) = 0.992.

   Why: Naming the two events clearly is the whole battle in complement questions.

2. Apply the complement rule: P(E) = 1 − P(not E) = 1 − 0.992 = **0.008**.

ANSWER: The probability that the 2 students have the same birthday is 0.008.

CBSE-STYLE MARKING SCHEME: [1 mark] Half a mark for identifying the complementary pair • half a mark for 0.008.

EX 14.1 Q8

A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is (i) red? (ii) not red?

THINK FIRST: Count balls (items), not colours. Part (ii) can be done two ways — do both and let them check each other.

1. Total outcomes: 3 + 5 = **8** balls, each equally likely to be drawn.
   Why: 'At random' means every single ball has the same chance.
2. (i) Favourable outcomes for red = 3, so P(red) = **3/8**.
3. (ii) Direct count: 'not red' means black, and there are 5 black balls, so P(not red) = **5/8**.
4. (ii) By complement: P(not red) = 1 − P(red) = 1 − 3/8 = 5/8. Both roads agree.
   Why: When two methods agree, your confidence should jump — use this habit in the exam.

ANSWER: (i) P(red) = 3/8 (ii) P(not red) = 5/8.

CBSE-STYLE MARKING SCHEME: [2 marks] 1 mark for the total and P(red) • 1 mark for P(not red) by either road.

EX 14.1 Q9

A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be (i) red? (ii) white? (iii) not green?

THINK FIRST: Part (iii) is a 'not' event: its outcomes are red OR white — count them directly, then cross-check with the complement.

1. Total outcomes: 5 + 8 + 4 = **17** marbles.
2. (i) P(red) = **5/17**. (ii) P(white) = **8/17**.
   Why: 17 is prime, so neither fraction reduces — leave them as they are.
3. (iii) Direct count: not green = red + white = 5 + 8 = 13 marbles, so P(not green) = **13/17**.
4. (iii) Cross-check by complement: P(green) = 4/17, so P(not green) = 1 − 4/17 = 13/17. Same answer.

ANSWER: (i) 5/17 (ii) 8/17 (iii) 13/17.

CBSE-STYLE MARKING SCHEME: [3 marks] 1 mark per part • the total 17 must be written before the parts begin.

EX 14.1 Q10

A piggy bank contains hundred 50p coins, fifty ₹1 coins, twenty ₹2 coins and ten ₹5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin (i) will be a 50p coin? (ii) will not be a ₹5 coin?

THINK FIRST: The total is a sum of four groups — write it as its own step; most errors in this question are born in the total.

1. Total coins = 100 + 50 + 20 + 10 = **180**.
   Why: Add group by group; do not rush this line.
2. (i) Favourable outcomes for a 50p coin = 100, so P(50p) = 100/180 = **5/9** (dividing by 20).
3. (ii) Direct count: coins that are NOT ₹5 = 100 + 50 + 20 = 170, so P(not ₹5) = 170/180 = **17/18** (dividing by 10).
4. (ii) Cross-check: P(₹5 coin) = 10/180 = 1/18, and 1 − 1/18 = 17/18. Agrees.

ANSWER: (i) 5/9 (ii) 17/18.

CBSE-STYLE MARKING SCHEME: [2 marks] 1 mark for the total 180 • half a mark per part, with reduction shown.

EX 14.1 Q11

Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish. What is the probability that the fish taken out is a male fish?

THINK FIRST: One line of counting — but the chapter's discipline still applies: total first.

1. Total outcomes: 5 + 8 = **13** fish.
2. Favourable outcomes for a male fish = 5, so P(male) = **5/13**.
   Why: 13 is prime; the fraction is already in lowest terms.

ANSWER: P(male fish) = 5/13.

CBSE-STYLE MARKING SCHEME: [1 mark] Half a mark for the total • half a mark for 5/13.

EX 14.1 Q12

A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, and these are equally likely outcomes. What is the probability that it will point at (i) 8? (ii) an odd number? (iii) a number greater than 2? (iv) a number less than 9?

THINK FIRST: Four parts, one total. Part (iv) is a sure event in disguise — spot it.

1. Total outcomes = **8** (the numbers 1 to 8, equally likely).
   Why: The equally-likely licence, in the question's own words.
2. (i) Only one sector shows 8: P(8) = **1/8**.
3. (ii) Odd numbers are 1, 3, 5, 7 — 4 outcomes: P(odd) = 4/8 = **1/2**.
4. (iii) Numbers greater than 2 are 3, 4, 5, 6, 7, 8 — 6 outcomes: P(> 2) = 6/8 = **3/4**.
5. (iv) EVERY number on the spinner is less than 9, so all 8 outcomes are favourable: P(< 9) = 8/8 = **1** — a sure event.

ANSWER: (i) 1/8 (ii) 1/2 (iii) 3/4 (iv) 1.

CBSE-STYLE MARKING SCHEME: [2 marks] Half per part • (iv) = sure event.

EX 14.1 Q13

A die is thrown once. Find the probability of getting (i) a prime number; (ii) a number lying between 2 and 6; (iii) an odd number.

THINK FIRST: For each part, LIST the favourable faces first.

1. Total outcomes = **6**: {1, 2, 3, 4, 5, 6}.
2. (i) Prime faces are {2, 3, 5} — 3 outcomes: P(prime) = 3/6 = **1/2**.
   Why: 1 is NOT prime – the classic trap here.
3. (ii) Numbers lying between 2 and 6 are {3, 4, 5} — 3 outcomes: P = 3/6 = **1/2**.
   Why: 'Between 2 and 6' excludes both ends.
4. (iii) Odd faces are {1, 3, 5} — 3 outcomes: P(odd) = 3/6 = **1/2**.

ANSWER: (i) 1/2 (ii) 1/2 (iii) 1/2.

CBSE-STYLE MARKING SCHEME: [3 marks] 1 per part as written out.

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EX 14.1 Q14

One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i) a king of red colour (ii) a face card (iii) a red face card (iv) the jack of Hearts (v) a spade (vi) the queen of Diamonds.

THINK FIRST: Every part is a counting question about the deck anatomy table of Section 04. Say each count in words before dividing.

1. Total outcomes = **52** cards, equally likely after well-shuffling.
2. (i) Red kings: the king of Hearts and the king of Diamonds = 2 cards. P = 2/52 = **1/26**.
   Why: Red suits are Hearts and Diamonds — one king in each.
3. (ii) Face cards: Jack, Queen, King in each of 4 suits = 12 cards. P = 12/52 = **3/13**.
   Why: Aces are NOT face cards.
4. (iii) Red face cards: J, Q, K of Hearts and of Diamonds = 6 cards. P = 6/52 = **3/26**.
5. (iv) The jack of Hearts is exactly ONE card: P = **1/52**.
   Why: A fully named card (rank + suit) is always a single outcome.
6. (v) Spades: one full suit = 13 cards. P = 13/52 = **1/4**.
7. (vi) The queen of Diamonds is exactly ONE card: P = **1/52**.

ANSWER: (i) 1/26 (ii) 3/13 (iii) 3/26 (iv) 1/52 (v) 1/4 (vi) 1/52.

CBSE-STYLE MARKING SCHEME: [3 marks] Half a mark per part • each favourable count (2, 12, 6, 1, 13, 1) must be justified in words.

EX 14.1 Q15

Five cards — the ten, jack, queen, king and ace of Diamonds — are well-shuffled with their face downwards. One card is then picked up at random. (i) What is the probability that the card is the queen? (ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?

THINK FIRST: Part (ii) is WITHOUT REPLACEMENT: after the queen is put aside, the experiment restarts with only 4 cards.

1. (i) Total outcomes = 5 cards; the queen is 1 of them: P(queen) = **1/5**.
2. Now the queen is drawn and PUT ASIDE. The remaining cards are ten, jack, king, ace — total outcomes for the second pick = **4**.
   Why: The total changed 5 → 4 — forgetting this is the most common error here.
3. (ii)(a) The ace is still among the 4 remaining cards: P(ace) = **1/4**.
4. (ii)(b) The only queen is already out of the game, so favourable outcomes = 0: P(queen) = 0/4 = **0** — now an impossible event.
   Why: An event can BECOME impossible after a removal.

ANSWER: (i) 1/5 (ii)(a) 1/4 (ii)(b) 0.

CBSE-STYLE MARKING SCHEME: [2 marks] 1 for (i) • 1 for (ii): new total 4 + both answers.

EX 14.1 Q16

12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

THINK FIRST: NCERT's 'not possible to tell by looking' is the equally-likely guarantee — quote that idea in Step 1.

1. Total pens = 12 + 132 = **144**. Since a defective pen cannot be identified by looking, every pen is equally likely to be taken out.
2. Favourable outcomes (good pen) = 132, so P(good) = 132/144 = **11/12** (dividing by 12).
3. Cross-check: P(defective) = 12/144 = 1/12, and 1 − 1/12 = 11/12. Agrees.

ANSWER: P(good pen) = 11/12.

CBSE-STYLE MARKING SCHEME: [2 marks] 1 for total 24 + equally-likely remark • 1 for 11/12.

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EX 14.1 Q17

(i) A lot of 20 bulbs contains 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective? (ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?

THINK FIRST: Part (ii) restarts the experiment: one good bulb has left the lot forever. Recount BOTH numbers — the total and the favourable count.

1. (i) Total = 20 bulbs; defective = 4: P(defective) = 4/20 = **1/5**.

2. The drawn bulb was NOT defective and is NOT replaced, so it leaves the lot. New total = 20 − 1 = **19** bulbs.

   Why: Without replacement, the sample space shrinks — this is the heart of the question.

3. The 4 defective bulbs are all still inside, so the non-defective bulbs now number 16 − 1 = **15**.

   Why: The removed bulb was a good one, so the good count falls too: 20 − 4 = 16 good bulbs originally, minus the 1 removed.

4. (ii) P(not defective) = **15/19**.

   Why: 19 is prime — the fraction is already in lowest terms.

ANSWER: (i) 1/5 (ii) 15/19.

CBSE-STYLE MARKING SCHEME: [3 marks] 1 mark for (i) • 2 marks for (ii): one for the new total 19, one for the new favourable count 15.

EX 14.1 Q18

A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5.

THINK FIRST: Three counting exercises on the numbers 1 to 90. In (i), count what you want by subtracting what you don't: two-digit = all − one-digit.

1. Total outcomes = **90** discs.

2. (i) One-digit numbers are 1 to 9: 9 discs. Two-digit numbers = 90 − 9 = **81**. P = 81/90 = **9/10**.

   Why: Subtracting the small unwanted group is faster and safer than counting 10, 11, ..., 90 one by one.

3.

EX 14.1 Q19

A child has a die whose six faces show the letters A, B, C, D, E, A. The die is thrown once. What is the probability of getting (i) A? (ii) D?

THINK FIRST: The faces are the outcomes — and the letter A owns TWO faces. Count faces, not distinct letters.

1. Total outcomes = **6** faces.
2. (i) The letter A appears on 2 faces, so favourable outcomes = 2: P(A) = 2/6 = **1/3**.
   Why: Outcomes are faces, not letter-types — the chapter's oldest trap.
3. (ii) The letter D appears on 1 face: P(D) = **1/6**.

ANSWER: (i) 1/3 (ii) 1/6.

CBSE-STYLE MARKING SCHEME: [1 mark] Half a mark per part • the count 'A on 2 faces' is the step that earns (i).

EX 14.1 Q20*

Suppose you drop a die at random on a rectangular region 3 m × 2 m which contains a circle of diameter 1 m. What is the probability that the die will land inside the circle? (NCERT marks this question with an asterisk: not from the examination point of view — but the idea is too beautiful to skip.)

THINK FIRST: When outcomes are points of a region, counting becomes MEASURING: probability = favourable AREA ÷ total AREA.

1. Total area of the rectangle = 3 × 2 = **6 m²**. This plays the role of 'total outcomes'.
   Why: The die is dropped at random, so every point of the rectangle is equally likely — the same licence, in area form.
2. The circle has diameter 1 m, so radius r = 1/2 m. Favourable area = $\pi r^2 = \pi \times (1/2)^2$ = **π/4 m²**.
3. Divide: P(lands inside the circle) = (π/4) ÷ 6 = **π/24**.
4. Sanity check: π/24 ≈ 0.13, which lies in [0, 1] and matches the picture — the small circle owns about one-eighth of the rectangle.
   Why: Even an answer with π in it must pass the 0-to-1 boundary check.

ANSWER: P(inside the circle) = π/24 ≈ 0.13.

CBSE-STYLE MARKING SCHEME: [Not examined, per NCERT] In school tests: 1 mark per area, 1 for the ratio.

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EX 14.1 Q21

A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy it if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that (i) she will buy it? (ii) she will not buy it?

THINK FIRST: Translate the story into events first: 'she will buy' = 'the pen is good'. The story words change; the counting does not.

1. Total pens = **144**. Good pens = 144 − 20 = **124**.
   Why: The good count is not given directly — produce it by subtraction and write it down.
2. (i) She buys exactly when the pen is good: P(buy) = 124/144 = **31/36** (dividing by 4).
3. (ii) She does not buy exactly when the pen is defective: P(not buy) = 20/144 = **5/36** (dividing by 4).
4. Cross-check: 31/36 + 5/36 = 36/36 = 1. The two story events are complements, as they must be.

ANSWER: (i) 31/36 (ii) 5/36.

CBSE-STYLE MARKING SCHEME: [2 marks] 1 mark for the good-pen count 124 • half a mark per probability, reduced.

EX 14.1 Q22 — The Two-Dice Table, Completed

EX 14.1 Q22

Two dice, one blue and one grey, are thrown at the same time. (i) Complete the table of probabilities for the event 'sum on 2 dice' (the counts for sums 2, 7 and 12 are started in NCERT as 1/36, 6/36, 1/36). (ii) A student argues that 'there are 11 possible outcomes 2, 3, 4, ..., 12, so each has probability 1/11'. Do you agree? Justify your answer.

THINK FIRST: Every count comes off the Section 04 grid — read down each diagonal. Part (ii) is the chapter's most important argument; learn to make it yourself.

1. The 36 pairs (blue, grey) are the equally likely outcomes. Count the pairs on each diagonal of the grid: sum 2 → 1, 3 → 2, 4 → 3, 5 → 4, 6 → 5, 7 → 6, 8 → 5, 9 → 4, 10 → 3, 11 → 2, 12 → 1.

   Why: The counts climb to the middle diagonal (sum 7) and fall symmetrically — the Section 04 picture.

2. Divide each count by 36 to complete the table (see below).

3. Sum-check: 1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1 = 36, so the probabilities add to 36/36 = 1, as they must.

   Why: The completed table passes the elementary-events check of Q1(iv) — in total form.

4. (ii) The student's argument is **WRONG**. The 11 sums are NOT equally likely: sum 7 owns 6 of the 36 pairs while sum 2 owns only 1, so P(7) = 6/36 ≠ P(2) = 1/36 ≠ 1/11.

   Why: The master formula may only be applied to equally likely outcomes. The 36 PAIRS are equally likely; the 11 SUMS built from them are not.

5. Correct statement: P(sum = s) = (number of pairs giving s) ÷ 36, read from the table.

ANSWER: Completed table below; the student's 1/11 argument is incorrect because the 11 sums are not equally likely.

CBSE-STYLE MARKING SCHEME: [3 marks] 1 mark for the completed counts • 2 marks for the justification in (ii), which must name the equally-likely condition.

Event: Sum on 2 dice	2	3	4	5	6	7	8	9	10	11	12
Probability	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

PIN POINT: The 36 pairs are equally likely; the 11 sums are NOT. Dividing by TYPES instead of OUTCOMES is always wrong — this one idea answers Q19, Q22 and Q25 together.

EX 14.1 Q23

A game consists of tossing a one-rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result, i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

THINK FIRST: 'Loses' is the complement of 'wins' — count the small side (winning) and subtract from 1.

1. List all outcomes of 3 coin tosses: HHH, HHT, HTH, THH, HTT, THT, TTH, TTT — total = **8** equally likely outcomes.

   Why: Each toss doubles the list: $2 \times 2 \times 2 = 8 = 2^3$. Write them in a fixed order so none is missed or repeated.

2. Hanif WINS on the outcomes where all three agree: HHH and TTT — just **2** outcomes. So P(win) = 2/8 = 1/4.

3. He LOSES on every other outcome. By the complement rule: P(lose) = 1 − P(win) = 1 − 1/4 = **3/4**.

   Why: Direct count agrees: HHT, HTH, THH, HTT, THT, TTH = 6 outcomes, and 6/8 = 3/4.

ANSWER: P(Hanif loses) = 3/4.

CBSE-STYLE MARKING SCHEME: [2 marks] 1 mark for the full list of 8 outcomes • 1 mark for using the complement to get 3/4.

EX 14.1 Q24

A die is thrown twice. What is the probability that (i) 5 will not come up either time? (ii) 5 will come up at least once? [Hint: throwing a die twice and throwing two dice simultaneously are treated as the same experiment.]

THINK FIRST: Work on the 36-pair grid. Part (i) is a clean count; part (ii) is the complement weapon of Derivation C.

1. Total outcomes = 36 pairs (first throw, second throw), all equally likely.

   Why: The hint lets us reuse the two-dice grid of Section 04.

2. (i) '5 will not come up either time' means BOTH throws land in {1, 2, 3, 4, 6} — 5 choices each. Favourable outcomes = 5 × 5 = **25**. So P(no 5 at all) = **25/36**.

   Why: Count the pairs that AVOID 5 entirely: the 5×5 sub-grid left after deleting the row and column of 5s.

3. (ii) '5 comes up at least once' is exactly the complement of '5 does not come up either time'. So P(at least once) = 1 − 25/36 = **11/36**.

   Why: 'At least once' is the complement rule's home ground — direct counting would need three separate cases.

EX 14.1 Q25

Which of the following arguments are correct and which are not correct? Give reasons. (i) If two coins are tossed simultaneously, there are three possible outcomes — two heads, two tails, or one of each. Therefore, for each of these outcomes, the probability is 1/3. (ii) If a die is thrown, there are two possible outcomes — an odd number or an even number. Therefore, the probability of getting an odd number is 1/2.

THINK FIRST: Both arguments make the SAME hidden assumption: that the named outcomes are equally likely. Test that assumption by dropping to the elementary outcomes underneath.

1. (i) The argument is **NOT correct**. The elementary outcomes of two coins are HH, HT, TH, TT — 4 equally likely outcomes. The group 'one of each' secretly contains TWO of them (HT and TH), so the three named groups own shares 1/4, 2/4, 1/4 — not 1/3 each.

   Why: Grouping outcomes does not make the groups equal. Always check what each group contains.

2. So P(two heads) = 1/4, P(two tails) = 1/4, P(one of each) = 2/4 = 1/2.

3. (ii) The argument is **NOT correct as a reason** — even though its final number, 1/2, happens to be right. 'Two possible outcomes' alone never justifies 1/2: a bag with 4 red balls and 1 blue ball also has two possible outcomes (red, blue), yet P(red) = 4/5, not 1/2.

   Why: A correct answer reached by a wrong road earns no marks for reasoning – and the road collapses on the very next question.

4. The CORRECT justification for the die: the elementary outcomes 1–6 are equally likely; the odd group {1, 3, 5} owns 3 of the 6, so P(odd) = 3/6 = 1/2. The value is right because the two groups happen to own EQUAL numbers of elementary outcomes — and that equality, not the count 'two', is the real reason.

ANSWER: (i) Incorrect — the three groups are not equally likely (shares 1/4, 1/2, 1/4). (ii) Incorrect reasoning — P(odd) = 1/2 is true only because the odd group owns 3 of the 6 equally likely faces.

CBSE-STYLE MARKING SCHEME: [2 marks] 1 mark per argument • each verdict must be backed by the elementary-outcome count.

TOPPER TIP: *Q22, Q25 and the letter-die Q19 are one family: the examiner is testing whether you count OUTCOMES or lazily count TYPES. Master this distinction and a full cluster of board questions becomes free marks.*

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08 Multiple Solution Methods

8D • CHALLENGE — ONE PROBLEM, TWO ROADS

A topper does not know more formulas — a topper knows **two roads** to the same answer and uses one to check the other. This chapter has one great pair of roads: **direct counting** and the **complement road**. Watch them race on two problems.

Problem A — P(at least one head when two coins are tossed)

METHOD 1 — DIRECT COUNT

Count the favourable outcomes themselves.

1. All outcomes: HH, HT, TH, TT — total 4.
2. 'At least one head' collects HH, HT, TH — 3 outcomes.

   Why: 'At least one' means one head OR two heads.
3. P = **3/4**.

ANSWER: 3/4.

METHOD 2 — COMPLEMENT ROAD

Count the opposite event and subtract from 1.

1. The opposite of 'at least one head' is 'NO head at all' — only TT: 1 outcome, so P(no head) = 1/4.

   Why: The opposite of a messy event is often a single clean outcome.
2. P(at least one head) = 1 − 1/4 = **3/4**. Same answer, half the listing.

ANSWER: 3/4 — both roads agree.

Problem B — the '5 at least once' race (NCERT Q24 revisited)

METHOD 1 — DIRECT COUNT (three cases)

Count every pair in which 5 appears.

1. Case 1: 5 on the first throw only: (5,1), (5,2), (5,3), (5,4), (5,6) — 5 pairs.
2. Case 2: 5 on the second throw only: (1,5), (2,5), (3,5), (4,5), (6,5) — 5 pairs.
3. Case 3: 5 on both throws: (5,5) — 1 pair. Total favourable = 5 + 5 + 1 = **11**, so P = **11/36**.
   Why: The pair (5,5) belongs to its own case — students who merge Cases 1 and 2 double-count it and get the wrong 12/36.

ANSWER: 11/36.

METHOD 2 — COMPLEMENT ROAD (one count)

Count the pairs with NO 5 at all.

1. No 5 on either throw: each throw has 5 allowed faces, giving 5 × 5 = 25 pairs. P(no 5) = 25/36.
2. P(at least one 5) = 1 − 25/36 = **11/36**. Same answer — in two lines.

ANSWER: 11/36 — both roads agree.

Choosing your road — the topper's rule

Direct count when the favourable outcomes are few and easy to list (a named card, a single total, one colour).
Complement road when you hear "at least one", "not", or when the favourable side has many cases but the opposite side has one clean count.
Both roads when the question carries 3+ marks and time permits — agreement between roads is the cheapest full-marks insurance in this chapter.

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09 Interactive Simulations & Hands-on Activities

TRY IT YOURSELF — LEARN BY DOING

Probability began as experiments, and it is learned best by running them. Do these four labs with a real coin, two real dice and a real deck — then compare your results with the theory of Sections 04–07.

Lab 1 — The 50-toss coin lab (empirical meets theoretical)

Toss one coin 50 times. Tally the heads in groups of 10 and fill the table. Watch the running fraction crawl towards 1/2 — it will wobble, and the wobble will shrink.

Tosses	10	20	30	40	50
Heads so far					
Heads ÷ tosses					

Theory says the last row should end near 0.5. If yours ends at 0.44 or 0.56, nothing is wrong — 50 trials is a small sample. Pool the whole class's results (1500 tosses) and watch the agreement sharpen.

Lab 2 — The two-dice tally (test grandfather's claim)

Throw two dice 72 times and tally the totals. Theory predicts the counts in the last column (72 × the grid probabilities). Compare your tallest bar with the prediction: total 7 should win, with about 12 hits.

Total	2	3	4	5	6	7	8	9	10	11	12
Your tally											
Predicted	2	4	6	8	10	12	10	8	6	4	2

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Activities (continued) — Labs 3 and 4

Lab 3 — Deck anatomy by hand

Take a real deck. Sort it into the 4 suits (Spades, Hearts, Diamonds, Clubs), then count aloud: 13 per suit, 12 face cards, 26 red, 26 black, 4 aces. Shuffle thoroughly, draw one card 26 times (replacing and reshuffling each time) and tally red vs black — NCERT Q14, rehearsed by hand.

Lab 4 — Design a spinner, then price it

Draw a spinner with 10 equal sectors: colour 3 red, 2 blue, 5 green. Before spinning, write the three theoretical probabilities (3/10, 2/10, 5/10). Spin 40 times and compare. Then redesign the spinner so that P(red) = 1/2 exactly — how many red sectors do you need? (Answer: 5 of 10.)

KALVIYOGI DIGITAL COMPANION: In the print edition, a QR code here opens the Chapter 14 simulations: the coin-toss animator, the two-dice grid explorer and the card-draw simulator. (Placeholder — activates with the KalviYogi app/website at print.)

PIN POINT: Your lab results (empirical probability) APPROACH the theory as trials grow — they never have to match it exactly in a small sample. CBSE Class 10 asks only for the theoretical formula, but the labs are why the formula deserves your trust.

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10 Common Mistakes

LEARN FROM OTHER STUDENTS' ERRORS — BEFORE THEY BECOME YOURS

Six errors account for nearly every mark lost in this chapter. Each is shown the way students actually write it — then repaired.

COMMON MISTAKE: Counting TYPES instead of OUTCOMES. **"A bag has 3 red balls and 1 blue ball, so P(red) = 1/2 — two colours, one is red."**
Right way: Count balls: 4 outcomes, 3 favour red, so P(red) = 3/4. Colours are types; balls are outcomes. Only outcomes are counted.

COMMON MISTAKE: Merging HT and TH. **"Two coins give 3 outcomes — both heads, both tails, one of each — so P(both heads) = 1/3."**
Right way: HT and TH are two different outcomes (first coin H or T matters). There are 4 outcomes and P(both heads) = 1/4. (NCERT Q25(i) tests exactly this.)

COMMON MISTAKE: Believing the 11 dice sums are equally likely. **"Totals run from 2 to 12, so P(total 7) = 1/11."**
Right way: Divide by the 36 PAIRS, never by the 11 sums: P(7) = 6/36 = 1/6, while P(2) = 1/36. The grid in Section 04 is the proof.

COMMON MISTAKE: Accepting an answer outside [0, 1]. **"P(not a king) = 1 − 4/52 = 52/52 − 4/52 = 56/52" — written, boxed, and submitted without a blink.**
Right way: 56/52 > 1 is impossible for a probability. The alarm should ring instantly: 52/52 − 4/52 = 48/52 = 12/13. Run the boundary check on EVERY final answer.

COMMON MISTAKE: Double-counting the overlap in 'at least once'. **"A 5 can come on the first die (6 pairs) or the second die (6 pairs), so P(5 at least once) = 12/36."**
Right way: The pair (5, 5) sits in BOTH groups and was counted twice: 6 + 6 − 1 = 11 pairs, P = 11/36. Or skip the trap entirely with the complement road: 1 − 25/36 = 11/36.

COMMON MISTAKE: Keeping the old total after a removal. **"The queen was drawn and put aside from 5 cards, so P(ace next) = 1/5."**
Right way: Without replacement, the second draw lives in a smaller world: 4 cards remain, so P(ace) = 1/4. Whenever something is 'put aside' or 'not replaced', recount the total before doing anything else. (NCERT Q15, Q17.)

PIN POINT: Before submitting any answer in this chapter, run the three-point check: (1) Did I count OUTCOMES, not types? (2) Is my total still correct after any removal? (3) Is the final fraction inside [0, 1] and in lowest terms?

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11 Graded Practice Problems

FIVE LEVELS — CLIMB ONE LEVEL AT A TIME • ANSWERS IN THE ANSWER KEY

Rules of the climb: finish a level with at most one error before moving up. Write the total outcomes and the favourable count for every question — answers alone earn nothing in the board exam.

LEVEL 1 — FOUNDATION (direct concepts)

L1.1 A coin is tossed once. What is the probability of getting a tail?

L1.2 A die is thrown once. What is the probability of getting the number 4?

L1.3 A bag contains 4 red balls and 6 blue balls. One ball is drawn at random. Find the probability that it is red.

L1.4 If P(E) = 0.35, find P(not E).

L1.5 A spinner is divided into 5 equal sectors numbered 1 to 5. What is the probability that the arrow points at 3?

L1.6 A die is thrown once. What is the probability of getting a number less than 7? What kind of event is this?

LEVEL 2 — SKILL BUILDER (textbook standard)

L2.1 A die is thrown once. Find the probability of getting an even number.

L2.2 A die is thrown once. Find the probability of getting a multiple of 3.

L2.3 A bag contains 7 white balls and 5 green balls. One ball is drawn at random. Find the probability that it is NOT green.

L2.4 Cards numbered 1 to 20 are placed in a box and mixed. One card is drawn at random. Find the probability that it bears an odd number.

L2.5 Two coins are tossed simultaneously. Find the probability of getting two heads.

L2.6 One card is drawn from a well-shuffled deck of 52 cards. Find the probability that it is an ace.

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Graded Practice — Levels 3, 4 and 5

LEVEL 3 — APPLICATION (real-life and competency)

L3.1 A class has 40 students, of whom 25 are girls. The teacher selects one student at random to be the class monitor. Find the probability that the monitor is a boy.

L3.2 In a lottery, 500 tickets are sold and there is a single first prize. Ravi buys 5 tickets. Find the probability that Ravi wins the first prize.

L3.3 A machine fills 200 bottles, of which 8 are defective. An inspector picks one bottle at random for testing. Find the probability that it is defective.

L3.4 A couple has two children. Assuming a boy and a girl are equally likely for each child, find the probability that both children are girls.

LEVEL 4 — ADVANCED REASONING (multi-concept)

L4.1 Two dice are thrown together. Find the probability that the sum of the numbers obtained is 7.

L4.2 Two dice are thrown together. Find the probability of getting a doublet (the same number on both dice).

L4.3 Cards numbered 11 to 50 are placed in a box and mixed thoroughly. One card is drawn at random. Find the probability that it bears a perfect square number.

LEVEL 5 — MATHEMATICAL GENIUS (olympiad and creative)

L5.1 Three coins are tossed simultaneously. Find the probability of getting exactly two heads.

L5.2 A bag contains 6 red balls and 4 white balls. One ball is drawn and NOT replaced; then a second ball is drawn. Find the probability that both balls drawn are red. (Hint: the second draw lives in a bag of 9 balls, of which 5 are red.)

> **TOPPER TIP:** *Stuck at Level 4 or 5? Return to the two-dice grid (Section 04) and the complement weapon (Section 08) — every Level 4–5 question in this chapter surrenders to one of those two pictures.*

12 Mathematical Reasoning

JUSTIFY AND PROVE — FULL SOLUTIONS

Reasoning questions pay for the ARGUMENT, not the number. Each solution below is written the way an examiner wants to read it: claim, evidence from the elementary outcomes, conclusion.

REASONING 1

A student argues: "When two coins are tossed, there are three possible outcomes — two heads, two tails, or one of each — so the probability of two heads is 1/3." Examine the argument and give the correct probability, with reasons.

THINK FIRST: This is NCERT Q25(i) in a mirror — the same family as the story's Lesson 2: count outcomes, not types.

1. The argument silently assumes the three named outcomes are equally likely. Test that assumption by listing the ELEMENTARY outcomes: HH, HT, TH, TT — 4 outcomes, each with the same chance.

   Why: Equally likely is a property to be CHECKED, never a gift of having few outcomes.

2. The group 'one of each' contains two elementary outcomes, HT and TH, while 'two heads' contains only HH.

3. So the three groups own 1, 2 and 1 of the 4 equal shares — they are NOT equally likely, and dividing by 3 is unjustified.

4. Correct value: P(two heads) = 1/4.

ANSWER: The argument is incorrect: P(two heads) = 1/4, because the elementary outcomes are HH, HT, TH, TT.

REASONING 2

Prove that the sum of the probabilities of all the elementary events of an experiment is always 1.

1. Let the experiment have n equally likely outcomes. Each single outcome is an elementary event.

2. The n elementary probabilities must be equal (the outcomes are equally likely) and together they describe EVERYTHING that can happen — one of them certainly occurs.

   Why: Certainty has probability 1 – Derivation B.

3. So if each elementary probability is p, then n × p = 1, giving p = 1/n.

4.

Reasoning 3 — a right answer with a wrong reason

REASONING 3

A student writes: "On a die there are only two kinds of outcomes, odd and even, so P(odd) = 1/2." The final value is correct, yet the reasoning is faulty. (a) Explain the fault, using a counterexample. (b) Give the correct justification for P(odd) = 1/2.

THINK FIRST: NCERT Q25(ii) is this exact trap. Examiners reward the student who attacks the REASON, not the answer.

1. (a) The fault: 'two kinds of outcomes' does NOT imply 'each kind has probability 1/2'. That follows only if the two kinds are equally likely — which must be proved, not assumed.

2. Counterexample: a bag holds 4 red balls and 1 blue ball. There are two kinds of outcomes (red, blue), but P(red) = 4/5 ≠ 1/2. The student's road collapses here.

   Why: One counterexample is enough to kill a false general claim — a favourite Olympiad and board-reasoning move.

3. (b) Correct justification: the die's elementary outcomes 1, 2, 3, 4, 5, 6 ARE equally likely. The odd kind owns {1, 3, 5} — 3 of the 6 — so P(odd) = 3/6 = 1/2.

   Why: The value 1/2 comes from EQUAL OWNERSHIP (3 faces each), not from the number of kinds (2).

4. Moral: in probability, always justify at the level of elementary outcomes. Kinds inherit their probabilities from the outcomes inside them.

ANSWER: (a) The reasoning is faulty — two kinds need not be equally likely (4 red, 1 blue gives P(red) = 4/5). (b) P(odd) = 3/6 = 1/2 because the odd faces own 3 of the 6 equally likely outcomes.

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13 Real-Life Applications

8D • APPLY — THE CHAPTER AT WORK IN THE WORLD

APPLICATION 1

Quality check: a factory packs 1000 pens in a carton, of which 25 are defective. A shopkeeper picks one pen at random from the carton to test it. What is the probability that the pen he picks is good?

1. Total outcomes = 1000 pens. Good pens = 1000 − 25 = 975.
   Why: Produce the good count by subtraction, as in NCERT Q21.
2. P(good) = 975/1000 = **39/40** (dividing by 25).
3. Interpretation: in the long run, about 39 of every 40 pens tested will be good — a 97.5% quality rate, stated as a probability.
   Why: A probability in the real world is a long-run share — say so when a question asks you to interpret.

ANSWER: P(good pen) = 39/40.

APPLICATION 2

School raffle: 800 tickets are sold for a fete raffle with exactly one prize. Meena buys 8 tickets. What is the probability that Meena wins the prize?

1. Total outcomes = 800 tickets, one of which will be drawn as the winner; every ticket is equally likely.
2. Meena owns 8 of the tickets, so her favourable outcomes = 8: P(win) = 8/800 = **1/100**.
   Why: Buying more tickets buys a bigger SHARE of the outcomes — the story's Lesson 1 again.
3. Interpretation: even with 8 tickets, Meena's chance is only 1 in 100 — probability keeps hope honest.

ANSWER: P(Meena wins) = 1/100.

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Applications 3 and 4

APPLICATION 3

Traffic signal: one cycle of a traffic light shows green for 40 seconds, yellow for 5 seconds and red for 35 seconds, repeating. A car reaches the signal at a completely random moment. What is the probability that the light is green when the car arrives?

1. One full cycle lasts 40 + 5 + 35 = **80 seconds** — this is the 'total' of this experiment.
   Why: Here the outcomes are moments in time, so we measure with seconds instead of counting items – the same idea as NCERT Q20.*
2. The light is green for 40 of those 80 seconds, so the favourable measure is 40.
3. P(green) = 40/80 = **1/2**.
   Why: Half of every cycle is green, so a random arrival finds green half the time.

ANSWER: P(green) = 1/2.

APPLICATION 4

Game night: in a board game, a player earns a bonus turn if the total on two dice is exactly 10. The dice are thrown once. What is the probability of earning the bonus?

1. Total outcomes = 36 pairs.
2. Read the total-10 diagonal of the grid: (4, 6), (5, 5), (6, 4) — 3 favourable outcomes.
   Why: The grid turns this into a counting glance – no fresh listing needed.
3. P(bonus) = 3/36 = **1/12**.
4. Interpretation: the bonus arrives about once in every 12 turns — rare enough to feel special, exactly as the game designer intended.

ANSWER: P(bonus turn) = 1/12.

TOPPER TIP: *Every application in this section was solved by the story's two lessons: find the whole, find the event's share of it, divide. If a real-life question ever looks new, ask: 'what is the whole here, and what share does my event own?'*

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14 Competency-Based Questions

CASE STUDIES & ASSERTION-REASON — BOARD EXAM STYLE, FULLY SOLVED

Case Study 1 — The Fete Lucky Dip

At the school fete, a stall runs a lucky dip: a box contains **30 tokens numbered 1 to 30**, well mixed. A visitor draws one token at random. Answer the four questions that follow.

CASE 1 (i)

Find the probability that the token bears a multiple of 5.

1. Total outcomes = 30 tokens.
2. Multiples of 5 up to 30: 5, 10, 15, 20, 25, 30 — 6 tokens. P = 6/30 = **1/5**.
   Why: Count = 30 ÷ 5 = 6, the Q18(iii) method.

ANSWER: 1/5.

CASE 1 (ii)

Find the probability that the token bears a prime number.

1. Primes up to 30: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 — count them: **10**.
   Why: Write the list; primes are the easiest count to get wrong from memory.
2. P(prime) = 10/30 = **1/3**.

ANSWER: 1/3.

Case Study 1 (continued)

CASE 1 (iii)

Find the probability that the token bears a two-digit number.

1. One-digit tokens are 1 to 9: 9 tokens. Two-digit tokens = 30 − 9 = **21**.
   Why: Subtract the small group, as in NCERT Q18(i).
2. P(two-digit) = 21/30 = **7/10**.

ANSWER: 7/10.

CASE 1 (iv)

Find the probability that the token bears a number divisible by 2 or by 3.

1. Divisible by 2: 15 tokens. Divisible by 3: 10 tokens.
2. Numbers divisible by BOTH 2 and 3 (i.e., by 6): 6, 12, 18, 24, 30 — 5 tokens. They were counted in both lists, so subtract the overlap once: 15 + 10 − 5 = **20**.
   Why: 'Or' questions always hide an overlap — remove the double count, like the (5,5) pair in Section 08.
3. P = 20/30 = **2/3**.

ANSWER: 2/3.

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Case Study 2 — The Class Card Trick

Case Study 2 — The Class Card Trick

A teacher uses a well-shuffled deck of **52 cards** for a probability demonstration. One card is drawn at random. Answer the four questions that follow.

CASE 2 (i)

Find the probability that the card is red.

1. Red cards are the Hearts and the Diamonds: 13 + 13 = **26**.
   Why: Suits are counted by their WORD names in this book — no symbols needed.
2. P(red) = 26/52 = **1/2**.

ANSWER: 1/2.

CASE 2 (ii)

Find the probability that the card is a king.

1. Kings: one in each of the 4 suits = **4**. P = 4/52 = **1/13**.
   Why: Every rank owns exactly 4 cards — one per suit.

ANSWER: 1/13.

Case Studies 2 (continued) and 3

CASE 2 (iii)

Find the probability that the card is a black queen.

1. Black queens: the queen of Spades and the queen of Clubs = **2**.
2. P = 2/52 = **1/26**.

ANSWER: 1/26.

CASE 2 (iv)

Find the probability that the card is NOT a face card.

1. Face cards = 12, so non-face cards = 52 − 12 = **40**. P = 40/52 = **10/13**.
2. Complement check: 1 − 12/52 = 40/52 = 10/13. Agrees.

ANSWER: 10/13.

Case Study 3 — The Two-Coin Game Stall

At a game stall, a player tosses **two coins together**. Prizes depend on the heads that appear.

CASE 3 (i)

Find the probability of getting at least one head.

1. Outcomes: HH, HT, TH, TT — total 4.
2. Complement road: P(no head) = P(TT) = 1/4, so P(at least one head) = 1 − 1/4 = **3/4**.

ANSWER: 3/4.

CASE 3 (ii)

Find the probability of getting no head.

1. Only TT has no head: P = **1/4**.

ANSWER: 1/4.

Case Studies 3 (continued) and 4

CASE 3 (iii)

Find the probability of getting exactly one tail.

1. Exactly one tail means exactly one head: HT and TH — 2 outcomes.
2. P = 2/4 = **1/2**.

ANSWER: 1/2.

CASE 3 (iv)

Find the probability of getting at most one head.

1. 'At most one head' means 0 heads or 1 head: TT, HT, TH — 3 outcomes.
   Why: 'At most 1' includes 0 — read these phrases literally.
2. P = **3/4**. (Complement check: 1 − P(two heads) = 1 − 1/4 = 3/4.)

ANSWER: 3/4.

Case Study 4 — The Marble Experiment

A bag contains **8 red, 6 blue and 6 green marbles** (20 in all), well mixed. One marble is drawn at random.

CASE 4 (i)

Find the probability that the marble is blue.

1. Total = 20 marbles; blue = 6. P = 6/20 = **3/10**.

ANSWER: 3/10.

CASE 4 (ii)

Find the probability that the marble is not red.

1. Not red = blue + green = 6 + 6 = 12 marbles. P = 12/20 = **3/5**.
   Why: Complement check: 1 − 8/20 = 12/20 = 3/5. Agrees.

ANSWER: 3/5.

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Case Studies 4 (continued) and 5

CASE 4 (iii)

Find the probability that the marble is red or green.

1 Red or green = 8 + 6 = 14 marbles — no overlap, since one marble has one colour. P = 14/20 = **7/10**.

Why: No overlap here – plain addition is safe.

ANSWER: 7/10.

CASE 4 (iv)

Find the probability that the marble is yellow.

1 There are NO yellow marbles in the bag: favourable outcomes = 0. P = 0/20 = **0** — an impossible event.

Why: A zero numerator is a complete answer.

ANSWER: 0.

Case Study 5 — The Maths-Lab Spinner

A maths-lab spinner has **12 equal sectors numbered 1 to 12**. It is spun once.

CASE 5 (i)

Find the probability of getting a multiple of 4.

1 Multiples of 4 up to 12: 4, 8, 12 — 3 sectors. P = 3/12 = **1/4**.

ANSWER: 1/4.

CASE 5 (ii)

Find the probability of getting a composite number.

1 Composites up to 12: 4, 6, 8, 9, 10, 12 — 6 sectors. (1 is neither prime nor composite; 2, 3, 5, 7, 11 are prime.)

2 P = 6/12 = **1/2**.

ANSWER: 1/2.

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Case Study 5 (continued) — and Assertion–Reason

CASE 5 (iii)

Find the probability of getting a number greater than 9.

1 Numbers greater than 9: 10, 11, 12 — 3 sectors. P = 3/12 = **1/4**.

ANSWER: 1/4.

CASE 5 (iv)

Find the probability of getting a perfect square.

1 Perfect squares up to 12: 1, 4, 9 — 3 sectors. P = 3/12 = **1/4**.

Why: Count squares by roots: 1^2, 2^2, 3^2 fit; $4^2 = 16$ does not.

ANSWER: 1/4.

Assertion–Reason Questions (10)

For each question, choose: **(a)** Both A and R are true and R is the correct explanation of A. **(b)** Both are true but R is NOT the correct explanation of A. **(c)** A is true, R is false. **(d)** A is false, R is true.

A-R 1. Assertion (A): The probability of getting the number 8 in a single throw of a die is 0.
Reason (R): An impossible event has probability 0.

Answer: (a) — No face shows 8, so the event is impossible, and R states exactly the rule that gives P = 0.

A-R 2. Assertion (A): When a coin is tossed twice, the probability of getting two tails is 1/4.
Reason (R): The total number of outcomes is 4: HH, HT, TH, TT.

Answer: (a) — Two tails is the single outcome TT out of the 4 listed in R, so P = 1/4, and R supplies the count.

A-R 3. Assertion (A): If P(E) = 0.7, then P(not E) = 0.3.
Reason (R): P(E) + P(not E) = 1.

Answer: (a) — R is the complement rule, and it directly forces P(not E) = 1 − 0.7 = 0.3.

A-R 4. Assertion (A): The probability of getting an even number when a die is thrown is 1/3.
Reason (R): The even numbers on a die are 2, 4 and 6.

Answer: (d) — R is true, but A is false: 3 favourable faces out of 6 give P = 3/6 = 1/2, not 1/3.

A-R 5. Assertion (A): The probability of getting a red face card from a deck of 52 cards is 3/26.
Reason (R): There are 6 red face cards in a deck.

Answer: (a) — 6/52 = 3/26, and R gives the favourable count that produces it.

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Assertion–Reason (6–10)

A-R 6. Assertion (A): The sum of the probabilities of all elementary events of an experiment is 1.
Reason (R): In a single throw of a die, each face has probability 1/6.

Answer: (b) — Both are true, but R is only one example of the general fact in A; it illustrates A rather than explaining it. (A itself was proved in Reasoning 2.)

A-R 7. Assertion (A): The probability of a sure event is 1.
Reason (R): A sure event contains every possible outcome of the experiment.

Answer: (a) — Owning all n outcomes gives P = n/n = 1 — R is precisely the reason.

A-R 8. Assertion (A): When two dice are thrown, P(sum 12) = P(sum 2).
Reason (R): Each of the sums 12 and 2 has exactly one favourable outcome.

Answer: (a) — Sum 2 comes only from (1, 1) and sum 12 only from (6, 6); equal counts over the same 36 give equal probabilities, 1/36 each.

A-R 9. Assertion (A): The probability that a leap year, chosen at random, has 53 Sundays is 2/7.
Reason (R): A leap year has 52 weeks and 2 extra days.

Answer: (a) — The 2 extra days form 7 equally likely pairs — (Sun, Mon), (Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat), (Sat, Sun) — and 2 of these pairs contain a Sunday, giving 2/7. R is the fact that starts the count.

A-R 10. Assertion (A): The probability of getting a head when a coin is tossed once is 1/2 because there are two outcomes.
Reason (R): The two outcomes, head and tail, are equally likely.

Answer: (b) — Both statements are true, but the correct explanation is R: two outcomes ALONE would not give 1/2 (recall the 4-red-1-blue bag). The 'because' in A names the wrong reason, so R does not correctly explain A as stated.

> **TOPPER TIP:** *Assertion–Reason tactic: judge A alone, judge R alone, and only then ask the third question — 'does R EXPLAIN A?' Most errors happen when students skip straight to the third question.*

15 Olympiad & Foundation Challenges

8D • CHALLENGE — FOR STUDENTS WHO FINISH EARLY AND THINK DEEP

Ten challenges, each fully solved. Every one of them is still the same master formula — the difficulty is in the COUNTING, which is exactly what Olympiads test.

CHALLENGE 1

Three coins are tossed simultaneously. Find the probability of getting at least two heads.

1. All 8 outcomes: HHH, HHT, HTH, THH, HTT, THT, TTH, TTT.
2. 'At least two heads' collects HHH, HHT, HTH, THH — 4 outcomes.
   Why: 'At least two' = exactly two OR exactly three — list by head-count to stay organised.
3. P = 4/8 = **1/2**.

ANSWER: 1/2.

CHALLENGE 2

Two dice are thrown together. Find the probability that the product of the numbers obtained is a perfect square.

1. Total outcomes = 36 pairs. Hunt the pairs whose product is a square, first die by first die: with 1: (1, 1) → 1, (1, 4) → 4. With 2: (2, 2) → 4. With 3: (3, 3) → 9. With 4: (4, 1) → 4, (4, 4) → 16. With 5: (5, 5) → 25. With 6: (6, 6) → 36.
   Why: A product a × b is a square exactly when a and b carry the same square-free part — the systematic row-by-row hunt finds them all without the theory.
2. Count the hunt: 2 + 1 + 1 + 2 + 1 + 1 = **8** favourable outcomes.
3. P = 8/36 = **2/9**.

ANSWER: 2/9.

Challenges 3 and 4

CHALLENGE 3

A bag contains 4 red balls and 4 white balls. Two balls are drawn one after another, without replacement. Find the probability that the two balls are of different colours.

1. Different colours happens in two orders: red-then-white (RW) or white-then-red (WR). Count each.

   Why: Splitting by ORDER keeps every path countable – the tree idea of Section 04.

2. P(RW): first ball red: 4/8. The red ball is gone, so the second draw lives in 7 balls with 4 white: 4/7. Path value = (4/8) × (4/7) = 16/56.

   Why: Without replacement, BOTH the total and the colour counts update after the first draw.

3. P(WR): by symmetry, (4/8) × (4/7) = 16/56.

   Why: The bag starts balanced (4 and 4), so the two orders must be equal.

4. Add the two paths: 16/56 + 16/56 = 32/56 = **4/7**.

ANSWER: 4/7.

CHALLENGE 4

A card is drawn at random from cards numbered 1 to 100. Find the probability that the number on the card is divisible by 3 or by 5.

1. Total outcomes = 100.

2. Divisible by 3: 99 ÷ 3 = 33 cards. Divisible by 5: 100 ÷ 5 = 20 cards.

3. Overlap — divisible by BOTH 3 and 5, i.e., by 15: 90 ÷ 15 = 6 cards (15, 30, 45, 60, 75, 90). Remove the double count: 33 + 20 − 6 = **47**.

   Why: The Case 1(iv) overlap move, at Olympiad scale.

4. P = **47/100**.

   Why: 47 is prime and 100 = $2^2 \times 5^2$ – already in lowest terms.

ANSWER: 47/100.

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Challenges 5 and 6

CHALLENGE 5

An integer is chosen at random from the integers −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5. Find the probability that its absolute value is at most 2 (that is, $|x| \leq 2$).

1. Total outcomes = 11 integers (five negative, zero, five positive).
   Why: Count the list itself — negative numbers make students drop outcomes.
2. $|x| \leq 2$ holds for x = −2, −1, 0, 1, 2 — **5** integers.
   Why: Absolute value folds the line at 0: both −2 and 2 qualify.
3. P = **5/11**.

ANSWER: 5/11.

CHALLENGE 6

Two dice are thrown together. Find the probability that the sum obtained is a prime number.

1. Prime totals possible on two dice: 2, 3, 5, 7, 11. (Totals 4, 6, 8, 9, 10, 12 are composite; total 1 is impossible.)
   Why: First decide WHICH totals qualify, then count each from the grid.
2. Counts from the grid: total 2 → 1, total 3 → 2, total 5 → 4, total 7 → 6, total 11 → 2. Sum: 1 + 2 + 4 + 6 + 2 = **15**.
3. P = 15/36 = **5/12**.

ANSWER: 5/12.

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Challenges 7 and 8

CHALLENGE 7

A non-leap year is chosen at random. Find the probability that it has 53 Sundays.

1. A non-leap year has 365 days = 52 full weeks + **1 extra day**. The 52 weeks already guarantee 52 Sundays.

   Why: Compare A-R 9: a leap year has TWO extra days — that single difference changes everything.

2. The extra day is equally likely to be any of the 7 weekdays. The year gets a 53rd Sunday exactly when the extra day IS a Sunday: 1 favourable weekday out of 7.

3. P = **1/7**.

   Why: Leap year: 2/7 (A-R 9). Non-leap year: 1/7. The pair makes a beautiful exam contrast.

ANSWER: 1/7.

CHALLENGE 8

A bag contains 5 red balls and 6 white balls. A ball is drawn, its colour noted, and it is REPLACED in the bag. A second ball is then drawn. Find the probability that both balls drawn are white.

1. Because the ball is replaced, the second draw faces the SAME bag as the first: 11 balls, 6 white, both times.

   Why: Replacement freezes the experiment; non-replacement shrinks it. Read the question for this one word before all others.

2. P(white on one draw) = 6/11. Both draws white: (6/11) × (6/11) = **36/121**.

   Why: Independent draws multiply along the path, as in Challenge 3.

ANSWER: 36/121.

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Challenges 9 and 10

CHALLENGE 9

A dart is thrown at a square board of side 10 cm and always hits the board. At the centre of the board is a square 'bullseye' region of side 4 cm. Assuming the dart is equally likely to land anywhere on the board, find the probability that it lands inside the bullseye.

1. This is NCERT Q20* grown up: with a region, probability = favourable area ÷ total area.
2. Total area = 10 × 10 = 100 cm^2. Bullseye area = 4 × 4 = 16 cm^2.
3. P(bullseye) = 16/100 = **4/25**.

ANSWER: 4/25.

CHALLENGE 10

In a single throw of two dice, find the probability that the sum of the numbers is a multiple of 4.

1. Multiples of 4 among the totals: 4, 8, 12.
2. Counts from the grid: total 4 → 3, total 8 → 5, total 12 → 1. Favourable = 3 + 5 + 1 = **9**.
3. P = 9/36 = **1/4**.

ANSWER: 1/4.

TOPPER TIP: *Every challenge fell to one of three moves: the grid, the overlap subtraction, or the path multiplication. When a hard probability question appears, ask which of the three moves it is hiding.*

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16

Chapter Mind Map

THE WHOLE CHAPTER ON ONE PAGE — REVISE FROM HERE

PROBABILITY

Heart of the chapter: **P(E) = favourable outcomes ÷ all possible outcomes** — valid only when all outcomes are EQUALLY LIKELY.

The Formula (derived)

n equally likely outcomes → each owns 1/n → an event with m outcomes owns m/n. Count OUTCOMES, never types.

→

Boundaries

0 ≤ P(E) ≤ 1. Impossible event: P = 0. Sure event: P = 1. Sum of all elementary probabilities = 1.

Complement

P(not E) = 1 − P(E). Weapon for 'at least once': 1 − P(not even once).

Coins

1 coin: 2 outcomes. 2 coins: 4 (HH, HT, TH, TT). 3 coins: 8. Each extra coin doubles: 2^n.

Dice

1 die: 6 outcomes. 2 dice: 36 pairs — use the grid. Total 7 is the most likely (6 pairs); 2 and 12 the least (1 pair).

Cards (52)

4 suits × 13: Spades, Hearts, Diamonds, Clubs. Red: Hearts, Diamonds (26). Face cards J, Q, K: 12. Aces are not face cards.

Without Replacement

The drawn item leaves the world: recount the TOTAL (and the favourable count) for the next draw. NCERT Q15 & Q17.

→

Golden Rules

Total first. List favourable outcomes. 'Or' → check overlap. Reduce the fraction. Boundary check [0, 1].

Key Results to Quote

P(7 on two dice) = 1/6 • Hanif loses with 3/4 • 5 at least once in two throws = 11/36 • P(red face card) = 3/26 • Bulbs Q17: 15/19 after removal

TOPPER TIP: *Revise actively: cover a box, speak its contents aloud from memory, then uncover and check. A mind map you can REPRODUCE from memory is worth ten readings.*

17 Pin-Point Flash Cards

THE WHOLE CHAPTER IN YOUR POCKET — CUT THEM OUT, CARRY THEM, MASTER THEM

How to use these cards

Cut along the dashed borders and keep the 16 cards in your pocket or a small box. On Smart Memory **Day 3, Day 7 and Day 21**, shuffle the cards and test yourself: read the title, say the pin-point aloud from memory, then flip the card to check. Any card you miss goes back into the box for tomorrow. Two minutes a day keeps this chapter alive all year.

CARD 1 — The Master Formula

P(E) = favourable outcomes ÷ all possible outcomes — ONLY when all outcomes are equally likely.

CARD 2 — Complement Rule

P(not E) = 1 − P(E). Hear 'at least one', think '1 minus none'.

CARD 3 — The Range

0 ≤ P(E) ≤ 1, always. An answer outside this range is automatically wrong — recount.

CARD 4 — The Two Boundaries

Impossible event: P = 0. Sure event: P = 1. Elementary events: their probabilities sum to 1.

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Flash Cards 5–10

CARD 5 — Coin Counts

1 coin: 2 outcomes. 2 coins: 4 — HH, HT, TH, TT. 3 coins: 8. HT and TH are DIFFERENT outcomes.

CARD 6 — Dice Counts

1 die: 6 outcomes. 2 dice: 36 ordered pairs — (2, 5) and (5, 2) are two outcomes, not one.

CARD 7 — The Two-Dice Grid

Totals are NOT equally likely. Total 7 owns 6 pairs (most); totals 2 and 12 own 1 pair each (least).

CARD 8 — Deck Anatomy

52 cards = 4 suits × 13. Suits: Spades, Hearts, Diamonds, Clubs. Red = Hearts + Diamonds (26).

CARD 9 — Face Cards

Jack, Queen, King — 3 per suit, 12 in the deck. Aces are NOT face cards. A named card (queen of Diamonds) is 1 outcome.

CARD 10 — Die Faces

Primes on a die: 2, 3, 5. Composites: 4, 6. The number 1 is NEITHER prime nor composite.

TOPPER TIP: Say each card aloud once before you test yourself on it — hearing your own voice say the fact makes the memory stick twice as fast.

Flash Cards 11–16

CARD 11 — 'At Least Once'

Never count it directly. P(at least once) = 1 − P(not even once). Example: a 5 in two throws: 1 − 25/36 = 11/36.

CARD 12 — Count Outcomes, Not Types

3 red + 1 blue balls = 4 outcomes, not '2 colours'. 11 dice totals are types; the 36 pairs are the outcomes.

CARD 13 — Equally Likely Is the Licence

Counting works only when every outcome has the same chance: fair coin, fair die, well-shuffled deck, well-mixed bag, 'at random'.

CARD 14 — Without Replacement

A drawn item that is NOT replaced leaves the world: the TOTAL drops for the next draw. Recount before you divide.

CARD 15 — The 'Or' Overlap

Divisible by 2 or 3? Count both lists, then SUBTRACT the overlap (divisible by 6) once — it was counted twice.

CARD 16 — The Final Check

Numerator ≤ denominator, fraction in lowest terms, value inside [0, 1]. Three checks, five seconds, zero silly errors.

MAKE YOUR OWN CARD

Write one pin-point YOU keep forgetting, in your own words. A card you write yourself is the card you remember best.

_____

_____

MAKE YOUR OWN CARD

One more: the mistake you made most often in this chapter, and the one-line rule that fixes it.

_____

18 Active Recall

8D • RECALL — CLOSE THE BOOK, THEN ANSWER

How this works: close every other page. Answer all ten questions on rough paper, from memory alone. Only then open the Answer Key. Anything you could not recall is exactly what your Day-3 revision must target — write it in the Error Tracker (Section 19).

Ten rapid questions

R1. State the definition of the probability P(E) of an event E, and the condition under which it is valid.

R2. If P(E) = 0.3, what is P(not E)?

R3. A die is thrown once. What is P(getting a 5)?

R4. Two coins are tossed. What is P(HH)?

R5. One card is drawn from a deck of 52. What is P(getting a card of Hearts)?

R6. Can the probability of an event be 1.5? Why, or why not?

R7. Two dice are thrown. What is P(getting a total of 12)?

R8. A bag contains 10 red balls and 10 blue balls. What is P(red)?

R9. What is the probability of an impossible event? Of a sure event?

R10. Three coins are tossed together. How many outcomes are there in all?

Five recall prompts — say each answer aloud, in a full sentence

P1. Define equally likely outcomes.

P2. Define an elementary event.

P3. State the complement rule and one situation where it is the fastest road.

P4. Define a sure event and an impossible event, with one example of each.

P5. Explain why (1, 2) and (2, 1) are two different outcomes when two dice are thrown.

> **TOPPER TIP:** *Scoring: 13–15 correct — chapter mastered, move on. 9–12 — revise the missed topics, retake in 3 days. Below 9 — re-study Sections 05–07, then retake. Record your score in the Mastery Tracker.*

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19 Self-Assessment & Mastery Tracker

KALVIYOGI SMART MEMORY SYSTEM — LEARN IT, THEN KEEP IT

Memory fades on a predictable curve — unless you revisit at the right moments. This schedule is your revisit plan for THIS chapter. Tick each day off when done.

Day	Your activity for Chapter 14	Done
Day 0	Pre-learning assessment: attempt the Quick Check (Section 02) and Rapid Questions R1–R10 (Section 18) BEFORE studying. Note your score.	☐
Day 1	Learn the chapter fully (Sections 01–10) and complete Level 1–3 practice.	☐
Day 3	Without-book recall: reproduce the Mind Map (Section 16) and the two-dice count row (1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1) on a blank page; finish Level 4.	☐
Day 7	Independent problem-solving: Level 5 + Olympiad Challenges, timed, book closed. Then take the **Day 7 Mini-Test** (Practice Plus) and run the Flash Cards (Section 17).	☐
Day 14	Mixed practice: re-attempt every question in your Error Tracker; solve Ex 14.1 Q14 and Q24 fresh.	☐
Day 21	Long-term recall test: write the derivation of the master formula and of the complement rule from memory, then verify line by line. Then take the **Day 21 Mini-Test** (Practice Plus).	☐
Day 30	Chapter mastery assessment: take the **Day 30 Mastery Test** (Practice Plus) under exam conditions; update your Concept Mastery Score below.	☐

Formula Recall Tracker — tick when you can say it with the book closed

Formula / Statement	Day 1	Day 3	Day 7	Day 21
P(E) = favourable ÷ total (equally likely outcomes only)	☐	☐	☐	☐
P(not E) = 1 − P(E)	☐	☐	☐	☐
$0 \le P(E) \le 1$; sure event = 1; impossible event = 0	☐	☐	☐	☐
Coin counts: 2, 4, 8 outcomes for 1, 2, 3 coins	☐	☐	☐	☐
Two-dice counts: 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1 for totals 2–12	☐	☐	☐	☐
Deck anatomy: 52 cards, 12 face cards, 26 red / 26 black	☐	☐	☐	☐

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Mastery Tracker — Errors and Concept Scores

Error Tracker — every mistake becomes a lesson

Date	Question	What went wrong	The correct idea

Concept Mastery Score — rate yourself honestly, out of 5

Concept	Score / 5	Status
Equally likely outcomes & the master formula	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Coins (one, two and three)	___ / 5	☐ Weak ☐ Okay ☐ Mastered
One die and the two-dice grid	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Cards — the 52-card deck	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Complement rule & 'at least once' questions	___ / 5	☐ Weak ☐ Okay ☐ Mastered
Draws without replacement	___ / 5	☐ Weak ☐ Okay ☐ Mastered

Learning Time Tracker

Day / Date	Minutes studied	What I studied

Mastery rule: a concept counts as MASTERED only when you score 4 or 5 AND can reproduce it on Day 21 without the book. Anything less — revisit, don't just re-read.

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Practice Plus — 25 Concept MCQs (1–13)

One mark each. Cover the Answer Key and choose the single best option — then check the key, where the tricky ones carry a one-line explanation.

Q1. If P(E) = 0.4, then P(not E) is
(a) 0.4 (b) 0.6 (c) 1.4 (d) 0

Q2. Which of the following cannot be the probability of an event?
(a) 0.99 (b) 17/16 (c) 0 (d) 5%

Q3. A die is thrown once. The probability of getting a prime number is
(a) 1/3 (b) 1/2 (c) 2/3 (d) 1/6

Q4. Two coins are tossed simultaneously. The probability of getting at least one tail is
(a) 1/4 (b) 1/2 (c) 3/4 (d) 1

Q5. One card is drawn from a deck of 52 cards. The probability of getting a black king is
(a) 1/52 (b) 1/26 (c) 1/13 (d) 2/13

Q6. A bag contains 3 red and 7 blue balls. The probability that a ball drawn at random is NOT blue is
(a) 3/10 (b) 7/10 (c) 3/7 (d) 1

Q7. The probability of a sure event is
(a) 0 (b) 1/2 (c) 1 (d) 2

Q8. A spinner has 8 equal sectors numbered 1 to 8. The probability that it points at a number greater than 6 is
(a) 1/8 (b) 1/4 (c) 3/8 (d) 1/2

Q9. A die is thrown once. The probability of getting a multiple of 6 is
(a) 1/6 (b) 1/3 (c) 1/2 (d) 5/6

Q10. If P(E) = 3/11, then P(not E) is
(a) 3/11 (b) 8/11 (c) 11/3 (d) 1

Q11. Cards numbered 1 to 15 are placed in a box. The probability that a card drawn at random bears an even number is
(a) 7/15 (b) 8/15 (c) 1/2 (d) 7/8

Q12. Two dice are thrown together. The probability of getting a total of 2 is
(a) 1/18 (b) 1/36 (c) 1/12 (d) 1/6

Q13. A coin is tossed 3 times. The probability of getting three tails is
(a) 1/3 (b) 1/6 (c) 1/8 (d) 3/8

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25 Concept MCQs (14–25)

Q14. 1000 tickets are sold in a lottery with one prize. Asha buys 4 tickets. Her probability of winning is
(a) 1/250 (b) 4/999 (c) 1/1000 (d) 996/1000

Q15. Which of the following is an impossible event?
(a) getting 7 in a single throw of a die (b) getting a head on a coin (c) getting an even number on a die (d) getting a number less than 7 on a die

Q16. One card is drawn from a deck of 52. The probability that it is NOT a king is
(a) 1/13 (b) 12/13 (c) 4/13 (d) 1/4

Q17. If P(E) = x and P(not E) = x + 0.2, then x equals
(a) 0.3 (b) 0.4 (c) 0.5 (d) 0.6

Q18. A die is thrown twice. The probability that 5 appears on both throws is
(a) 1/36 (b) 1/6 (c) 1/18 (d) 11/36

Q19. A letter is chosen at random from the word PROBABILITY. The probability that it is the letter I is
(a) 1/11 (b) 2/11 (c) 3/11 (d) 2/9

Q20. A bag contains 6 orange candies and 5 apple candies. The probability that a candy taken out at random is orange is
(a) 5/11 (b) 6/11 (c) 1/2 (d) 6/5

Q21. Two dice are thrown together. The probability of getting a total of 1 is
(a) 0 (b) 1/36 (c) 1/18 (d) 1/12

Q22. If P(not E) = 0.05, then P(E) is
(a) 0.05 (b) 0.5 (c) 0.95 (d) 1.05

Q23. A die is thrown once. The probability of getting a composite number is
(a) 1/6 (b) 1/3 (c) 1/2 (d) 2/3

Q24. Cards numbered 10 to 19 are placed in a box. The probability that a card drawn bears a prime number is
(a) 3/10 (b) 2/5 (c) 1/2 (d) 4/19

Q25. The sum of the probabilities of all the elementary events of an experiment is
(a) 0 (b) 1/2 (c) 1 (d) the number of outcomes

> **TOPPER TIP:** *Scored below 20? Do not just re-read the key — find WHICH pin-point card covers each missed question and revise that card first.*

Find the Error — Questions 1–5

Each piece of working below contains exactly one planted mistake — the kind real students make in board exams. Spot it before you check the key.

ERROR 1

"A bag has 2 red balls and 3 blue balls. P(red) = 2/3, because there are 2 red balls and 3 blue balls."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 2

"Two coins are tossed. There are 3 outcomes: two heads, two tails, or one of each. So P(two heads) = 1/3."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 3

"P(E) = 0.3, so P(not E) = 1 − 0.3 = 0.07."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 4

"The prime numbers on a die are 2 and 5, because 3 is odd and odd numbers are not prime. So P(prime) = 2/6 = 1/3."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 5

"P(king) = 4/52 = 1/13. So P(not a king) = 1 − 1/13 = 13/13 = 1."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

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Find the Error — Questions 6–10

ERROR 6

"Two dice are thrown. The totals run from 2 to 12, which is 11 totals. So P(total 7) = 1/11."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 7

"A spinner has sectors 1 to 8. P(getting 9) = 1/8, because 9 is one number, like any other."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 8

"A bag has 5 red balls and 5 white balls. P(red) = 5/5 = 1."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 9

"An event can have probability 5/4 if it is extra sure — even more certain than a sure event."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

ERROR 10

"Two dice are thrown together. Total outcomes = 6 + 6 = 12, because each die has 6 faces."

Your task: find the mistake and write the correct solution. (Corrections are in the Answer Key.)

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Previous Year Questions — Real CBSE Board Questions (I)

These are genuine CBSE board questions, with the year verified. Solve them exactly as you would in the exam hall — full steps earn full marks.

CBSE 2023 — 1 MARK (MCQ)

Two dice are thrown together. The probability of getting the difference of numbers on their upper faces equals to 3 is: (a) 1/9 (b) 2/9 (c) 1/6 (d) 1/12

1. Total outcomes = 36 pairs.
2. Difference 3 pairs: (1, 4), (2, 5), (3, 6), (4, 1), (5, 2), (6, 3) — 6 pairs. Read them off two diagonals of the grid: first die 3 more, or second die 3 more.

   Why: Difference ignores order, so BOTH diagonals count – forgetting one direction halves the answer.
3. P = 6/36 = 1/6 — option **(c)**.

ANSWER: Option (c): 1/6.

CBSE 2023 — 1 MARK (MCQ)

A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has she bought? (a) 40 (b) 240 (c) 480 (d) 750

1. Her probability = (tickets she bought) ÷ (total tickets sold). Let her tickets be x: x/6000 = 0.08.

   Why: This is the master formula run BACKWARDS – the probability is given, a count is asked.
2. x = 0.08 × 6000 = **480** — option **(c)**.

ANSWER: Option (c): 480 tickets.

Previous Year Questions (2)

CBSE 2024 — 1 MARK (MCQ)

A bag contains 3 red balls, 5 white balls and 7 black balls. The probability that a ball drawn from the bag at random will be neither red nor black is: (a) 1/3 (b) 1/5 (c) 7/15 (d) 8/15

1. Total balls = 3 + 5 + 7 = 15.
2. 'Neither red nor black' leaves exactly one colour: white, 5 balls.
   Why: Translate 'neither–nor' into the surviving category before counting.
3. P = 5/15 = 1/3 — option **(a)**.

ANSWER: Option (a): 1/3.

CBSE 2024 — 1 MARK (MCQ)

Two dice are thrown together. The probability that they show different numbers is: (a) 1/6 (b) 5/6 (c) 1/3 (d) 2/3

1. 'Different numbers' is the complement of 'same number' (a doublet).
   Why: Direct counting of 'different' means 30 pairs – the complement is 6. Choose the small side.
2. Doublets: (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) — 6 outcomes, so P(same) = 6/36 = 1/6.
3. P(different) = 1 − 1/6 = 5/6 — option **(b)**.

ANSWER: Option (b): 5/6.

CBSE 2024 — 1 MARK (MCQ)

The probability of getting a bad egg in a lot of 400 eggs is 0.045. The number of good eggs in the lot is: (a) 18 (b) 180 (c) 382 (d) 220

1. Bad eggs = 0.045 × 400 = 18.
   Why: The formula run backwards again: count = probability × total.
2. Good eggs = 400 − 18 = **382** — option **(c)**.
   Why: The question asks for GOOD eggs – the last subtraction is where this mark is won or lost.

ANSWER: Option (c): 382.

Previous Year Questions (3)

CBSE 2022 — 2 MARKS

Two different dice are thrown together. Find the probability that the numbers obtained have (i) an even sum (ii) an even product.

1. Total outcomes = 36 pairs.

2. (i) Even sum: the sum is even exactly when both dice agree in parity — both odd or both even. Odd faces {1, 3, 5} and even faces {2, 4, 6}: pairs = 3 × 3 + 3 × 3 = 18. P = 18/36 = **1/2**.

   Why: Counting by parity blocks is faster than listing 18 pairs — and the grid confirms it: the even-sum cells form a chessboard of 18.

3. (ii) Even product: the product is ODD only when both dice are odd: 3 × 3 = 9 pairs. So even-product pairs = 36 − 9 = 27. P = 27/36 = **3/4**.

   Why: Complement road again: 'even product' fails only in the all-odd corner.

ANSWER: (i) 1/2 (ii) 3/4.

CBSE 2023 — 2 MARKS

A bag contains 4 red, 3 blue and 2 yellow balls. One ball is drawn at random from the bag. Find the probability that the drawn ball is (i) red (ii) yellow.

1. Total balls = 4 + 3 + 2 = **9**.

2. (i) P(red) = **4/9**.

3. (ii) P(yellow) = **2/9**.

   Why: 9 shares no factor with 4 or 2 — both fractions are already in lowest terms.

ANSWER: (i) 4/9 (ii) 2/9.

TOPPER TIP: *Board pattern to notice: the examiners return again and again to the same five machines — two dice on the grid, complements, deck anatomy, backwards formula (count = probability × total), and without-replacement recounts. Master the machines, not the questions.*

Chapter Model Test A — Basics (20 Marks)

Time: 40 minutes. Attempt all questions. Show every step — the marking key in the Answer Key shows where each mark lives.

Q1. If P(E) = 0.19, find P(not E). **[2 marks]**

Q2. A die is thrown once. Find the probability of getting an even number. **[2 marks]**

Q3. A bag contains 6 red balls and 4 black balls. One ball is drawn at random. Find the probability that it is (i) black (ii) not black. **[3 marks]**

Q4. Two coins are tossed simultaneously. Find the probability of getting at least one head. **[3 marks]**

Q5. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i) a queen (ii) a red card. **[4 marks]**

Q6. Cards numbered 1 to 25 are placed in a box and mixed. One card is drawn at random. Find the probability that it bears (i) a prime number (ii) a multiple of 5 (iii) a perfect square number. **[6 marks]**

Chapter Model Test B — Standard (20 Marks)

Time: 40 minutes. Board-pattern questions at full Class 10 level.

Q1. Two dice are thrown together. Find the probability that the sum of the numbers obtained is 8. **[3 marks]**

Q2. A bag contains 4 red balls and some blue balls. If the probability of drawing a blue ball is three times the probability of drawing a red ball, find the number of blue balls in the bag. **[3 marks]**

Q3. 8 defective bulbs are mixed with 92 good bulbs. One bulb is drawn at random. Find the probability that it is a good bulb. Also find the probability that it is defective. **[4 marks]**

Q4. A box contains 50 discs numbered 1 to 50. One disc is drawn at random. Find the probability that it bears (i) a two-digit number (ii) a number divisible by 7. **[4 marks]**

Q5. A jar contains 30 marbles, some green and the others blue. If the probability of drawing a green marble is 3/5, find (i) the number of green marbles (ii) the number of blue marbles and the probability of drawing a blue marble. **[6 marks]**

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Chapter Model Test C — Advanced (20 Marks)

Time: 45 minutes. Counting-heavy, HOTS-level — this is the paper that separates 95 from 100.

Q1. Three coins are tossed simultaneously. Find the probability of getting at least two tails. **[4 marks]**

Q2. Two dice are thrown together. Find the probability that the product of the numbers obtained is 12. **[4 marks]**

Q3. A box contains 15 balls, of which x are red. One ball is drawn at random. If 5 more red balls are put into the box, the probability of drawing a red ball becomes double of what it was before. Find x. **[5 marks]**

Q4. Cards numbered 21 to 60 are placed in a box. One card is drawn at random. Find the probability that it bears a number divisible by 4 or by 6. **[3 marks]**

Q5. Two dice are thrown together. Find the probability that the sum of the numbers obtained is a multiple of 3. **[4 marks]**

> **TOPPER TIP:** *Take these three tests on three different days — Test A after Section 11, Test B after Section 14, Test C in the final revision week. Record every score in your Mastery Tracker.*

My Test Record — Chapter 14

Test	Date	Score	Weakest topic — revise this first
Model Test A — Basics		/ 20	
Model Test B — Standard		/ 20	
Model Test C — Advanced		/ 20	
Day 7 Mini-Test		/ 10	
Day 21 Mini-Test		/ 10	
Day 30 Mastery Test		/ 10	

A rising line of scores is the real certificate of this chapter — not any single day's mark. If a score dips, the 'weakest topic' column tells you exactly which section to revisit before moving on.

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Smart Memory Mini-Tests — Day 7 and Day 21

Ten quick questions each, closed book, five minutes. Answers are in the Answer Key. These are the Day 7 and Day 21 stops of your Smart Memory schedule (Section 19).

DAY 7 TEST

1. If P(E) = 0.42, find P(not E).

2. A die is thrown once. Find the probability of getting an odd number.

3. A bag contains 9 balls, of which 4 are green. Find the probability of drawing a green ball.

4. Two coins are tossed. Find the probability of getting no tail.

5. One card is drawn from a deck of 52. Find the probability of getting a jack.

6. A spinner has 10 equal sectors numbered 1 to 10. Find the probability that it points at 10.

7. A die is thrown once. Find the probability of getting a number greater than 4.

8. Which of these cannot be a probability: 0.9, 1.2, 1, 0? Write the value.

9. A bag contains 3 red balls and 3 white balls. Find the probability of drawing a red ball.

10. Two dice are thrown together. How many outcomes are there in all?

DAY 21 TEST

1. Two dice are thrown. Find the probability of getting (6, 6).

2. Three coins are tossed. Find the probability of getting all heads.

3. One card is drawn from a deck of 52. Find the probability of getting a red queen.

4. Cards are numbered 1 to 30. Find the probability that a card drawn at random bears a multiple of 10.

5. If P(E) = 5/9, find P(not E).

6. A die is thrown once. Find the probability of getting a composite number.

7. A bag contains 12 balls. The probability of drawing a red ball is 1/4. How many red balls are in the bag?

8. Two coins are tossed. Find the probability of getting exactly one head.

9. A die is thrown twice. Find the probability that 5 does not come up on either throw.

10. What is the probability of a sure event?

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Smart Memory Mini-Test — Day 30 Mastery Test

The final checkpoint. Ten questions, closed book — if you score 9 or 10, this chapter is yours for the whole year.

DAY 30 MASTERY TEST

1. Two dice are thrown. Find the probability that the sum is 9.

2. Three coins are tossed. Find the probability of getting at least one tail.

3. One card is drawn from a deck of 52. Find the probability of getting an ace or a king.

4. Discs are numbered 1 to 100. Find the probability that a disc drawn at random bears a number divisible by 8.

5. In one line, explain why the probability of an event can never be greater than 1.

6. Two dice are thrown. Find the probability that the two numbers are equal.

7. A leap year is chosen at random. Find the probability that it has 53 Sundays.

8. A bag contains 20 balls, of which x are red. If P(red) = 2/5, find x.

9. A spinner has 12 equal sectors numbered 1 to 12. Find the probability that it points at a prime number.

10. If P(not E) = 0.875, find P(E).

> **TOPPER TIP:** *Missed any Day 30 question? That question's Flash Card goes back into your pocket box for one more week.*

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Answer Key — Graded Practice, Levels 1 and 2

Level 1 — Foundation

L1.1 1/2. **L1.2** 1/6. **L1.3** 4/10 = 2/5. **L1.4** 1 − 0.35 = 0.65. **L1.5** 1/5. **L1.6** 6/6 = 1 — a sure event, since every face of a die shows a number less than 7.

Level 2 — Skill Builder

L2.1 Even faces {2, 4, 6}: 3/6 = 1/2. **L2.2** Multiples of 3 are {3, 6}: 2/6 = 1/3. **L2.3** Not green = the 7 white balls out of 12: 7/12. **L2.4** Odd numbers in 1–20: 10 cards, so 10/20 = 1/2. **L2.5** HH is 1 of the 4 outcomes: 1/4. **L2.6** 4 aces out of 52: 4/52 = 1/13.

Level 3 — Application

L3.1 Boys = 40 − 25 = 15, so P(boy) = 15/40 = 3/8. **L3.2** 5/500 = 1/100. **L3.3** 8/200 = 1/25. **L3.4** Outcomes BB, BG, GB, GG (each child equally likely a boy or a girl): both girls is 1 of 4 = 1/4.

TOPPER TIP: *Marked yourself wrong at Level 2? Compare your counting with the twin solution in Section 07 — find the FIRST count where you differ. That count is the lesson.*

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Answer Key — Levels 4, 5 and Active Recall

Level 4 — Advanced Reasoning

L4.1 Sum 7 owns 6 of the 36 pairs: 6/36 = 1/6. **L4.2** Doublets are (1,1), (2,2), (3,3), (4,4), (5,5), (6,6): 6/36 = 1/6. **L4.3** Cards from 11 to 50: 40 cards. Perfect squares among them: 16, 25, 36, 49 — 4 cards. P = 4/40 = 1/10.

Level 5 — Mathematical Genius

L5.1 Exactly two heads among HHH, HHT, HTH, THH, HTT, THT, TTH, TTT: the outcomes HHT, HTH, THH — 3 of 8 = 3/8. **L5.2** First ball red: 6/10. Not replaced, the bag now holds 9 balls with 5 red: second ball red: 5/9. Both red = (6/10) × (5/9) = 30/90 = 1/3.

Active Recall (Section 18)

R1 P(E) = (number of outcomes favourable to E) ÷ (number of all possible outcomes), valid when all outcomes are equally likely. **R2** 0.7. **R3** 1/6. **R4** 1/4. **R5** 13/52 = 1/4. **R6** No — every probability satisfies $0 \leq P(E) \leq 1$. **R7** 1/36 (only (6, 6)). **R8** 10/20 = 1/2. **R9** 0; 1. **R10** 8.

P1 Outcomes are equally likely when each has the same chance of occurring as every other. **P2** An elementary event is an event having exactly one outcome. **P3** P(not E) = 1 − P(E); fastest for 'at least once' questions, where 'not even once' is one clean count. **P4** A sure event is certain to happen (P = 1), e.g. a number less than 7 on a die; an impossible event can never happen (P = 0), e.g. an 8 on a die. **P5** The dice are two different objects: (1, 2) means first die 1 and second die 2, while (2, 1) is the reverse — two separate outcomes of the experiment.

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Answer Key — Practice Plus

25 Concept MCQs

1 (b) 0.6 **2** (b) 17/16 **3** (b) 1/2 **4** (c) 3/4 **5** (b) 1/26 **6** (a) 3/10 **7** (c) 1 **8** (b) 1/4 **9** (a) 1/6 **10** (b) 8/11 **11** (a) 7/15 **12** (b) 1/36 **13** (c) 1/8

14 (a) 1/250 **15** (a) getting 7 on a die **16** (b) 12/13 **17** (b) 0.4 **18** (a) 1/36 **19** (b) 2/11 **20** (b) 6/11 **21** (a) 0 **22** (c) 0.95 **23** (b) 1/3 **24** (b) 2/5 **25** (c) 1

Explanations for the tricky ones. Q2: 17/16 > 1 — no probability may exceed 1. Q5: there are 2 black kings (Spades and Clubs), so 2/52 = 1/26. Q11: in 1–15 the even numbers are 2, 4, ..., 14 — only 7 of them; the odd numbers are 8. Q14: 4/1000 = 1/250. Q17: x + (x + 0.2) = 1 gives 2x = 0.8, so x = 0.4. Q19: PROBABILITY has 11 letters, and I appears twice (B also appears twice — count carefully). Q21: the smallest possible total on two dice is 2, so a total of 1 is impossible: probability 0. Q24: primes between 10 and 19 are 11, 13, 17, 19 — 4 of the 10 cards = 2/5.

Find the Error — Corrections

E1 The denominator must be ALL the balls, not the blue ones: total = 2 + 3 = 5, so P(red) = **2/5**.

E2 'One of each' hides two outcomes, HT and TH. There are 4 equally likely outcomes, so P(two heads) = **1/4**.

E3 Arithmetic slip: 1 − 0.3 = **0.7**, not 0.07. (A decimal-place error the boundary check cannot catch — recompute complements digit by digit.)

E4 3 IS prime — odd numbers can be prime; only the even number 2 is prime among evens, but 3, 5 are odd primes. Primes on a die: {2, 3, 5}, so P = 3/6 = **1/2**.

E5 1 − 1/13 = **12/13**, not 13/13. The working subtracted wrongly: 1 is 13/13, and 13/13 − 1/13 = 12/13.

E6 The 11 totals are not equally likely. Divide by the 36 pairs: P(total 7) = 6/36 = **1/6**.

E7 9 is not on the spinner at all — the event is impossible: P = **0**. 'One number like any other' only works for numbers that exist among the outcomes.

E8 The denominator is the total number of balls, 5 + 5 = 10: P(red) = 5/10 = **1/2**.

E9 Impossible: a probability can never exceed 1. A sure event has probability exactly 1 — there is no 'more than sure'.

E10 Outcomes multiply, they do not add: each of the 6 faces of the first die pairs with each of the 6 faces of the second, giving 6 × 6 = **36** outcomes.

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Answer Key — Model Tests and Day Tests

Model Test A — Basics (20)

Q1 P(not E) = 1 − 0.19 = **0.81** [rule 1, value 1]. **Q2** Even faces {2, 4, 6}: 3/6 = **1/2** [list 1, value 1]. **Q3** Total 10 [1]; P(black) = 4/10 = **2/5** [1]; P(not black) = 6/10 = **3/5** [1]. **Q4** Outcomes HH, HT, TH, TT [1]; at least one head: 3 outcomes [1]; **3/4** [1]. **Q5** P(queen) = 4/52 = **1/13** [2]; P(red) = 26/52 = **1/2** [2]. **Q6** Total 25 [1]; primes {2, 3, 5, 7, 11, 13, 17, 19, 23} = 9 → **9/25** [2]; multiples of 5: 5 → **1/5** [1]; squares {1, 4, 9, 16, 25} = 5 → **1/5** [2].

Model Test B — Standard (20)

Q1 Sum-8 pairs (2,6), (3,5), (4,4), (5,3), (6,2) = 5 [2]; **5/36** [1]. **Q2** Let blue balls = x. x/(x + 4) = 3 × 4/(x + 4) [1]; so x = 3 × 4 = **12 blue balls** [2]. **Q3** Total 100 [1]; good bulbs 92 [1]; P(good) = 92/100 = **23/25** [1]; P(defective) = 8/100 = **2/25** [1]. **Q4** Total 50 [1]; two-digit numbers: 10–50 = 41 → **41/50** [2]; divisible by 7: {7, 14, 21, 28, 35, 42, 49} = 7 → **7/50** [1]. **Q5** Green = (3/5) × 30 = **18** [3]; blue = 30 − 18 = **12**, and P(blue) = 12/30 = **2/5** [3].

Model Test C — Advanced (20)

Q1 8 outcomes [1]; at least two tails: TTH, THT, HTT, TTT = 4 [2]; **1/2** [1]. **Q2** Product 12: (2,6), (3,4), (4,3), (6,2) = 4 [3]; 4/36 = **1/9** [1]. **Q3** (x + 5)/20 = 2 × x/15 [2]; 15x + 75 = 40x, so 25x = 75 and **x = 3** [2]; check: before 3/15 = 1/5, after 8/20 = 2/5 — double [1]. **Q4** Cards 21–60: 40 [1]; divisible by 4: 10 numbers, by 6: 7 numbers, by 12 (overlap): 4 numbers → 10 + 7 − 4 = 13 [1]; **13/40** [1]. **Q5** Sums 3, 6, 9, 12 have counts 2, 5, 4, 1 = 12 [3]; 12/36 = **1/3** [1].

Day 7 Test

1 0.58 **2** 1/2 **3** 4/9 **4** 1/4 (only HH has no tail) **5** 4/52 = 1/13 **6** 1/10 **7** {5, 6}: 2/6 = 1/3 **8** 1.2 **9** 3/6 = 1/2 **10** 36

Day 21 Test

1 1/36 **2** 1/8 **3** 2/52 = 1/26 **4** Multiples of 10 in 1–30: 3, so 3/30 = 1/10 **5** 4/9 **6** Composites {4, 6}: 1/3 **7** (1/4) × 12 = 3 red balls **8** HT, TH: 2/4 = 1/2 **9** 25/36 **10** 1

Day 30 Mastery Test

1 Sum 9: 4 pairs, 4/36 = 1/9 **2** 1 − P(HHH) = 1 − 1/8 = 7/8 **3** (4 + 4)/52 = 8/52 = 2/13 **4** Multiples of 8 in 1–100: 12, so 12/100 = 3/25 **5** Favourable outcomes can never exceed total outcomes, so their ratio can never exceed 1 (and counts are never negative, so it cannot fall below 0) **6** 6 doublets: 6/36 = 1/6 **7** 2/7 **8** x = (2/5) × 20 = 8 **9** Primes up to 12: {2, 3, 5, 7, 11} = 5, so 5/12 **10** 1 − 0.875 = 0.125

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Revision Card — Chapter 14 on One Page

The one formula

P(E) = (number of outcomes favourable to E) ÷ (number of all possible outcomes) — valid only when all outcomes are equally likely. Derived: n equal shares of 1/n; an event with m outcomes owns m/n.

The complement rule

P(not E) = 1 − P(E). Home ground: 'at least once' questions — P(at least once) = 1 − P(not even once). Boundaries: impossible event P = 0; sure event P = 1; always $0 \leq P(E) \leq 1$; elementary probabilities sum to 1.

The counting tables

Coins: 2, 4, 8 outcomes (1, 2, 3 coins) — HT ≠ TH. **Dice:** 6 outcomes; two dice 36 pairs with total counts 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1 for totals 2–12. **Cards:** 52 = 4 suits (Spades, Hearts, Diamonds, Clubs) × 13; face cards 12; red 26, black 26.

The five machines

Grid: every two-dice question. **Complement:** 'not' and 'at least' questions. **Anatomy:** every card question. **Backwards:** count = probability × total (lottery tickets, bad eggs). **Recount:** without replacement, the total drops before the second draw.

Exam-eve checklist

☐ Total outcomes written first, with its reason ☐ Favourable outcomes LISTED, not just counted ☐ Outcomes counted, never types ☐ Overlap subtracted in 'or' questions ☐ Fraction reduced, and inside [0, 1]

Chapter 14 complete — and with it, the whole book. You did not memorise Probability — you discovered, visualised, derived and mastered it.

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